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Amortization by a life annuity

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In a paper, "Annuities for the Aged," presented at the 26th Annual Actuarial Research Conference at the University of Illinois-Urbana in August 1991, we briefly introduced the concept of amortization by a life annuity. We called the concept "survivorship amortization," and we remain fascinated by the concept. This article presents a setting for, and a numerical illustration of, a simple survivorship amortization. Several ways of viewing the process exist. Some of these may be discarded for lack of practical relevance or for the extra complexity that results.

A simple form of survivorship amortization is given by a whole life immediate annuity of 1 payable at the end of each year while (x) survives. The actuarial present value of such an annuity at issue age x is a_x.

For ages $y \geq x$, we have the recurrence relation

$$a_y = vp_y(1 + a_{y+1}) \qquad \text{(a)}$$

which may be rearranged as

$$a_y \frac{1+i}{p_y} = 1 + a_{y+1}$$

or

$$1 = \left[\frac{1+i}{p_y} - 1\right] a_y + (a_y - a_{y+1}) \qquad \text{(b)}$$

The interest and survivorship rate between the first square brackets in (b) may be written as

$$\frac{i + q_y}{p_y} \qquad \text{(c)}$$

which is enlightening, but the format in (b) is convenient for calculation.

Formula (b) indicates that the payment at age $y + 1$ if (x) survives consists of two components:

1) The interest and survivorship rate

$$\left[\frac{1+i}{p_y} - 1\right] = \frac{i + q_y}{p_y}$$

applied to a_y, the actuarial present value of the annuity as of attained age y. The product is the amount of increase, under interest and survivorship, of the actuarial present value before the year-end payment of 1 is made.

2) The repayment of principal at age $y + 1$, namely,

$$1 - \left[\frac{1+i}{p_y} - 1\right] a_y = a_y - a_{y+1}$$

This brings the outstanding principal down from a_y to a_{y+1} after the payment is made.

We illustrate this survivorship amortization by exhibiting selected lines from an amortization schedule for a life annuity issued initially at age 65. Survivorship is according to the Blended 1983 *a*-D-Mortality Table, and interest is at the effective annual rate of 4%.

One notable feature is that the rate of increase (under interest and survivorship) is 5.06% for the initial year on a principal of 12.735. For the year of age (85, 86), the rate is 12.59% on a principal of 43% of the original. But for the year of age (114, 115), the rate is 946% on a principal of less than 1% of the original.

The amount of increase (under interest and survivorship) is lower for the year beginning at age 75 than for the years beginning at ages 65 and 85. Correspondingly, the repayment of principal is higher for the year beginning at age 75 than for the other two years.

It appears that survivorship amortization analysis can be applied broadly to most forms of life annuities. It will take much actuarial study and communication to develop all the practical ramifications such as extension to monthly payment life annuities, the treatment of expenses, adjustment to experience by dividend distributions in the form of graded benefits, and a new view of federal income tax liabilities under life annuities. Will such a new annuity theory be adapted to practice, or will it remain as a mathematical diversion?

Note that the foregoing model is deterministic but has the possibility of periodic adjustments to emerging experience.

A final observation considers compound interest amortization, established by the relation $A = Ra_{\overline{n}|i}$ where A denotes the initial principal and R the level payment at the end of each interest interval. Splitting the payments R into principal and interest components can be somewhat arbitrary. For example, the payment of R at the end of the jth interest period may have the components P_j and $(R - P_j)$, respectively, for principal and interest, subject to the condition that $P_1 + P_2 + \ldots P_n = A$. By actuarial reasoning, or as an interesting mathematical exercise, one can show that the present value of the

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Illustration of Survivorship Amortization

(1) Attained Age	(2) Outstanding principal at beginning of year of age $(y, y+1)$, i.e. actuarial present value	(3) Rate of increase under interest and survivorship over year of age $(y, y+1)$ $\left[\frac{1.04}{p_y} - 1\right] =$	(4) Amount of increase under interest and survivorship over year of age	(5) Repayment of principal at end of year of age
y	a_y	$\frac{.04 + q_y}{p_y}$	(2) x (3)	1 - (4) = $a_y - a_{y+1}$
65	12.735	.0506	.6444	.3556
75	9.038	.0691	.6245	.3755
85	5.516	.1259	.6945	.3055
95	3.029	.2689	.8145	.1855
105	1.303	.6445	.8398	.1602
110	.559	1.5597	.8719	.1281
114	.09556	9.4649	.9045	.0955

Guides cont'd

of any company marketing this highly focused product.

