ACTUARIAL RESEARCH CLEARING HOUSE
1997 VOL. 1

Economic Scenario Generator for Insurance and Pension Rational Decision Making Under Uncertainty

by

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Abstract: We develop a stochastic generator for the generation of scenarios of the S & P 500 index, dividend yield, consumer price index, and U.S. Treasury yields. We first create a set of "stylized facts" for these series. We estimate statistical models for these series. These in-sample statistical models are themselves not suitable for generation of scenarios for decision making, but instead are additional "stylized facts" that assist in model development. The "best" statistical model according to standard statistical model selection criteria can easily lead to a model that is highly unsuitable for generation of scenarios for decision making. We develop a stochastic generator that is suitable for decision making under uncertainty.

Key Words: Stylized Facts, Double Mean Reverting Process™, ARIMA models, transfer functions, Green's Functions, diffusion models, Rational Decision Making, Insurance, Pension, Uncertainty.

Economic Scenario Generation

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General Observations

- A joint process on the yield curve, and additional economic state variables is needed in many applications
- Examples of additional economic state variables are inflation, stock price index, dividend yield on this index, and currency exchange rates.

Examples

- To evaluate an equity linked annuity, one needs a joint process on the yield curve and the equity index's price and dividend returns. The pricing and the hedge strategy depend on this process.
- For proper asset-liability projection of an entire company, inflation is usually required. Also exchange rates are required if the company is international.

Examples, Continued

- Economic scenarios could be used in the bond and stock asset mix analysis in pension plans and casualty insurance companies.

Qualitative Stylized Facts

- Qualitative
  - These would be qualitative observations that are true over long time interval and across economies. They are not episode dependent.
  - We believe that these facts should be contained in the equilibrium model for all industrial economies.

Quantitative Stylized Facts

- These are specific numerical observations of specific market processes.
- These are less stable than qualitative facts, and can change through time.

Phenomenon

- Interesting relationships observed.
- These still need to be explained.

Philosophy

- Two approaches to statistical modeling.
  1. We find the true model and find its parameters by maximum likelihood estimation. This model is selected to pass all in-sample statistical tests no matter how strange the resulting model.

Philosophy, Continued

2. We find an approximate model. We constrain the functional form of the model to conform to qualitative stylized facts. We may constrain the parameters of the model to satisfy the quantitative stylized facts. We may estimate the model with a criterion function based on multiple measures of in-sample fit....

Philosophy, Continued

2. Continued: In the above estimations we may create rough quantitative stylized facts from qualitative stylized facts. However, the approximate model may not pass all statistical tests.

Tenney's Interest Rate Razor

- Interest rates do not go to zero or infinity but stay within a reasonable range.
- Interest rates can spend up to several years within a narrow band or trading range.
- Short- and long-term rates are correlated but not perfectly.

Tenney's Razor, Continued

- The volatility of long-term rates is less than that of short-term rates.
- Yield curves can have a variety of shapes
- Volatility is higher for higher levels of rates.

Modified Becker's Interest Rate Razor

- Interest rates are nonnegative
- Interest rates do not go to zero nor do they go 'low' and stay low indefinitely.
- Interest rates do not go to 'infinity' nor do they grow 'large' and stay large indefinitely.

Becker's Razor, Continued

- Interest rates neither increase or decrease rapidly with significant frequency.
- Interest rate have an appearance of mean reversion

Becker's Razor, Continued

- Interest rates tend to cluster in 'trading ranges' or narrow bands (sometimes for extended periods) before breaking out to a higher or lower range.

