

# The Principal-Agent Relationship Between the Actuary and the Pension Administrator

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## 1 Introduction

The purpose of this study is to examine, in an agency theory context, the relationship between a pension fund manager and an actuary. The distinguishing characteristic of the agency model is that the agent provides information to the principal; the principal then can take remedial action based on that information. Our study concentrates on how the actuary plays a role as an information providers on the employers' benefit plans issue.

The Employee Retirement Income Security Act of 1974 (ERISA) attempts to safeguard an employee's pension by mandating many pension plan requirements, including participation rates and minimum funding. These requirements affect an employer's costs significantly. Under this legislation, an employer must annually fund the plan in accordance with the actuarial funding method to ensure that there is a sufficient fund to pay for pension obligations. If funding is not carried out in a reasonable manner, tax deductions will be denied and fines will be imposed.<sup>2</sup>

Under the terms of ERISA, enrolled actuaries determining minimum pension plan funding must be engaged on behalf of all pension plan participants by the administrator of the pension plan. The actuary plays an important role in structuring the defined benefit pension plan. The actuary combines prior experience with respect to inflation, salaries, and interest rates to produce funding patterns that result in costs that are somewhat constant as a percentage of payroll. The assumptions chosen are the actuary's best estimate of future happenings over the lifetime of the plan and do not take into account short-term influences. The actuary must also make sure the

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<sup>2</sup>The plan administrator will face other kinds of penalties imposed by the IRS (see 1997 Instruction for Form 5500, p.2.)

operations of pension plans conform to accounting requirements.

In order to fulfill legal requirements, administrators have to interact with actuaries. Since the plan administrators lack specialized skills or knowledge in actuarial fields, they need to use the work of an actuary. Without the support from the actuary and the administrator of pension plan would not run the pension plan well and the plan participants will be exposed to unreasonable financial risks.

The basic model in agency theory can be summarized as follows: The principal signs an employment contract with an agent because the agent's effort will produce benefits for the principal. The agent has specialized ability that the principal cannot duplicate. Therefore, the principal wants to guarantee that the contract aligns the incentives of the agent with those of the principal (incentive compatibility); in addition, the principal cannot in general observe the actions of the agent. Finally, the agent must be willing to work for the principal (adverse selection).

In the standard principal-agent models, working harder means more productive output (see Holmstrom (1979), Shavell (1979), Antel (1982), Demski, Patell and Wolfson (1984), and Christensen and Feltham (193)). Our model is designed differently. In our model, working harder means more accurate information provided by the agent. If the agent works harder to provide the principal with more accurate information about pension funding level, then the principal could save some remedial action costs.

## **2 The Pension Fund Administrator – Actuary Model**

Here we would like to introduce the game that is the focus of our analysis and to provide some preliminary results. Our game model is based on an assumption that the agent's payoff will depend on the quality of information, not the quantity of the production output; this distinguishes our model from the usual principal-agent models. The role of the actuary in our model is as an agent providing information about the pension fund level; in particular, the pension fund. We introduce a game model of a pension fund administrator-actuary model.

The pension fund administrator hires an actuary to reasonably estimate the pension fund level. According to the actuary's recommendation, the

administrator takes remedial action to (1) adjust the pension fund level to comply with the regulation requirements <sup>3</sup> (2) to avoid failing to pay off employees' pension, and (3) to avoid potential penalties imposed by IRS or the Department of Labor.

When the pension fund administrator hires the actuary to evaluate the pension fund level, the actuary is to take an action  $a \in R$  that influences the quality of information  $U \in R$ . The quality of information,  $U$ , may be interpreted as the cost saving in the remedial action from the validity of information received by the administrator. We admit uncertainty, so  $U$  may be dependent on the state of world,  $\theta \in \Theta$ , where  $\Theta$  is some abstract state space. The level of the actuary's action will affect the costs of the administrator's remedial action. The administrator is to set up a contract to induce the actuary to provide more accurate information about the pension fund managed by the pension administrator.

An important feature of our model is the fact the neither the actuary nor the administration knows the state of the pension fund with certainty. The actuary observes a signal about the level of funding, and relays that signal to the administrator.

In a single period model, we are going to eliminate the need to consider reputation issues. Interaction between the administrator and the actuary could be depicted in the Figure 1.

