ACTUARIAL RESEARCH CLEARING HOUSE
1995 VOL. 2

AN ALGEBRAIC RESERVING METHOD FOR PAID LOSS DATA

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Sooner or later a casualty actuary is confronted by the question, "Given a history of paid loss amounts by calendar year, what should reserves be?" Often, it is not possible to accurately gather and analyze additional data within the time constraints for the reserving decision. The algebraic reserving method presented in this paper offers one approach to rapidly addressing this problem. The paper consists of four sections - General Considerations, Formulas, Examples, and Conclusion.

General Considerations

In general, reserve estimates will prove more accurate to the extent that they reflect information from a variety of sources and several actuarial methods. In any reserving situation, available data and information is limited by practical constraints (e.g., design of systems) and time constraints (e.g., financial reporting deadlines). In addition, the question of whether the benefits of better actuarial estimates are worth the costs of gathering better information is implicit in any reserving situation.

The situation to which the algebraic reserving method applies is one in which available information is paid losses by calendar year and there is some basis on which the actuary can assess the annual change in the level of incurred losses by accident year.

For example, a history of earned premiums might be used to create an index of loss levels by accident year. Or, the assumption regarding loss levels by accident year might even be weaker. For example, losses for similar business might have increased an average of 10% per year for the period for which paid losses are available.

The information on loss levels need not be detailed to afford an algebraic solution to the reserving problem. However, in general the more accurate the assumed relative loss levels, the more accurate the estimated reserves will be.

In addition to requiring an assumption regarding relative loss levels by accident year, the method assumes that there is a stable development pattern across all accident years. Thus, as the number of calendar years increases or the numbers of claims whose payments comprise calendar year paid amounts decreases, the possibility of fluctuations in actual payment patterns becomes more important in evaluating the results of the algebraic method.

The information on paid losses should cover all calendar years from the inception of the program. Otherwise, the method cannot estimate reserves without ad hoc adjustments. For example, if data started with the third year of the program, the method would estimate the portion of accident year losses paid through 36 months maturity instead of 12 months. Since the most recent accident year would be at 12 months maturity, the estimate through 36 months would have to be allocated to the maturities 0-12 months, 12-24 months, and 24-36 months using other techniques in order to derive reserve estimates for all accident years.

Tail factors are beyond the scope of the algebraic method. For n calendar year periods, the algebraic method derives development through maturity n years and leaves the tail factor to further analysis. Unless a parameterized payment pattern is assumed and the structure of the equations changed, the tail factor will require separate actuarial analysis.

Finally, the method is called the "algebraic method" because it is based on the algebraic solution of n linear equations in n unknowns. Thus, for any set of assumed relative loss levels, there is a unique solution for unpaid (unreported) losses that will be paid (reported) on or before accident years attain maturity n years. Reserve estimates based on successful mathematical solutions of the equations may differ from reasonable actuarial estimates. The algebraic method can provide useful input into actuarial decisions on appropriate reserves, but should not be used as an algorithm without professional scrutiny.

Formulas

The following equations define the "algebraic method."

I_j = *Incurred amount for accident year j.* (1)

n = *Number of calendar years for which data is available*
= *Number of accident years affecting data* (2)

$$f_i = \textit{Fraction of accident year loss paid during year } i \textit{ after start of accident year.} \qquad (3)$$

Because the algebraic method estimates development through maturity n and because the sum of the fractions of losses at maturity n paid in each calendar year must total unity (i.e., 100%),

$$f_n = 1 - \sum_{i=1}^{n-1} f_i \qquad (4)$$

Calendar year payments can now be expressed in terms of accident year components.

$$\begin{aligned} P_j &= \textit{Amount paid during calendar year } j \textit{ for all accident years} \\ &= \sum_{i=1}^{j} f_i \, I_{j+1-i} \qquad (5) \\ \textit{so that, if } j=n,\ P_j &= \sum_{i=1}^{n-1} f_i \, I_{n+1-i} + \left(1 - \sum_{i=1}^{n-1} f_i\right) I_1 \end{aligned}$$

Introducing loss level indices facilitates solving equation (5). We define indices as follows:

$$\begin{aligned} g_j &= \textit{Index for accident year } j \textit{ loss level} \\ &= \frac{I_j}{I_1} \qquad (6) \\ \textit{so that } g_1 &= 1.000 \\ g_j &= g_2^{(j-1)} \quad \textit{for uniform growth} \end{aligned}$$

