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Better Late than Never: The Case of the Rollover Option

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Abstract

In addition to death and maturity guarantees on the mutual funds they sell, some insurance companies make it possible for the investor to extend the guarantees for a fixed number of years. In this paper, we consider the case when that option is of the European type; that is, the investor, at maturity, can either close out the contract or extend it for the stated, fixed term. When extended, the guarantee is on the value of the fund at the original maturity date. The fund is assumed to be fully invested in common stock.
The value of that option, the rollover option, is derived in a risk-neutral environment. Mortality is also taken into account when calculating the value of the option. The formulas obtained are of the Black-Scholes type.

1 Introduction

Computerization and customer sophistication have rendered possible and necessary financial innovations which insurance companies probably would not even have considered thirty years ago.
As a result, over the last decade or two, to make their products more attractive, insurance companies have added investment features to their insurance products (*e.g.*, variable life insurance). Likewise, they have enhanced their

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investment products by adding insurance features onto them (*e.g.*, maturity guarantees). (For an overview of the later developments in that arena, see Mavrogenes and Verrier [7].)

The latter kind of innovation is of interest here. In particular, maturity guarantees ensure that the payoff to the investor, at maturity, will be of at least a certain stated amount. They are valuable in case of downward market fluctuations. In Section 2, the results already found in the literature in the case of the basic maturity guarantee will be looked at.

Then, in Section 3, we will define the rollover option. We will also derive the pricing formula for that option under the same assumptions as in Section 2. As will then be seen, an assumption with respect to the investor's behavior is necessary. In that section, the investor will be naive.

In Section 4, the results of Section 3 will be modified to accommodate the case of the investor with optimal behavior. Section 5 will compare the two formulae obtained, both theoretically and numerically, with one another as well as with the basic guarantee. Section 6 will contain the conclusion to this paper.

2 Basic Maturity Guarantees

Brennan and Schwartz [4, 5] have done the pioneering work for the valuation of these guarantees. The Maturity Guarantees Working Party [6] also studied their value, but their approach, based on simulations with the Wilkie model, will not be adopted here. Bacinello and Ortu [1, 2] further generalized Brennan and Schwartz' work by endogenizing the guarantees.

Since we will only consider the single premium case with exogenous guarantees for the rollover option, we will limit ourselves to that case as well in reviewing the results found in the literature.

The basic maturity guarantee is priced easily when its characteristics are recognized to be the same as that of a European put option on common stock. Indeed, on a single amount invested S_0 at time 0, the investor is guaranteed to receive at time T at least K, the guaranteed amount. This means that, at time T, the investor receives $\max[S_T,\ K]$, where S_T is the value of the fund at time T. Another way to write the same expression, so as to single out the guarantee, is $S_T\ +\ \max[0,\ K - S_T]$.

Hence, the value of the guarantee at maturity is the second part of the second expression. This element being identical to that of the payoff of a put option, its expected value at time 0 is given by the following:

$$P_b(S_0, K, r, \delta, \sigma, T) = K \cdot e^{-r \cdot T} \cdot N(-d_2) - S_0 \cdot e^{-\delta \cdot T} \cdot N(-d_1). \qquad (1)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(\frac{S_0}{K}) + (r - \delta + .5 \cdot \sigma^2) \cdot T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \end{aligned}$$

P_b denotes the price of a basic maturity guarantee, r is the riskfree force of interest, δ is the annual dividend payout rate, and σ is the annual volatility of the stock fund.

It is important to understand what δ actually stands for. It is *not* the annual stock dividend rate, *i.e.* the rate at which dividends are paid by the company to the fund. Rather, it the rate at which dividends are paid by the fund to the investor. Hence, if all dividends received by the fund are reinvested (*i.e.*, not distributed), δ is 0. However, if all dividends are distributed, the two concepts of "dividend rates" are equivalent and δ equals the annual stock dividend rate.

The above result is valid in a risk-neutral world, where markets are perfect and frictionless. It is, obviously, the celebrated Black-Scholes result.

