ACTUARIAL RESEARCH CLEARING HOUSE
1999 VOL. 1

The Variance Premium Principle: A Bayesian Robustness Analysis*

[Actuarial Research Clearing House, 1, 1999]

E. Gómez-Déniz, F.J. Vázquez-Polo
University of Las Palmas de Gran Canaria. Dpt. of Quantitative Methods. Fac. CC. Económicas. Campus de Tafira. 35017 Las Palmas de Gran Canaria. Spain.
(emilio@empresariales.ulpgc.es, polo@empresariales.ulpgc.es)

A. Hernández-Bastida
University of Granada. Dpt. of Applied Economics. Fac. CC. Económicas. Campus de Cartuja. 18011 Granada. Spain.
(bastida@goliat.ugr.es)

Abstract. The Bayesian model of collective risk theory is extended in the sense that a large class of distributions is used instead of a single one. We then investigate the sensitivity of variance premium principle when the structure function belongs to that class.

According to robust Bayesian methodology, the uncertainty in the prior can be modeled by specifying a class Γ of priors instead of a single one. We examine the ranges of the Bayesian premium, also called experience rated premium, when the priors belong to that class. Most of the robust Bayesian procedures developed measures of sensitivity of quantities which can be expressed in terms of posterior expectation (e.g. mean, variance and probability of given sets). Nevertheless, relatively few papers have been related to measure Bayesian sensitivity of quantities which can be expressed in terms of ratio of two posterior expectations, as occurs in the variance premium principle. Approapiated tecniques to solve this situation are considered. Unimodality turns out to be very convenient for modelling subjective beliefs about the risk parameter.

The very common Poisson-Gamma model is developed in depth using our methodology.

Key words: Variance premium principle, Bayesian robustness, Classes of priors.

AMS classification: 62F15,62P05.

1. Introduction

A premium calculation principle is a functional that assigns a usually loaded premium to any distribution of claims. The most useful and famous principle are, among others, the net premium, exponential, Esscher and variance premium principle, obtained from different underliying loss functions.

The use of standard Bayesian analysis in risk theory has been considered in several actuarial applications (Makov, et. al., 1996, Eichenauer et.al., 1988, Heilmann, 1989, Klugman, 1992 among others). To use Bayesian analysis to model insurance loss, the practitioner usually

* A preliminary version of this paper was presented by authors at 33rd. Actuarial Research Conference. Atlanta, 1998.

chooses a prior distribution (structure function in risk theory). Prior distribution can be difficult to choose in actuarial context. Bayesian robustness analysis has received substantial attention and a noumerous list of authors have been proposed many solutions for this problem (Berger, 1985; Berger and O'Hagan, 1988; Lavine, 1991 Sivaganesan, 1988, among others. An excellent revision of this topic is contained in Berger, 1994).

Our approach is to assume that practitioner is unwilling or unable to choose a functional form of the structure function, π, but we may be able to restrict the possible prior to belong to a class suitable to quantify actuary's uncertainty. Then it becomes of interest to study how the premium for priors in that class behaves. We shall use classical ε-contamination class of priors. The Bayesian premium for variance principle can be written as the ratio of two particular posterior expectations. We present a basic result for studying the range of variation of the Bayesian variance premium as the prior distribution varies over an ε-contamination class $\Gamma_\varepsilon = \{\pi(\theta) = (1-\varepsilon)\pi_0(\theta) + \varepsilon q(\theta) \mid q \in \mathcal{Q}\}$, where ε reflects the amount of probabilistic uncertainty in a base prior π_0 and $\mathcal{Q}$ is a class of allowable contaminations. For $\mathcal{Q}_1 = \{\text{All probability distributions}\}$ and $\mathcal{Q}_2 = \{\text{All unimodal distributions}\}$ we determine the range of the Bayesian premium as π varies over Γ_ε.

The remainder of the paper is as follows. In section 2 we describe the variance premium principle in a classical and Bayesian analysis. Section 3 presents a brief introduction to the robust Bayesian analysis and provides a technical result for a Bayesian sensitivity analysis. In section 4 we calculate the risk and Bayesian premium in the no compound collective model Poisson-Gamma. Upper and lower bounds for variance principle in this model is obtained. Finally, sections 5 and 6 conclude with some examples and final remarks on related work.

2. Premium Calculation

2.1. Classical Analysis

We assume that the claim of a risk, or policyholder, per policy period is a random variable X with probability function $f(x \mid \theta)$, and that value θ is fixed for a given risk, although it is commonly unknown.

