2018 Investment Symposium

Session 3A: The Market Impact of Dynamic Hedging on Hedging Program Peformance

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Major shifts in offering within rapidly changing economic and regulatory landscape have increased the use of derivatives by the insurance industry

- Rapidly changing economic and regulatory landscape
  - Regulatory (Solvency II, Dodd-Frank, EMIR,DOL), increased the cost of capital in favor of hedging: more market consistent valuation, risk sensitive approach,credit given to advanced risk mitigation techniques
  - Historically prolonged period of low interest rates, which reduces diversification b/w assets and business effectiveness, puts pressure on asset management fees, calls for flexible product design
  - Ultra-easy monetary policy may risk to create market exuberance, Ultra-easy monetary policy may risk to create market exuberance

- Limits of Reinsurance (capacity, cost)
  - Reinsurance was popular in the US throughout the 1990's, with almost half of GMDB and GMIB exposure being reinsured, which almost disappeared post equity market falls of 2000-03
  - Traditional Reinsurance is expensive in low rates environment and subject to regulatory review / approval
  - It is impossible and/or uneconomical to fully hedge market risk in all scenarios especially as it relates to varying policyholder behaviour, while extreme market conditions can also lead to ineffective hedging which leads to hedge breakage
  - Capital Markets reinsurance is limited in terms of capacity: transfer of market risks only, retain Policyholder Behavior, Basis and Longevity risks up to a guaranteed value

- Major shifts in offering need hedging risks
  - Growing unit-linked with guarantees / Growing protection & Health with Savings components

- "Liquidity is the new leverage"
  - Pockets of "late cycle excess" due to high tolerance for risk in search of yield were less likely to arise from credit and leverage strategies than from yield-harvesting strategies that were effectively short liquidity.
  - The nature and magnitude of mortgages, structured products and Variable Annuities hedging activities have raised the issue of its market impact, where disruptions to liquidity can affect the short term dynamics of the underlying, independently of fundamentals but related to the growth of the options market
  - Feb 2018: Demand from leveraged VIX exchanged-traded products was "the major driver for the move post the cash close. Two exchange-traded products that democratized access to one of Wall Street's most tried-and-true strategies -- selling volatility -- had just $3.6 billion in assets on Monday. That's a tiny fraction of the roughly $2 trillion estimated to be linked to short-volatility strategies -- and a speck of dust compared to the $23 trillion in market value of S&P 500 companies. Yet the popularity of these vehicles might have contributed to one of the most violent moves in U.S. equities in history: the Dow Jones Industrial Average slump more than 6 percent in a span of six minutes. After the dust settled, the combined assets in the two exchange-traded products shrank to $135 million. One of them -- the VelocityShares Daily Inverse VIX Short-Term ETN, known as XIV -- will soon be extinct

Agenda

- **Derivatives use by insurance within a low volatility combined with spikes environment**
  - **Derivatives use by insurance**
  - **Liability hedging principles and mechanics**
  - **From linear risks to non linear risks embedded within liabilities**
  - **From delta hedging limitations to volatility tail and gap risks**
  - **Market context: low volatility levels combined with spikes**

- **Hedging Market impact for life insurance liabilities**
  - **Derivatives supply-demand imbalance**
  - **From market impact pricing to optimal trading transactions**
  - **From market impact to amplifying delta hedging feedback effects**
  - **Vol Control Products as major Driver of Feedback Loops**
  - **Monitoring Hedging Feedback Loops**

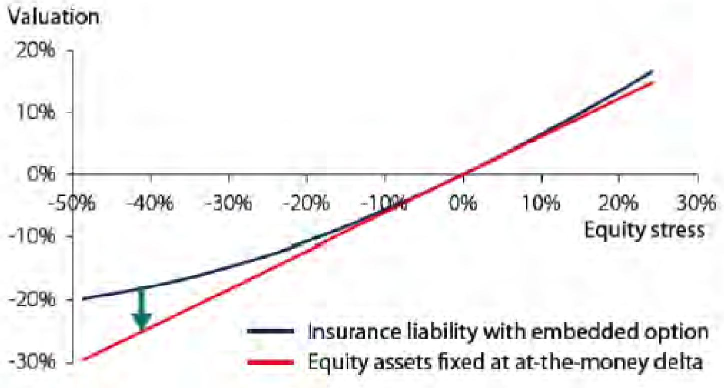
- **Hedging Solutions**
  - **Pricing & Hedging Feedback effects**
  - **Mitigating Tail Risks**
  - **Reducing hedging costs**

Derivatives use by Life insurance companies

- 96% derivatives are used by life insurance among corporates
  - Out of which 78% dedicated to hedging (49% swaps, 45% options)
  - From Traditional ALM to dynamic & static hedging
  - from $786bn as of FY 2010 to $1885bn as of FY2014

- Three main features: long term duration, large volumes and significant market risk exposure, pct for Variable Annuities

- Need to buy convex equity hedges such as put options to match the liability risk profile -> illiquidity, which Solvency II will strengthen, in order to meet capital requirements during periods of market stress



Derivatives use by insurance

EARNINGS

Hedging reduces earnings sensitivity to market risks

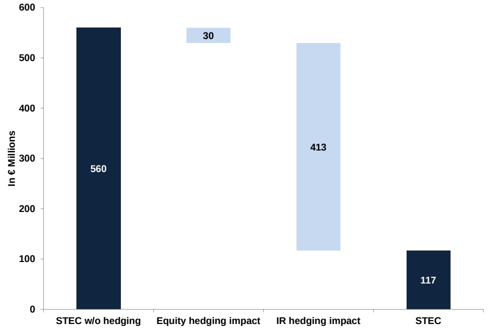
- Equity sensitivity
- Interest rate sensitivity

$$\begin{array}{c}\text{Dollar amount of repriceable}\\ \text{(interest-sensitive)}\\ \text{assets}\end{array} = \begin{array}{c}\text{Dollar amount of repriceable}\\ \text{(interest-sensitive)}\\ \text{liabilities}\end{array}$$

$$Duration\ gap = duration\ of\ earning\ assets - duration\ of\ paying\ liabilities \times \frac{paying\ liabilities}{earning\ assets}$$

CAPITAL

Hedging reduces Required Economic Capital



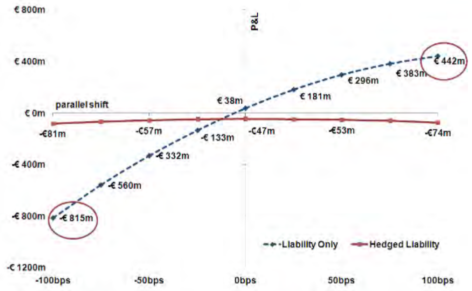
LIQUIDITY

Hedging long term liabilities needs liquidity

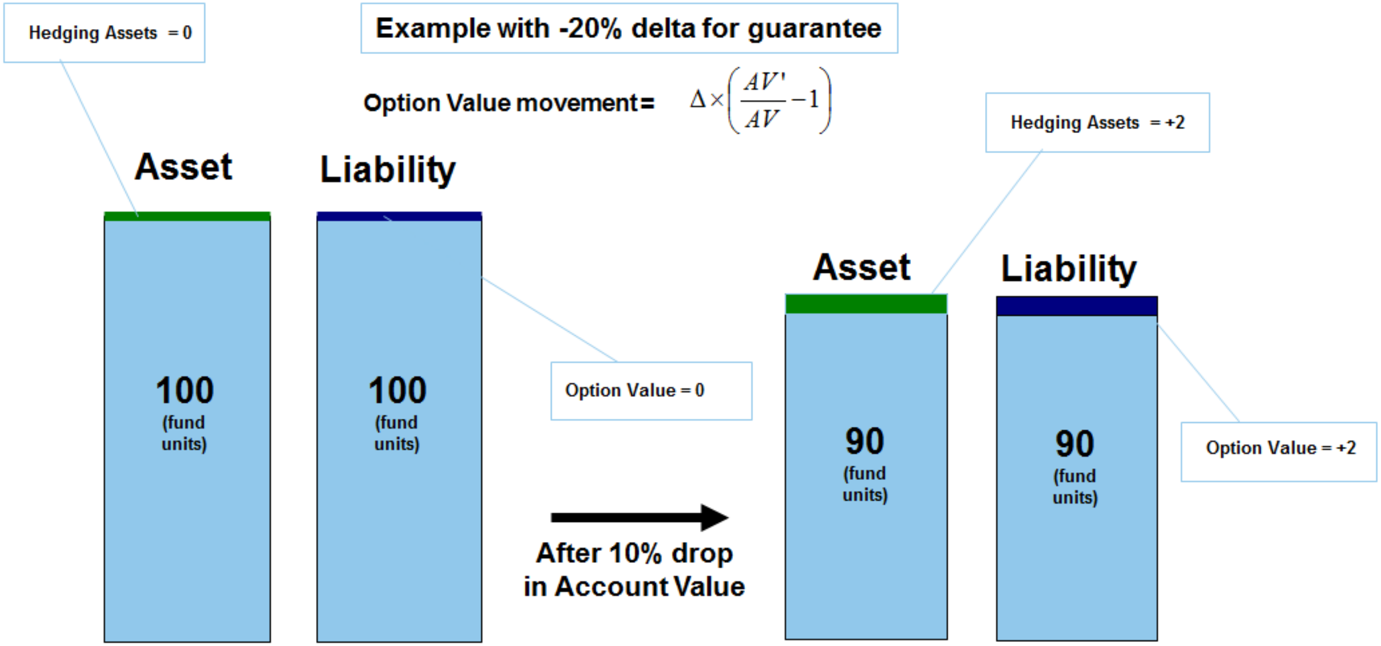
Source: Goldman Sachs

VALUE

Hedging locks-in significant part of Business Value



Liabilities Market Risks Hedging Principle

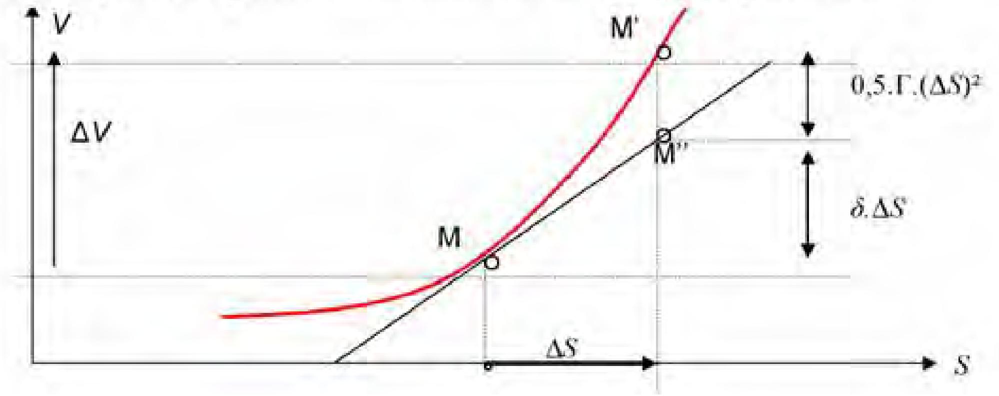


AXA invests the same number of units as the amount for which it is liable to the policyholder

After a 10% drop in the value of the Account Value with hedging, assets and liabilities are matched → the P&L of the hedging assets is equal to the Option Value

From linear risks to non linear risks embedded within liabilities

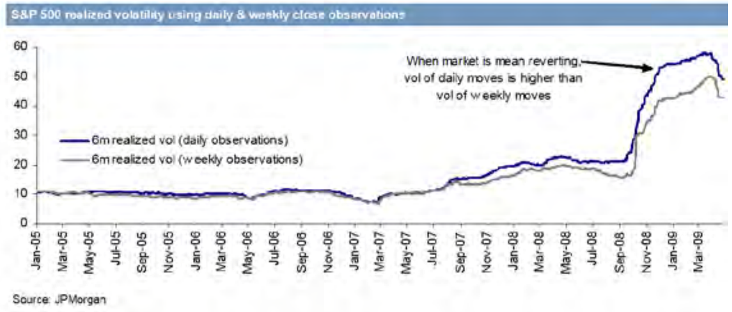
Sous l'effet de ΔS, le point M se déplace en M'. Si la convexité était nulle (Γ=0), il se déplacerait en M''