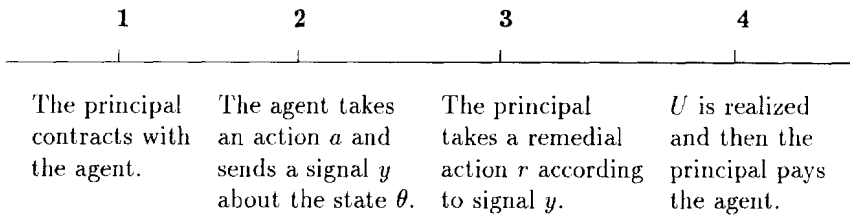


Figure 1: Time Line of The Pension Fund Administrator – Actuary Model

The administrator would like to hire the actuary to evaluate the pension fund and offers the actuary a contract. The prior distribution of information  $\theta \in \{\beta, 1\}$  is known, where  $\beta \in [0,1]$  is the percentage of funded level. The actuary then decides to accept or reject the administrator's contract. The

<sup>3</sup>ERISA, FASB 87, 106

actuary takes an action  $a \in \mathbb{R}^+ \cup \{0\}$  and a signal  $y \in \{0, 1\}$  is realized and is sent to the administrator.  $y = 0$  means the pension is underfunded while  $y = 1$  means the pension is fully funded. Once the administrator receives the agent's signal  $y$ , the administrator then takes a remedial action  $r \in \mathcal{R}$  to adjust pension fund level, if necessary. When the state of world is realized, the actuary is then paid according to the contract. As  $a$  increases, the accuracy of the information about the state improves. The actuary and the administrator act as Bayesian when updating their beliefs about the state of the pension fund.

In this study, we assume<sup>4</sup> that the prior probability of  $\theta$  is  $Pr(\theta = \beta) = Pr(\theta = 1) = \frac{1}{2}$  and define the probability of signal  $y$  as follows:

$$\begin{aligned} Pr(y = 0|\theta = \beta, a) &= 1 - \frac{1}{2}e^{-a} \\ Pr(y = 1|\theta = \beta, a) &= \frac{1}{2}e^{-a} \\ Pr(y = 0|\theta = 1, a) &= \frac{1}{2}e^{-a} \\ Pr(y = 1|\theta = 1, a) &= 1 - \frac{1}{2}e^{-a} \end{aligned}$$

If  $a = 0$ , then  $Pr(y = 0|\theta = \beta, a) = Pr(y = 1|\theta = \beta, a) = Pr(y = 0|\theta = 1, a) = Pr(y = 1|\theta = 1, a) = \frac{1}{2}$ . The information is pure noise, and the posterior probability will be the same as the prior probability.

In contrast, if  $a \rightarrow \infty$ , then  $Pr(y = 0|\theta = \beta, a) = Pr(y = 1|\theta = 1, a) \rightarrow 1$  and  $Pr(y = 1|\theta = \beta, a) = Pr(y = 0|\theta = 1, a) \rightarrow 0$ . This is the perfect information case. It means if an agent works very hard, his signal should be very highly correlated with the true state of world.

The posterior probability of  $\theta$  is computed using Bayesian updating, and can be easily be shown to be as follows:

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<sup>4</sup>This is done for computational ease and could easily be relaxed.

$$Pr(\theta = \beta|y = 0, a) = 1 - \frac{1}{2}e^{-a}$$

$$Pr(\theta = 1|y = 0, a) = \frac{1}{2}e^{-a}$$

$$Pr(\theta = \beta|y = 1, a) = \frac{1}{2}e^{-a}$$

$$Pr(\theta = 1|y = 1, a) = 1 - \frac{1}{2}e^{-a}$$

When  $a = 0$ , the prior probability is equal to the posterior probability.

$W(\cdot)$  is defined as the agent's wage function where  $W'(\cdot) > 0$ . The agent's utility function is  $W(\cdot) - C(a)$  where  $C(a)$  is the agent's cost of action. The principal utility function is  $U(\cdot) - W(\cdot)$ .

## 2.1 The Actuary's Action is Observable by the Administrator

Suppose that the administrator could observe the agent's actual level action taken to verify the level of the pension.