One way to make this formula more applicable to the insurance industry is to incorporate the mortality risk into it. It is typical that the guarantee upon death will be 100% of the amount invested, even when the guarantee at maturity is less than 100%. Assuming deaths occur at end of year and the amount guaranteed on death is equal to that guaranteed at maturity, the price of the guarantee becomes

$$\begin{aligned} P_b^m(S_0, K, r, \delta, \sigma, T, \{_{j-1|}q_x\}_{j=1}^T) = \quad & \sum_{j=1}^{T} {}_{j-1|}q_x \cdot P_b(S_0, K, r, \delta, \sigma, j) \\ + \quad & {}_Tp_x \cdot P_b(S_0, K, r, \delta, \sigma, T), \end{aligned}$$

where x is the age of the insured at time 0, ${}_{j-1|}q_x$ is the probability that that individual dies in the j^{th} year, and ${}_Tp_x$ is the probability that he will survive the next T years. The superscript m indicates that mortality is accounted for.

3 Rollover Option – Naive Behavior

Now that we have the cornerstone, we are ready to build on it and consider the rollover option. That option has other names in the insurance market.

However, we have chosen to call it "rollover option" as, at the original maturity, the investor has the option to *roll over* his investment.
Hence, under the terms of the option studied here, at time T, the investor has the choice between two courses of action:

- Exercise the guarantee; *i.e.*, receive $\max[S_T, K]$ at time T.
- Roll over the guarantee; *i.e.*, renew the guarantee with new face amount S_T and new guaranteed amount $K \cdot \frac{S_T}{S_0}$, and receive $\max[S_T, K \cdot \frac{S_T}{S_0}]$ at time $2T$.

Of course, the particular terms of that option vary from company to company. On the one hand, it may be possible for the investor to roll over the guarantee at any time before time T, thereby making the option of the American type. On the other hand, the investor may have some discretion as to the new maturity of the guarantee when rolling over. However, the new guaranteed amount, should the guarantee be rolled over, is well-defined in the contract and is not an element over which the investor has any control.
For the sake of simplicity, we deal only with the European option whose extended maturity is equal to the original one. We also assume that the new guaranteed amount represents the same percentage of S_T as K did of S_0. For instance, if the original guarantee is 100% and the single premium is \$100, then the new guarantee, upon exercise, would be 100% of S_T.
In order to value the guarantee, one first has to understand the decision facing the investor at time T, that time at which the guarantee may be renewed or the contract closed out. On the one hand, it is clear that if closing the contract results in the guarantee having no value (*i.e.*, $S_T \geq K$), then the guarantee should be renewed as the new guarantee has a nonnegative value. On the other hand, one view, somewhat naive, is that if the guarantee is of value at time T (*i.e.*, $S_T < K$), the guarantee should be taken advantage of, and the contract terminated.
According to this naive behavior, the value of the rollover option is given by the following equation:

$$\begin{aligned} P_n(S_0, K, r, \delta, \sigma, T) = \quad & K \cdot e^{-r \cdot T} \cdot N(-d_2) - S_0 \cdot e^{-\delta \cdot T} \cdot N(-d_1) \\ + \quad & e^{-\delta \cdot T} \cdot N(d_1) \\ & \cdot [K \cdot e^{-r \cdot T} \cdot N(-d_2) - S_0 \cdot e^{-\delta \cdot T} \cdot N(-d_1)], \end{aligned} \tag{2}$$

where d_1 and d_2 are as in Equation 1. The first two terms on the right-hand side are the same as in Equation 1, since the payoff at time T is the same for the rollover option as for the basic maturity guarantee. The multiplicative term on the second line, $e^{-\delta \cdot T} \cdot N(d_1)$ represents the expected value of \$1 (invested in stock at time 0) at time T, discounted back to time 0, given that the accumulated value turns out to be more than $\frac{K}{S_0}$. The other multiplicative term, on the third line, is the value of the basic guarantee since, when renewed, the guarantee takes on the features of a newly issued basic guarantee.

From Equation 2, it is easy to see what the extra value of the rollover option is, compared to the basic one. The second and third lines of the equation capture the nature of the added feature. However, it is important to remember that that is the naive approach, and, in the next section, a better result will be derived.