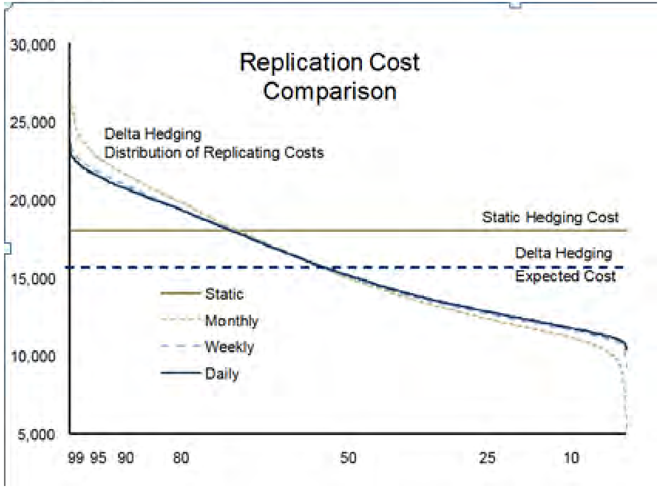


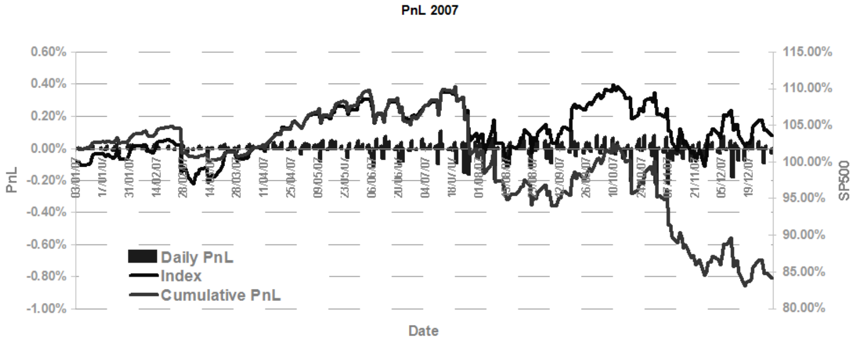
- **Dynamic delta hedging can not lock in the guarantees perfectly: 60-70% only of the variation of the liability price is explained by the first order delta: the mismatch linked to volatility risk arises as the delta hedging is not continuous in practice as illustrated by discrete rebalancing frequency. As the insurer is negative convexity, dynamic delta hedging is buy-high-and sell-low -> she loses whenever markets move**
- **Such loss increases during periods when underlying funds moves sharply, i.e. with higher realized volatility, as it results in increased "buy-high-and-sell-low" round trip trades**
- **Those mismatches are even further exacerbated by market gaps as illustrated by jumps in realized volatility, which requires dedicated gamma hedging strategies by using costly convex derivatives such as options and variance swaps that are impacted by implied volatility skew and term structures**

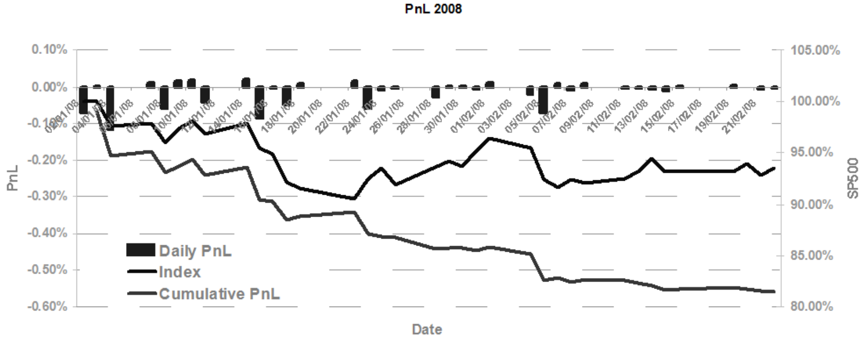
From delta hedging limitations to volatility tail and gap risks

- **E.g. discrete rebalancing frequency, transaction costs and non constant volatility**
- **Usually the higher the rebalancing frequency, the lower the replication gap; however the cost is also proportional in the volatility which increases with large mean-reverting moves at the end of day,**

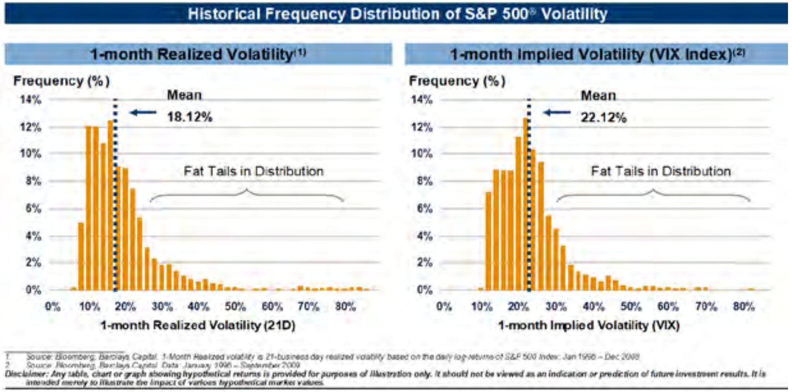


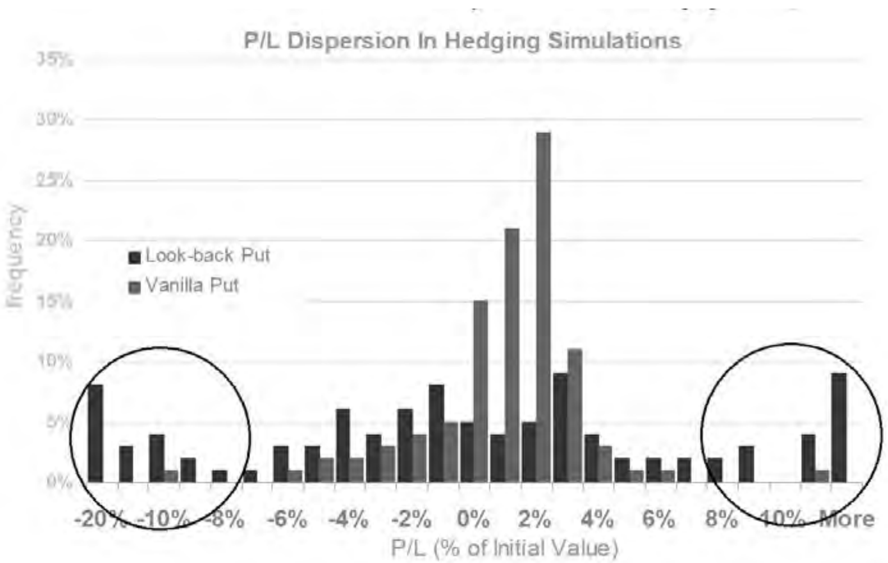






From delta hedging limitations to volatility tail and gap risks





Strategy	Average Payoff	Tail Ranking	Hedge Effectiveness
Naked	Best	4 (worst)	N/A
Delta only	Lower	3	~ 50%
Delta / Rho	Lowest	2	~ 70%
Delta / Rho / Vega	Lower (same as delta)	1 (best)	~ 90%

Market context: low volatility levels combined with spikes

- Events produced very little vol: the VIX averaged just above 11 through November – lowest since 1990, "spiked" to over 15% only a handful of times, only eight 1% moves and no 2% moves in the SPX

- As fundamental factors like growth and earnings kept vol low:
  - Macroeconomic cycle: continued global growth, solid-to positive earnings in the US, falling unemployment and growth pick-up
  - Easy monetary policy : central Banks added $2 trillion to their balance sheets in 2017 further depressing volatility.
  - Stock Buybacks

- Low volatility regime looks stable
  - Most of vol regime metrics (Fundamental, Technical) suggest the low vol regime is set to continue.
  - Vol risk premiums expected to stay elevated, as investors remain cautious. longer-dated IV is high vs. short-dated IV, making term structure steep, which reinforces the potential performance of vol selling strategies: short VIX futures strategy is up ~150%, making money every single month of the year at an average of ~3 vols per month.
  - Short-dated IV skew structurally steepened since 2012, which increases the attractiveness of selling puts.

- Still, given the combination of low vol, and low vol-of-vol, even a small vol move represents a large percentage change and can cause managed volatility funds / derivatives to exacerbate a vol event.
  - Mean-reversion of volatility is as fast as we have ever observed it, and happening more frequently. These potential spikes, rather than slowing economic growth, will likely be the cause of higher average volatility.
  - Over the past 2 years many instances when market volatility suddenly increased even when there were no well known catalysts.
    - On the one hand as many transitions from very low vol to much higher vol in the last 6Y as in the prior two decades -> new volatility-of-volatility regime; On the other hand Implied Vol of vol has drifted UP (though IV has fallen), while upside skew is very low vs. very high downside skew -> very high vol convexity
    - Selloffs that unfold faster and rebounds that grind along slowly are consistent with high skew. Furthermore, sizable put and risk reversal buying by vol control funds drives up skew -> the positioning of dealers has an effect on realized vol then implied vol.

- A number of factors have changed throughout 2017 that point to greater volatility spike potential going forward
  - Greater risk of pullbacks for the SPX, which can be intensified by crowded carry trade unwinds, as “Short volatility” positioning has grown and rapid de-risking will exacerbate a downturn
  - all the more than ~50% trading concentrate on shorter dated options, close to the money (30% in strikes within 2% of spot and less than 2 weeks to expiry) as the decline in implied volatility has made further OTM options less attractive from a premium selling perspective.
  - Traders wait until a well flagged catalyst is close at hand, especially if dealers/delta-hedgers are "one-way“. Single stocks realized volatilities increasingly driven by corporate events like earnings -> focus on trading catalysts with short-dated options.
  - Leverage continues to build in the long-running bull market with low levels of cash in retail accounts
  - Investors less portfolio protection on to limit losses, after the frustration of burning options premium in the undisturbed rally
  - With the announced Taper by the FED and a slowdown in the pace of balance sheet expansion by the ECB, the later half of 2018 can see some market stress as the accommodation is expected to slow.

Vol Control embedded within VA as major Driver of Market Feedback Loops

- Volatility control products are funds that promise their investors a specific realized volatility
  - Aiming to achieve an exact number (e.g. 10%), a specific range (e.g 8-12%), or a limit (no more than 12%).
  - Achieved through dynamic asset allocation, where the percentage of assets invested in equities varies over time based on volatility levels
  - High vol-of-vol and low interest rates thus costly insurers' hedging costs -> vol control funds valuable as providing stable volatility profile, making the insurer's cost of hedging stable over time which can allow insurers to offer product guarantees that might not otherwise be possible -> **Reducing the demand for long-dated vol.**

- Over the past 3 years short-dated realized volatilities have fallen into the 10-12 range where many vol target strategies using these volatility measures as signals **become close to fully allocated to equity, implying a greater amount of deleveraging (through forced selling of SPX futures) if volatility were to increase markedly.**

- **Market feedback loops & managed volatility funds**
  - Vol control products sell equities when volatility is rising and buy equities when volatility is falling -> market feedback loop -> exacerbates both selloffs (volatility typically rises when markets are falling) and rallies (volatility typically falls as markets rise) -> one key driver of the repeated pattern of sharp selloffs followed by consistent rebounds seen, and contributes to high skew and vol-of-vol in derivative market.
  - A vol spike from a low starting vol level drives the most aggressive de-allocations - particularly if the starting vol level is just low enough for a given fund to be at its maximum allowable equity allocation, where a selloff can drive realized vol materially higher -> the selling by vol control funds can drive the selloff to strengthen. a “volatility-control fund” with a 12 percent target, 70 percent average exposure to stocks would sell equity futures totaling 15 percent of their assets, while a sudden 4% global equity selloff would drive $20bn of selling by volatility control funds.
  - Then it can take weeks or even months to replace positions that have been sold, because In the aftermath of a significant selloff realized vol slowly "forgets" a big market move.

- **Management volatility funds were important contributors to the severity of selloffs**
  - August 2015: one of the most severe shifts in SPX volatility, switching from low volatility (10% - just a bit lower than the maximum sensitivity point for many vol control) to extremely high (11% selloff in a week) almost instantaneously, during which vol control funds sold around $50bln equities.
  - June 2016: in the aftermath of the Brexit vote sold around $25bln, with target allocations dropped by tens of percents overnight
  - February 2018: the VIX had the largest one-day move in its history (20 points to 37, the highest since August 2015), almost entirely technical, not fundamental: the move was roughly 5x larger than expected vs, a beta to the 4.2% move in the S&P.

Market context: Non-fundamental market sell-off and “volatility feedback loops” in Feb 2018 1/2

Few days after a sudden spike in volatility sparked a stock-market crash, market participants are left to ponder the wreckage of the sell-off and the mysterious dynamics that caused it, all the more than the VIX averaged 11 through 2017 – the lowest since 1990 (with a short VIX futures strategy P&L at +150% in 2017, making money every single month of the year 2017), in the context of solid fundamental factors (continued global growth, solid-to positive earnings, falling unemployment), easy monetary policy and share buybacks. January’s stock-market euphoria and strong returns likely prompted pensions to reduce risk after a strong start to the year, setting the stage for a grind higher in realized equity volatility that would hit a crescendo.