Now, we define our principal-agent model by the following:

$$\text{Max}_{r,a,W(\cdot)} E_{\theta,y} [U(r, \theta) - W(y, \theta)] \quad (1)$$

Subject to :

$$E_{\theta,y} [W(y, \theta)] - C(a) \geq \bar{U} \quad (\text{IR})$$

$$r_1 \in \text{ArgMax } E_{\theta} [U|y = 1] \quad (\text{IC}_{p1})$$

$$r_0 \in \text{ArgMax } E_{\theta} [U|y = 0] \quad (\text{IC}_{p2})$$

Equation (IR) represents the individual rationality constraint or the participation constraint for the actuary (agent). The agent's expected payoff from the principal minus the cost of action should be at least as great as the agent's reservation wage. Equation (IC<sub>p1</sub>) and (IC<sub>p2</sub>) are like "incentive compatibility" constraints for the principal. Receiving the signals  $y = 0$  and  $y = 1$  from the agent, the principal will take the best action for each signal.

The objective function can be written explicitly in terms of the posterior probability distributions as follows:

$$\begin{aligned}
& E_{\theta,y}[U(r, \theta) - W(y, \theta)] \\
&= Pr(y = 0)\{E_{\theta}[U|y = 0] - W(y = 0, \theta)\} \\
&\quad + Pr(y = 1)\{E_{\theta}[U|y = 1] - W(y = 1, \theta)\} \\
&= \frac{1}{2}\left\{\frac{1}{2}e^{-a}(U_{\ell,h} - W_{\ell,h}) + (1 - \frac{1}{2}e^{-a})(U_{\ell,\ell} - W_{\ell,\ell})\right\} \\
&\quad + \frac{1}{2}\left\{\frac{1}{2}e^{-a}(U_{h,\ell} - W_{h,\ell}) + (1 - \frac{1}{2}e^{-a})(U_{h,h} - W_{h,h})\right\}
\end{aligned}$$

Where

$$\begin{aligned}
U_{\ell,h} &= U(r_0, \theta = 1), & U_{\ell,\ell} &= U(r_0, \theta = \beta), \\
U_{h,h} &= U(r_1, \theta = 1), & U_{h,\ell} &= U(r_1, \theta = \beta), \\
W_{\ell,\ell} &= W(y = 0, \theta = \beta), & W_{\ell,h} &= W(y = 0, \theta = 1) \\
W_{h,\ell} &= W(y = 1, \theta = \beta), & W_{h,h} &= W(y = 1, \theta = 1)
\end{aligned}$$

In contrast, (IR), (IC<sub>p1</sub>) and (IC<sub>p2</sub>) can be written as follows:

Constraint (IR)

$$\begin{aligned}
& E_{\theta,y}[W(y, \theta)] - C(a) \geq \bar{U} \\
&= \frac{1}{2}\left\{\frac{1}{2}e^{-a}[W_{h,\ell} + W_{\ell,h}] + (1 - \frac{1}{2}e^{-a})(W_{h,h} + W_{\ell,\ell})\right\} - C(a) \geq \bar{U}
\end{aligned}$$

Constraint (IC<sub>p1</sub>)

$$\begin{aligned}
r_1 &\in \text{ArgMax } E_{\theta}[U|y = 1] \\
r_1 &\in \text{ArgMax}\left\{(1 - \frac{1}{2}e^{-a})U_{h,h} + \frac{1}{2}e^{-a}U_{h,\ell}\right\} \\
(1 - \frac{1}{2}e^{-a})U'_{h,h} + \frac{1}{2}e^{-a}U'_{h,\ell} &= 0
\end{aligned}$$

Constraint (IC<sub>p2</sub>)

$$\begin{aligned}
r_0 &\in \text{ArgMax } E_{\theta}[U|y = 0] \\
r_0 &\in \text{ArgMax}\left\{(1 - \frac{1}{2}e^{-a})U_{\ell,\ell} + \frac{1}{2}e^{-a}U_{\ell,h}\right\} \\
(1 - \frac{1}{2}e^{-a})U'_{\ell,\ell} + \frac{1}{2}e^{-a}U'_{\ell,h} &= 0
\end{aligned}$$

Therefore, we could reorganize the programming problem as follows:

$$\begin{aligned} \text{Max}_{r,a,W(\cdot)} \quad & \frac{1}{2} \left\{ \frac{1}{2} e^{-a} [U_{\ell,h} - W_{\ell,h} + U_{h,\ell} - W_{h,\ell}] + \right. \\ & \left. \left( 1 - \frac{1}{2} e^{-a} \right) [U_{\ell,\ell} - W_{\ell,\ell} + U_{h,h} - W_{h,h}] \right\} \end{aligned}$$

