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A Cash Flow Approach To Teaching Actuarial Science

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Abstract

The complex world of life insurance, historically the domain of those gifted in integral calculus and probabilistic and present value mathematics, is explored using Excel spreadsheets and the Solver macro. The cash flow approach to learning life insurance examines how the cash flows through an insurance company. By avoiding the need for integral calculus, probabilities, and present value mathematics, the fascinating world of life insurance is opened up for study to those students who possess only modest math backgrounds.

The cash flow approach to teaching actuarial science avoids the use of both probabilities and present values, and allows students with very modest math skills to learn how life insurance works. This approach involves the same assumptions and the same data as in the traditional approach. The traditional assumptions are:

(1) Premiums are payable at the beginning of the year;
(2) Premiums are level in amount; and
(3) Claims are paid at the end of the year.

The required data are:

(1) The age of the insured when the policy begins;
(2) The age of the insured when premiums end;
(3) The age of the insured when the benefit ends;
(4) A mortality table;
(5) An interest rate; and
(6) A death benefit. [Crabb (1979, p. 716)]

The cash value approach examines how the cash flows through an insurance company for a group of insureds. See Figure One for the equations underlying those cash flows. Prior to the payment of the first premium, the Beginning Reserve has a zero dollar balance. Premium Income equals the number of insureds alive at a given age multiplied times the premium payable that year. (Since the premium is unknown, it is temporarily set equal to zero.) Interest Income is computed on the sum of the Beginning Reserve plus the Premium Income. Claims Paid are equal to the number of insureds dying in a given year multiplied times the death benefit. The Ending Reserve is the sum of the Beginning Reserve plus the Premium Income plus the Interest Income less the Claims Paid. The Beginning Reserve in the next contract year is equal to the Ending Reserve from the previous year.

Figure One depicts the cash flow for a group of insureds age 21 who are purchasing a ten year term life insurance policy. In a mathematical sense, there is one equation in one unknown, with that unknown being the term life premium. Using the Solver function (a built-in macro present on most spreadsheets), the problem is to find a term premium such that the ending reserve for the policy is zero; that is, find the value for E14 such that G30 has a value of zero. See Figure Two.

The Solver function would be found under Tools in an Excel environment. Once the problem is specified (see Figure Two), the student clicks on the [Solve] button, and the Excel Solver macro iterates towards a solution. See Figure Three.

However, there is a problem with the solution. Although the ending reserve is zero at the end of age 30, at ages 24, 25, 26, and 27, the ending reserves are negative. The insurer can not allow this to happen, and hence the number of premium payments must be reduced. This creates a trial and error situation. Reduce the number of premium payments by one payment. Bring up Solver. Check to see if the negative reserves have become positive. In the example in this paper, a reduction to nine payments is sufficient to solve the negative reserve problem. See Figure Four. For longer contracts and for contracts at different issue ages, more payments may need to be eliminated.

A reduction from ten payments to nine payments causes the premium to rise from \$12.49 to \$13.56. What if the number of payments was reduced to five payments? Then the premium increases to \$22.13. What if the insured made only one payment? Then the premium increases to \$107.27. The student now sees how a single premium contract works. The cash flow shows that the amount of interest increases significantly as the premium paying period is shortened.

What if the interest rate was 6% instead of 5%? Change the interest rate; bring up Solver; and learn the answer. The student can thoroughly investigate the relationships between and among the number of premium payments and/or potential interest rates for the ten year term contract. By extending the cash flow formulae to the end of the mortality table and resetting Solver to make the ending reserve at the last age in the mortality table to be equal to zero, the student can explore the whole life insurance product.

Whole life products have cash surrender values that steadily increase over the length of the contract, approaching the face amount of that contract as the end of the mortality table nears. [Black & Skipper (1994, p. 990] In the spreadsheet environment, this characteristic is explored by adding a single equation to the set of equations already present on the spreadsheet. In Column H in row 21, enter [=G21/A22] (the Ending Reserve at the end of age 21 is divided by the insureds who survived to age 22). Copy this formula down the page to the end of the mortality table. See Figure Five.

In addition to its cash surrender value, the whole life insurance product also has two other nonforfeiture options: Extended Term insurance and Paid-Up insurance. What if the insured, after making the first two whole life insurance premium payments, decided to make no more premium payments? Then the insurance company would use the money in the Ending Reserve to buy as much insurance as it could with that amount of money. Set all future premiums to zero, and look for a negative Ending Reserve. For the whole life policy issued at age 21, the \$81 cash value which exists when the makes only premium payments will provide a death benefit of \$10,000 for 7 years and 196 days. The Extended Term option is laid out in Figure Six.

The Paid-Up option requires a second cash flow algorithm. Duplicate the whole life cash flow algorithm. On the copy of the original algorithm, set the number of whole life premiums to one. The Solver computed premium of \$945.52 is the net single premium for age 21. The resulting set of cash values for this algorithm yields the net single life premiums for ages 22 through the end of the mortality table.