If we wish to distinguish in which year, or policy period, the claims X occur, we write X_i for the claims in year $i = 1, 2, ..., t$. For a fixed value θ we assume that the random variables X are independent for different values of θ.

A premium calculation principle (Heilmann, 1989) is a functional H that assigns to any risk X (with probability function $f(x \mid \theta)$, where x

takes values in the sample space $\mathcal{X}$ and θ is thought of being a realization of some parameter space Θ) a real number $H(X)$, the premium. Let $L : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a loss function that assigns to any $(x, P) \in \mathbb{R}^2$ the loss sustained by a decision maker who takes the action P, the premium charged, and is faced with the outcome x of some random experiment. $H(X)$ has to be determined such that the expected loss is minimized, i.e. as the minimum point of the mapping $P \longrightarrow E\left[L(X, P)\right]$. In risk theory many loss functions have been used (Heilmann, 1989): the loss cuadratic function gives the net premium principle, the loss exponential function results in the exponential principle, etc.

Let $L(x, P) = x(x - P)^2$, then

$$H(X) \equiv P(\theta) = \frac{E_f\left[X^2 \mid \theta\right]}{E_f\left[X \mid \theta\right]} \tag{1}$$

is the *variance premium principle.*

As in Gerber (1979, p. 66), a principle of premium calculation is a rule that assigns a usually loaded premium to any distribution of claims. Each one of the most important principles of premium calculation can have attractive and desirable properties such as nonnegative safety loading and consistency, among others.

Obviously the premium calculation principles developed above can only be applied if the distribution of the risk X is known. $P(\theta)$, is called the *true individual premium* or *risk premium.*

2.2. Bayesian Analysis

In this paper we consider the case in which the distribution of X is specified up to an unknown parameter, and where ratemaking incorporates individual claim experience. Therefore we now consider the case that the risk X within a given collective of risks is characterized by an unknown parameter θ which is thought to be a realization of some parameter space Θ.

We shall assume that given the risk X there is some claim experience $M = m$. Let $f(x \mid \theta)$ be the density function of X; given a proper prior on Θ, say $\pi_0(\theta)$, the posterior distribution, $\pi_0(\theta \mid m)$, is obtained by using Bayes'theorem as the product of the prior and the likelihood function, $f(x \mid m)$, divided by the predictive distribution of the data, $p(m \mid \pi_0) = \int_\Theta f(m \mid \theta)\pi_0(\theta)d\theta$. This is,

$$\pi_0(\theta \mid m) = \frac{f(m \mid \theta)\pi_0(\theta)}{\int_\Theta f(m \mid \theta)\pi_0(\theta)d\theta} \propto f(m \mid \theta)\pi_0(\theta).$$

The *Bayesian premium* of X is then defined (Heilmann, 1989) to be the real number $P^*_{\pi_0}(m)$ minimizing the posterior expected loss

$E_{\pi_0(\theta|m)}\left[L(P(\theta), P^*_{\pi_0}(m)\right]$; the posterior expected loss sustained by a practitioner who takes the action $P^*(m)$ in place of $P(\theta)$ that is unknown.

Let now $L\left(P(\theta), P^*_{\pi_0}(m)\right) = P(\theta)\left(P(\theta) - P^*_{\pi_0}(m)\right)^2$, then

$$P^*_{\pi_0}(m) = \frac{\int_\Theta P(\theta)^2 \pi_0(\theta \mid m) d\theta}{\int_\Theta P(\theta) \pi_0(\theta \mid m) d\theta} = \frac{E_{\pi_0(\theta|m)}\left[P(\theta)^2\right]}{E_{\pi_0(\theta|m)}\left[P(\theta)\right]}, \qquad (2)$$

with $P(\theta)$ as in (1), is the Bayesian premium for the variance principle.

3. Robust Bayesian analysis

In the standard Bayesian analysis the difficulty lies in the right choice of the prior distribution. The actuary may have some difficulties in providing suitable information about an uniquely specified prior of θ, perhaps because has not enough information for identyfying one single prior. It is then natural to question the robustness of the analysis to this specification.

Robust Bayesian analysis consist of the sensitivity of Bayesian answers to uncertain inputs. As we have metioned above, this paper develops Bayesian robust tools for making inferences about premiums using the variance premium principle. According to robust Bayesian methodology, the uncertainty in the prior can be modeled by specifying a class Γ of priors instead of a single one. Thus, one must study the robustness or sensitivity of Bayesian analysis by considering a class of priors that are reasonable representation of prior beliefs and we examine the ranges of Bayesian premium when the priors belong to that class. Most of the robust Bayesian procedures developed measures of sensitivity of quantities which can be expressed in terms of posterior expectations (e.g. mean, variance and probability of sets). Nevertheless, a main difference appears in the actuarial context considered here. Expression in (2) suggests that the quantity of interest can be expressed in terms of the ratio of posterior expectations. Appropiated techniques are considered in order to analyse the sensitivity of the premiums charged to changes in priors.

