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Distribution and Quantile Estimates for Parametric and Non-parametric Models on Value at Risk

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Abstract

The data on cash-flow-testing in Robbins, Cox, and Phillips (1997) was given a parametric fit by the authors, and the confidence regions of various models were studied in a discussion by Chan (1998). In this paper, we compare percentile estimates by parametric and non-parametric methods.

1 Introduction

Robbins, Cox, and Phillips (1997) asked the question: Given a large number of cash-flow-testing scenarios resulting in only a very small number of scenarios landing in the adverse area (ruin tail), how can the results of the entire set of observations be used to better estimate the area under the ruin tail? They fit parametric models to the 500 point data in the paper. Among these parametric models, Chan (1998) chose the Gamma and the DB2 models to work through the confidence regions of the parametric space to obtain the confidence bands of the percentiles.

We give a summary of the above by three figures. Figure 1 is a histogram of the 500 point data, Figure 2 shows the fitted Gamma models' distribution functions, and Figure 3 shows the fitted DB2 models' distribution functions. The 95% confidence band of the 99th percentile, for example, is read off by a horizontal line through 0.99 in Figures 2 or 3.

Figure 1

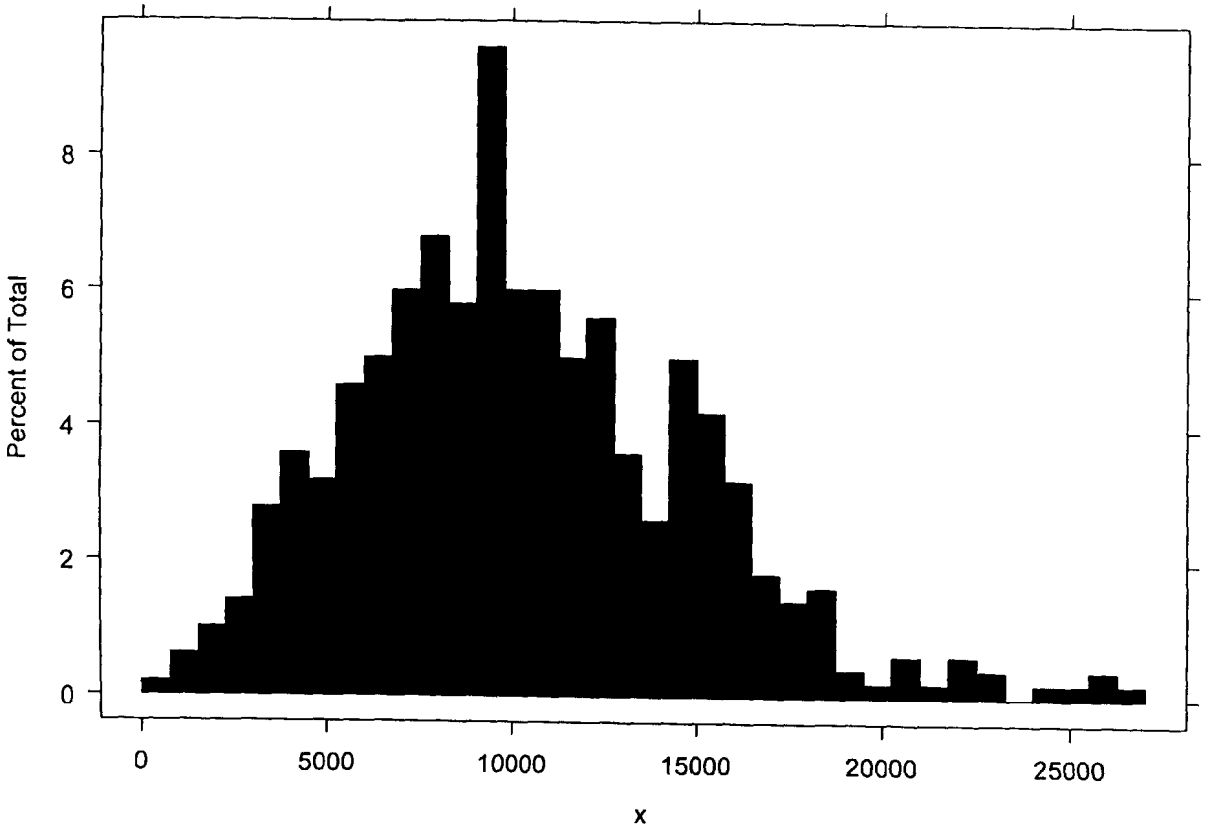


Figure 2

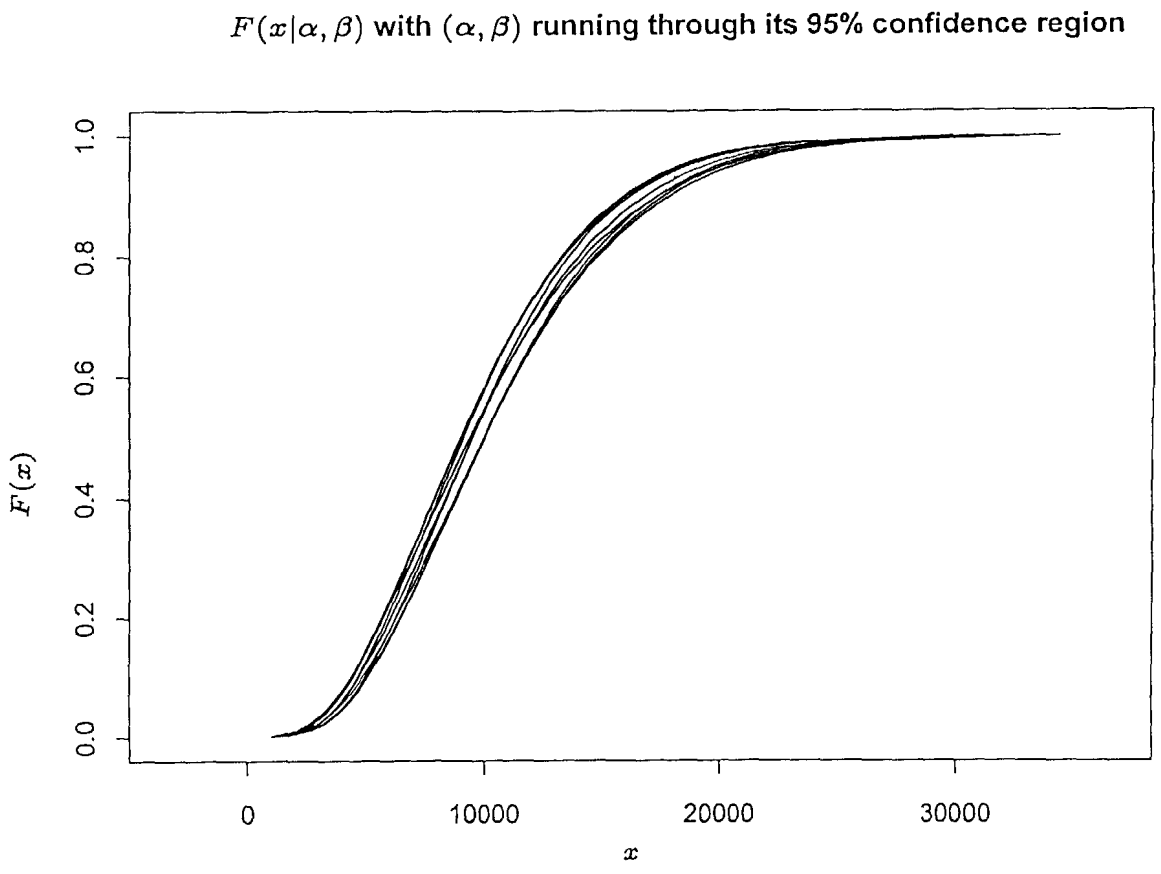
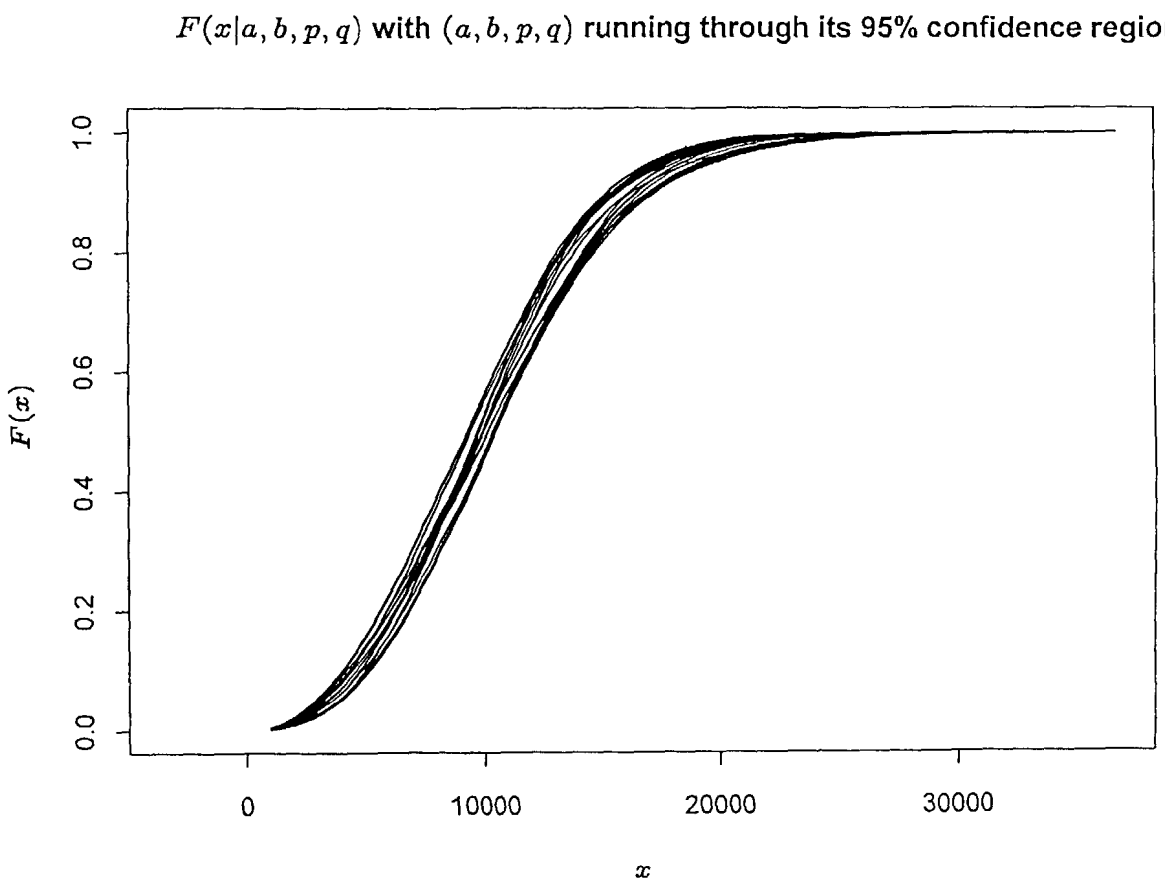

