"Representative Interest Rate Scenarios"

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This paper suggests a possible flexible solution to the time and resource problems of running a large number of stochastic interest rate scenarios, by selecting a representative subset. Each interest rate scenario consists of 30 future spot yield curves, where a reasonable number of points are specified on each curve (such as 12). The distribution of the scenarios is approximated by the subset and each scenario in the subset has equal weight. The method is independent of the inter rate generator used.

REPRESENTATIVE INTEREST RATE SCENARIOS

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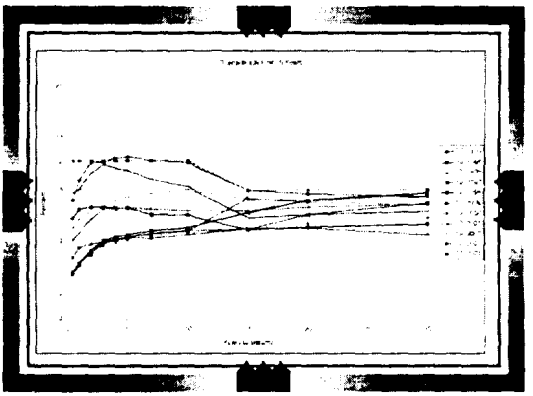
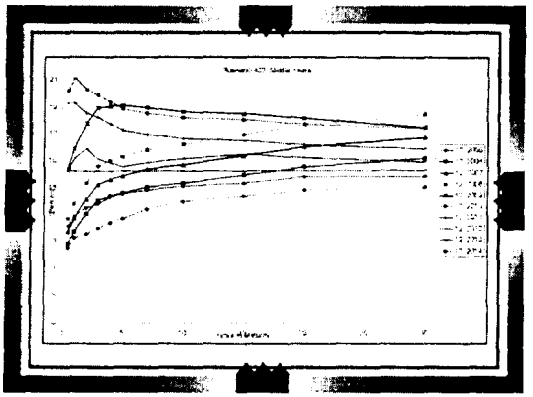
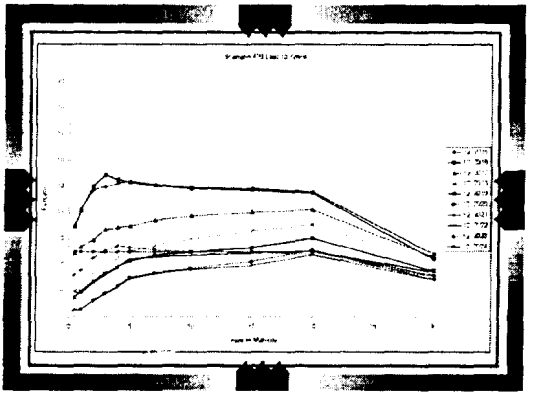
What is an Interest Rate Scenario ?

- A set of spot (zero coupon) yield curves
- One curve for each of the next 30 years
- Each curve has rates specified at .25, .5, 1, 2, 3, 4, 5, 7,10,15, 20 and 30 years







Who and Why?

- Regulators
- Management
- Rating Agencies
- Need confidence in ability to handle future
- Foundation for many models

For What Purpose?

- Cashflow testing for
  - New York Regulation 126
  - SVL Section 8 Asset Adequacy Analysis
  - Surplus or reserve adequacy
- Pricing
  - Profitability
- Portfolio Management

Why not lots of scenarios?

Limited
Time and
Resources

Real vs Ideal

- 50 stochastic scenarios
- 7 NY scenarios
- 2 shock scenarios

- 1000 stochastic scenarios
- 7 NY scenarios
- 2 shock scenarios

How to reduce??

The number of scenarios

Solution

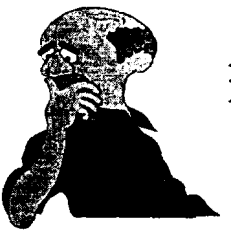
Find a representative subset

How many subsets are there?

$$\binom{1000}{50} = 9.640461 \times 10^{84}$$

WOW What a Number!!

- Look at each one?
- This is worse, not better.
- What is a representative subset, anyway?