Again, it is possible to generalize the equation so as to account for mortality. It should be relatively straightforward and, using the notation already introduced, the price of the rollover option then turns out to be

$$
\begin{aligned}
P_n^m(S_0, K, r, \delta, \sigma, T, \{{}_{j-1|}q_x\}_{j=1}^{2T}) \\
= \quad & \sum_{j=1}^{T} {}_{j-1|}q_x \cdot P_b(S_0, K, r, \delta, \sigma, j) \\
+ \quad & {}_Tp_x \cdot P_b(S_0, K, r, \delta, \sigma, T), \\
+ \quad & {}_Tp_x \cdot e^{-\delta \cdot T} \cdot N(d_1) \cdot [\sum_{j=1}^{T} {}_{j-1|}q_{x+T} P_b(S_0, K, r, \delta, \sigma, j) \\
& \qquad + {}_Tp_{x+T} \cdot P_b(S_0, K, r, \delta, \sigma, T)].
\end{aligned}
$$

As before, the right-hand side can be decomposed into meaningful parts. The first two lines recover the expression found for the price of the basic guarantee in Section 2. The third and fourth lines have the expected unit value given the guarantee is renewed, multiplied by the price of the basic guarantee at time T.

4 Rollover Option – Optimal Behavior

Now, although it may not be apparent at first sight, the result in Equation 2 is an underestimation of the true value of the rollover option. The reason is that, at time T, the naive behavior is suboptimal.

Of course, as was noted earlier, renewing the guarantee when it would otherwise be of no value makes sense. However, it is not necessarily a good strategy, in a risk-neutral environment, to exercise the guarantee at time

T instead of renewing it when it is only slightly in the money. (The same kind of argument is used for the exercise policy under the American stock options.)

What the investor really should do at time T is compare the value of the exercised guarantee to the expected value of the renewed guarantee. From that comparison, a breaking point can be determined; that point will divide the distribution of S_T into its "exercise" and "renew" regions. At the breaking point K^*, where indifference between the two actions is reached, the following equality holds:

$$max[0, K - K^*] = K^* \cdot P_b(1, \frac{K}{S_0}, r, \delta, \sigma, T).$$

Noting that P_b is always positive and rearranging the above equation, we obtain the following expression for K^*:

$$K^*(S, K, r, \delta, \sigma, T) = \frac{K}{1 + P_b(1, \frac{K}{S_0}, r, \delta, \sigma, T)}. \tag{3}$$

Knowing the breaking point, we then can calculate the value of the rollover option when the investor is behaving optimally. Hence, if $S_T \leq K^*$, the investor terminates the contract and takes advantage of the guarantee. So, in that case, we are back to the basic guarantee, with the difference that, although exercise is determined by K^*, the payoff still depends on K. If $S_T > K^*$, we need to find the expected discounted value of S_T conditional on that event, and multiply it by the value of a basic guarantee at time T. Following that line of reasoning produces the equation below.

$$\begin{aligned} P_o(S_0, K, r, \delta, \sigma, T) = & \quad K^* \cdot e^{-r \cdot T} \cdot N(-d_2^*) - S_0 \cdot e^{-\delta \cdot T} \cdot N(-d_1^*) \\ & + \; N(-d_2^*) \cdot (K - K^*) \cdot e^{-r \cdot T} \\ & + \; e^{-\delta \cdot T} \cdot N(d_1^*) \\ & \quad \cdot [K \cdot e^{-r \cdot T} \cdot N(-d_2) - S_0 \cdot e^{-\delta \cdot T} \cdot N(-d_1)], \end{aligned} \tag{4}$$

where

$$\begin{aligned} d_1^* &= \frac{\ln(\frac{S_0}{K^*}) + (r - \delta + .5 \cdot \sigma^2) \cdot T}{\sigma\sqrt{T}}, \\ d_2^* &= d_1^* - \sigma\sqrt{T}, \end{aligned}$$

d_1 and d_2 are as in Equation 1, and K^* is as defined in Equation 3. The first line on the right-hand side is the value of a basic guarantee with guaranteed amount K^*. The second line accounts for the fact that, as a matter of fact, K is the amount guaranteed, even though K^* defines the exercise policy. The last two lines give the value of the option conditional on $S_T > K^*$. There is a similarity between that term and the corresponding one in Equation 2; however, here, $N(d_1^*)$ is used instead of $N(d_1)$ as the first multiplicative term. As will be seen in the next section, through numerical values, Equation 4 yields larger values than Equation 2, all other things being equal. On the one hand, the value attached to the optimal behavior is decreased by the more stringent exercise policy at time T. On the other hand, that loss is more than recouped by the additional expected gain from holding the guarantee longer. Once more, it is possible to generalize the previous equation to accommodate mortality. However, unlike in the previous cases, incorporating the mortality rates is slightly more complicated, since the breaking point will also depend on them. So, that is, again, the first object to calculate. K^* is given by