Subject to :

$$\frac{1}{2} \left\{ \frac{1}{2} e^{-a} [W_{h,\ell} + W_{\ell,h}] + \left( 1 - \frac{1}{2} e^{-a} \right) [W_{h,h} + W_{\ell,\ell}] \right\} - C(a) \geq \bar{U} \quad (\text{IR})$$

$$\left( 1 - \frac{1}{2} e^{-a} \right) U'_{h,h} + \frac{1}{2} e^{-a} U'_{h,\ell} = 0 \quad (\text{IC}_{p1})$$

$$\frac{1}{2} e^{-a} U'_{\ell,h} + \left( 1 - \frac{1}{2} e^{-a} \right) U'_{\ell,\ell} = 0 \quad (\text{IC}_{p2})$$

Since the agent's action is observable by the principal, you expect to obtain first best solution by using the first order condition method.<sup>5</sup>

### Example 1.

Let's define a quadratic function

$$U(r, \theta) = -r^2 + 2(1 - \theta)r + C \quad (2)$$

where C is a constant, and define the cost of agent's action  $C(a) = \alpha a$ , where  $\alpha > 0$ .

We then want to solve for  $a, r_0, r_1$ , and  $W(\cdot)$ .

Given the function  $U(r, \theta)$ , we could obtain the following results:

$$\begin{aligned} U_{\ell,h} &= U(r_0, \theta = 1) = -r_0^2 + C, & U'_{\ell,h} &= -2r_0 \\ U_{\ell,\ell} &= U(r_0, \theta = \beta) = -r_0^2 + 2(1 - \beta) + c, & U'_{\ell,h} &= -2r_0 + 2(1 - \beta) \\ U_{h,h} &= U(r_1, \theta = 1) = -r_1^2 + c, & U'_{\ell,h} &= -2r_1 \\ U_{h,\ell} &= U(r_1, \theta = \beta) = -r_1^2 + 2(1 - \beta) + C, & U'_{\ell,h} &= -2r_1 + 2(1 - \beta). \end{aligned}$$

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<sup>5</sup>A different approach, developed by Grossman and Hart (1983), is to focus on contracts that induce the agent to choose a particular action, rather than to directly attack the problem of maximizing profits.

We are going to employ the following steps to solve this mathematical programming problem.

1. Given  $U(r, \theta)$ , we first focus on the constraint equations  $(IC_{p1})$  and  $(IC_{p2})$  and solve for the optimal value of  $r_0$  and  $r_1$ .
2. Once we obtain these optimal values of  $r_0$  and  $r_1$  from the first step, we then substitute  $r_0$  and  $r_1$  in constraint  $(IR)$  and the objective function by these two optimal values.
3. Use the Karush-Kuhn-Tucker Theorem to define a Lagrangian function  $L$  and solve for variables  $a$  and  $W(\cdot)$ .

From the constraint equations  $(IC_{p1})$  and  $(IC_{p2})$ , we obtain

$$r_0 = (1 - \beta)(1 - \frac{1}{2}e^{-a}) \tag{3}$$

$$r_1 = (\frac{1 - \beta}{2})e^{-a}, \tag{4}$$

respectively.

Let's define  $W_{h,\ell} + W_{\ell,h} = W_2$  and  $W_{h,h} + W_{\ell,\ell} = W_1$  and replace  $r_0$  and  $r_1$  in the constraint  $(IR)$  and the objective function by (3) and (4). Then, our programming problem can be written as follows:

$$\text{Max}_{a, W(\cdot)} \frac{1}{2}(1 - \beta)^2 \{1 - e^{-a} + \frac{1}{2}e^{-2a}\} - \frac{1}{2} \{ \frac{1}{2}e^{-a}W_2 - (1 - \frac{1}{2}e^{-a})W_1 \}$$

Subject to

$$\frac{1}{2} \{ \frac{1}{2}e^{-a}W_2 + (1 - \frac{1}{2}e^{-a})W_1 \} - \alpha a \geq \bar{U}$$

Then, we could have the Lagrangian function:

$$L = \frac{1}{2} \{ (1 - \beta)^2 \{1 - e^{-a} + \frac{1}{2}e^{-2a}\} - \frac{1}{2}e^{-a}W_2 - (1 - \frac{1}{2}e^{-a})W_1 \} + \lambda \{ \frac{1}{2} \{ \frac{1}{2}e^{-a}W_2 + (1 - \frac{1}{2}e^{-a})W_1 \} - \alpha a - \bar{U} \} \tag{5}$$



where  $\lambda \geq 0$ .