The recent market sell-off has stemmed from 3 major drivers:

- A demand from leveraged VIX exchanged-traded products (“ETPs”) which was the trigger of the jump in volatility
- The selling of Equities by “managed volatility funds” post the jump in volatility, whose Asset Under Management increased greatly over the past decade
- The hedging flows for the trillion dollar S&P 500 index put options and for the trillion dollar Variable Annuities

Demand from leveraged VIX exchanged-traded products (“ETPs”) was the trigger of the jump in volatility (the VIX jumped 20 points to 37 in one day then as high as 50 the following day

- Market technicals were flashing warning signs, where positioning in “short volatility” had become extremely crowded. As a result the rebalancing needs of VIX ETPs following a spike in volatility was near all time highs, implying that short VIX ETPs had more vega to buy on a given vol spike than ever before, creating the potential for an outsized increase in volatility should the S&P sell off sharply: a 3-point spike in VIX futures would drive VIX ETP issuers to buy $110mm vega - double the highest ever seen before 2017
- Only $3.6 billion transactions in reversing 2 specific short VIX strategies have triggered jump in volatility - a tiny fraction of the roughly $2 trillion estimated to be linked to short-volatility strategies and of the $23 trillion in market value of S&P 500 companies..
- As volatility-related products buy VIX futures in order to rebalance ahead of their 4:15 p.m. daily deadline to calculate the value of their underlying assets -- they effectively pushed up the price of the contracts and eventually the index in order to rebalance positions quickly to avoid unhedged overnight risk (ETN issuers) or excessive tracking error (ETF issuers).

Market context: Non-fundamental market sell-off and “volatility feedback loops” in Feb 2018 2/2

Once realized volatility rose, various forms of systematic volatility control funds added outflows, initiating “volatility feedback loops”

- Since the Brexit low realized volatility left volatility control funds with well below “volatility targets”, implying high Equity allocation. The sudden selloff brought then realized volatility to “volatility targets”– implying to decrease their leverage to Equities., translating into market “volatility feedback loops” that exacerbate both selloffs and rallies. It is one key driver of the repeated pattern of sharp selloffs followed by consistent rebounds as experienced.
- $500 billion of assets are tied to those “volatility control funds” that target a given level of volatility -- two-thirds of which are traded by algorithms, while 275bn$ are held in variable annuity (VA) Subaccounts

Similar to 24th August 2015, the largest and quickest punch still came from “feedback effects” of hedging flows for the trillion dollar+ S&P 500 index put options and for the Variable Annuities

- “Hedging Feedback effects” mostly stemmed from the buyback of significant short call options consistent with the rallye, as illustrated by the massive long gamma positions held by market makers before the market crash. The hedging feedback effects were all the more impactful than ~50% trading concentrated on shorter dated options and close to the money (30% in strikes within 2% of spot and less than 2 weeks to expiry).
- “Hedging Feedback effects” were further strengthened from the significant short puts positions held from market makers (30-60% short option convexity imbalance) who then delta-hedged with the index, *i.e.* sold more index as the market dropped or bought more as the market rallied, which exacerbated the initial move and realized volatility.
- Total risk exposure may have amounted up to 40% of the total market gamma. This was reinforced with varswaps bought back by investors, where market makers sold vanilla options (particularly OTM puts) to hedge their long varswaps and then delta-hedged those vanilla options using cash equities, pushing further down the Equity market as it slumped, specifically at the market close as the variance swap payoff is calculated based on the closing levels of the index.

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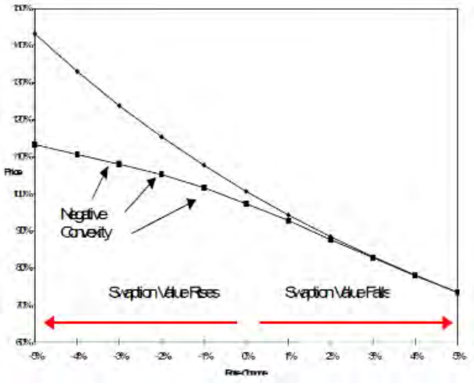
Derivatives supply-demand imbalance

- $5 trillion outstanding equity derivatives in the US: $3 trillion listed, x20 since 2000, > cash since 2005; $2 trillion in the OTC, but 40% drop since 2008
- Although fears about growth or sovereign debts sustainability were drivers of significant declines in equities as experienced in May 2006, May 2010 or August 2011, they did not fully explain the 1) extreme magnitude of the shocks and 2) the repeated occurrence at the close in European and US markets.

- **Structural supply-demand imbalance in short-dated puts**
  - 65-80% index puts by directional investors for portfolio protection; investors also sell variance swaps then hedged by market makers through puts
  - 70% more put options outstanding than call outstanding (whose flows are typically b/w buyers and sellers), mostly caused by the demand for protection -> ~40% (= 70% /1,7) of the puts are bought outright thus need to be delta- hedged by market makers

- **Structural supply-demand imbalance in the long-dated volatility market**
  - Equities Variable annuities and structured products impact
    - In the US, S&P realized vol at ~17% vs. 19% for 1- year options since 1983, 22% for 5-year, 26% for 10-year implieds: premium due to lack of supply for protection and hedging issuance of variable annuities ($1.7 trillon, 60% in Equities), $60-$120M vega *p.a.*
    - In Europe, Solvency II may boost long-term index implied volatilities, while Basel III will limit banks' ability to sell long-term vega.
    - In Asia, supply from issued retail structured products / demand institutional investors. With rising volatility and correlation, drop by 65%
  - Fixed income swaptions for hedging mortgages convexity: homeowners own the option to prepay when interest rates fall, leaving the market net short volatility -> 30% of the 30-yr market = $900 bn options hedge in 2005 (vs. $200 billion in 1993)



From market impact pricing to optimal trading transactions

- **Convex liabilities need convex derivatives to match their risk profile**

- **Supply & demand imbalance calls for minimizing market impact of hedging**
  - **Explicit modeling through the use of a specific market impact function**

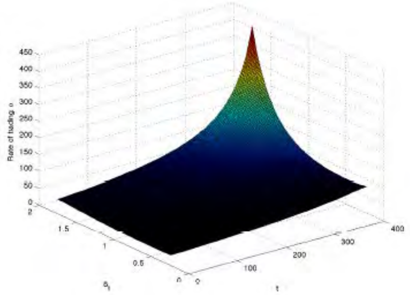
$$\tilde{P}(t,S_t,\alpha_t) \approx P(t,S_t) + f(t,S_t,\alpha_t) \qquad \text{where } f(t,S_t,\alpha) = \gamma S_t \alpha_t \Delta(t,S_t).$$

  - **Option price thus depends on temporary impact strength stemming from the pace of trading, the equity level, the option delta**

$$\tilde{P}(t,S_t,\dot{x}_t) = P(t,S_t) - \eta \dot{x}_t S_t \Delta(t,S_t)$$

where:

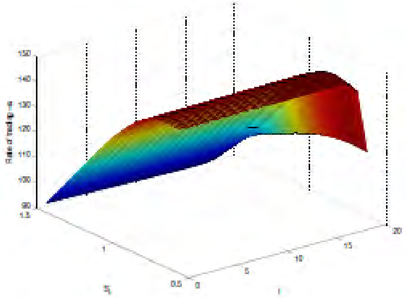
- *η controls the temporary impact strength and is in \$ $\times$ hour/N shares.*
- *$\dot{x}_t$ is the speed of trading in number of options per time unit ($\dot{x}_t > 0$).*
- *Δ is the put option sensitivity w.r.t the underlying asset ($\Delta(t,S_t) < 0$).*



  - **Hedge effectiveness strongly depends on the Risk Appetite**
    - **Maximization of the mean profit, or minimization of the mean cost -> optimal trading pace still depends only mildly on the stock price path and increases as expiry approaches**

$$\begin{aligned} \mathcal{C}(x) &= \int_0^T \tilde{P}_t \dot{x}_t dt = \int_0^T P_t \dot{x}_t dt + \eta \int_0^T \dot{x}_t S_t(-\Delta(t,S_t))dt \\ &= -XP_0 - \int_0^T x_t dP_t + \eta \int_0^T \dot{x}_t^2 S_t(-\Delta(t,S_t))dt \end{aligned}$$

$$\min_x \mathbb{E}[\mathcal{C}(x)] = -XP_0 + \eta \min_x \mathbb{E}\left[\int_0^T \dot{x}_t^2 S_t\left(-\Delta_{adj}(t,S_t)\right)dt\right]$$



    - **Mean-variance including the minimization of the dispersion of the profit and loss -> optimal execution hardly depends on time passing , but significantly depends on stock path: faster pace when the stock falls (in order to offset rising option cost**

$$\mathbb{V}\left[\int_0^T x_t dP_t\right] = \mathbb{E}\left[\int_0^T \sigma^2(S_t) x_t^2 S_t^2 \partial_S P^2(t,S_t)dt\right]$$

$$\begin{aligned} &\mathbb{E}\left[\mathcal{C}(x)\right] + \tilde{\lambda}\mathbb{E}\left[\int_0^T x_t^2 \sigma^2(S_t) S_t^2 \Delta^2(t,S_t)dt\right] = \\ &-XP_0 + \mathbb{E}\left[\eta \int_0^T \dot{x}_t^2 S_t\left(-\Delta_{adj}(t,S_t)\right)dt + \tilde{\lambda}\int_0^T x_t^2 \sigma^2(S_t) S_t^2 \Delta^2(t,S_t)dt\right] \end{aligned}$$

From market impact to amplifying delta hedging feedback effects

Delta-hedging of options positions can also be major driver of such market feedback loops

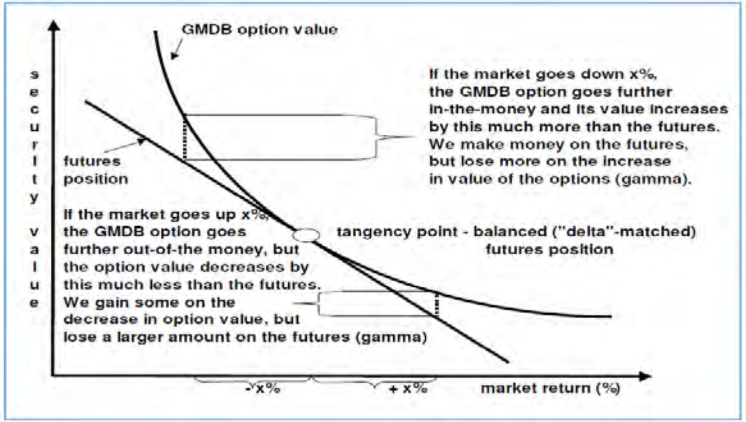
- Since most investors buy puts for portfolio protection while sell variance swaps –> significant 30-60% short option convexity imbalance for market makers then delta-hedged with large transactions in the spot -> sell more index as the market drops or buy more as the market rallies -> exacerbating the initial move and realized volatility, illustrative of pro-cyclical feedback hedging effects. Total risk exposure = up to 40% of the total gamma.

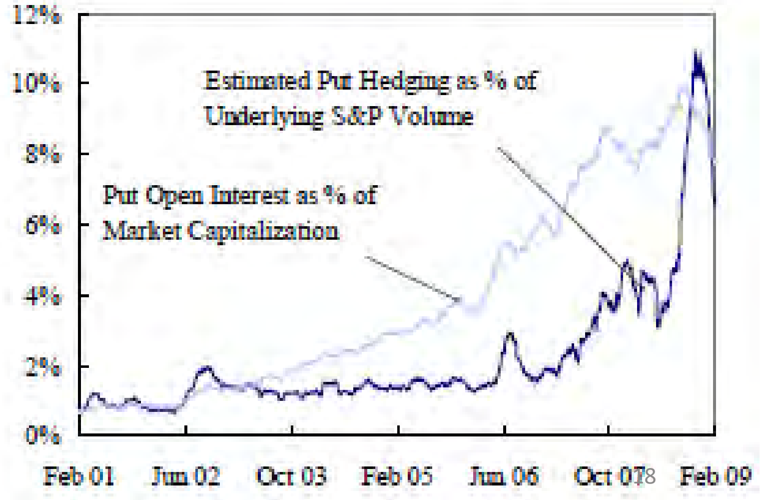
- If these options were be transacted in a two-way market by directional participants, rebalancing flow would be reduced, BUT if dealers HOLD the put options SHORT in their trading books, their gamma trading may ultimately add to trading pressure in a severe selloff. All the more that hedgers of short option positions usually transact at the market close and that the market volatility has increased initially.

- This is reinforced with varswaps where market makers sell vanilla options (particularly OTM puts) to hedge their varswaps then delta hedge these vanilla options using cash equities, specifically at the market close as the variance swap payoff is calculated based on the closing levels of the index.