$$
\begin{aligned}
&K^{*,m}(S,K,r,\delta,\sigma,T,\{{}_{j-1|}q_{x+T}\}_{j=1}^{T}) \\
&= \frac{K}{1+\sum_{j=1}^{T} {}_{j-1|}q_{x+T}\cdot P_b(S_0,K,r,\delta,\sigma,j) + {}_{T}p_{x+T}\cdot P_b(S_0,K,r,\delta,\sigma,T)} \\
&= \frac{K}{1+P_b^m(S,K,r,\delta,\sigma,T,\{{}_{j-1|}q_{x+T}\}_{j=1}^{T})},
\end{aligned}
$$

where x is the age of the investor at time 0. That formula is obtained by an argument similar to that used to get Equation 3.

Now that we have K^*, it is possible to find the price of the rollover option to be

$$
\begin{aligned}
&P_o^m(S,K,r,\delta,\sigma,T,\{{}_{j-1|}q_x\}_{j=1}^{2T}) \\
&= \sum_{j=1}^{T} {}_{j-1|}q_x\cdot P_b(S_0,K,r,\delta,\sigma,j) \\
&\quad + {}_{T}p_x\cdot[K^*\cdot e^{-r\cdot T}\cdot N(-d_2^*) - S_0\cdot e^{-\delta\cdot T}\cdot N(-d_1^*) \\
&\qquad\qquad + N(-d_2^*)\cdot(K-K^*)\cdot e^{-r\cdot T}] \\
&\quad + {}_{T}p_x\cdot e^{-\delta T}\cdot N(d_1^*)\cdot[\sum_{j=1}^{T} {}_{j-1|}q_{x+T}\cdot P_b(S_0,K,r,\delta,\sigma,j) \\
&\qquad\qquad + {}_{T}p_{x+T}\cdot P_b(S_0,K,r,\delta,\sigma,T)],
\end{aligned}
$$

where it is important to note that d_1, d_2, d_1^* and d_2^* all are functions of S, K, r, δ, σ and T.

5 Numerical Results

This section features numerical results based on the equations presented in the previous sections. We have ignored mortality, but we think that the arguments to be presented should convince the reader that similar conclusions would be reached when taking mortality into account.
All results are based on $S_0 = 100$, $r = .06$ and $\sigma = .20$. The choice of S_0 is unimportant - in fact, the ratio $S_0 : K$ is what matters. However, r and σ are important assumptions and should be viewed here as totally arbitrary choices.
The time to maturity will take on a range of values. As for the dividend payout rate, δ, it will either be 0 or .03. In the first case, all dividends received by the fund are reinvested. In the second case, under the assumption that the stock dividend rate also is .03, all dividends received by the fund are distributed to the investors.
Table 1 contains the values obtained when $K = 100$, while Table 2 contains those obtained when $K = 75$, both for δ=0. Figures 1 and 2 are meant to help visualize the contents of those tables. (All tables and figures are found at the end of the paper, after the references.)
In addition, Table 3 and Table 4 are the counterparts to Tables 1 and 2 in the case δ=.03. Again, Figures 3 and 4 are meant to depict the numbers found in the corresponding tables.
In all four tables, the third and fourth columns contain the values obtained when using Equations 2 and 4, respectively. As can be seen in the tables and on the graphs, these values are very close to one another, especially for later maturities. More comments will be made later on that initial observation.
The second column features the value of the basic guarantee. It is only meant to provide an idea of the difference in value between that and the rollover option. As seen on the graphs, there is an appreciable difference between the two. In view of that, ignoring the added value of the rollover option would appear to be a significant mistake.
As for the fifth column, it simply is twice the second one when δ=0. Otherwise, it is $1 + e^{-\delta \cdot T}$ times the second one. Interestingly enough, the longer the maturity, the more the values in the fourth and fifth columns seem to agree. So, one may wonder what kind of option the fifth column represents, if any. As a matter of fact, these costs are those of an option which would allow the investor to cash in the value of the guarantee at time T, as well as at time $2T$ based on the revised amount set at time T. We will refer to it as a "tandem put". (See Blazenko, Boyle and Newport [3] for more details on the tandem option, which is an option similar to the one just described,