$F(x|\alpha,\beta)$ with (α,β) running through its 95% confidence region
$F(x)$
0.0
0.2
0.4
0.6
0.8
1.0
0
10000
20000
30000
x

Figure 3


$F(x|a,b,p,q)$ with (a,b,p,q) running through its 95% confidence region
$F(x)$
0.0
0.2
0.4
0.6
0.8
1.0
0
10000
20000
30000
x

2. Non-parametric method

We take 1000 bootstrap replications of the 99th percentile. The same is repeated five more times to produce Figure 4. Each of the six tells that 1000 replications captured the features of the 99th percentile. Figures 5, 6, and 7 do the same for the 95th, the 5th, and the 1st percentiles.

The non-parametric method here do not take advantage of the entire set of observations to estimate the tail as was done in the parametric method before. As we shall see later, it uses observations near the tail to estimate the tail. The 99th percentile and the 95th percentile appear more unstable than the 1st and the 5th. This is a direct consequence of the nature of the 500 point data. The lowest 25 points thin out nicely while the highest 25 points spread across a wide range. While the parametric percentiles (Figures 2 and 3) show the difference between the extreme low and the extreme high end, it is less sensitive to local tail behaviour as the non-parametric percentiles (Figures 4 to 7). If the percentile of interest is on one end of the distribution, such as the Value at Risk, the non-parametric method given here appears to be more responsive to observations near that end.

