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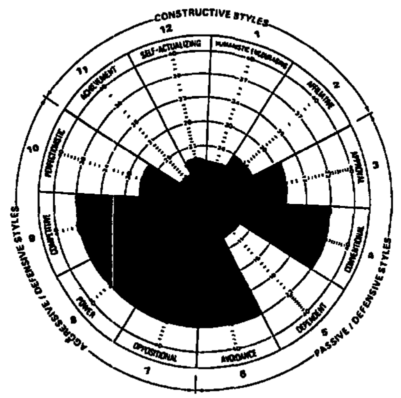
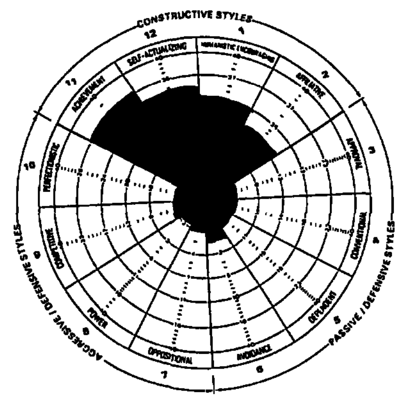


The above profile depicts a very ineffective manager -- one whose company was losing $400;000 a month.



This profile shows a highly effective key-level manager whose company recently experienced one of the most profitable years in its history.

Plymouth, Michigan, has developed a tool known as the Life Styles Inventory (LSI). The leader's guide states, "The LSI provides you with a valuable opportunity to look at your thinking and behavior — to recognize your specific strengths, as well as any 'stumbling blocks' that may be standing in your way. You can use what you learn to initiate positive changes in how you think and act and increase your personal and professional effectiveness."

The self-development guide uses the following steps:

The Life Styles Inventory
You are asked for candid responses to 240 phrases: "0" indicates the phrase is unlike you; "1" indicates the phrase is like you quite often; and "2" indicates the phrase is like you most of the time. Your scores are then tabulated on a scoring sheet.

The Life Styles Circumplex
Your scores in the first step are transferred to a circular diagram resembling a clock. (Shown on the previous page.) The 12 positions on the circumplex correspond to the life styles previously mentioned. Note the scales are not uniform. For example, a score of 24 falls within the 25th percentile for "Affiliative" (2 o'clock), but the same score falls within the 91st-99th percentile for "Power" (8 o'clock). The six concentric rings radiating from the center of the circumplex correspond to the following percentiles:

(a) Low - 10th
(b) Low - 25th
(c) Medium - 50th
(d) Medium - 75th
(e) High - 90th
(f) High - 99th

The two profiles on this page are shaded to show how scores radiate from the center.

Interpreting the scores
The self-development guide explains in detail the characteristics of each life style, with variations depending on whether your score fell into the low, medium, or high range. In addition, the guide shows how one life style relates to other styles.

For example, a low score on the "Perfectionistic" style (10 o'clock) indicates you are relatively free from perfectionistic drives and are probably realistic about what you can accomplish. However, a very low score indicates you are working below your potential and may have difficulty setting and maintaining appropriate performance standards.

The self-development guide cites many examples where the life styles either complement or contradict one another. The guide also includes a chart titled, "The LSI Thinking Styles and Effective Management," a self-improvement plan, and a bibliography of suggested reading.

The guide also suggests that you retake the Life Styles Inventory in three to six months to identify specific changes in your thinking and behavior and to determine your progress.

Those interested can contact Human Synergistics at 38819 Plymouth Road, Plymouth, MI 48170-4290, telephone: 313/459-1030.

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A heuristic approach to solving probability problems

by Jonathan Balsam

Actuaries are, first and foremost, problem solvers. The Society of Actuaries recently gave this opinion official sanction by formalizing a course requirement in problem solving (part of the two-and-a-half-day Fellowship Admissions Course).

Like problem solvers in any field, we face the danger that, burdened with scores of well-studied solution techniques, we may lose sight of a problem's structure and simplicity. Risk and probability are areas especially fraught with this danger. To see a problem's simplicity, we often must step back and try to see the forest despite its proliferation of distracting trees.

Fortunately, we are not alone. A formal science beginning in the early part of this century attempts to characterize and analyze problem solving techniques. This science is called heuristics. Its name stems from the Greek word for "find," familiar to us in the present perfect tense as "eureka," which we associate with the blinding inspiration felt by a naked Archimedes dashing through the streets of ancient Syracuse in triumph. Heuristics codifies the thought processes latent in a flash of inspiration, providing those of us who lack Archimedes' gifts with problem-solving techniques that are more likely to be effective than giving up and taking a hot bath.

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Heuristics cont'd

Heuristic techniques (heuristics, for short) are many and varied. The interested reader should consult George Polya's two-volume masterwork, *Mathematics and Plausible Reasoning* (Princeton University Press).

By using examples, this article will discuss a heuristic I have found useful in solving probability problems. Many actuaries may already, consciously or unconsciously, use this heuristic, but crystallizing it and spelling it out may prove helpful.

The heuristic comprises two contrasting rules: the first involves symmetry; the second, new information.