The Paid-Up option death benefit can be computed for ages 22 through 109 by using the cash values from Figure Five and the net single premiums from Figure Seven. Note that those net single premiums are located in column H, the Cash Value column.

The quotient of the whole life cash value (from Figure Five) to the net single premium (from Figure Seven) yields the amount of the death benefit payable under the Paid-Up option. See Figure Seven for the set of net single premiums. Interested readers can make the computations on their own.

The spreadsheet and Solver allow the student to visually see how cash flows into an insurance company and how cash flows out of an insurance company. By using the spreadsheet for the purpose for which it was designed (the What if? purpose) and combining the "What if?" with the Solver macro, students with only modest mathematical backgrounds can learn how life insurance products work.

References

Black, K. & Skipper, H.D. (1994) *Life Insurance*. Prentice Hall

Crabb, Ronald R. (1979) A Computer Approach To Teaching Life Insurance Mathematics. *The Journal of Risk and Insurance*, pp. 715-725.

Figure One. The Equations Underlying The Cash Flows For A Insurance Company

Microsoft Excel - CASHFLOW.XLS

	A	B	C	D	E	F	G
14				Term Premium	0		
15				Interest Rate	0.05		
16				Benefit	10000		
17							
18	Number	Number	Beginning	Premium	Interest	Claims	Ending
19	Living	Dying	Reserve	Income	Income	Paid	Reserve
20							
21	97623	124	0	=E$14*A21	=E$15*(D21+C21)	=E$16*B21	=C21+D21+E21-F21
22	=A21-B21	129	=G21	=E$14*A22	=E$15*(D22+C22)	=E$16*B22	=C22+D22+E22-F22
23	=A22-B22	130	=G22	=E$14*A23	=E$15*(D23+C23)	=E$16*B23	=C23+D23+E23-F23
24	=A23-B23	130	=G23	=E$14*A24	=E$15*(D24+C24)	=E$16*B24	=C24+D24+E24-F24
25	=A24-B24	128	=G24	=E$14*A25	=E$15*(D25+C25)	=E$16*B25	=C25+D25+E25-F25
26	=A25-B25	126	=G25	=E$14*A26	=E$15*(D26+C26)	=E$16*B26	=C26+D26+E26-F26
27	=A26-B26	126	=G26	=E$14*A27	=E$15*(D27+C27)	=E$16*B27	=C27+D27+E27-F27
28	=A27-B27	126	=G27	=E$14*A28	=E$15*(D28+C28)	=E$16*B28	=C28+D28+E28-F28
29	=A28-B28	127	=G28	=E$14*A29	=E$15*(D29+C29)	=E$16*B29	=C29+D29+E29-F29
30	=A29-B29	127	=G29	=E$14*A30	=E$15*(D30+C30)	=E$16*B30	=C30+D30+E30-F30
31	=A30-B30	130					