except that it is based on calls and does not allow for dividends.)
In other words, the tandem put is like a rollover option which does not require the investor to terminate the contract when exercising the guarantee at time T. Since the rollover option only allows exercise once, its value is bounded above by double the basic guarantee when δ=0 or, more generally, by that of the tandem put.
As was noted earlier, the third and fourth columns are in close agreement. A look at Equations 2 and 4 confirms that the basic difference between the naive and optimal behaviors is the value of the breaking point. As seen in Table 5, K and K^* eventually get closer and closer to one another, as maturity increases. As a result, the values under the two exercise policies also get closer and closer to one another, as maturity increases.
It was also noted that the fourth and fifth columns are not too far from one another, although not the same at any maturity considered. Since both the naive and optimal behaviors eventually yield sensibly the same results, Equations 2 and 1 will be compared. This considerably simplifies the analysis. P_n and P_b are related in the following way:

$$P_n(S, K, r, \delta, \sigma, T) = [1 + e^{-\delta \cdot T} N(d_1)] \cdot P_b(S, K, r, \delta, \sigma, T). \qquad (5)$$

Hence, the closer $N(d_1)$ is to 1, the closer P_n is to $(1 + e^{-\delta \cdot T}) \times P_b$, which the value of the tandem put. Table 6 confirms that $N(d_1)$ increases towards 1 for larger maturities. This results in the restriction of a single exercise resulting in less and less of a loss.
Based on these earlier observations, it then seems that, for large maturities, using the simpler result provided by P_n would provide a satisfactory approximation. Likewise, $(1 + e^{-\delta \cdot T}) \times P_b$ may also provide a ballpark figure.
When δ=0, the factor $e^{-\delta \cdot T}$ disappears in Equation 5. As a result, past a certain T, the ratio of P_n to P_b strictly increases towards 2.
However, when $\delta > 0$, that exponential factor goes to 0 faster than $N(d_1)$ goes to 1. Hence, for strictly positive values of δ, the value of the rollover option also eventually gets close to that of the basic maturity guarantee. That means that, eventually, not only does the value of the rollover option get closer, from below, to that of the tandem put, but also that the rollover option is worth little more than the basic maturity guarantee in the long run. That fact is due to the readjustment of the guaranteed amount at time T.
One final observation may be worth making. For the basic maturity guarantee, the larger δ is, the more valuable the guarantee becomes. That is so because of the risk-neutrality assumption: the expected return must be the same regardless of the value of δ. This implies that the larger δ is, the smaller the expected capital gain is.

However, the same comparison is not so straightforward in the case of the rollover option. Again, because the expected capital gain decreases when δ increases, larger values of δ have a positive impact on the value of the rollover option. The readjustment of the guaranteed amount at time T, though, has the opposite effect, as larger values of δ lead to smaller expected renewal guaranteed amounts. Based on the values found in Tables 1 to 4, it appears that the former effect prevails and, thus, that a full reinvestment of dividends makes the rollover option most valuable.

Of course, that argument only looks at the value of the guarantee being offered. From the investor's point of view, all cash flows should be taken into account. However, all cash flows other than the guarantee payoff add up to the same expected value at time 0 no matter what δ is equal to. That statement also holds at time T. Hence, the same conclusions are reached even when considering all cash flows.

6 Conclusion

In this paper, we have derived formulae to value the rollover option in a risk-neutral environment. We have done so under two types of behavior: naive and optimal. Differences between the two behaviors have been analyzed qualitatively and quantitatively. The rollover option has also been compared to the basic guarantee, thereby showing how much more valuable it really is.

Although these results are interesting in themselves, determining how to hedge the rollover option, if feasible, would be of value. One could then study the impact of discretization as well as of transaction costs. Of course, whether or not hedging is possible would be the first issue to address. While it is clear that hedging could be done at time T if and when the guarantee is renewed, it is not clear that hedging could effectively be done starting at time 0.