Becker's Razor, Continued

- Higher absolute interest rate levels are often associated with higher absolute interest rate volatility
- Short- and long-term rates are not perfectly correlated, but do often move together.
- Short rate volatility is higher than long rate volatility

Becker's Razor, Continued

- Yield curves can have a variety of shapes
  - Significant inversions are infrequent (less than 13%) and of relatively limited durations (less than 27 months).
  - Yield curves are normally positively sloped, but can have 'humps'

Mark Tenney's DMRP™

- Mark has effectively modeled the above two Razors using his Double Mean Reverting Process™.
- His use of Green's Functions and the efficient solution of stochastic differential equations, allows the DMRP™ model to be a good starting point for an economic generator.

Wilkie's Economic Generator

- Developed in mid 1980's by David Wilkie .
- Uses transfer models to create a cascade model projecting inflation, stock dividend yield, stock price return and long term interest rates.
- Recently modified (1995) to model wages, property indices, and currency exchange rates.

Some Problems with Wilkie's Generator(s)

- Annual model only
- All the processes depend on the inflation process.
- The probability of stable inflation rates over several years in his U.S. model is low.

Some Problems, Continued

- Long-term interest rates are dependent upon dividend yields. This does not make sense.

Our Sword - Qualitative Stylized Facts

- Tenney's Razor
- Modified Becker's Razor
  - The above is implemented with Tenney's DMRP™.
- Selections from Wilkie's comments.

Our Sword, Continued

- The rate of inflation has trended upward from the bottom of the depression to the early 80's and then significantly lower up to 1996. (Modified from Wilkie)
- Higher absolute inflation rate levels are often associated with higher absolute inflation rate volatility. (Modified from Wilkie)

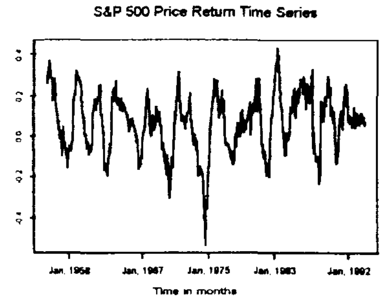
Our Sword, Continued

- The stock price return is weakly correlated with interest rates.*
- The stock index tends to trend up.
- Stock prices can cluster in 'trading ranges' or narrow bands (sometimes for extended periods)

*This similar to Vishwanath Tirupattur's results.

Phenomenon

- Inverted Yield Curves and S&P 500
- Interest Rate Volatility

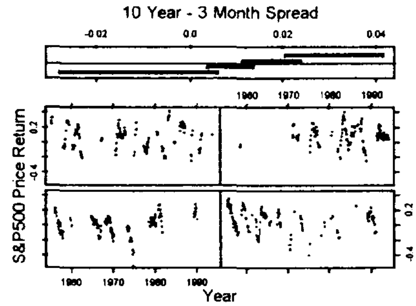


Coplot

- The first interval has min(spread) as its left end point.
- The last interval has max(spread) as its right end point.
- All other endpoints are values in spread.

Coplot, Continued

- The number of values of spread in the intervals are as nearly equal as possible
- For two successive intervals, the fraction of points shared by the intervals is as close as 25% as possible.
- See Cleveland's "Visualizing Data."



S&P Price Return Statistics -.0282<= Spread <=.0058

Min	-53.44%
1'st Qu.	-5.46%
Median	4.00%
Mean	2.40%
3'rd Qu.	11.41%
Max	29.56%
Std Dev	13.87%

S&P Price Return Statistics .0037<= Spread <=.0135

Min	-38.21%
1'st Qu.	-3.91%
Median	11.58%
Mean	8.01%
3'rd Qu.	18.31%
Max	36.99%
Std Dev	15.03%

S&P Price Return Statistics .0112<= Spread <=.0237

Min	-23.20%
1'st Qu.	-6.09%
Median	10.27%
Mean	8.08%
3'rd Qu.	20.92%
Max	42.77%
Std Dev	15.95%

S&P Price Return Statistics .0204<= Spread <=.0414

Min	-23.20%
1'st Qu.	3.26%
Median	9.48%
Mean	9.94%
3'rd Qu.	18.12%
Max	41.74%
Std Dev	12.19%