When we take the partial derivative with respect to  $W_1$  and  $W_2$ , we find that the value of  $\lambda$  is 1. It implies that the constraint in our program is binding. In other words, the net income obtained by the agent will be equal to his reservation wage, i.e.

$$\frac{1}{2}e^{-a}W_2 + (1 - \frac{1}{2}e^{-a})W_1 = 2\{\alpha a + \bar{U}\}. \quad (6)$$

Now, we use (6) to simplify the Lagrangian function and take the partial derivative with respect to  $a$  to find the optimal value of  $a$ :

$$\begin{aligned} \frac{\partial L}{\partial a} &= 0 \\ \frac{1}{2}e^{-a}\{2(1 - \beta)^2(1 - e^{-a})\} &= \alpha \\ e^{-2a} - e^{-a} + \frac{\alpha}{(1 - \beta)^2} &= 0. \end{aligned}$$

Then, we solve for  $e^{-a}$  and get the following possible solutions:

$$e^{-a} = \frac{1 \pm \sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{2} \quad (7)$$

We here would like to check the 2nd order condition to find out the right solution for  $a$ .

$$\begin{aligned} \frac{\partial^2 L}{\partial a^2} &= (1 - \beta)^2(-e^{-a} + 2e^{-2a}) \\ &= (1 - \beta)^2 e^{-a}\{2e^{-a} - 1\}. \end{aligned}$$

We find that

$$\begin{aligned} \frac{\partial^2 L}{\partial a^2} \left( \frac{1 + \sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{2} \right) &> 0 \\ \frac{\partial^2 L}{\partial a^2} \left( \frac{1 - \sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{2} \right) &< 0. \end{aligned}$$

Thus, the optimal value of  $a$  solves:

$$e^{-a} = \frac{1 - \sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{2}. \quad (8)$$

Then, we use equation (6) and (8) to find out the principal's optimal expected payoff is:

$$(1 - \beta)^2 \left( \frac{3}{8} + \frac{\sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{8} - \frac{\alpha}{4(1 - \beta)^2} \right) - \alpha a - \bar{U} \quad (9)$$

**Remark 1.** *In our example 1, we could not find the closed form solutions for  $W(\cdot)$ . But, the optimal value of  $a$  (see (8)) and the principal's optimal expected payoff (9) will be used to compare with those results generated by the situation where the agent's action is unobservable by the principal.*

**Remark 2.** *From (8), we find that the value of  $a$  is greater than 0. It means that since the principal can observe the agent's action, the agent has to make certain amount of effort in order to obtain enough payoff  $W$  to cover his cost of action  $C(a)$  and earn his minimum wage  $\bar{U}$ . There is no "Free Lunch" existing in this example.*

## 2.2 The Actuary's Action is not Observable by the Administrator

The agent's incentive compatibility constraint is eliminated for the principal's optimization problem in last section. Since the actuary's action is not observable here, we need to consider the agent's incentive compatibility constraint while solving this optimization problem to obtain the second best solution.

Now, we define our new principal-agent model by the following:

$$\begin{aligned} & \text{Max}_{r,a,W(\cdot)} E_{\theta,y} [U(r, \theta, r) - W(y, \theta)] \\ & \text{Subject to :} \\ & E_{\theta,y} [W(y, \theta)] - C(a) \geq \bar{U} \quad (\text{IR}) \\ & a \in \text{ArgMax} \{E_{\theta}(W(y, \theta) - C(a))\} \quad (\text{IC}_a) \\ & r_1 \in \text{ArgMax} E_{\theta}[U|y = 1] \quad (\text{IC}_{p1}) \\ & r_0 \in \text{ArgMax} E_{\theta}[U|y = 0] \quad (\text{IC}_{p2}) \end{aligned}$$

Equation (IC<sub>a</sub>) represents the agent's incentive compatibility constraint in this model. It means that the agent will take his best action for his own interests.