Likewise, calculating the value of the rollover option under other hypotheses regarding the stock market and the investor's behavior might yield more relevant results. In any case, pinpointing the optimal course of action at time T remains central to the solution.

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Table 1: Prices of Different Options
S_0=100, K=100, r=.06, δ=0, σ=.20

Original Maturity	Basic Guarantee	Naive Rollover	Optimal Rollover	2 x Basic Guarantee
1	5.166	8.552	8.784	10.332
2	5.890	10.096	10.292	11.779
3	6.026	10.580	10.735	12.052
4	5.920	10.586	10.705	11.840
5	5.697	10.337	10.428	11.394
6	5.415	9.945	10.015	10.831
7	5.107	9.474	9.527	10.214
8	4.790	8.963	9.003	9.580
9	4.475	8.435	8.466	8.950
10	4.168	7.908	7.931	8.337
11	3.874	7.391	7.409	7.748
12	3.594	6.890	6.904	7.188
13	3.330	6.411	6.422	6.660
14	3.081	5.956	5.964	6.163
15	2.849	5.525	5.531	5.698
16	2.632	5.120	5.124	5.264
17	2.430	4.739	4.743	4.860
18	2.242	4.384	4.386	4.484
19	2.068	4.052	4.054	4.136
20	1.907	3.743	3.745	3.813

Table 2: Prices of Different Options
S_0=100, K=75, r=.06, δ=0, σ=.20

Original Maturity	Basic Guarantee	Naive Rollover	Optimal Rollover	2 x Basic Guarantee
1	0.279	0.548	0.548	0.557
2	0.766	1.489	1.490	1.532
3	1.109	2.148	2.150	2.219
4	1.324	2.563	2.566	2.648
5	1.447	2.805	2.808	2.895
6	1.507	2.927	2.929	3.015
7	1.523	2.964	2.966	3.047
8	1.509	2.942	2.944	3.018
9	1.474	2.880	2.881	2.948
10	1.425	2.789	2.790	2.850
11	1.367	2.680	2.681	2.733
12	1.303	2.559	2.560	2.606
13	1.236	2.431	2.432	2.472
14	1.168	2.300	2.301	2.335
15	1.100	2.170	2.170	2.200
16	1.033	2.040	2.041	2.066
17	0.968	1.914	1.915	1.936
18	0.906	1.792	1.793	1.811
19	0.846	1.676	1.676	1.692
20	0.789	1.564	1.564	1.578

Figure 1: Comparison of Different Option Prices
S_0=100, K=100, r=.06, δ=0, σ=.20