Trading Strategy I

- The first month that the spread was less than 58 bp, invest $100 in S&P 500 and $100 in T-bills.
- Stop the strategy the first month the spread exceeds the 58 bp spread. Compare the returns

Trading Example-Basic Data

Date	S&P Index	Div Index	3 Mon Rate	10 Year Rate
11/55	42.34			
12/55	45.48	0.18	.0256	.0296
01/56	43.82	0.13	.0246	.0290
02/56	45.34	0.13	.0237	.0284
03/56	48.48	0.13	.0231	.0296

Trading Example

- Calculate S&P 500 total monthly return by adding the month end S&P index and the Dividend index and divide the total by the prior month end S&P index value.

Documention

- See Excel T:\craighs\snp500\arcdata.xls

Trading Example, Continued

Date	Spread	Total Return	S&P Invest	T-Bill Invest
12/55	40bp		100.00	100.00
01/56	44bp	0.9664	96.64	100.21
02/56	47bp	1.0377	100.27	100.41
03/56	65bp	1.0721	107.51	100.60

Dates when spread < 58bp

STARTING DATE	ENDING DATE
12/1955	3/1956
4/1956	7/1956
9/1956	7/1957
8/1957	2/1958
10/1959	3/1960
11/1964	4/1967
1/1968	2/1968

Dates when spread < 58bp

STARTING DATE	ENDING DATE
4/1968	4/1970
4/1973	10/1974
11/1974	1/1975
10/1978	5/1980
10/1980	12/1980
1/1981	10/1981
3/1989	2/1990

Trading Strategy Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
12/1955	107.51	100.60
4/1956	103.10	100.63
9/1956	109.06	102.64
8/1957	92.21	101.49
10/1959	97.62	101.73
11/1964	120.43	111.17
1/1968	97.15	100.41

Trading Strategy Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
4/1968	89.14	113.47
4/1973	73.24	112.41
11/1974	110.96	101.14
10/1978	130.30	118.24
10/1980	107.31	102.48
1/1981	97.80	111.55
3/1989	116.15	107.51

Trading Strategy Rates of Return

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
12/1955	2.44%	.20%
4/1956	1.02%	.21%
9/1956	0.87%	.26%
8/1957	-1.34%	.25%
10/1959	-0.48%	.34%
11/1964	0.64%	.37%
1/1968	-2.86%	.41%

Trading Strategy Rate of Return

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
4/1968	-.48%	.53%
4/1973	-1.72%	.65%
11/1974	5.34%	.57%
10/1978	1.40%	.89%
10/1980	3.59%	1.23%
1/1981	-.24%	1.22%
3/1989	1.37%	.66%

Trading Strategy Performance

	AVERAGE	STD DEV
S&P ARI	103.71	13.73
TBILL ARI	106.10	5.86
DELTA	-2.39	14.32
S&P ROR	.68%	2.08%
TBILL ROR	.56%	0.33%

Trading Strategy II

- The first month that the spread is between 37bp and 135bp, invest $100 in S&P 500 and $100 in T-bills.
- Stop the strategy the first month the spread moves outside of the above spread interval. Compare the returns

Dates 37bp<spread<135bp

STARTING DATE	ENDING DATE
4/1953	2/1954
3/1954	5/1954
12/1954	1/1955
3/1955	6/1955
7/1955	12/1956
5/1957	9/1957
10/1957	12/1957
1/1958	2/1958

Dates 37bp<spread<135bp

STARTING DATE	ENDING DATE
9/1958	5/1959
6/1959	11/1959
2/1960	6/1960
9/1960	10/1960
3/1961	4/1961
1/1962	12/1964
6/1965	7/1965
8/1965	10/1965

Dates 37bp<spread<135bp

STARTING DATE	ENDING DATE
4/1967	6/1967
7/1967	4/1968
4/1970	10/1970
7/1971	8/1971
12/1972	6/1973
10/1974	11/1974
1/1975	2/1975
10/1977	11/1977