Taking advantage of mathematical symbols and assumptions in the previous sections, we re-write the optimization problem as follows:

$$\begin{aligned} & \text{Max}_{r,a,W(\cdot)} \frac{1}{2} \left\{ \frac{1}{2} e^{-a} [U_{\ell,h} - W_{\ell,h} + U_{h,\ell} - W_{h,\ell}] + \right. \\ & \left. (1 - \frac{1}{2} e^{-a}) [U_{\ell,\ell} - W_{\ell,\ell} + U_{h,h} - W_{h,h}] \right\} \end{aligned}$$

Subject to :

$$\frac{1}{2} \left\{ \frac{1}{2} e^{-a} [W_{h,\ell} + W_{\ell,h}] + (1 - \frac{1}{2} e^{-a}) [W_{h,h} + W_{\ell,\ell}] \right\} - C(a) \geq \bar{U} \quad (\text{IR})$$

$$\frac{1}{4} e^{-a} [W_{h,h} + W_{\ell,\ell} - W_{\ell,h} - W_{h,\ell}] - C'(a) = 0 \quad (\text{IC}_a)$$

$$(1 - \frac{1}{2} e^{-a}) U'_{h,h} + \frac{1}{2} e^{-a} U'_{h,\ell} = 0 \quad (\text{IC}_{p1})$$

$$\frac{1}{2} e^{-a} U'_{\ell,h} + (1 - \frac{1}{2} e^{-a}) U'_{\ell,\ell} = 0 \quad (\text{IC}_{p2})$$

Since the agent's actions are unobservable by the principal, we could not obtain the first best optimal solution from this problem. But, we are expecting to obtain the second best solution by some optimization mathematical

methods.<sup>6</sup>

**Example 2.**

We use the same quadratic function  $U(r, \theta)$  in Example 1. But, we here change one of our assumptions that the agent's action is unobservable by the principal. In order to capture this change, we add one constraint ( $IC_a$ ) into our programming. We then want to solve for  $a, r_0, r_1$ , and  $W(\cdot)$ .

In this example, we are going to use some symbols defined in the Example 1.

From the constraint equations ( $IC_a$ ), the optimal value of  $a$  solves:

$$e^{-a} = \frac{4\alpha}{W_1 - W_2}. \tag{10}$$

Now, we have the optimal value of  $r_0$  (see equation (3)),  $r_1$  (see equation (4)), and  $a$  (see equation (10)) and replace  $r_0, r_1$ , and  $a$  in the constraint equation (IR) and the objective function by those optimal values. Then, our programming problem can be written as follows:

$$\begin{aligned} \text{Max}_{W(\cdot)} \quad & \frac{1}{2}(1 - \beta)^2 \left\{ 1 - \frac{4\alpha}{W_1 - W_2} + \frac{8\alpha^2\alpha}{(W_1 - W_2)^2} \right\} - \frac{1}{2} \left\{ \frac{2\alpha}{W_1 - W_2} W_2 \right. \\ & \left. + \left( 1 - \frac{2\alpha}{W_1 - W_2} \right) W_1 \right\} \end{aligned}$$

Subject to

$$\frac{1}{2} \left\{ \frac{2\alpha}{W_1 - W_2} W_2 + \left( 1 - \frac{2\alpha}{W_1 - W_2} \right) W_1 \right\} - \alpha \ln \left( \frac{W_1 - W_2}{4\alpha} \right) \geq \bar{U}$$

Since we have

$$\frac{2\alpha}{W_1 - W_2} W_2 + \left( 1 - \frac{2\alpha}{W_1 - W_2} \right) W_1 = W_1 - 2\alpha,$$

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<sup>6</sup>We could use the mechanism design method (Fudenberg and Tirol, 1991) to reduce the number of constraints.

we could simplify our programming problem as follows:

$$\begin{aligned} & \text{Max}_{W(\cdot)} \quad \frac{1}{2}(1-\beta)^2 \left\{ 1 - \frac{4\alpha}{W_1 - W_2} + \frac{8\alpha^2\alpha}{(W_1 - W_2)^2} \right\} - \frac{1}{2}\{W_1 - 2\alpha\} \\ & \text{Subject to} \\ & \quad \frac{1}{2}\{W_1 - 2\alpha\} - \alpha \ln \left( \frac{W_1 - W_2}{4\alpha} \right) \geq \bar{U} \end{aligned}$$

Then, we could define our Lagrangian function:

$$\begin{aligned} L = & \frac{1}{2}(1-\beta)^2 \left\{ 1 - \frac{4\alpha}{W_1 - W_2} + \frac{8\alpha^2\alpha}{(W_1 - W_2)^2} \right\} - \frac{1}{2}\{W_1 - 2\alpha\} \\ & + \lambda \left\{ \frac{1}{2}(W_1 - 2\alpha) - \alpha \ln \left( \frac{W_1 - W_2}{4\alpha} \right) - \bar{U} \right\} \end{aligned} \quad (11)$$

where  $\lambda \geq 0$ .