Figure 4

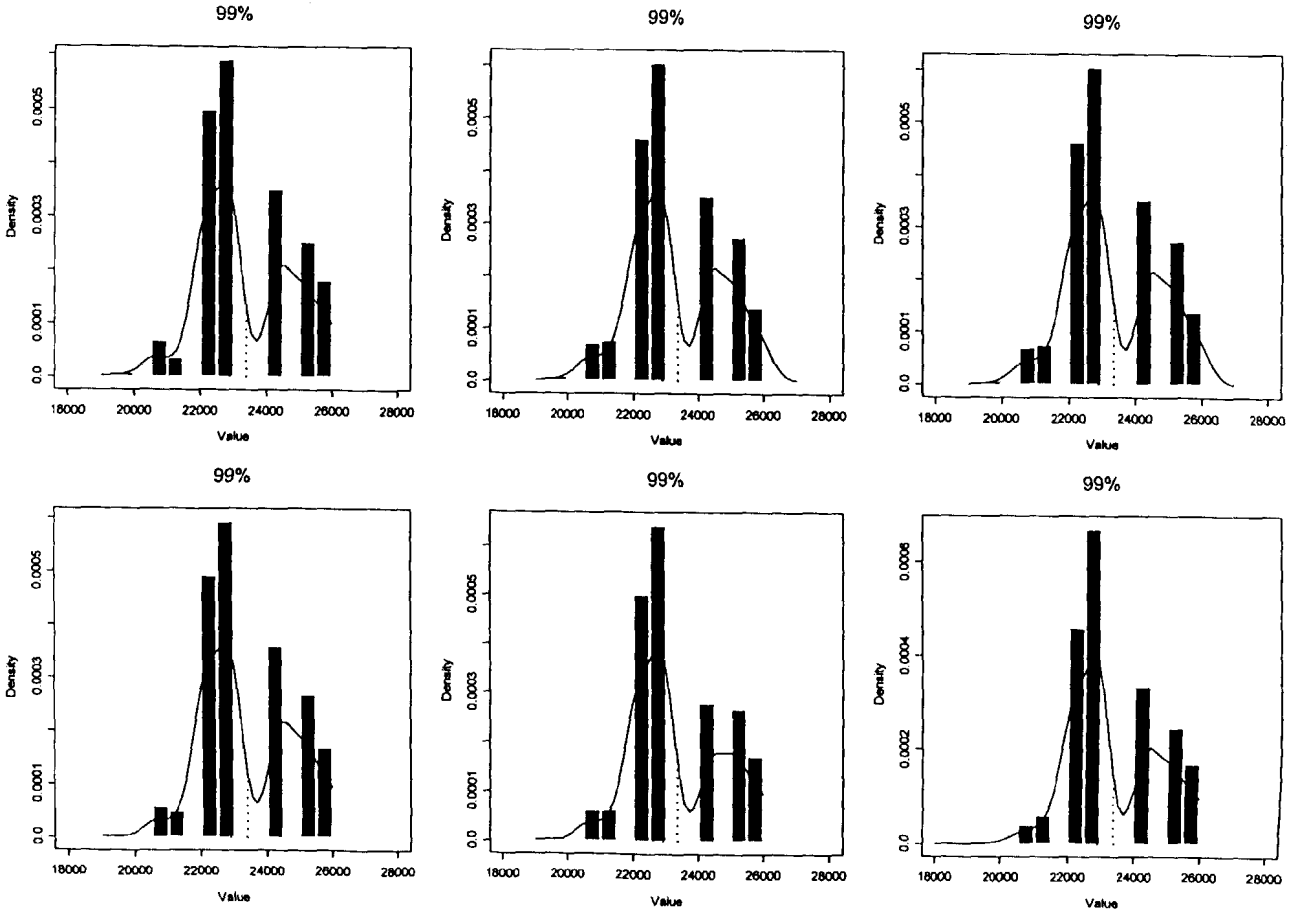

99%
99%
99%
99%
99%
99%
Density
Value
0.0
0.0001
0.0003
0.0005
0.0002
0.0004
0.0006
18000
20000
22000
24000
26000
28000