The most attractive ε-contaminated class of prior is given by (Sivaganesan, 1988 and Sivaganesan and Berger, 1989, among others),

$$\Gamma_\varepsilon = \{\pi(\theta) = (1-\varepsilon)\, \pi_0(\theta) + \varepsilon q(\theta) \mid q \in \mathcal{Q}\}, \qquad (3)$$

where $\pi_0(\theta)$ is a base prior that one would use in a single prior Bayesian analysis, coming from past experience, control studies, and/or subjective judgement. $\varepsilon \in [0,1]$ is the amount of error that one attaches to $\pi_0(\theta)$, and $\mathcal{Q}$ is a large class of plausible contaminations.

A natural goal of a robustness investigation is to find the range of the posterior quantity, $P^*_\pi(m)$ in our case, when π runs over Γ_ε. Thus our attention will be focused in $\inf_{\pi\in\Gamma_\varepsilon} P^*_\pi(m)$ and $\sup_{\pi\in\Gamma_\varepsilon} P^*_\pi(m)$.

A common first class to check local sensitivity to prior is the class of all possible distributions. In some sense we can say that the practitioner is indifferent to the choice of prior distribution. Obviusly, this class contains some unrealistic distributions. However when robustness is present one may feel comfortable with its conclusions. On the other hand, a large range means that the results are meaningfully different, and matter what prior is chosen. In this case, a more realistic and smaller class may be considered. For instance, one may show some shape preferences for the prior. Since mode is very intuitive statistical concept, the actuary who has a good statistical trainning should have no problem to assess it, based on historical data or any other procedure, the unimodality of risk parameter and its numerical value. We are then speaking about the consideration of all unimodal distributions with a given mode. The range of $P^*_\pi(m)$ is found over the two following classes,

$$\Gamma^i_\varepsilon = \{\pi(\theta) = (1-\varepsilon)\,\pi_0(\theta) + \varepsilon q(\theta) \mid q \in \mathcal{Q}_i\}, \qquad (i=1,2)$$

where

$$\mathcal{Q}_1 = \{\text{ All distributions }\},$$

and

$$\mathcal{Q}_2 = \{\text{ All unimodal distributions with the same mode, } \theta_0, \text{ as } \pi_0 \}.$$

We shall prove that the $\sup_{\pi\in\Gamma^i_\varepsilon} P^*_\pi$ and $\inf_{\pi\in\Gamma^i_\varepsilon} P^*_\pi$ as π varies over the class Γ^i_ε for $\mathcal{Q}_1$, and $\mathcal{Q}_2$ can be obtained by minimizing and maximizing a function of one variable, as we show in the following theorems.

Theorem 1.

$$\inf_{\pi\in\Gamma^1_\varepsilon}\left[\sup_{\pi\in\Gamma^1_\varepsilon}\right] P^*_\pi(m) = \inf_{\theta\in\Theta}\left[\sup_{\theta\in\Theta}\right] \frac{R_1 P^*_{\pi_0}(m) + R_2(\theta)}{R_1 + R_3(\theta)},$$

where $R_1 = (1-\varepsilon)p(m \mid \pi_0)\int_\Theta P(\theta)\pi_0(\theta \mid m)d\theta$, $R_2(\theta) = \varepsilon P(\theta)^2 f(m \mid \theta)$, $R_3(\theta) = \varepsilon P(\theta) f(m \mid \theta)$, $P(\theta)$ as in (1) and $P^*_{\pi_0}(m)$ as in (2).

Proof. For $\pi(\theta)$ as in (4) $\pi(\theta \mid m)$ is given by

$$\pi(\theta \mid m) = \lambda(m)\pi_0(\theta \mid m) + [1-\lambda(m)]\, q(\theta \mid m),$$