Symmetry

Let us use a subjective definition of probability: the probability of an event is a measure of the degree of certainty an observer has that the event will occur. The probability is numerically equal to the maximum amount that a risk-neutral person would pay to play a game with a payoff of a dollar if the event occurs, and no payoff if it does not. (This definition is in contrast to the oft-cited relative-frequency definition of probability, which leaves us without a definition of the probability of an unrepeatable event, say a rainstorm or an earthquake. Actuaries must, therefore, subscribe at least in part to the subjective definition.)

In the early stages of analyzing a problem, its symmetry often leaves us little or no information to distinguish between the possible outcomes. Without such information, we have no impetus to assign different (subjective) probabilities to the outcomes. This leads to the Symmetry Rule: Events in a state of symmetry must have equal probability.

The power of this rule is illustrated in this example:

> A bookstore has in stock 10 signed copies of the Bowers, Gerber, Hickman, Jones, and Nesbitt text. Unknown to management, one of the texts has forged signatures and will therefore have no collectible value. An eager actuarial student buys a copy. A week later, a second student buys a copy, and a week after that, a third student buys one. No copies are sold in between. What is the probability that the third student received an authentic copy?

(The reader may want to work out a solution before reading on.)

Solution: Let A_i be the event that student i gets an authentic copy. A standard approach uses conditional probabilities. Thus

$$\begin{aligned} Pr(A_3) &= Pr(A_1A_2A_3 \cup \bar{A}_1A_2A_3 \cup A_1\bar{A}_2A_3 \cup \bar{A}_1\bar{A}_2A_3) \\ &= Pr(A_1A_2A_3) + Pr(\bar{A}_1A_2A_3) + Pr(A_1\bar{A}_2A_3) + Pr(\bar{A}_1\bar{A}_2A_3) \\ &= \tfrac{9}{10} \bullet \tfrac{8}{9} \bullet \tfrac{7}{8} + \tfrac{1}{10} \bullet 1 \bullet 1 + \tfrac{9}{10} \bullet \tfrac{1}{9} + 0 = \tfrac{9}{10} \end{aligned}$$

Use of the Symmetry Rule obviates all this algebra. We simply note that the problem as stated does not distinguish among the three students. The fact that they make their purchases in succession rather than all at once does not matter, since each purchase is random. So A_2 and A_3 must equal A_1, easily seen to have a value of 9/10.

Note also that use of the Symmetry Rule invokes the same solution process for any number of students, while the work using conditional probabilities increases exponentially with the student count.

New information

As actuaries, we seldom are satisfied with probabilities as originally determined. Rather, we continually revise them in the face of new information. Some subtlety is required to decide how relevant the new information is to the existing model. This is the subject of the New Information Rule: When new information arises, consider whether it relates to the probability model previously developed. If it does, revision may be necessary. If it does not, then the previously assessed probabilities must still hold.

Again, this is best illustrated by example. The "Let's Make a Deal" problem, originally printed in Marilyn vos Savant's *Parade* magazine column, aroused much controversy in *The Actuary* and elsewhere. (See March, April, June, November 1991 *Actuary* "Lighter Side" columns.)

For those readers who missed it in *Parade*, the problems was:

> On a game show, you are offered your choice of one of three boxes. One of the boxes contains a car, while the other two contain goats. The host of the show knows the contents of the boxes, but you (of course) do not. You choose Box 1. Before you open it, the host opens Box 3, shows you that it contains a goat, and invites you to switch your selection to Box 2. Should you switch?

Solution: vos Savant correctly noted in her column that you should switch. (This raises your probability of winning from 1/3 to 1/2.) A firestorm of controversy arose. Respondents, including mathematicians with Ph.D.s, castigated vos Savant for mathematical illiteracy.

What provoked such strong resistance to accepting the correct answer to a question every college math student has the tools to solve? Their resistance, it seems, derived from application of the Symmetry Rule without countervailing application of the New Information Rule. Readers correctly noted that before the host opened Box 3, the Symmetry Rule dictated the equivalence of all the boxes. Therefore, switching carried no benefit. By showing you the contents of Box 3, has the host provided any new information about the relative merits of boxes 1 and 2? On the surface, he has not, so switching seems useless.

Looking beneath the surface, though, he has provided new information. To see this, first consider the slightly different scenario in which Box 3 is opened by mechanical error and reveals a goat. Such an error could not convey useful information, since a random event affecting Box 3 can tell us nothing that distinguishes the other two boxes. But in our scenario, the host chose to open Box 3, knowing it contained the goat. It is possible he chose 3 over 2 precisely because 2 contained the car. In that possibility lies the new information content of the host's action, which makes switching to Box 2 a prudent choice.

Conclusion

The two portions of this heuristic, the Symmetry Rule and the New Information Rule, mirror processes we follow naturally in solving probability problems. Their explicit use has proven helpful in organizing one's thoughts when approaching the problems. As an additional benefit, the heuristic provides a common vocabulary, a means to avoid the miscommunications and misunderstandings so common in probabilistic discourse.

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