Figure 5

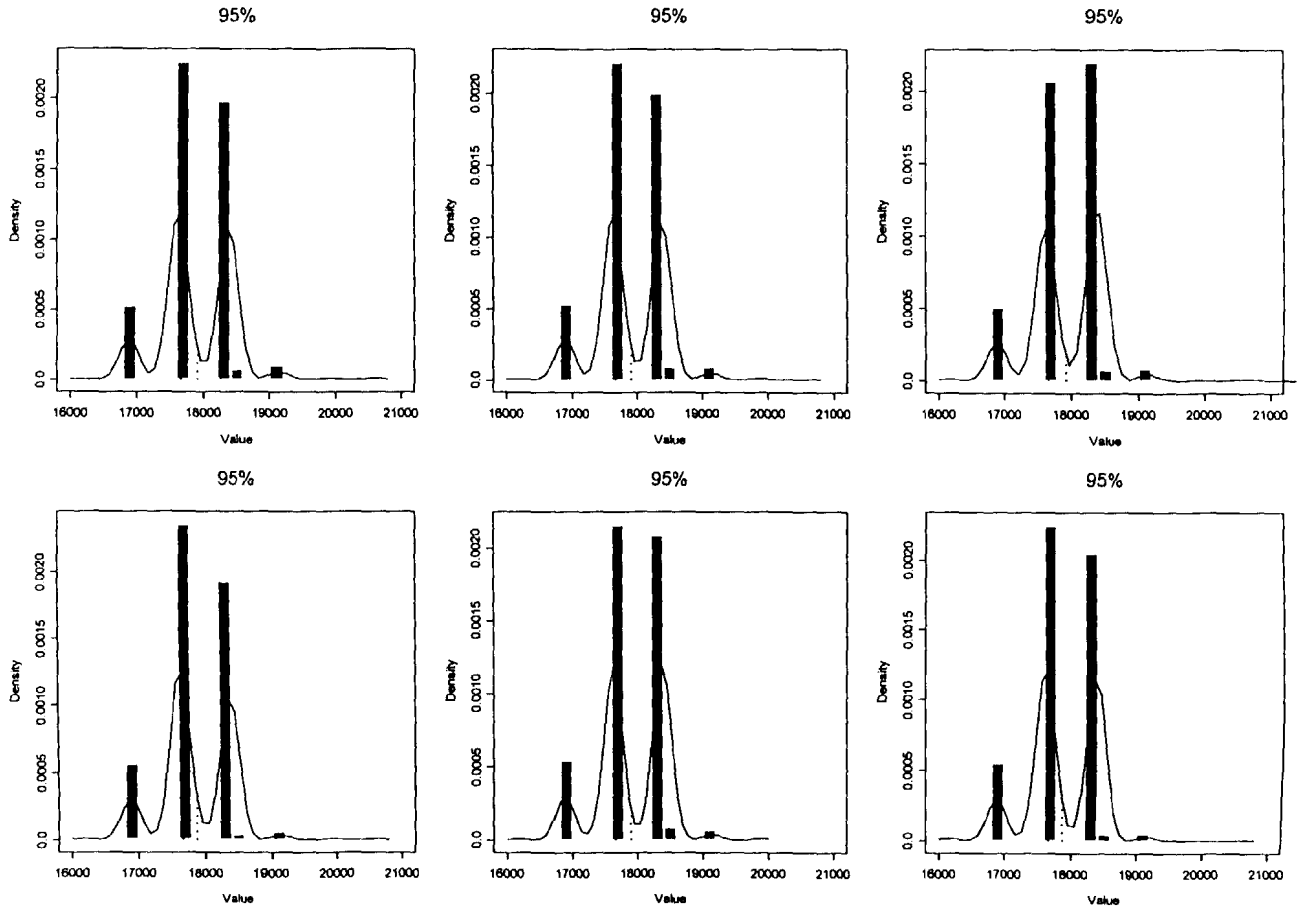