Dates 37bp<spread<135bp

STARTING DATE	ENDING DATE
9/1978	11/1978
5/1980	6/1980
9/1980	10/1980
10/1981	11/1981
2/1982	3/1982
4/1982	5/1982
3/1986	5/1986
1/1988	6/1989

Dates 37bp<spread<135bp

STARTING DATE	ENDING DATE
9/1989	11/1989
1/1990	9/1990
11/1990	1/1991

Trading Strategy II Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
4/1953	111.67	101.44
3/1954	109.39	100.15
12/1954	102.08	100.11
3/1955	113.38	100.38
7/1955	113.34	103.64
5/1957	90.64	101.13
10/1957	98.14	100.54

Trading Strategy II Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
1/1958	98.23	100.13
9/1958	119.76	101.90
6/1959	101.06	101.59
2/1960	102.60	101.06
9/1960	100.07	100.20
3/1961	100.63	100.19
1/1962	135.40	109.69

Trading Strategy II Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
6/1965	101.59	100.32
8/1965	106.58	100.66
4/1967	96.92	100.59
7/1967	105.50	103.70
4/1970	104.15	103.28
7/1971	103.86	100.42
12/1972	89.62	103.10

Trading Strategy II Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
10/1974	95.12	100.63
1/1975	106.38	100.47
10/1977	103.16	100.51
9/1978	93.24	101.41
5/1980	103.16	100.58
9/1980	102.02	100.97
10/1981	104.13	100.94

Trading Strategy II Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
2/1982	99.48	101.04
4/1982	96.59	101.01
3/1986	104.13	101.02
11/1988	118.55	105.02
9/1989	99.66	101.28
1/1990	95.33	105.23
11/1990	107.25	101.10

Trading Strategy II Performance

	AVERAGE	STD DEV
S&P ARI	103.80	8.70
TBILL ARI	101.58	1.92
DELTA	2.21	7.83
S&P ROR	.94%	2.50%
TBILL ROR	.44%	0.26%

Trading Strategy III

- The first month that the spread is between 112bp and 237bp, invest $100 in S&P 500 and $100 in T-bills.
- Stop the strategy the first month the spread moves outside of the above spread interval. Compare the returns

Dates 112bp<spread<237bp

STARTING DATE	ENDING DATE
10/1953	12/1953
1/1954	8/1955
2/1958	10/1958
1/1959	6/1959
7/1959	8/1959
6/1960	4/1962
5/1962	7/1962
8/1962	11/1962

Dates 112bp<spread<237bp

STARTING DATE	ENDING DATE
5/1967	7/1967
9/1970	3/1971
4/1971	1/1972
3/1972	4/1972
6/1972	2/1973
2/1975	4/1975
7/1975	11/1975
6/1977	9/1978

Dates 112p<spread<237bp

STARTING DATE	ENDING DATE
7/1980	10/1980
10/1981	12/1981
1/1982	2/1982
3/1982	4/1982
5/1982	8/1982
3/1983	8/1983
8/1984	11/1984
12/1985	5/1987

Dates 112p<spread<237bp

STARTING DATE	ENDING DATE
7/1988	12/1988
8/1990	4/1991
10/1993	11/1993

Trading Strategy III Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
10/1953	102.33	100.26
01/1954	177.91	101.85
02/1958	128.93	101.03
01/1959	106.84	101.22
07/1959	98.76	100.28
06/1960	121.42	104.57
05/1962	98.24	100.47
08/1962	106.37	100.70
05/1967	106.91	100.65

Trading Strategy III Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
9/1970	121.11	102.33
4/1971	102.45	103.39
3/1972	100.68	100.31
6/1972	106.23	103.22
2/1975	107.76	100.94
7/1975	104.26	102.05
6/1977	108.84	108.26
7/1980	106.09	102.62