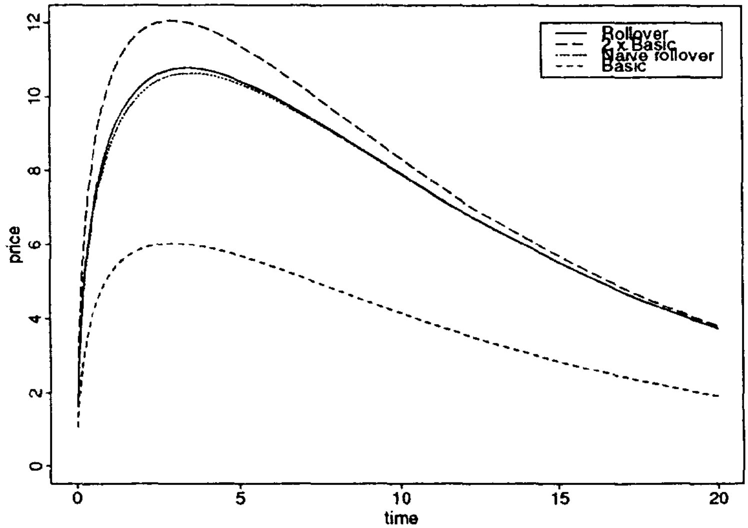

Figure 2: Comparison of Different Option Prices
S_0=100, K=75, r=.06, δ=0, σ=.20

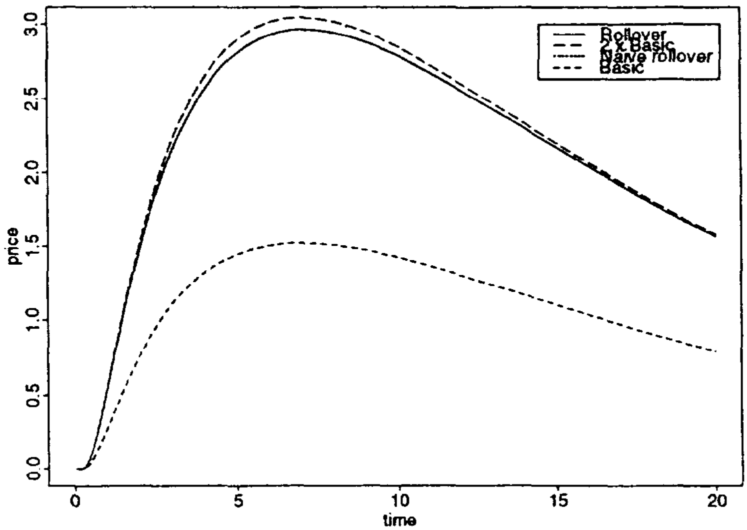

Table 3: Prices of Different Options
S_0=100, K=100, r=.06, δ=.03, σ=.20

Original Maturity	Basic Guarantee	Naive Rollover	Optimal Rollover	$(1+e^{-\delta \cdot T})$ x Basic Guarantee
1	6.267	9.908	10.258	12.349
2	7.770	12.439	12.793	15.087
3	8.493	13.674	13.997	16.255
4	8.834	14.251	14.536	16.668
5	8.949	14.432	14.678	16.651
6	8.919	14.356	14.566	16.369
7	8.793	14.108	14.286	15.920
8	8.600	13.743	13.893	15.365
9	8.363	13.300	13.426	14.747
10	8.095	12.806	12.911	14.093
11	7.808	12.279	12.367	13.422
12	7.509	11.736	11.809	12.748
13	7.204	11.186	11.247	12.082
14	6.898	10.638	10.689	11.430
15	6.593	10.097	10.140	10.797
16	6.293	9.568	9.604	10.186
17	5.998	9.055	9.084	9.600
18	5.711	8.559	8.583	9.039
19	5.432	8.081	8.101	8.505
20	5.163	7.623	7.640	7.997

Table 4: Prices of Different Options
S_0=100, K=75, r=.06, δ=.03, σ=.20

Original Maturity	Basic Guarantee	Naive Rollover	Optimal Rollover	$(1+e^{-\delta \cdot T})$ x Basic Guarantee
1	0.394	0.759	0.759	0.776
2	1.174	2.186	2.190	2.281
3	1.816	3.304	3.311	3.475
4	2.294	4.103	4.113	4.329
5	2.641	4.654	4.666	4.914
6	2.885	5.017	5.031	5.295
7	3.049	5.239	5.253	5.521
8	3.152	5.354	5.366	5.631
9	3.206	5.386	5.398	5.654
10	3.222	5.356	5.366	5.610
11	3.209	5.278	5.288	5.516
12	3.173	5.165	5.174	5.386
13	3.118	5.025	5.033	5.230
14	3.050	4.867	4.874	5.054
15	2.971	4.695	4.701	4.866
16	2.885	4.515	4.520	4.670
17	2.793	4.329	4.333	4.470
18	2.697	4.141	4.145	4.268
19	2.599	3.954	3.957	4.068
20	2.499	3.768	3.770	3.871

Figure 3: Comparison of Different Option Prices
S_0=100, K=100, r=.06, δ=.03, σ=.20