Cash Flow 10 Year Term

Figure Two. Preparing Solver To Solve For The Ten Year Term Premium

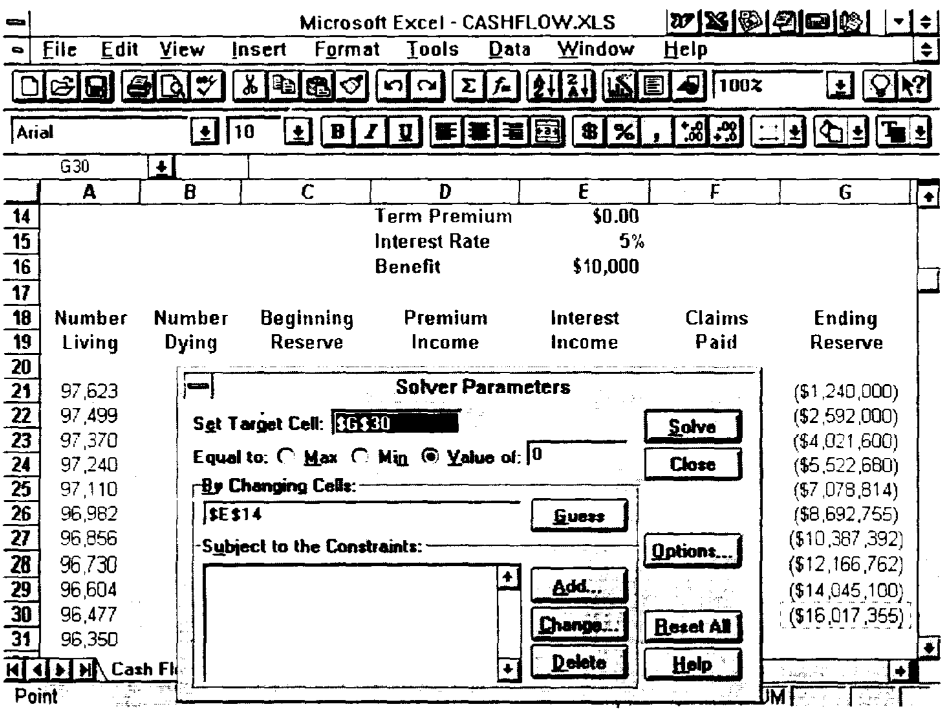

Figure Three. The Solution To The Ten Year Term Premium Problem

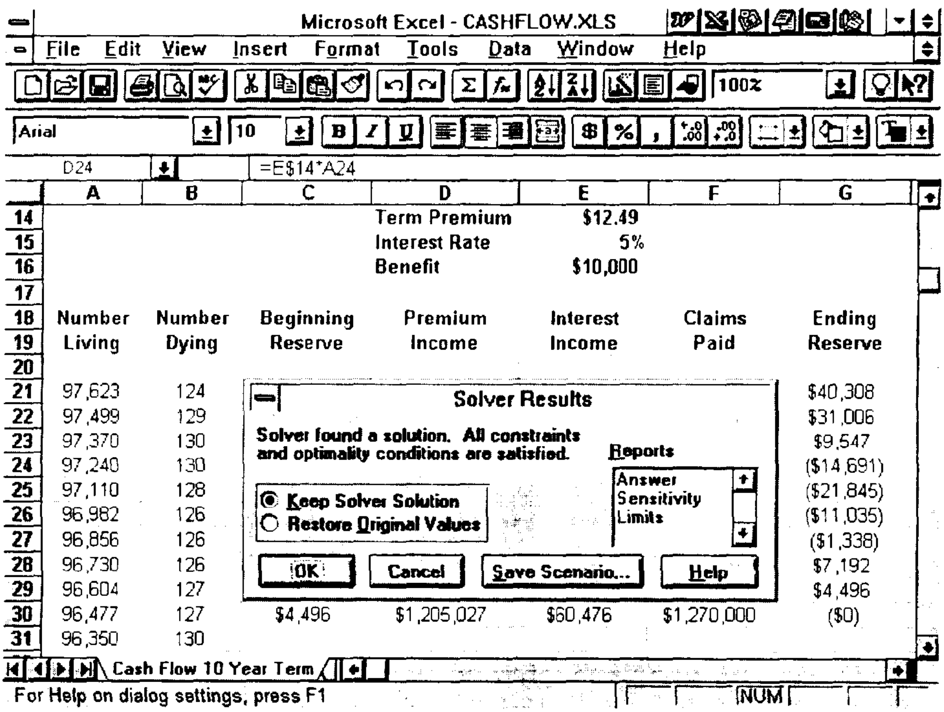

Figure Four. The Cash Flow For A Nine Pay Ten Year Term Policy

Microsoft Excel - CASHFLOW.XLS

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	A	B	C	D	E	F	G
14				Term Premium	$13.56		
15				*Interest Rate*	5%		
16				Benefit	$10,000		
17							
18	Number	Number	Beginning	Premium	Interest	Claims	Ending
19	Living	Dying	Reserve	Income	Income	Paid	Reserve
20							
21	97,623	124	$0	$1,323,924	$66,196	$1,240,000	$150,120
22	97,499	129	$150,120	$1,322,242	$73,618	$1,290,000	$255,980
23	97,370	130	$255,980	$1,320,493	$78,824	$1,300,000	$355,296
24	97,240	130	$355,296	$1,318,730	$83,701	$1,300,000	$457,727
25	97,110	128	$457,727	$1,316,967	$88,735	$1,280,000	$583,428
26	96,982	126	$583,428	$1,315,231	$94,933	$1,260,000	$733,592
27	96,856	126	$733,592	$1,313,522	$102,356	$1,260,000	$889,470
28	96,730	126	$889,470	$1,311,813	$110,064	$1,260,000	$1,051,347
29	96,604	127	$1,051,347	$1,310,104	$118,073	$1,270,000	$1,209,524
30	96,477	127	$1,209,524	$0	$60,476	$1,270,000	($0)
31	96,350	130					