Trading Strategy III Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
1/1982	94.41	101.15
3/1982	104.52	101.07
5/1982	108.49	102.78
3/1983	109.20	103.70
8/1984	99.28	102.45
12/1985	143.96	108.65

Trading Strategy III Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
12/1985	143.96	108.65
07/1988	103.83	103.15
08/1990	118.97	104.44
10/1993	99.05	100.26

Trading Strategy III Performance

	AVERAGE	STD DEV
S&P ARI	110.68	16.82
TBILL ARI	102.36	2.14
DELTA	8.32	16.23
S&P ROR	1.24%	1.98%
TBILL ROR	0.49%	0.29%

Trading Strategy IV

- The first month that the spread is above 204bp, invest $100 in S&P 500 and $100 in T-bills.
- Stop the strategy the first month the spread moves below 204bp. Compare the returns

Dates 204<spread<414bp

STARTING DATE	ENDING DATE
06/1958	08/1958
02/1971	06/1971
01/1972	09/1972
03/1975	07/1975
09/1975	08/1977
06/1980	08/1980
11/1981	02/1982
06/1982	07/1982

Dates 204<spread<414bp

STARTING DATE	ENDING DATE
08/1982	09/1982
10/1982	06/1983
07/1983	02/1986
09/1986	11/1986
04/1987	09/1988
03/1991	02/1994

Trading Strategy IV Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
06/1958	106.22	100.22
02/1971	104.11	101.34
01/1972	108.40	102.60
03/1975	107.93	101.88
09/1975	124.96	110.13
06/1980	108.04	101.45
11/1981	90.79	103.13
06/1982	98.22	100.99

Trading Strategy IV Performance

DATE	STOCK PORTFOLIO	T-BILL PORTFOLIO
08/1982	101.25	100.68
10/1982	129.62	105.60
07/1983	156.31	124.47
09/1986	108.34	100.88
04/1987	98.95	109.07
03/1991	135.67	111.48

Trading Strategy IV Performance

	AVERAGE	STD DEV
S&P ARI	112.77	17.06
TBILL ARI	105.28	6.45
DELTA	7.49	12.21
S&P ROR	1.28%	1.96%
TBILL ROR	0.55%	0.25%

Trading Strategy V

- The first month that the 3 Month Bill Rate exceeds the 10 year Bond Rate by 3%, invest $100 in S&P 500 and $100 in T-bills.
- Stop the investment the first month when the above condition changes.

Trading Strategy V Performance

Starting Date	Stock Portfolio	T-bill Portfolio
09/66	104.24	102.47
06/69	100.43	105.54
06/73	116.81	111.15
11/78	95.07	139.53

Trading Strategy V Performance

	Stock Portfolio	T-Bill Portfolio
Average	104.14	114.67
Std. Dev	8.00	14.69

Simple Regression Results

- Let S&P 500 Total Monthly return be regressed against the following:
  1. 3 Month T-Bill Rate
  2. 10 Year T-Bond Rate
  3. 10 Year to 3 Month spread
  4. 1 and 3 above
  5. 2 and 3 above

3 Month T-Bill Rate

- S&P tot return = a * T3Mon + b
- a = -.121 (.0623) (std err of coef)
- b = 1.017
- R^2 = .0077
- Std err of Y Est = .0414

10 Year T-Bond Rate

- S&P tot return = a * T10Year + b
- a = -.052 (.0636)
- b = 1.013
- R^2 = .0415
- Std err of Y Est = .0415

10 Year to 3 Month Spread

- S&P tot return = a * Spread + b
- a = 0.444 (.1554)
- b = 1.004
- R^2 = .0164
- Std err of Y Est = .0412

3 Month T-Bill and Spread

- S&P tot ret = a * T3Mon + b*Spread + c
- a = -.0829 (.0639)
- b = .3930 (.1601)
- c = 1.0010
- R^2 = .0198
- Std err of Y Est = .0412

10 Year T-Note and Spread

- S&P tot ret = a * T10Year + b*Spread + c
- a = -.0829 (.0639)
- b = .4760 (.1573)
- c = 1.0010
- R^2 = .0198
- Std err of Y Est = .0412

Interest Rate Volatility

- Examining the daily yield curves from 5/1/81 to 4/30/92, we defined volatility as a 10 trading day rolling standard deviation.
- We then conducted a simple linear regression at each maturity with the independent variable being the level of the rate and the dependent variable being our estimate of volatility, i.e. Vol = a*rate + b.

