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Stochastic pension fund modelling

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Abstract

In this paper we consider a simple model for a defined benefit pension plan under stochastic rates of return.

First we describe the general framework and the model which will be used in our analysis. Using this model we are interested in deriving expressions for the moments of the fund size and of the contribution rate when the spread method of amortization is used.

Under the model used, the fund size follows the recurrence equation

$$F(t+1) = (1 + i(t+1))(F(t) + C(t) - B),$$

where $i(t+1)$ is the effective rate of interest earned on the fund during the period t up to $t+1$, $C(t)$ is the contribution rate at time t and B is the constant benefit outgo.

Note that under the spread method of amortization, the contribution rate is

$$C(t) = NC + (1/\ddot{a}_{\overline{m}|})(AL - F(t)),$$

where AL is the actuarial liability and m is the amortization period.

We consider the case where rates of return in successive years are independent and identically distributed and investigate the relationship between the first two moments of the contribution rate, the valuation rate of interest and the period of amortization used. This leads to the introduction of the efficient frontier which describes the minimum level of variability which is attainable for a given mean contribution rate, μ_C.

We show that this minimum variability is a quadratic function of μ_C and is given by

$$m(\mu_C) = \frac{(v_1^2 - v_2)(1 - v_2)(B - \mu_C)^2}{(1 - v_1)^2 v_2}.$$

Here v_1 and v_2 are adjusted valuation rates which depend on the random rate of return.

We then consider models in which rates of return in successive years are not independent. We describe the general approach we use for dependent rates and include a result which

provides a sufficient condition for the fund size and the contribution rate to be ergodic. This then permits the use of long simulation runs as a means of deriving approximate summaries of the stationary distribution of the process.

In order to obtain more explicit results we focus our attention on Gaussian processes, with particular attention being paid to the AR(1) process as a description for the force of interest. Exact expressions as well as bounds for the first two moments of the fund size are derived and their relationship with the first order autocorrelation coefficient is considered.

We present a procedure that can be used to find an efficient frontier for the variability in the contribution rates when the AR(1) process is used.

Finally, conditional moments are studied and a recursive method for deriving the conditional distribution of the fund size is described.