Definition: Representative Subset

For each maturity,

the subset and the set have the same:

- *Mean*
- *Range*
- *Variance*

The Algorithm

- Set up for 1000 reproducible scenarios
- Keep random number generator seed
- Run 200 scenarios at a time
- Want to choose 10 representative ones

For each maturity rate we have:

$$i_{s_1,0}, i_{s_1,1}, \ldots i_{s_1,30}$$

$$i_{s_2,0}, i_{s_2,1}, \ldots i_{s_2,30}$$

.....

$$i_{s_{200},0}, i_{s_{200},1}, \ldots i_{s_{200},30}$$

Start with the 3 month rate and find

- Min, Max, Mean, Median, and Standard deviation
- For each scenario (m,s) and the set of all scenarios (μ,σ).

Complete the list

- Repeat for all of the maturity rates creating one list.
- Only the subsets on this list will be considered.
- The list has at least 12 subsets

The Goal

match the run statistics with those for the subset, for all rates, simultaneously!

Evaluating the candidates

- Weight the maturities from 4 most important to 1 least important
- Determine statistics for subset and run

Sshhh--The Secret is

- Do not consider every possible subset
- That's good ...there are still $2.245100431 \times 10^{16}$ subsets
- *The secret is ...A candidate list*

The candidate list contains:

- Subsets of scenario numbers
- Each subset begins with matched extremes
- Consider all combinations without repetition

Now add those scenarios whose average rate

- approximates to μ-.85σ, μ+.85σ, μ-.65σ, and μ+.65σ to each subset.
- Now each subset has 6 elements, and mean m_s.
- Choose the four scenarios that are closest to $(10\mu - 6m_s)/4$, and
- Add their numbers to the subset

Selecting *the* subset

- Find the weighted least squares difference, D, of the means between the overall set and subset.
- Choose the candidate from the list with whose D value is the minimum.

Finishing

- Repeat for the other runs
- Keep track of the scenario numbers for the representative scenarios.
- Re-create the 50 representative scenarios
- Compare descriptive statistics

Well, does it work?

- Sample results from June 1995
- Comparison automatically produced

June 1995

Data for 1000 scenarios

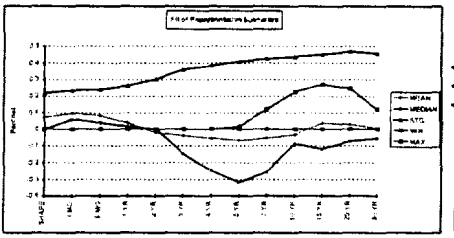
	MEAN	MEDIAN	STD	MIN	MAX
SHAPE	4.528	4.000	2.431	1.000	11.000
3 MO	6.214	5.761	2.375	3.500	25.000
6 MO	6.294	5.898	2.337	3.500	25.000
1 YR	6.603	6.278	2.307	3.500	25.000
2 YR	7.071	6.704	2.275	3.500	25.000
3 YR	7.363	7.014	2.212	3.500	25.000
4 YR	7.585	7.264	2.162	3.500	25.000
5 YR	7.757	7.450	2.115	3.500	25.000
7 YR	7.937	7.668	2.071	3.500	25.000
10 YR	8.071	7.805	2.045	3.500	25.000
15 YR	8.241	8.131	2.032	3.525	24.994
20 YR	8.475	8.345	2.044	3.611	24.393
30 YR	8.671	8.544	2.099	3.748	24.886

June 1995

Data for Representative Scenarios

	MEAN	MEDIAN	STD	MIN	MAX
SHAPE	4.002	4.000	2.030	1.000	11.000
3 MO	6.312	5.823	2.011	3.500	25.000
6 MO	6.380	5.940	2.379	3.500	25.000
1 YR	6.845	6.296	2.370	3.500	25.000
2 YR	7.048	6.704	2.379	3.500	25.000
3 YR	7.324	6.887	2.371	3.500	25.000
4 YR	7.530	7.017	2.348	3.500	25.000
5 YR	7.689	7.134	2.322	3.518	25.000
7 YR	7.882	7.412	2.498	3.627	25.000
10 YR	8.037	7.710	2.482	3.727	25.000
15 YR	8.283	8.011	2.484	3.796	24.994
20 YR	8.505	8.275	2.515	3.858	24.393
30 YR	8.699	8.489	2.557	3.862	24.886

Difference



Conclusion

- New Algorithm
- Reproduces Probability distribution
  - of 1000 scenarios
  - in a subset of 50
- Met Target