Examples

- late 1990's bull market: overwriting supplied vol from pressure for income generation
- 1997-2000: bond + call participation rates with high interest rates and low vols demand
- The '02 sell-off large-scale hedging amid extraordinary volatility and a spike in option prices
- The sudden collapse of implied vol in 2003 from an overwhelming issuance of structured products in Asia reacting to a huge rise, for yield enhancement or upside participation
- April 2005 large fall in equity markets caused significant hedging issues for the short skew position held by banks, lifting longer-dated implieds hence term structure
- October 2008 triggered the short 50%-70% knock-in barrier put within autocallables on Nikkei & Kospi, which required to sell the index -> exacerbating the market by -15%



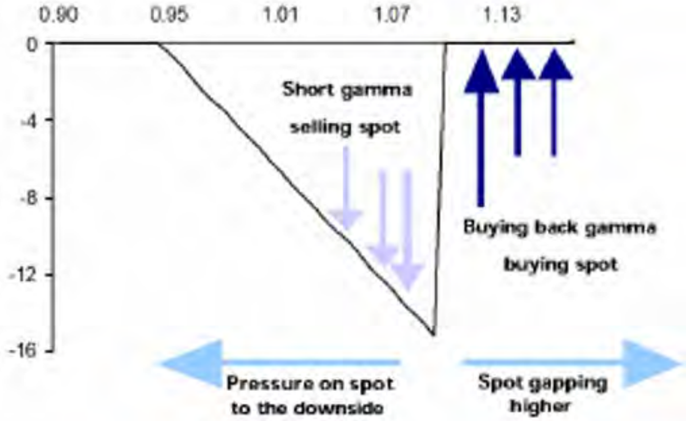


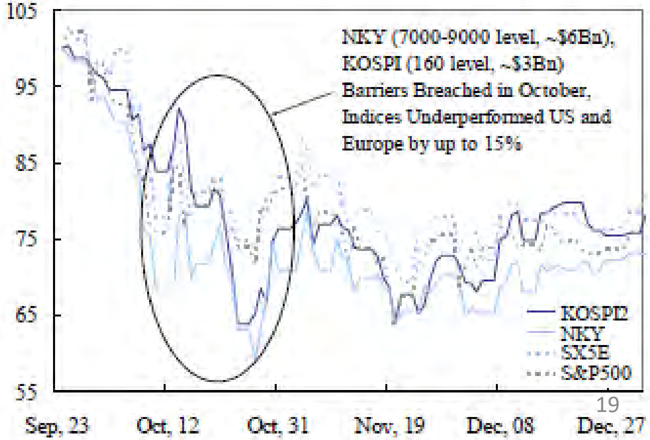
From market impact to amplifying delta hedging feedback effects: from spot to volatility 2/3

- **Although gamma hedging flows impacts were secondary as a consequence of fundamentally driven sell-off and surge in market volatility, the reversion of the temporary impact of those flows largely contributes to daily reversion patterns (up to 15% in the last 30')**

- **How does Trading distort the spot distribution ?**
  - With maturity gamma concentrates around the strike -> barriers can cause gamma to reverse in large size across very small movements as spot rallies towards a KO barrier-> large hedgers will sell large amounts of spot as it approaches the barrier, preventing the spot from hitting the barrier
  - Still the barrier may break at some point -> the option disappears, and the trader is left only with naked hedges, which he must cover by buying back spot and gamma, driving the spot even higher -> choppy trading and market gaps

- **How does Trading distort the implied vol distribution ?**
  - To hedge the long vega exposure from the autocallable issuance, the issuers short volatility using variance swaps and put options
  - When the structure is knocked-in on the downside, the embedded knock-in put becomes a vanilla that is very much ITM with very little vega, forcing the issuers to buy back their short volatility positions to re-hedge





From market impact to amplifying delta hedging feedback effects: from short term to long term 3/3

- **How delta-hedging flows exacerbate short-dated vol**
  - 65-80 % Investors buy index put as downside protection -> market makers short put options hedged by selling futures: if the market suddenly drops, need to sell further to adjust -> amplify the down market move
  - Risk exposure = 40% gamma = $7Bn sold near market close for 1% move per day

- **How structured products exaggerate long-dated implied volatility**
  - As structured products are often upside (capped) participation with capital guaranteed (floor), all market makers are long ATM vol /short OTM vol -> short long-dated skew and short long-dated vega convexity -> buy long-dated volatility when it rises and vice versa, exaggerating any move and starting a vicious circle -> imbalance due to the lack of natural counterparties, up to 10-year maturity, concentrated around the 5-year

- **With a few exceptions, the positions arising from insurance products guarantees can tend to leave market makers 'the same way around' – either buying or selling particular types of hedging instruments**
  - As insurance guarantees are often upside (capped) participation with capital guaranteed (floor), market makers are long ATM vol and short OTM vol at both wings -> short long-dated skew and short long-dated vega convexity -> market makers (through hedging) buy long-dated volatility when it rises and vice versa, exaggerating any move, all the more than hedge funds are more reluctant to remove any imbalance for maturities longer than 2-3 years, therefore causes a large movement in long-dated implied volatility.

- **Delta-hedging of Variable Annuities can also be major driver of such market feedback loops**

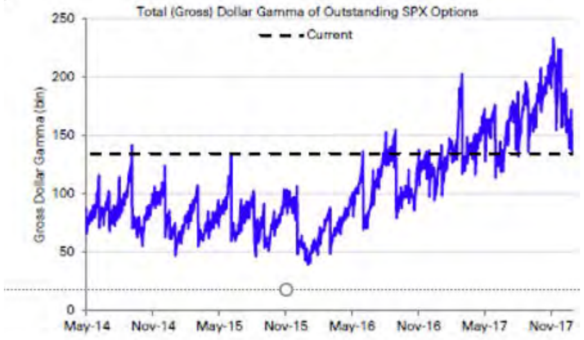
Monitoring Hedging Feedback Loops

- **SPX Gamma net long and may dampen realized vol by subduing moves locally**
  - Currently dealers are likely long ~$8-12 bln dollar gamma, consistent with the recent rally used for selling options. Low implied vol and high spot -> gross gamma near highs : 30% of SPX traded within 2 weeks of expiration and within 2% of spot - i.e. gamma-heavy options.
  - Since 2013, SPX 1-m realized vol has only seen spikes over 20 when dealers were either relatively flat or short gamma.
  - Vol selling strategies may be causing dealers to be even longer gamma, as dealers may be pushing realized vol lower as they delta hedge those short term options: sell equities when equities are rising but buy them when they're falling -> long gamma dampens realized vol

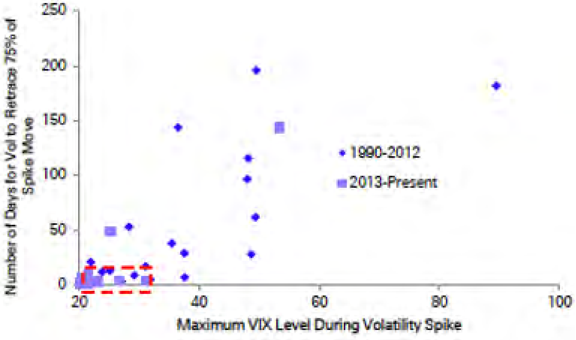
- **However the short-dated, strike-dependent nature of the gamma profile means this can disappear quickly, all the more than the equity exposure is close to 5Y highs while gross SPX put open interest is near 5Y lows -> investors may be quicker to unwind positions in a selloff with these lower levels of protection, as illustrated by the increase in the "volatility of volatility", and the frequent and short-lived spikes in implied vol followed by quick reversals.**

- **The growing popularity of short vol strategies through the VIX or delta-hedged options has tended to exacerbate the original spike making what may have been smaller increases in volatility more pronounced. A market decline would be exacerbated given positioning and reduced hedging notional around macro events.**

Source: Deutsche Bank Equity Derivatives Strategy, OptionMetrics

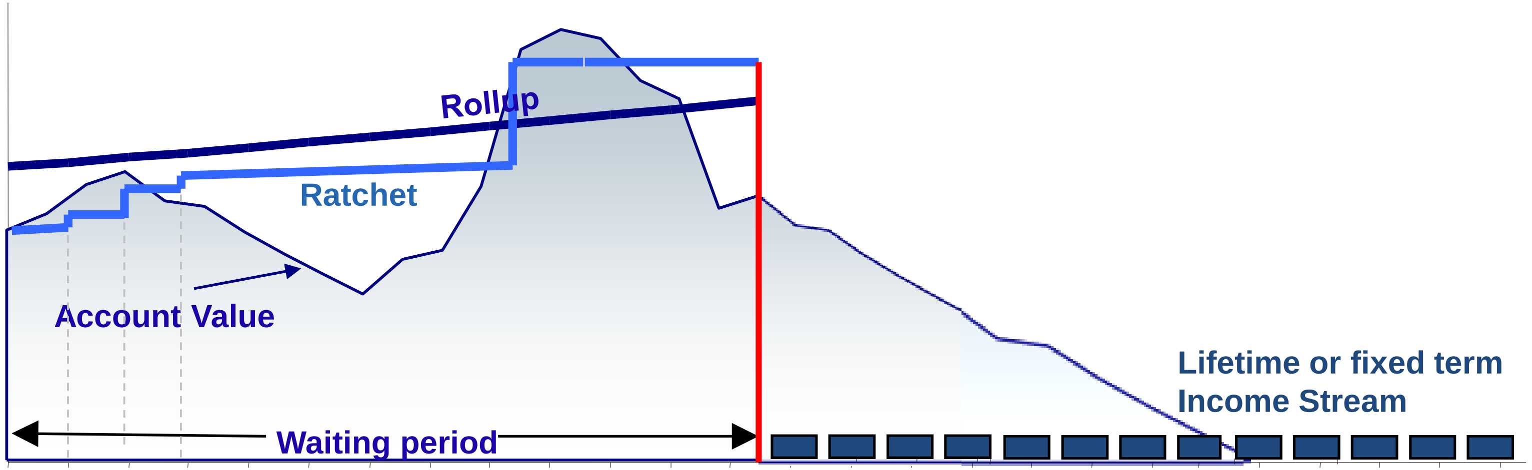


Source: Deutsche Bank Equity Derivatives Strategy, OptionMetrics



Source: Deutsche Bank, Bloomberg Finance L.P.

Delta-hedging of Variable Annuities can also be major driver of such market feedback loops due to Options Characteristics of GMWBs



- Asian Averaging: 5% withdrawal per year until account value exhausted
- Basket Option: Multiple asset classes with one guarantee
- Lookback on the Strike Put: Max Anniversary Value Benefit
- Down-and-In and Up-and-Out Barrier Put: Driven by Policyholder Behavior

- Greeks are similar to long dated index options
- Gamma and convexity demonstrate that the first order greeks are less stable, resulting in more "dynamic" hedge ratios
- Higher Order Greeks also demonstrate greater "dynamics" of hedge ratios

GMWB versus Long Dated SPX Put		
	GMWB	SPX Put
Type	65 Male	15-yr ATM
AV/Notional	100,000	89,910
PV(Claims)	-20,739	-20,739
Delta	11.6	11.7
Gamma	-0.032	-0.012
Vega	-665	-613
Rho	5,986	4,812
Convexity	-1,792	-983
Correlation	-1.9	-1.8

Impact of GMWBs on Derivatives Markets

- VA Sales are \$150 billion per year - About 40% of sales have GMWBs, or \$60 billion

- Impact on Derivatives Markets Vega
  - Vega is about \$4.2 per \$1K of Account Value -> Annual sales have about \$250 M of Vega
  - -> **Vega hedging needs are huge** *vs.* ~\$300-400 M short Vega positions managed by market-makers
    - Even prior to the crisis implied vols in the long dated only a fraction of the vega was being hedged
    - Crisis has resulted in lower supply
      - Less risk capital is available
      - Perceived attractiveness of short vega strategies has deteriorated
    - Market has become significantly less liquid as prices exceed those insurers are willing to pay

- Impact on Derivatives Markets Gamma
  - \$250 M Vega implies ~1,2 Bn **Gamma hedging needs, can be met** with listed options given the ~ short \$7bn Gamma managed by market makers

Agenda

- **Derivatives use by insurance within a low volatility combined with spikes environment**
  - **Derivatives use by insurance**
  - **Liability hedging principles and mechanics**
  - **From linear risks to non linear risks embedded within liabilities**
  - **From delta hedging limitations to volatility tail and gap risks**
  - **Market context: low volatility levels combined with spikes**

- **Hedging Market impact for life insurance liabilities**
  - **Derivatives supply-demand imbalance**
  - **From market impact pricing to optimal trading transactions**
  - **From market impact to amplifying delta hedging feedback effects**
  - **Vol Control Products as major Driver of Feedback Loops**
  - **Monitoring Hedging Feedback Loops**

- **Hedging Solutions**
  - **Pricing & Hedging Feedback effects**
  - **Mitigating Tail Risks**
  - **Reducing hedging costs**

Pricing & Hedging Feedback effects 1/2

- Interaction of one "large trader" whose action affects prices and many price taking "small traders"
- The usual no arbitrage condition (Delbaen and Schachermayer (1994)) doesn't apply.