When we take the partial derivative with respect to  $W_1$  and  $W_2$ , we find that

$$\frac{\partial L}{\partial W_1} = \frac{1}{2}(1-\beta)^2 \left\{ \frac{4\alpha}{(W_1 - W_2)^2} - \frac{16\alpha^2}{(W_1 - W_2)^3} \right\} - \frac{1}{2} + \lambda \left\{ \frac{1}{2} - \frac{\alpha}{W_1 - W_2} \right\} = 0 \quad (12)$$

$$\frac{\partial L}{\partial W_2} = \frac{1}{2}(1-\beta)^2 \left\{ \frac{-4\alpha}{(W_1 - W_2)^2} + \frac{16\alpha^2}{(W_1 - W_2)^3} \right\} - \frac{1}{2} + \lambda \left\{ \frac{\alpha}{W_1 - W_2} \right\} = 0 \quad (13)$$

From equation (12) and (13), we have

$$\begin{aligned} -\frac{1}{2} + \lambda \left\{ \frac{1}{2} - \frac{\alpha}{W_1 - W_2} \right\} &= \lambda \left\{ \frac{\alpha}{W_1 - W_2} \right\} \\ \Rightarrow \lambda \left\{ \frac{1}{2} - \frac{2\alpha}{W_1 - W_2} \right\} &= \frac{1}{2} \\ \Rightarrow \lambda &= \frac{1}{1 - \frac{4\alpha}{W_1 - W_2}} \end{aligned}$$

Since we have  $W_1 - W_2 \geq 4\alpha$  from equation (10), we conclude that  $\lambda > 0$ . It implies that the constraint in our program is binding, i.e.

$$W_1 - 2\alpha = 2(\alpha a - \bar{U}) \quad (14)$$

Use equation (14) to simplify the Lagrangian equation

$$L = \frac{1}{2}(1 - \beta)^2 \left\{ 1 - \frac{4\alpha}{W_1 - W_2} + \frac{8\alpha^2}{(W_1 - W_2)^2} \right\} - \alpha \ln \left( \frac{W_1 - W_2}{4\alpha} \right) - \bar{U}$$

and, in fact, our programming problem now becomes an unconstrained programming. We simply take derivative with respect to  $W_1$  and  $W_2$  to search for their optimal values.

$$\begin{aligned} \frac{\partial L}{\partial W_1} &= \frac{\partial L}{\partial W_2} \\ &= \frac{1}{2}(1 - \beta)^2 \left\{ \frac{4\alpha}{(W_1 - W_2)^2} - \frac{16\alpha^2}{(W_1 - W_2)^3} \right\} - \frac{\alpha}{W_1 - W_2} = 0 \end{aligned}$$

We solve for  $W_1 - W_2$  and obtain

$$W_1 - W_2 = \frac{2(1 - \beta)^2 \pm \sqrt{4(1 - \beta)^4 - 32\alpha(1 - \beta)^2}}{2} \quad (15)$$

$\because W_1 > W_2$

$$\begin{aligned} W_1 - W_2 &= \frac{2(1 - \beta)^2 - \sqrt{4(1 - \beta)^4 - 32\alpha(1 - \beta)^2}}{2} \\ &= (1 - \beta)^2 - \sqrt{(1 - \beta)^4 - 8\alpha(1 - \beta)^2} \quad (16) \end{aligned}$$

Now, we use equation (16) to obtain the following results:

$$\begin{aligned} W_1 &= 2\left\{ \alpha + \bar{U} + \ln \left( \frac{(1 - \beta)^2 - \sqrt{(1 - \beta)^4 - 8\alpha(1 - \beta)^2}}{4\alpha} \right) \right\} \\ W_2 &= 2\left\{ \alpha + \bar{U} + \ln \left( \frac{(1 - \beta)^2 - \sqrt{(1 - \beta)^4 - 8\alpha(1 - \beta)^2}}{4\alpha} \right) \right\} \quad (17) \end{aligned}$$

$$e^{-\alpha} = \frac{4\alpha}{(1 - \beta)^2 - \sqrt{(1 - \beta)^4 - 8\alpha(1 - \beta)^2}} \quad (18)$$