Interest Rate Volatility

- We then used the R^2 value to indicate the percentage of volatility explained purely by the level of the rate.

Regression Results

Name	a	b	R^2
90 Day	.0249	-.00079	26.6%
180 Day	.0220	-.00061	27.5%
1 Year	.0214	-.00064	28.7%
2 Year	.0164	-.00035	21.1%
3 Year	.0149	-.00024	20.4%
5 Year	.0146	-.00028	20.9%
7 Year	.0150	-.00037	22.0%
10 Year	.0147	-.00039	20.6%
30 Year	.0150	-.00051	23.1%

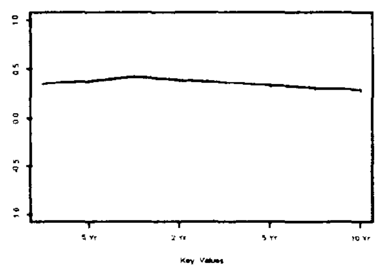
Interest Rate Volatility

- Using a more complex multivariate linear regression for each maturity, the R^2 values increased, however the overall appearance between each maturity was similar to the prior slide.
- With this primitive test, we observe that volatility is effected by the level of the rates, but the effect is not that strong.

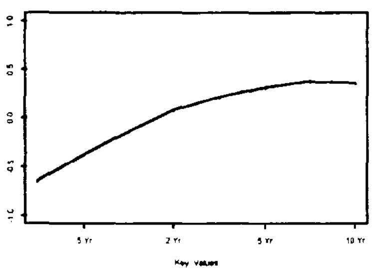
Interest Rate Volatility

- Using Principal Component Analysis on the volatility key rate changes, we obtain the covariance matrix of these changes. In identifying the eigenvectors of the matrix, we find that most of the volatility movements can be explained by the following three movements. Note the relative influence of each eigenvector.

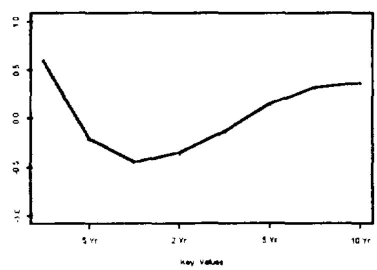
Eigenvector 1 78.69%



Eigenvector 2 14.30%



Eigenvector 3 3.68%

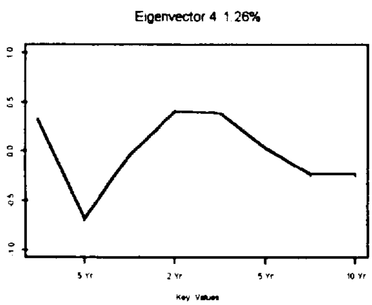


Interest Rate Volatility

- Eigenvector 1 (shift), demonstrates empirically say that 79% of the change in volatility is parallel across the yield curve.
- Eigenvector 2 (tilt) is a seesaw effect, where 14% of the change is related to if the 10 year rate volatility goes up the 90 day will go down and vice versa.

Interest Rate Volatility

- Eigenvector 3 (flex) explains that 4% of the change is where the volatility for the 90 day and the 10 year rate goes up and the 1 year volatility goes down (and vice versa).
- The remaining 3% of the change is explained by 5 more complex eigenvectors with no easy interpretation.



Next Steps

- Specify future actions