  -> continuous time version of Jarrow (JFQA 1994): "No Market Manipulation Strategies"

  -> our approach also differs by the fact that the large trader is better informed.

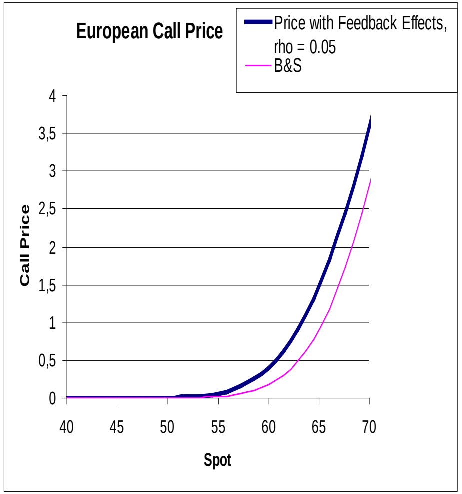
- *This requires additional but relevant assumptions:*
  - The asset price is independent of the large trader's past holdings.
  - Real wealth (as if the holdings liquidated)
  - Synchronous Markets Condition: prices adjust instantaneously across underlying and derivatives.
  - Absence of corners

- **Large dealers = net writers of options -> neutralize risk by replicating synthetically options -> additional process = number of underlying assets held by the large trader**
- **Non linear feedback effects** (Frey, 1998)

$$d\tilde{S}_t = \sigma_t \tilde{S}_t dW_t + \rho_t \lambda\left(\tilde{S}_t\right)\tilde{S}_t d\alpha_t$$

$$\begin{cases} u_t\left(t,\tilde{S},\gamma\right)+\frac{1}{2}\frac{1}{\left(1+\rho\lambda\left(\tilde{S}\right)\tilde{S}u_{\tilde{S}\tilde{S}}\left(t,\tilde{S},\gamma\right)\right)^2}\sigma^2\tilde{S}u_{\tilde{S}\tilde{S}}\left(t,\tilde{S},\gamma\right)=0 \\ u\left(T,\tilde{S},\gamma\right)=nh\left(\tilde{S}\right) \end{cases}$$

Ex: large trader sells a European call -> buys S to hedge -> S rises -> short delta more negative -> gamma < 0 -> feedback volatility rises -> call price rises



Pricing & Hedging Feedback effects 2/2

- **Tracking error is always positive**
  - so the B&S delta hedging strategy always implies a systematic loss, directly linked to the higher vol
  - growing with the gamma, i.e. with large trader's hedging activity, and with lower liquidity (higher ρ)

- **Delta sensitivity analysis**

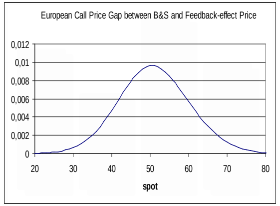
  Moneyness effect ++ The large trader buys more underlying assets for in the money calls

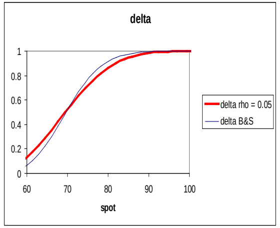
  Volatility effect - For in the money calls, a higher volatility implies a higher probability to leave out of the money, which reduces the delta

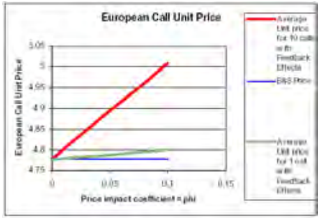
  Time to Maturity effect - - As residual time to maturity decreases, the optimal quantity to hedge is more predictable, which reduces the delta

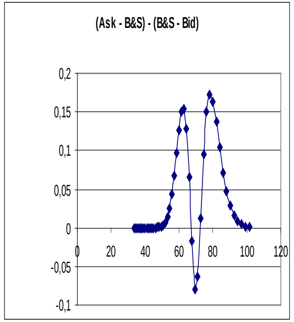
- The replicating cost grows more than linearly with respect to the number of options held (Carr, Geman, Madan, JFE, 2001),and **naturally generates empirical and consistent bid-offer spread**
  - Asymmetry (option price closer to the bid) increases in and out the money
  - The ask changes more than the bid, same for their stdev, and the stdev of their changes
  - Both Quotes std decrease w.r.t. the moneyness for a call (reverse for a put)

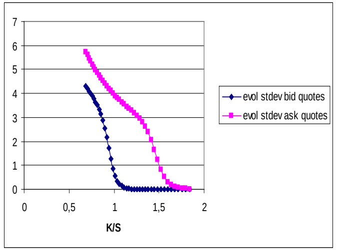
- **Naturally generates empirical and consistent implied volatility skew**

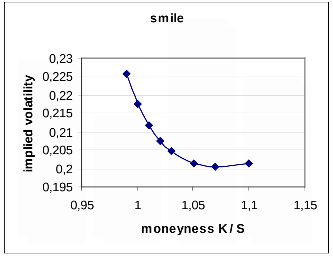






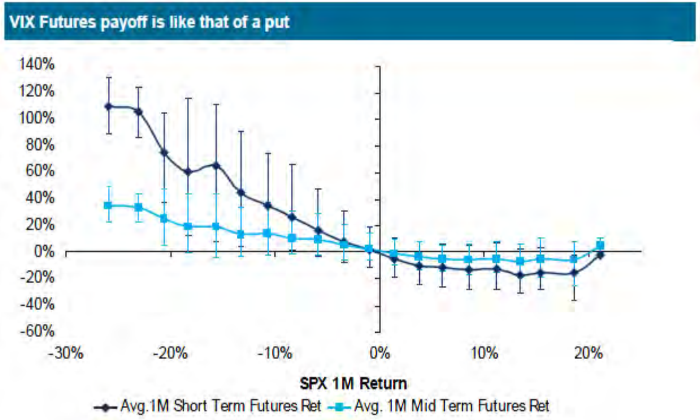






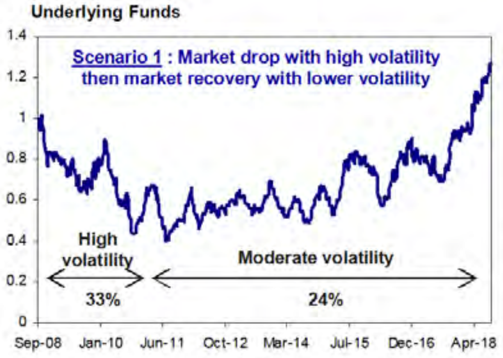
Mitigating Tail Risks with equity index & vol options

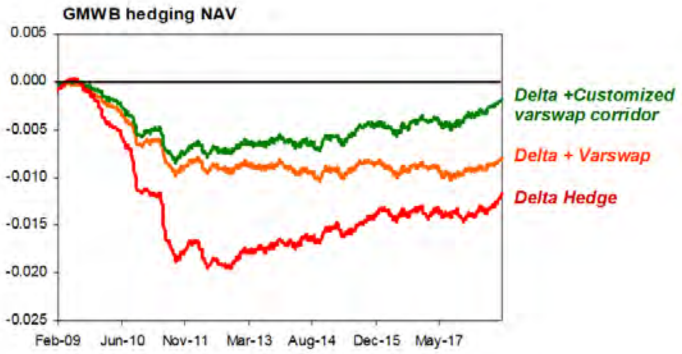
- The growth of vol control products has, at least at the margins, the potential to re-frame how volatility sellers manage their positions: the potential for market feedback loops like vol control funds to intensify selloffs that start at low vol levels makes short vol strategies riskier at low starting vol levels - increasing the need for tail protection as an overlay.
- One trend that has already begun is the use of equity index options inside volatility control products. $37bln of large VA funds already using options, generally in one of three structures:
  - Short-dated (several-month) collars. The gamma profile of a long put helps mitigate the equity selling to be done in a severe selloff, and the short call position can reduce the negative carry of the put in flat markets even though the vol control fund would likely be buying equities as the call gets closer to the money both to replace delta lost to the call and also because vol has likely been falling.
  - Long 12-24 month puts. VA issuers have often been buyers of long dated put options outside of subaccounts. Including these somewhat long-dated puts inside a VA fund changes the profile of the end investor's investment – reducing losses in a V-shaped market, but adding some negative carry.
  - VIX products, whose liquidity continues to increase, where VIX future returns are convex thus reactive relative to SPX returns thus higher than VIX…but not for free. Short-term VIX futures have more convexity than medium term

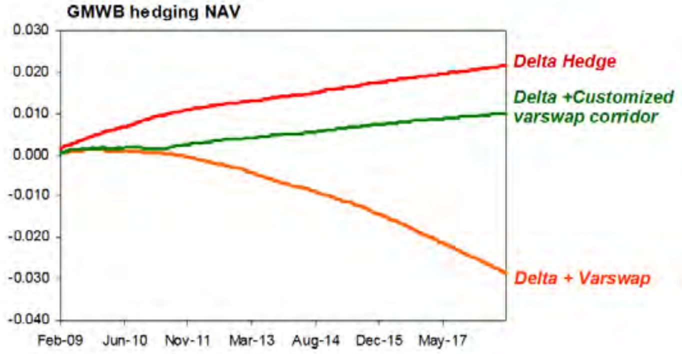


Mitigating Tail Risks with long-dated Varswaps

- Efficient way to gain exposure to long-dated implied volatility (up to 10 years) pct skew structure
- Constant dollar gamma & vega profile
- Deep enough for the largest investors and low bid-ask spreads



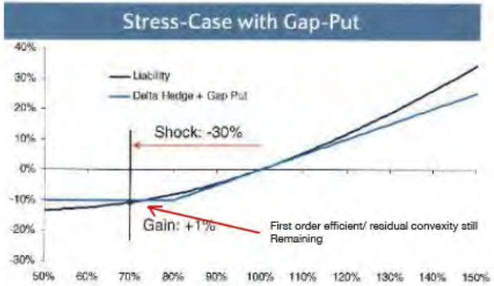


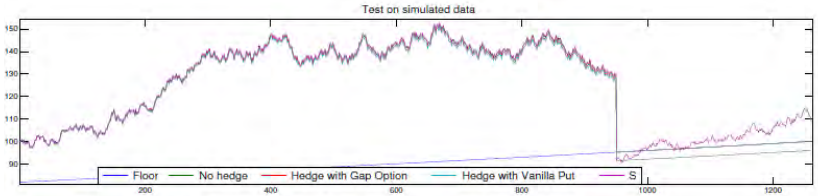


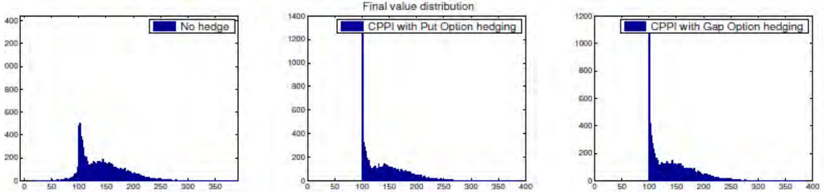
Mitigating Tail Risks with Gap options

- As the underlying portfolio drops substantially from the previous day's closing price

For illustration only
Source: Bloomberg, March 2009







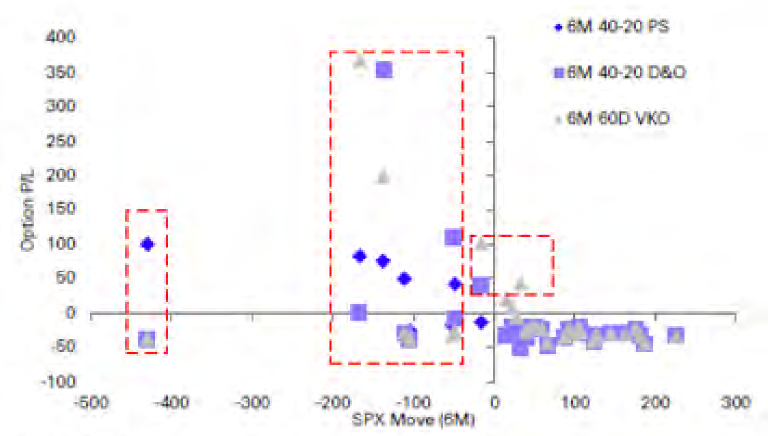
Quantile	Put Option	Gap Option
Median	133.35	134.98
Hedging cost	2.26	1.08

- In both cases, the aim is to prevent a a quick decay of more than $1/m$ in the risky asset and to recover the guarantee whenever the iCPPI breaches the bond floor
- The Gap option is significantly cheeper than Put option (1.08 vs 2.26) while providing similar returns
- The results are model dependent

Mitigating Tail Risks at lower cost

- With SPX spot at all time highs, buy SPX 6-month ~60-delta strike **variance knockout puts**. These puts knockout only if realized volatility is higher than the realized vol barrier set at trade initiation. In the event of a small market pullback (~5%), one that has not occurred for some time now, with realized volatility lower than the variance barrier, these VKO trades should perform well. The VKO is ~1/3rd the cost of a same strike and maturity plain vanilla put.