Then, the principal's optimal expected payoff is:

$$\begin{aligned} & \frac{1}{2}(1-\beta)^2 \left( 1 - \frac{4\alpha}{(1-\beta)^2 - \sqrt{(1-\beta)^4 - 8\alpha(1-\beta)^2}} + \right. \\ & \left. \frac{8\alpha^2}{((1-\beta)^2 - \sqrt{(1-\beta)^4 - 8\alpha(1-\beta)^2})^2} \right) \\ & - \alpha \ln \left( \frac{(1-\beta)^2 - \sqrt{(1-\beta)^4 - 8\alpha(1-\beta)^2}}{4\alpha} \right) - \bar{U}. \end{aligned} \quad (19)$$

**Remark 3.** In this example, we find that the sum of wage for right information  $W_1$  is bigger than that for wrong information  $W_2$ , i.e.  $W_1 - W_2 \geq 4\alpha > 0$ . If  $W_1 - W_2 = 4\alpha$ , then the optimal agent's action is doing nothing ( $a = 0$ ) and the agent's expected payoff is the average of all possible payment schemes, i.e.  $\frac{(W_1+W_2)}{4} = \frac{(W_{l,t}+W_{h,h}+W_{t,h}+W_{h,t})}{4}$ . Since the principal can not observe the agent's action, the agent still might get paid for doing nothing.

**Lemma 2.1.** The action  $a_1$  which the agent will take when the principal can observe the agent's action is larger than the action  $a_2$  which the agent will take when the principal can not observe the agent's action.

**Proof.**

From Example 1 and Example 2, we have

$$e^{-a_1} = \frac{1 - \sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{2} \quad e^{-a_2} = \frac{4\alpha}{(1-\beta)^2 - \sqrt{(1-\beta)^4 - 8\alpha(1-\beta)^2}}$$

In order to prove that  $a_1 > a_2$ , we need to show that  $e^{-a_1} < e^{-a_2}$ .

Let  $\frac{4\alpha}{(1-\beta)^2} = X$ , where  $X < 1$ . Then, we have

$$\begin{aligned} e^{-a_1} &= \frac{1 - \sqrt{1 - \frac{4\alpha}{(1-\beta)^2}}}{2} \\ &= \frac{1 - \sqrt{1 - X}}{2} = \frac{1}{2}(1 - \sqrt{1 - X}) < \frac{1}{2} \end{aligned} \quad (20)$$

and

$$\begin{aligned}
 e^{-a_2} &= \frac{4\alpha}{(1-\beta)^2 - \sqrt{(1-\beta)^4 - 8\alpha(1-\beta)^2}} \\
 &= \frac{X}{1 - \sqrt{1-2X}} = \frac{1}{2}(1 + \sqrt{1-2X}) > \frac{1}{2}
 \end{aligned}
 \tag{21}$$

From (20) and (21), we conclude that

$$e^{-a_2} > \frac{1}{2} > e^{-a_1}.$$

■

**Lemma 2.2.** *The principal's expected payoff  $F_1$  when the principal can observe the agent's action is larger than the principal expected payoff  $F_2$  when the principal can not observe the agent's action.*

**Proof.**

Let the feasible set for the action  $a_1$  in the Example 1 be **A** and the feasible set for the action  $a_2$  in the Example 2 be **B**.

Since we show that the individual rationality constraint is binding in both Example 1 and Example 2 and there is one more constraint ( $IC_a$ ) in Example 2, we can conclude that the feasible set **B** is included in the feasible set **A**, i.e.  $\mathbf{B} \subset \mathbf{A}$ . In other words, whatever  $(\alpha, \beta)$  in **B** will be also in **A**.

In Lemma (2.1), we show that  $a_1 > a_2$ . It also implies that  $a_1 \neq a_2$ . Since  $a_1$  and  $a_2$  both are the unique solution in Example 1 and Example 2, respectively, and  $F_1(a_1)$  is the optimal value in the feasible set **A**, we conclude that  $F_1(a_1) > F_2(a_2)$ . ■

### 3 Conclusions and Future Research

This study examines the relationship between the administrator of a pension plan ( the principal ) and the actuary (the agent ). The objective is to design a contract for both the plan administrator and the actuary so that the actuary provides the appropriate level of information to the plan administrator. We consider a situation in which the pension plan managed by the administrator is a defined contribution plan.

We begin with a simple model -one principal (the pension fund administrator) and one agent (the actuary) in one-period model. In standard



principal-agent models, working harder means higher productive output; in our model, it means more accurate information. Using moral hazard model, we would like to capture the interaction between the principal and the agent. Extending our simple to the multiperiod model then will be our next step. We are interested in investigating the impact of multiperiod on the incentive contract.

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