Cash Flow 10 Year Term

Ready

Figure Five. The Cash Flow For A Whole Life Policy

Microsoft Excel - CASHFLOW.XLS

	A	B	C	D	E	F	G	H
18				$49.73	5%	$10,000		
19	Number	Number	Beginning	Premium	Interest	Claims	Ending	Cash
20	Living	Dying	Reserve	Income	Income	Paid	Reserve	Value
21	97,623	124	$0	$4,854,453	$242,723	$1,240,000	$3,857,175	$40
31	96,350	130	$47,751,083	$4,791,151	$2,627,112	$1,300,000	$53,869,346	$560
41	94,706	241	$120,492,093	$4,709,400	$6,260,075	$2,410,000	$129,051,569	$1,366
51	90,986	584	$212,496,705	$4,524,418	$10,851,056	$5,840,000	$222,032,179	$2,456
61	82,581	1233	$301,329,880	$4,106,466	$15,271,817	$12,330,000	$308,378,163	$3,791
71	66,165	2193	$339,319,193	$3,290,156	$17,130,467	$21,930,000	$337,809,816	$5,281
81	40,208	3036	$267,436,791	$1,999,404	$13,471,810	$30,360,000	$252,548,005	$6,794
91	11,908	2045	$93,672,829	$592,143	$4,713,249	$20,450,000	$78,528,221	$7,962
101	815	245	$6,939,206	$40,527	$348,987	$2,450,000	$4,878,720	$8,559
102	570	177	$4,878,720	$28,344	$245,353	$1,770,000	$3,382,417	$8,607
105	179	60	$1,561,601	$8,901	$78,525	$600,000	$1,049,027	$8,815
106	119	41	$1,049,027	$5,917	$52,747	$410,000	$697,692	$8,945
107	78	27	$697,692	$3,879	$35,079	$270,000	$466,649	$9,150
108	51	18	$466,649	$2,536	$23,459	$180,000	$312,645	$9,474
109	33	33	$312,645	$1,641	$15,714	$330,000	($0)	undefined

Cash Flow Whole Life

Ready

Figure Six. The Cash Flow Solution For The Extended Term Option At Age 23

Microsoft Excel - CASHFLOW.XLS

G51 =(C30+E30)/F30*365

	B	C	D	E	F	G	H
17							
18			$49.73	5%	$10,000		
19	Number	Beginning	Premium	Interest	Claims	Ending	Cash
20	Dying	Reserve	Income	Income	Paid	Reserve	Value
21	124	$0	$4,854,453	$242,723	$1,240,000	$3,857,175	$40
22	129	$3,857,175	$4,848,287	$435,273	$1,290,000	$7,850,735	$81
23	130	$7,850,735	$0	$392,537	$1,300,000	$6,943,272	$71
24	130	$6,943,272	$0	$347,164	$1,300,000	$5,990,435	$62
25	128	$5,990,435	$0	$299,522	$1,280,000	$5,009,957	$52
26	126	$5,009,957	$0	$250,498	$1,260,000	$4,000,455	$41
27	126	$4,000,455	$0	$200,023	$1,260,000	$2,940,478	$30
28	126	$2,940,478	$0	$147,024	$1,260,000	$1,827,502	$19
29	127	$1,827,502	$0	$91,375	$1,270,000	$648,877	$7
30	127	$648,877	$0	$32,444	$1,270,000	($588,680)	($6)
31	130						
41							
51	Years of Coverage		7		Days of Coverage	196	
61							

Cash Flow Whole Life

Ready NUM

Figure Seven. Computing A Set Of Net Single Premiums For Ages 21 to 109

Microsoft Excel - CASHFLOW.XLS

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	A	B	C	D	E	F	G	H
18				$945.52	5%	$10,000		
19	Number	Number	Beginning	Premium	Interest	Claims	Ending	Cash
20	Living	Dying	Reserve	Income	Income	Paid	Reserve	Value
21	97,623	124	$0	$92,304,539	$4,615,227	$1,240,000	$95,679,766	$981
31	96,350	130	$134,337,013	$0	$6,716,851	$1,300,000	$139,753,863	$1,452
41	94,706	241	$198,645,776	$0	$9,932,289	$2,410,000	$206,168,065	$2,182
51	90,986	584	$278,433,828	$0	$13,921,691	$5,840,000	$286,515,519	$3,169
61	82,581	1233	$350,920,546	$0	$17,546,027	$12,330,000	$356,136,573	$4,378
71	66,165	2193	$369,796,229	$0	$18,489,811	$21,930,000	$366,356,040	$5,727
81	40,208	3036	$280,167,581	$0	$14,008,379	$30,360,000	$263,815,960	$7,097
91	11,908	2045	$96,075,129	$0	$4,803,756	$20,450,000	$80,428,885	$8,155
101	815	245	$7,053,669	$0	$352,684	$2,450,000	$4,956,374	$8,695
102	570	177	$4,956,374	$0	$247,819	$1,770,000	$3,434,192	$8,738
105	179	60	$1,583,197	$0	$79,160	$600,000	$1,062,357	$8,927
106	119	41	$1,062,357	$0	$53,118	$410,000	$705,475	$9,045
107	78	27	$705,475	$0	$35,274	$270,000	$470,748	$9,230
108	51	18	$470,748	$0	$23,537	$180,000	$314,286	$9,524
109	33	33	$314,286	$0	$15,714	$330,000	($0)	undefined

Cash Flow Whole Life

Ready NUM