- For more muted sell-offs, the VKO had much better performance per premium spent than a vanilla put-spread in most cases shown in the figure below. Note the positive p/l from the VKO in instances where spot is down slightly or even up a little. This is due to the ITM strike on these options and an underlying that finishes flat without much realized volatility.



Source: Deutsche Bank

SOCIETY OF ACTUARIES®

Risk Managed Strategies and Their Impact on Markets

Oksana Cherniavsky, Managing Director, Prudential

SOA Investment Symposium, March 8, 2018

Disclaimer:

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Introduction

- For many years there has been a growing need for individuals to manage their own retirement income
  - Pensions no longer offered by most employers
  - More and more individuals are participating in the “Gig Economy”
- Saver’s dilemma: Market volatility can be scary, but interest rates are too low to meet income goals
  - Increased need for income or principal protection products that provide equity exposure while seeking to limit downside risk
- Implication: Someone needs to underwrite and manage that risk
- Market risk is diversifiable *to a much lower extent* than traditional insurance risks
  - Direct market risk management strategies are necessary to partially or fully immunize the risk
- What implications would a growing “protected” retirement savings pool have on overall market dynamics?

Variable Annuity Embedded Options (Death Benefits)

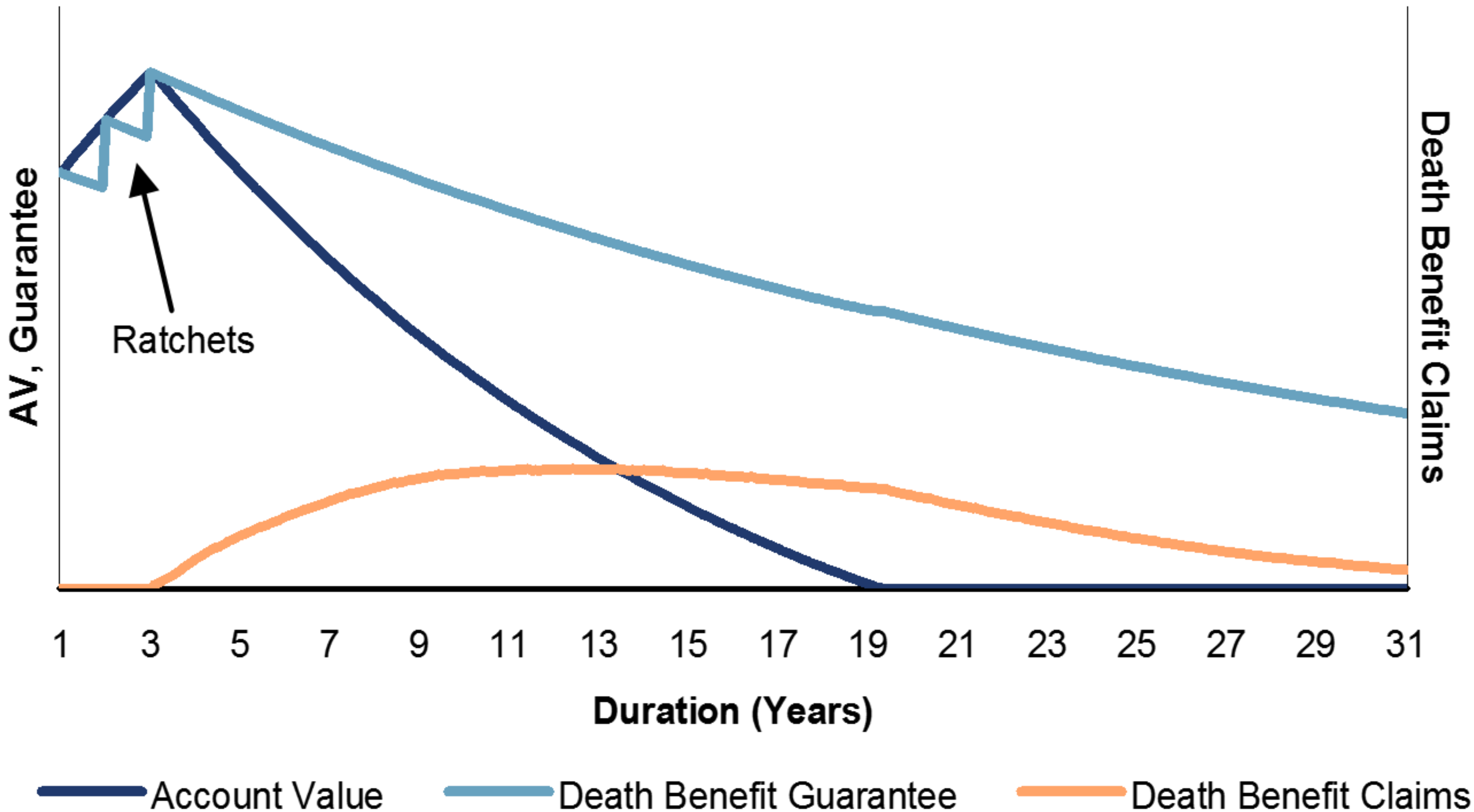
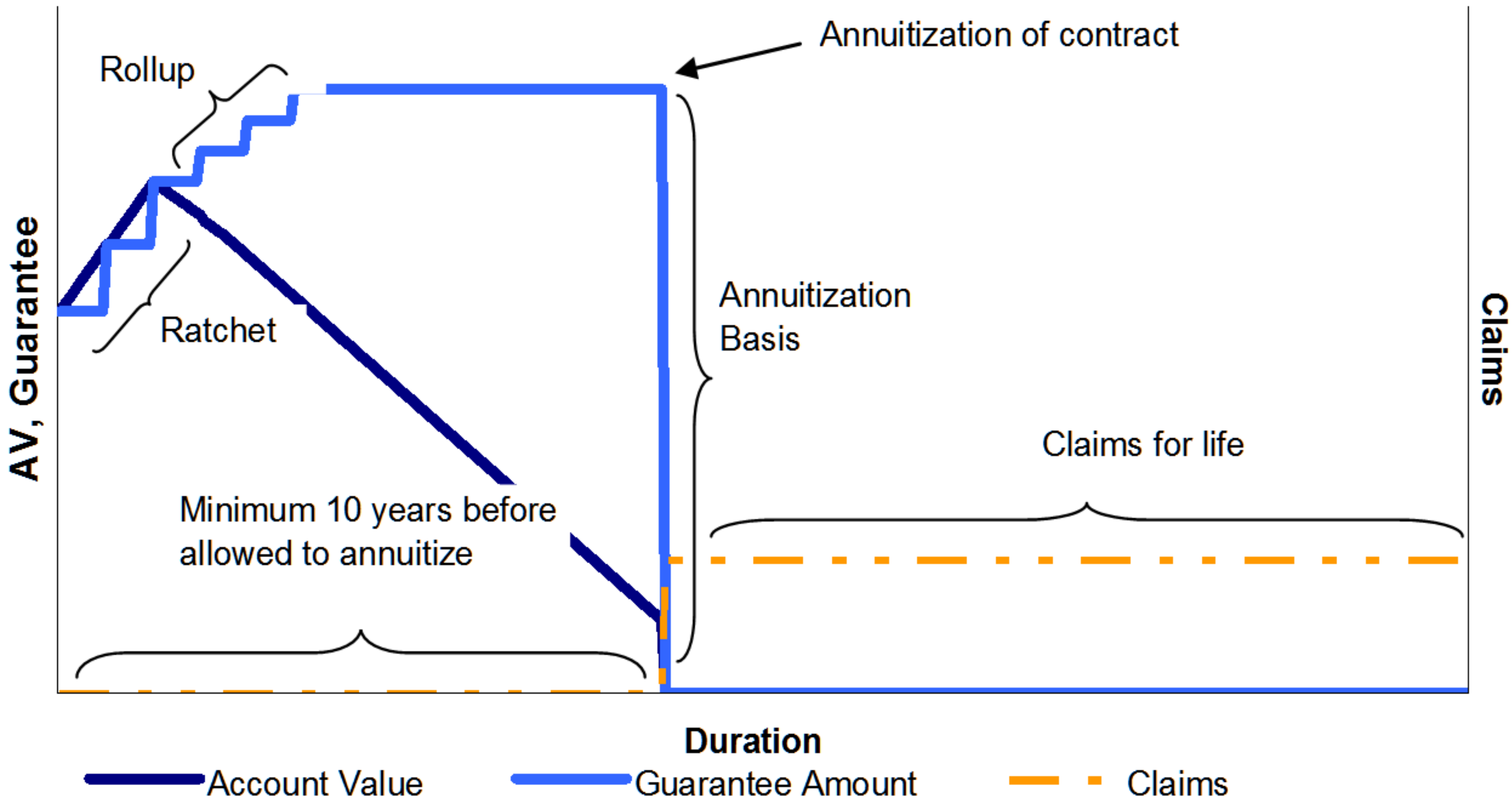


Diagram provided for illustration purposes only. Not actual performance of any product

Variable Annuity Embedded Options (Income Benefits)



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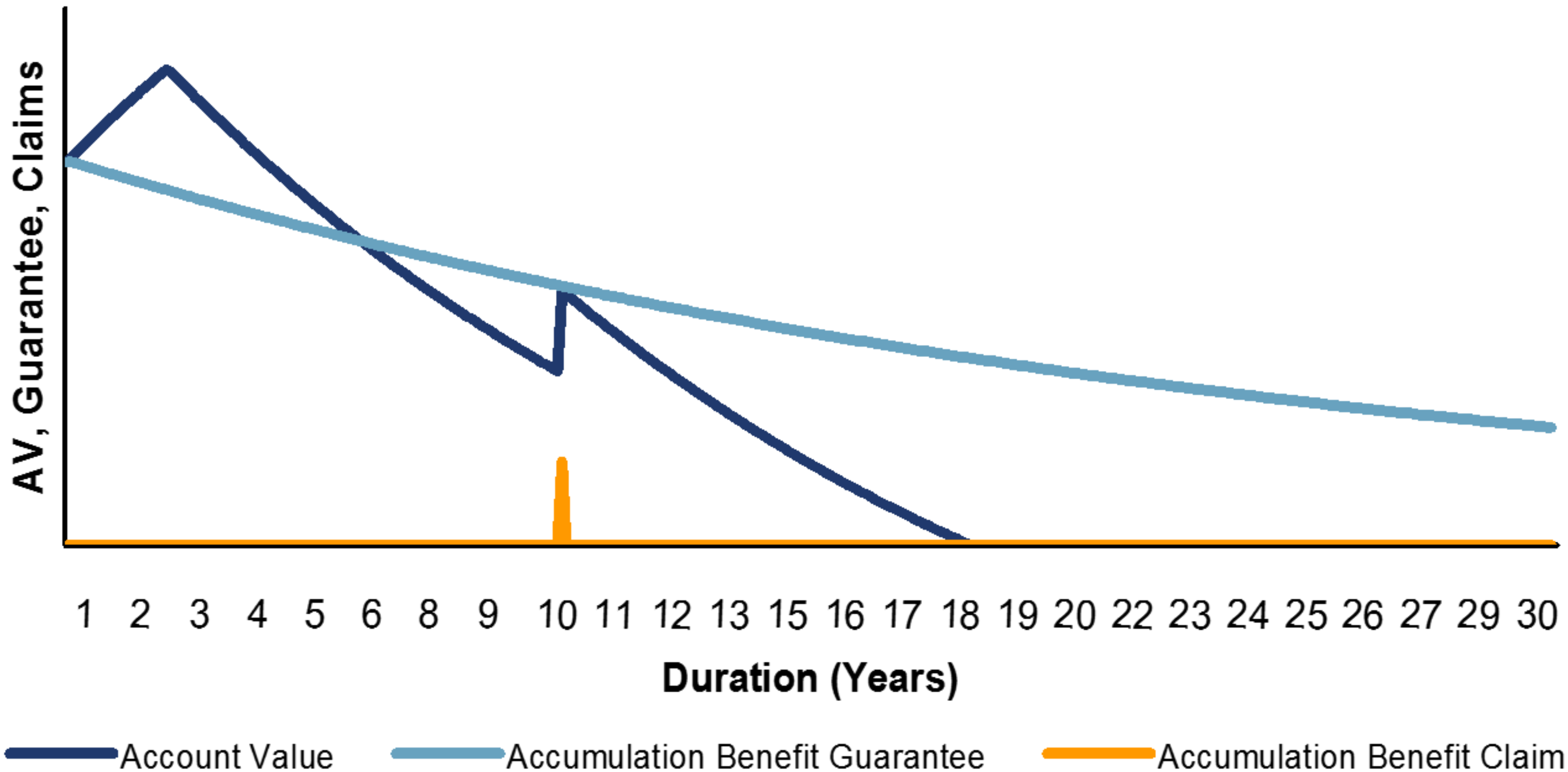
Variable Annuity Embedded Options (Accumulation Benefits)