According to Beal, "The complexity of the business and the potential volatility of the results demand that the actuary be proactively involved in most aspects of the business, including marketing, underwriting, claim management, product design, pricing, valuation, financial reporting, and analysis."

Readings are divided into sections related to the actuary's role, individual markets, product design and pricing, experience, reserves, underwriting and claim management, and reinsurance.

Individual Medical Expense Insurance Editor Anthony Houghton focuses on pricing, reserving, and complying with regulations for health insurance.

The SOA Continuing Education's Professional Actuarial Specialty Guide Committee is charged with developing guides for all major actuarial specialties. All guides and updates are routinely sent to actuaries in related specialties.

SOA members with topic ideas or who are interested in editing a guide should contact Louis M. Weisz, chairperson. SOA members are already editing other specialty guides for release in the near future.

William H. Lewis, Jr., is president, Lewis & Ellis Inc., and a member of the Professional Actuarial Specialty Guides Committee.

New journal seeks authors

SOA member Colin M. Ramsay, associate professor of actuarial science at the University of Nebraska-Lincoln, will serve as editor of a new journal to begin publication in April 1993. *The Journal of Actuarial Practice* will be published twice a year.

Ramsay is asking for "practical and readable" technical and nontechnical articles on all aspects of actuarial practice. Submissions will be reviewed for content, originality, and clarity. For more information, write *The Journal of Actuarial Practice*, c/o Absalom Press, P.O. Box 67175, Lincoln, NE 68506, or call Ramsay at 402-472-5823.

Election results cont'd

Insurance and Pension Research, University of Waterloo; Paul R. Fleischacker, vice president with Towers Perrin, New York; John H. Harding, president and chief operating officer with National Life Insurance Company, Montpelier, Vermont; Daniel J. McCarthy, consulting actuary with Milliman & Robertson, Inc., New York; Patricia L. Scahill, principal with William M. Mercer, Inc., Baltimore, Maryland; and Robert D. Shapiro, president of The Shapiro Network, Inc., Milwaukee, Wisconsin.

As a result of Section elections, nine special interest Sections have added new Council members with three-year terms:

Education & Research Section — Sarah L. M. Christiansen, Charles S. Fuhrer, Aaron Tenenbein

Financial Reporting Section —Frank J. Buck, Bradley M. Smith, Michael L. Stickney

Futurism Section — Paul A. Gewirtz, Kenneth E. Polk, Dennis R. Barry

Health Section — David S. Helwig, Joan E. Herman, Nancy F. Nelson

Investment Section — Cindy L. Forbes, Robert R. Reitano, Bruce F. Vane

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Product Development Section — Bradley E. Barks, Timothy C. Pfeifer, Klaus O. Shigley

Reinsurance Section — Wayne D. Bidelman, Kin K. Gee, John E. Tiller, Jr.

Amortization cont'd

$(R - P_j)$ interest payments equals the present value of the interest payments required on outstanding principal. Such payment for the end of interval h is $i\,[A - (P_1 + P_2 + \ldots P_{h-1})]$.

Despite the wealth of choices, we believe the actuarial view would be to require that each payment R be first applied to pay interest on outstanding principal and the balance of the payment be applied to principal repayment. This is a well-defined and widely understood amortization process, because it synchronizes the interest payments with the outstanding principal. A process using P_j's is more complex but seems to be acceptable to the accounting profession.

Analogously, one can decompose life annuity payments into a principal repayment amount and a residual growth amount. Our actuarial view would lean to first determining the growth (under interest and survivorship) of the outstanding principal and using the remainder of the annuity payment to repay principal. Such a process has been followed in our life annuity illustration to provide the analogue of our actuarial view of compound interest amortization.

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Seminar Calendar

Cash Balance Plans (8 hours of EA credit)	October 21-22	Arizona Biltmore Phoenix
Annual Meeting (12 hours of EA Credit)	October 26-28	Washington Hilton Washington, D.C.
Symposium on Current Issues under IRC Regulations 7702 and 7702A	November 5-6	The Willard Inter-Continental Washington, D.C.
Critical Issues in Contemporary Risk Selection	November 16-17	San Antonio Hyatt San Antonio
Teleconference: 401(a)(4): Amended Regulations (4 hours of EA Credit)	November 19	Various locations

For registration information, please call the Society of Actuaries Continuing Education Department at 708-706-3545.