Equation (5) can now be rewritten as:

$$P_j = \sum_{i=1}^{j} f_i \, g_{j+1-i} \, I_1 \qquad \text{if } j<n$$

$$P_j = I_1 + \sum_{i=1}^{n-1} f_i \, (g_{n-i}-1) \, I_1 \quad \text{if } j=n$$

In order to generate n linear equations in n unknowns, we introduce a variable equal to the reciprocal of incurred losses.

$$R_j = \frac{1}{I_j} \tag{8}$$
$$= \textit{Reciprocal of incurred loss for accident year } j$$

The resulting n linear equations are:

$$0 = -P_j R_1 + \sum_{i=1}^{j} f_i \, g_{j+1-i} \qquad \text{if } j<n$$

$$-1 = -P_j R_1 + \sum_{i=1}^{n-1} f_i \, (g_{n-i}-1) \qquad \text{if } j=n$$

Thus, the algebraic reserving method solves the n equations

$$\begin{array}{rcl}
0 & = & -P_1R_1 + 1\ f_1 + 0\ f_2 + 0\ f_3 + \ldots + 0\ f_{n-1} \\
0 & = & -P_2R_1 + g_2\ f_1 + 1\ f_2 + 0\ f_3 + \ldots + 0\ f_{n-1} \\
0 & = & -P_3R_1 + g_3\ f_1 + g_2\ f_2 + 1\ f_3 + \ldots + 0\ f_{n-1} \\
\ldots & = & \ldots \\
-1 & = & -P_nR_1 + (g_n-1)f_1 + (g_{n-1}-1)f_2 + (g_{n-3}-1)f_3 + \ldots + (g_2-1)f_{n-1}
\end{array} \tag{10}$$

for R_1, f_1, ..., f_{n-1}.

Examples

In the attached exhibits, data for private passenger automobile liability/medical from pages 63 and 79 of the 1993 edition of Best's Aggregates & Averages is used to illustrate the algebraic method. For convenience, loss and allocated loss adjustment expense is called "loss" in this discussion.

Exhibit I presents a link ratio approach to establish a benchmark for comparison to the results of the algebraic method. Weighted three point average development factors are employed. Other link ratio calculations are possible, but only one is used for comparison purposes in this paper. Exhibit I-1 presents raw data. Exhibit I-2 derives development factors. Exhibit I-3 derives reserve estimates using the development pattern from Exhibit I-2.

Exhibit II derives values for use in subsequent algebraic method calculations. Exhibit II-1 derives calendar year paid loss as if accident year 1983 were the first year of a program. Exhibit II-2 uses earned premiums to estimate loss level indices. Distinct indices by year are used in Exhibit III and a rough average annual growth rate is used in Exhibit IV.

Exhibit III applies the algebraic method using distinct indices by year. Exhibit III-1 presents the matrix defining the simultaneous equations. Exhibit III-2 presents the inverted matrix and the estimated parameters R_1, f_1, ..., f_{n-1}. Exhibit III-3 compares the paid amounts based on the parameters to the actual paid amounts by accident year component as well as by calendar year total. Exhibit III-4 adjusts the development pattern for negative values and derives corresponding reserve estimates.

Negative values might be attributable to several causes (e.g., influence of particular large claims, shifts in development patterns over time). Consideration of alternative possible adjustments will vary with available data and reserving context, and is, therefore, beyond the scope of this paper.

Exhibits IV are organized identically to Exhibits III. The difference is that a uniform annual change in loss level is used in lieu of individual annual indices.

Following Exhibits IV are four graphs. Graph 1 presents the three cumulative development patterns fit using the above techniques. Graph 2 presents the same development patterns on an interval basis. Graph 3 compares reserves estimates by accident year. Graph 4 presents the components of accident year losses using the three methods.

Conclusion

For the data used in the example, the algebraic method presented above produced reserve estimates quite close (within 10%) to reserve estimates based on a link ratio method. Therefore, it might prove useful in situations in which detailed data is unavailable. In particular, it might prove useful in reserving situations for which only calendar year paid loss data is available.

For the example, the method required elimination of some negative values from the development pattern. Also, the algebraic reserving method is quite sensitive to the selection of loss level indices. Therefore, although it can prove useful in particular situations, it is not well suited to use as an algorithm without professional scrutiny by a casualty actuary.