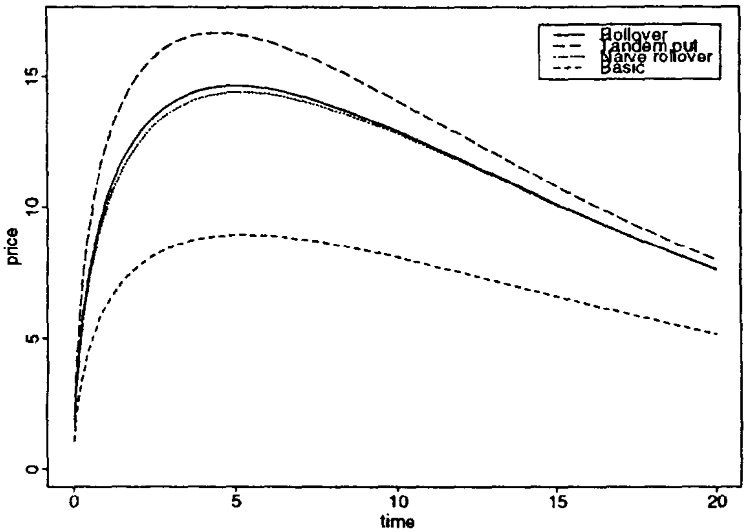

Figure 4: Comparison of Different Option Prices
S_0=100, K=75, r=.06, δ=.03, σ=.20

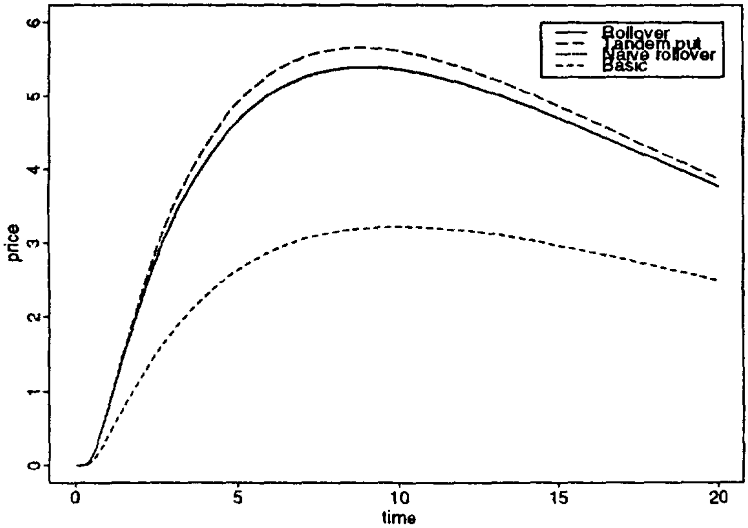

Table 5: Breaking Points

T	$\delta = 0$		$\delta = .03$	
	K = 100	K = 75	K = 100	K = 75
1	95.088	74.792	94.103	74.706
2	94.438	74.430	92.791	74.129
3	94.317	74.177	92.172	73.663
4	94.411	74.020	91.883	73.318
5	94.610	73.930	91.786	73.070
6	94.863	73.886	91.811	72.897
7	95.141	73.875	91.918	72.781
8	95.429	73.885	92.081	72.708
9	95.717	73.910	92.282	72.670
10	95.998	73.946	92.511	72.659
11	96.270	73.989	92.757	72.668
12	96.531	74.035	93.015	72.694
13	96.777	74.084	93.280	72.732
14	97.011	74.134	93.547	72.780
15	97.230	74.184	93.815	72.836
16	97.436	74.233	94.080	72.897
17	97.628	74.281	94.341	72.962
18	97.807	74.327	94.597	73.031
19	97.974	74.371	94.847	73.100
20	98.129	74.413	95.090	73.171

Table 6: Values of $N(d_1)$

	$\delta = 0$		$\delta = .03$	
T	K = 100	K = 75	K = 100	K = 75
1	0.65542	0.96700	0.59871	0.95433
2	0.71420	0.94327	0.63816	0.91476
3	0.75579	0.93616	0.66750	0.89679
4	0.78814	0.93564	0.69146	0.88862
5	0.81445	0.93794	0.71192	0.88538
6	0.83641	0.94145	0.72985	0.88485
7	0.85504	0.94542	0.74583	0.88592
8	0.87105	0.94949	0.76025	0.88794
9	0.88493	0.95347	0.77337	0.89055
10	0.89705	0.95726	0.78540	0.89351
11	0.90769	0.96083	0.79649	0.89668
12	0.91707	0.96414	0.80676	0.89995
13	0.92538	0.96720	0.81631	0.90326
14	0.93276	0.97002	0.82521	0.90656
15	0.93933	0.97261	0.83354	0.90982
16	0.94520	0.97498	0.84134	0.91302
17	0.95045	0.97715	0.84868	0.91615
18	0.95516	0.97913	0.85558	0.91920
19	0.95938	0.98094	0.86208	0.92216
20	0.96318	0.98259	0.86822	0.92502