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Variable Annuity Embedded Options (Withdrawal Benefits)

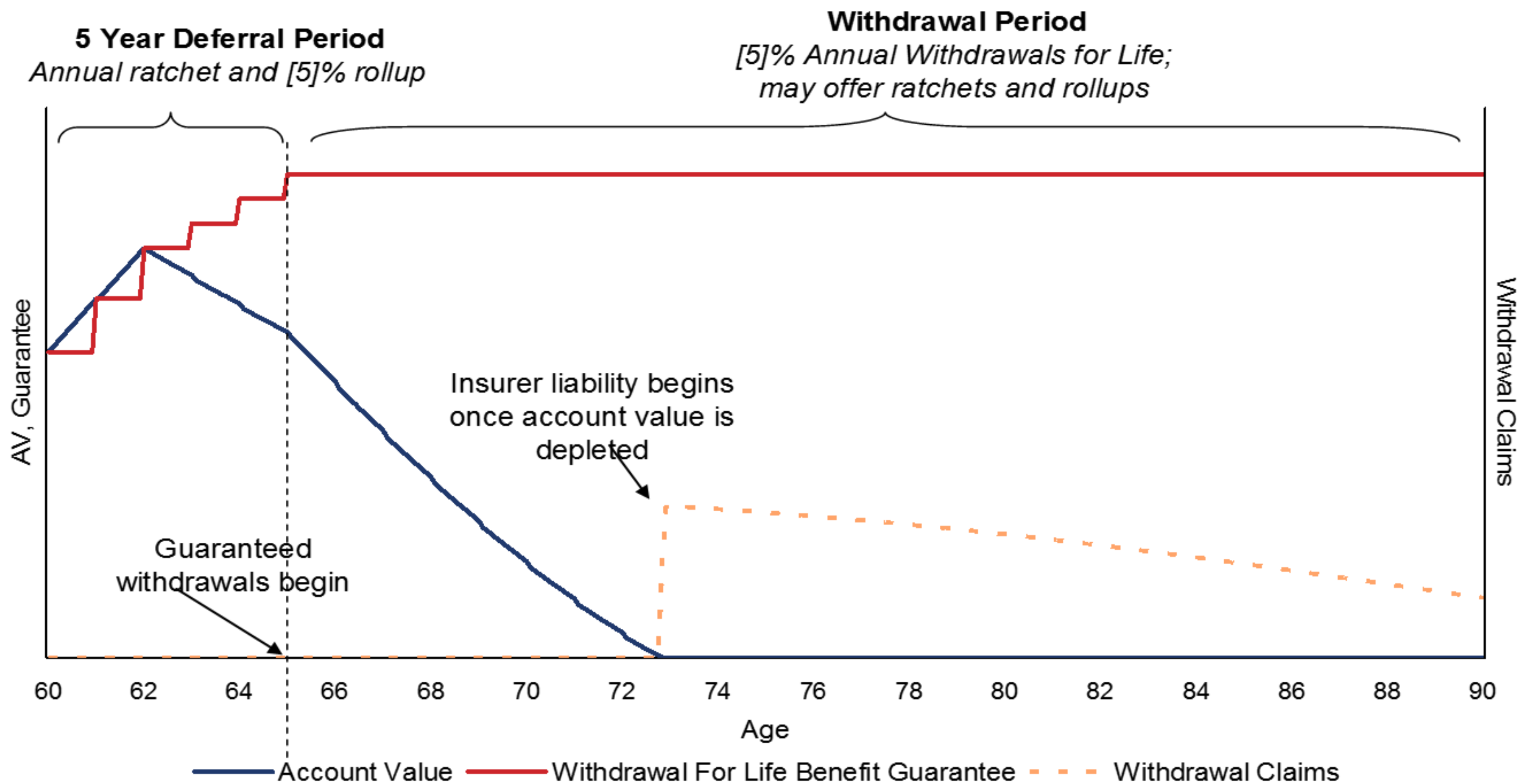
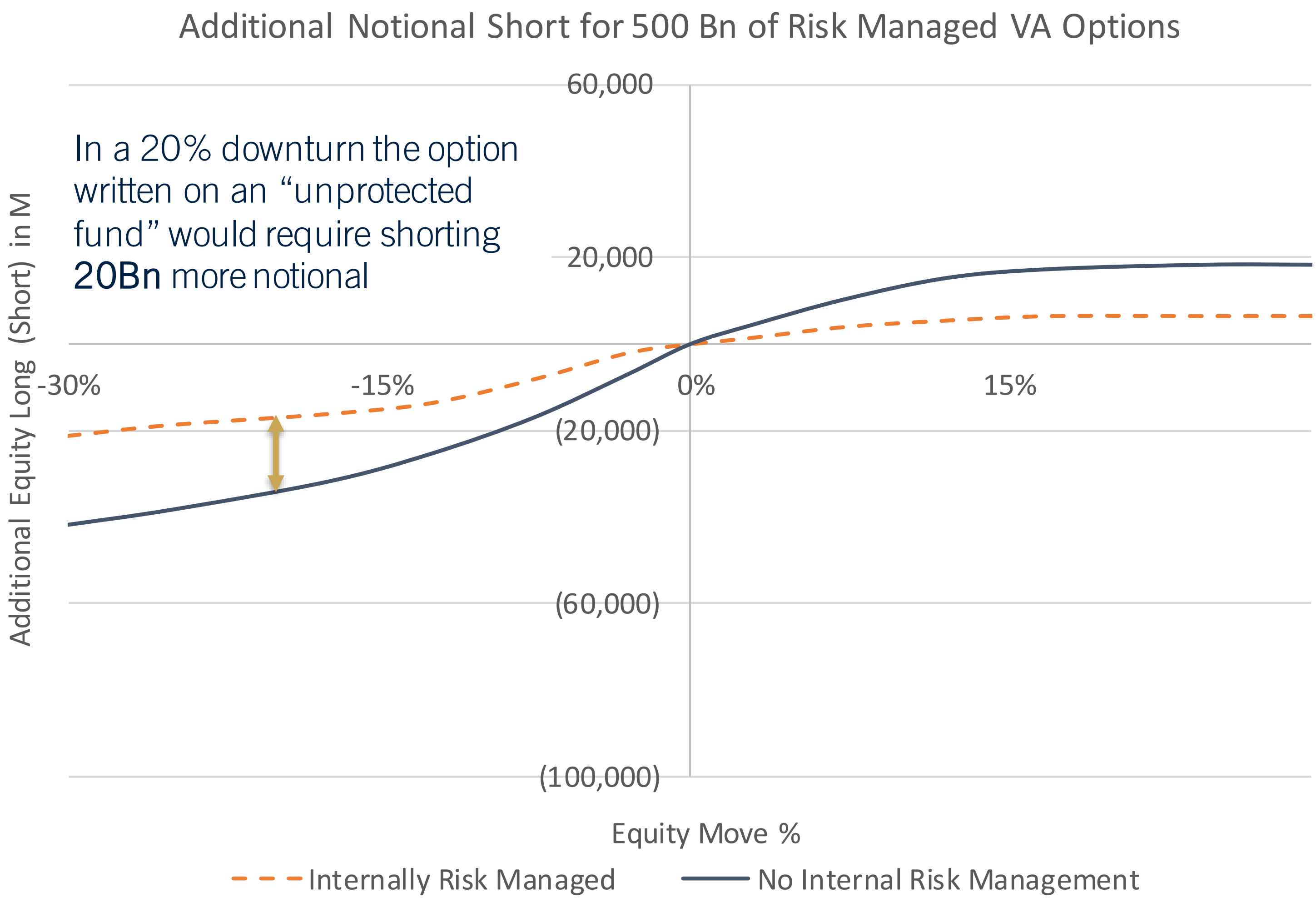


Diagram provided for illustration purposes only. Not actual performance of any product

Market Size and Implications

- Total **Variable Annuity Market Size** is **~ 2Tr** in the US
- Of that **500 Bn – 1Tr has some embedded options**
  - For Accounting and Statutory Capital drivers, for the purpose of the discussion we assume that ~ 50% of the exposure is being somehow directly hedged using market instruments
- The Options are Very long dated (exposures much lower than 1 Delta)
  - Expensive to purchase static protection
- Exposures tend to have material basis risk to underlying investments and Insured's decisions
  - Even "static protection" would have to be frequently adjusted
- Most participants have engaged in dynamic hedging strategies using derivatives on liquid indices (~ 50% in S&P 500)
- After experiencing material increased costs during the financial crisis there has been increased interest to shift risk management directly to investment options
- How does this shift impact the overall market risk profile or the market (if at all)?

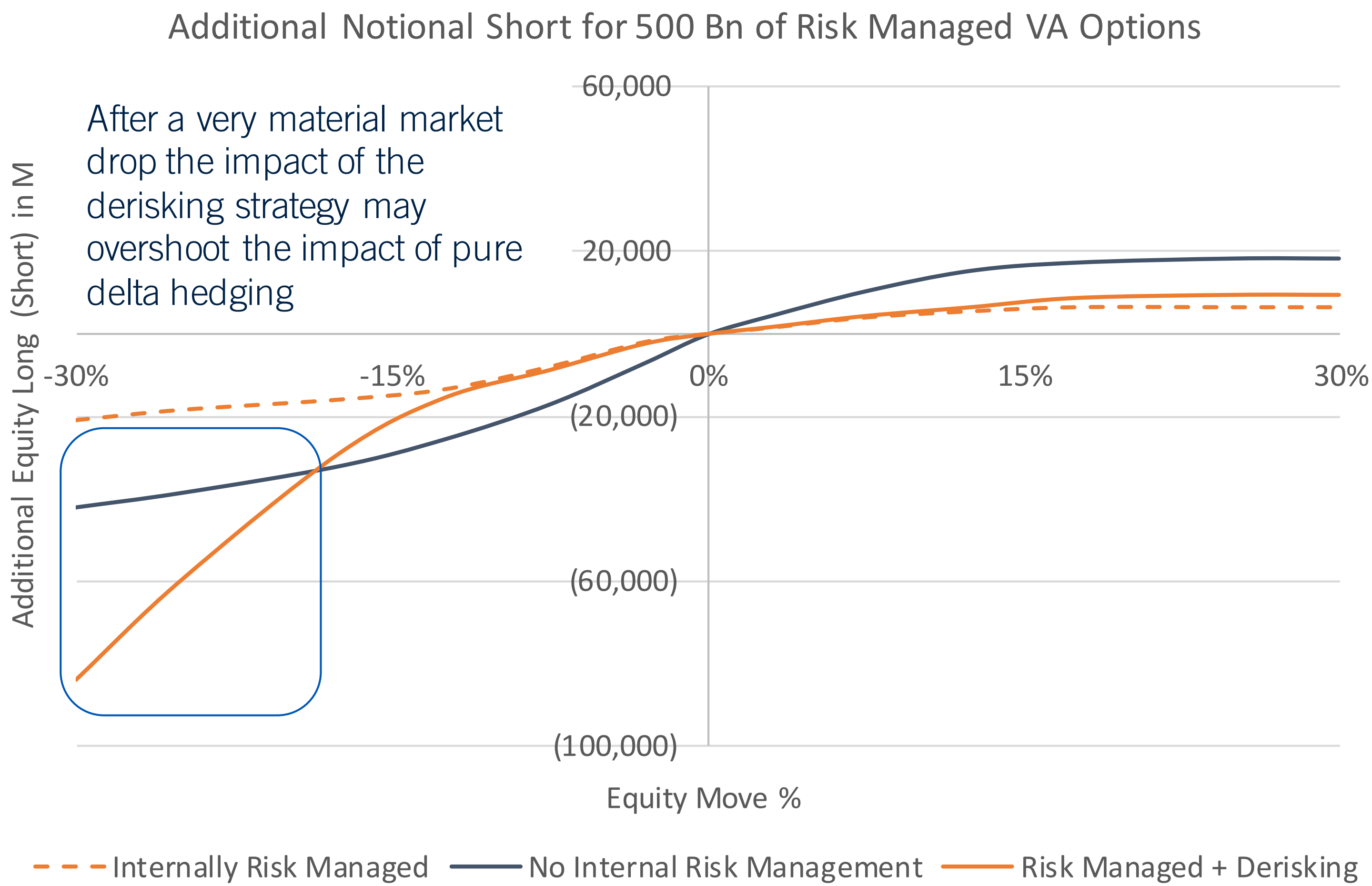
Example of a Typical Product With and Without an Embedded (RM) Strategy