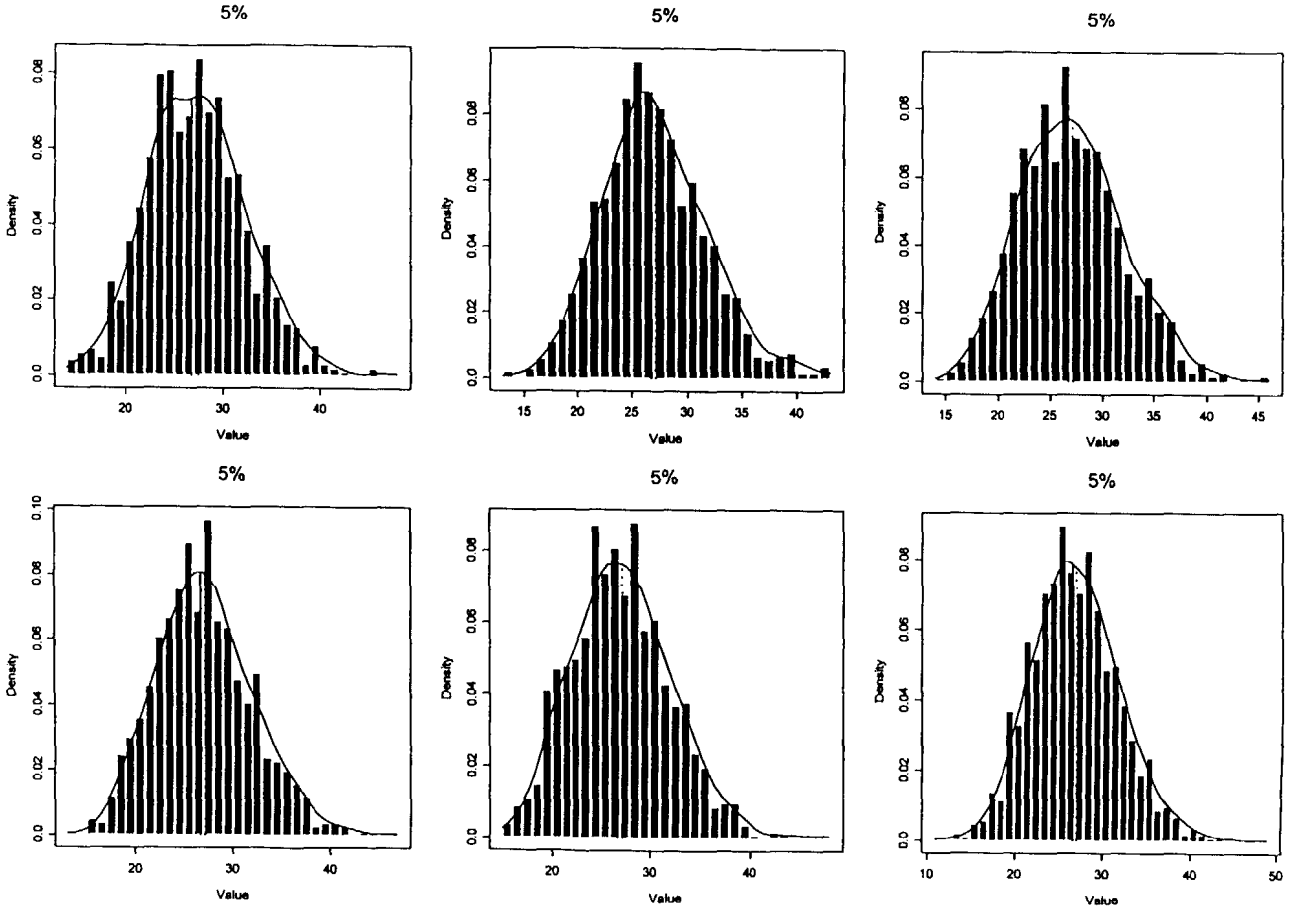

95%
95%
95%
95%
95%
95%
Density
Value
0.0
0.0005
0.0010
0.0015
0.0020
16000
17000
18000
19000
20000
21000

Figure 6


5%
Density
Value
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Value
5%
Density
Value
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Value
5%
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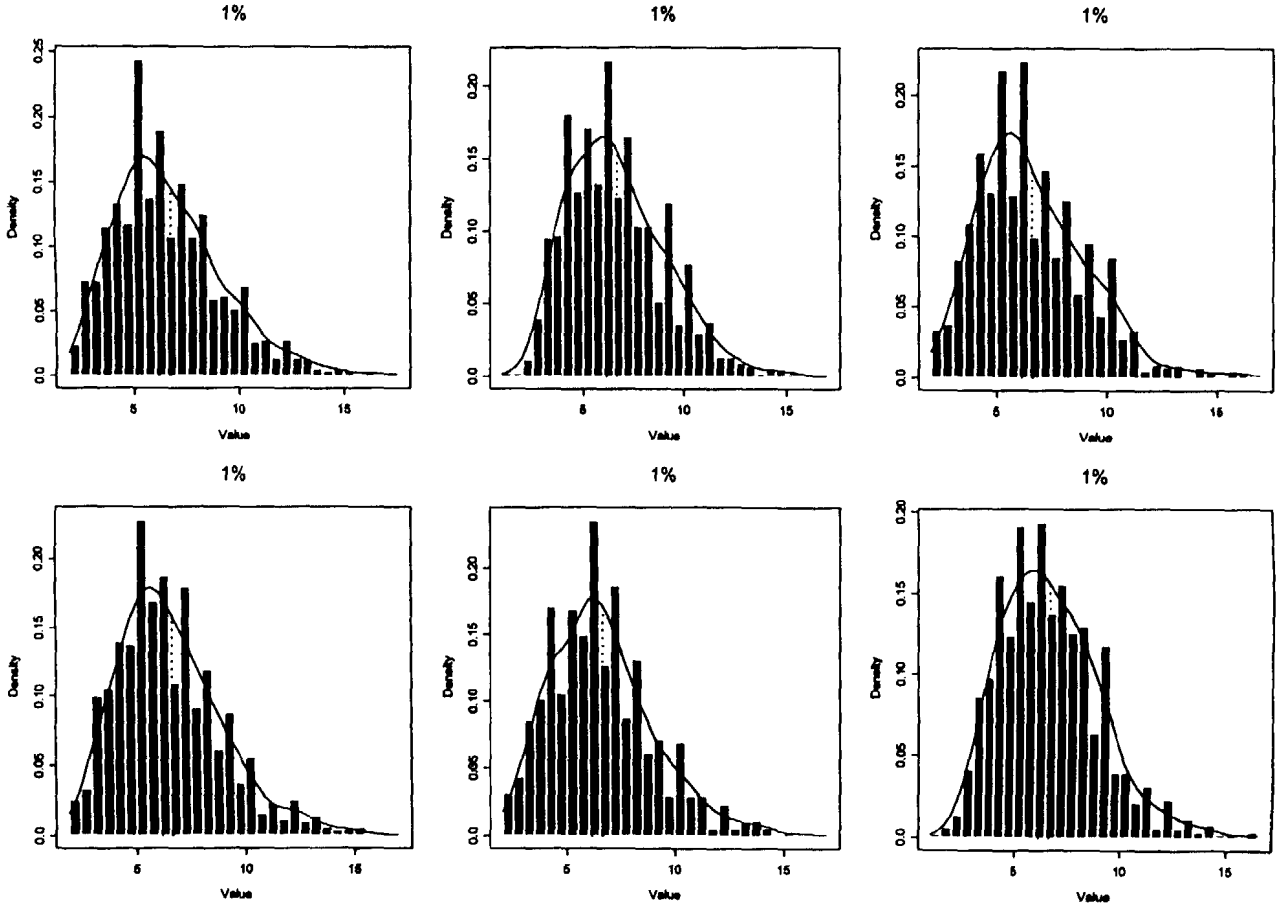
Density
Value

Figure 7



References

Robbins, E.L., Cox, S. H., and Phillips, R.D. 1997. "Applications of Risk Theory to Interpretation of Cash-Flow-Testing Results," *North American Actuarial Journal* 1:85-98.

Chan, B. 1998. Discussion of "Applications of Risk Theory to Interpretation of Cash-Flow-Testing Results," *North American Actuarial Journal* , to appear