Assumes a simple income protection product 70% equities, with and without a CPPI like derisking strategy embedded in the underlying funds, and 500 Bn of underlying notional with 100% of delta coverage

Example above is intended for illustrative purposes only

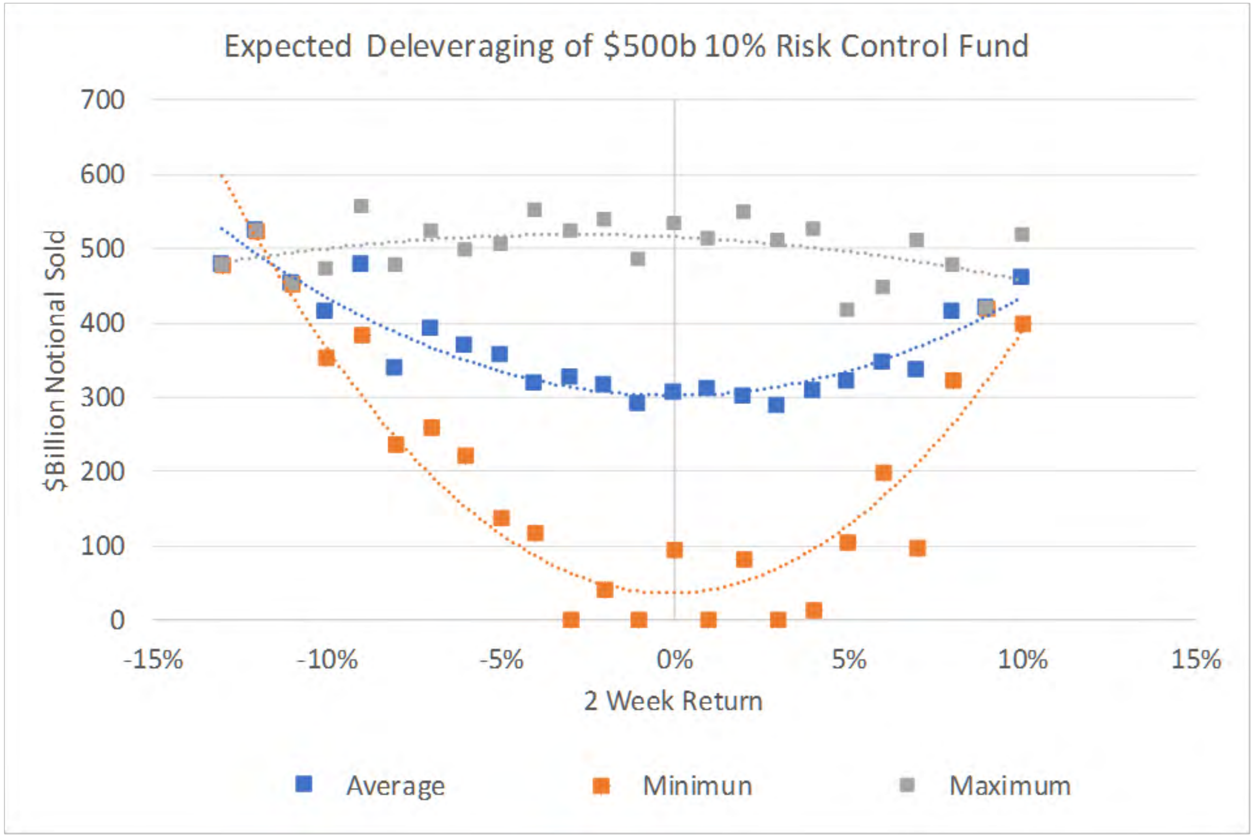
What if we Add the Impact of the Risk Management Algorithm?



Assumes a simple GLBW product 70% equities, with and without a CPPI like derisking strategy embedded in the underlying funds
We Assume 500 Bn of underlying notional with 100% of delta coverage

Example above is intended for illustrative purposes only

Impact on Trading Volume from a Target Vol Fund

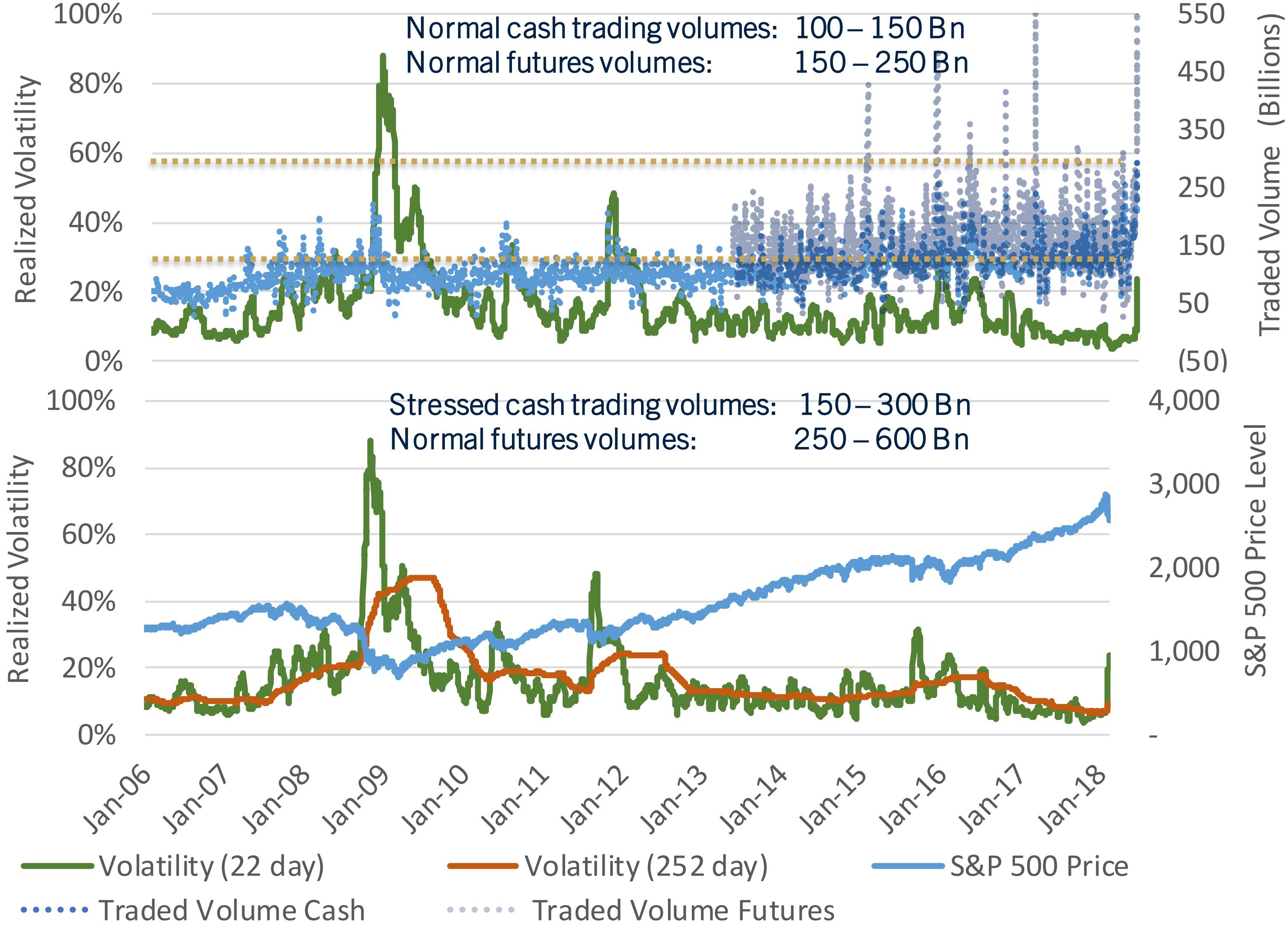


- Simulated the S&P Daily Risk Control 10% target volatility Index over 1000 scenarios generated using real world stochastic volatility jump diffusion model

- Index starts at maximum leverage of 150%
  - Simulation begins in January 2018 when volatility was low and Index was fully levered

Index based on SYP 500 Daily Risk Control 10%https://us.spindices.com/indices/strategy/sp-500-daily-risk-control-10-usd-total-return-index

Are any of these Impacts material?
(background data)



Data sources: Barclays, Bloomberg

Are any of these Impacts Material?

- Market Cap S&P 500 ~ 24 Trillion
- US Equity Total Market ~30 Trillion
- Global Market ~ 80 Trillion

Risk managing 500 Bn notional in long dated options (with or without portfolio embedded risk management) on a given day with a ~5% drawdown, can trigger 3 – 7 Bn in Equity deleveraging

Of the anticipated deleveraging we would expect 2 – 5 Bn to be in Large Cap stocks

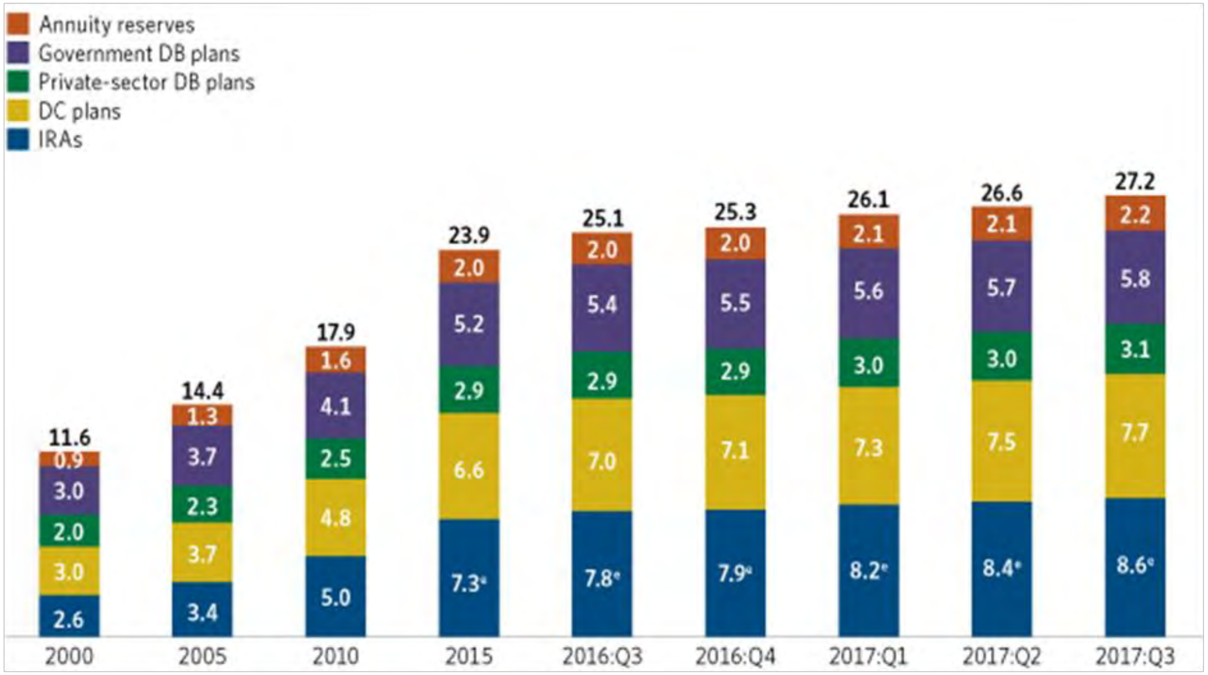
Target Vol Funds are expected to have larger impacts at the beginning of the downturns

Normal Market Traded volumes in S&P today are 200 – 500 Bn

A 5 Bn move would not appear to be material even at current levels…. But a 100 – 200 Bn most likely would ….

?

Retirement Market is Growing Fast (>27 Trillion)



Only a small fraction of these assets are currently in protected strategies (we're assuming 0.5 Tn – 1 Tn)

https://www.ici.org/research/stats/retirement/ret_17_q3

Conclusion

- Impact of excess demand for puts (especially in S&P), and subsequent hedging has been repeatedly observed and discussed*
  - Either you have to sell down your own stocks in a down market, or you pay someone else to do it for you. Either way the stock gets sold
- Writing and hedging options on funds that have embedded risk management appears to be no worse for the market than direct hedging, especially for smaller corrections
  - Target Vol strategies are expected to cause more material (short term) impact (depending on size of total Target Vol market and starting conditions)
- If the low rate environment continues and future retirees seek protected equity strategies to enhance their potential for retirement income, the ability of the market to absorb the risk management activities may become limited
- This suggests that a complement of products, not dependent on some version of put options, is needed to address the current growing retirement income needs

* M. Kolanovc; JP Morgan. North American Equity Research (22 Oct 2008)