where

$$\lambda(m) = \frac{(1-\varepsilon)p(m \mid \pi_0)}{p(m \mid \pi)},$$

and

$$1 - \lambda(m) = \frac{\varepsilon p(m \mid q)}{p(m \mid \pi)}.$$

Then,

$$\begin{aligned}
P_\pi^*(m) &= \frac{\int_\Theta P(\theta)^2 \pi(\theta \mid m) d\theta}{\int_\Theta P(\theta)\pi(\theta \mid m) d\theta} \\
&= \frac{\lambda(m) \int_\Theta P(\theta)^2 \pi_0(\theta \mid m) d\theta + [1 - \lambda(m)] \int_\Theta P(\theta)^2 q(\theta \mid m) d\theta}{\lambda(m) \int_\Theta P(\theta) \pi_0(\theta \mid m) d\theta + [1 - \lambda(m)] \int_\Theta P(\theta) q(\theta \mid m) d\theta} \\
&= \frac{(1-\varepsilon) p(m \mid \pi_0) \int_\Theta P(\theta)^2 \pi_0(\theta \mid m) d\theta + \varepsilon p(m \mid q) \int_\Theta P(\theta)^2 q(\theta \mid m) d\theta}{(1-\varepsilon) p(m \mid \pi_0) \int_\Theta P(\theta) \pi_0(\theta \mid m) d\theta + \varepsilon p(m \mid q) \int_\Theta P(\theta) q(\theta \mid m) d\theta} \\
&= \frac{(1-\varepsilon) p(m \mid \pi_0) P_{\pi_0}^*(m) \int_\Theta P(\theta) \pi_0(\theta \mid m) d\theta + \varepsilon p(m \mid q) \int_\Theta P(\theta)^2 q(\theta \mid m) d\theta}{(1-\varepsilon) p(m \mid \pi_0) \int_\Theta P(\theta) \pi_0(\theta \mid m) d\theta + \varepsilon p(m \mid q) \int_\Theta P(\theta) q(\theta \mid m) d\theta}.
\end{aligned}$$

Now, interchanging $q(\theta \mid m)$ by $f(m \mid \theta) q(\theta)/p(m \mid q)$, the Bayesian premium in the contamination class can be rewritten as

$$P_\pi^*(m) = \frac{R_1 P_{\pi_0}^*(m) + \int_\Theta R_2(\theta) q(\theta) d\theta}{R_1 + \int_\Theta R_3(\theta) q(\theta) d\theta}.$$

Now $\inf\limits_{\pi \in \Gamma_\varepsilon^1} P_\pi^*(m)$ and $\sup\limits_{\pi \in \Gamma_\varepsilon^1} P_\pi^*(m)$ follows by an application of Lemma 1 of Sivaganesan and Berger (1987). □

Theorem 2.

$$\inf_{\pi \in \Gamma_\varepsilon^2} \left[\sup_{\pi \in \Gamma_\varepsilon^2} \right] P_\pi^*(m) = \inf_{z \geq 0} \left[\sup_{z \geq 0} \right] R(z),$$

where

$$R(z) = \frac{R_1 P_{\pi_0}^*(m) + \frac{1}{z} \int_{\theta_0}^{\theta_0 + z} R_2(\theta) d\theta}{R_1 + \frac{1}{z} \int_{\theta_0}^{\theta_0 + z} R_3(\theta) d\theta} \text{ if } z > 0,$$

and

$$R(0) = \frac{R_1 P_{\pi_0}^*(m) + R_2(\theta_0)}{R_1 + R_3(\theta_0)},$$

with R_1, $R_2(\theta)$ and $R_3(\theta)$ how in the Theorem 1 respectively.

Proof. The proof is similar using now Lemma 3.2.1 of Sivaganesan and Berger (1989) and Lemma 1 of Sivaganesan and Berger (1987).□

4. Application to Poisson-Gamma model

Consider a portfolio of insurance business where the number of claims X has a Poisson distribution with parameter θ. This parameter θ can represent the propensity to have a claim and $\pi(\theta)$ indicates how that propensity is distributed throughout of the insured population. One of the most useful no compound collective risk model consists in assuming Gamma prior distribution over the risk parameter θ. Also, assume that given a parameter θ, the observations $X_1, ..., X_t$ are independently and identically distributed with Poisson probability mass function, $f(x \mid \theta)$. Then prior and likelihood function are given by

$$\pi_0(\theta) \propto \theta^{b-1} e^{-a\theta},$$

$$f(x_1, ..., x_t \mid \theta) \equiv f(m \mid \theta) \propto \theta^{tm} e^{-t\theta},$$

where a and b are fixed prior parameters, and $m = \frac{1}{t}\sum_{i=1}^{t} x_i$. According to Bayes' theorem the posterior density $\pi_0(\theta \mid m)$ is also a Gamma distribution with parameters $a+t$ and $b+tm$, respectively.

Now, using (1), $P(\theta) = \theta + 1$ is the true individual premium for the variance principle and from (2),

$$P^*_{\pi_0}(m) = \frac{(b+tm)(b+tm+1) + 2(a+t)(b+tm) + (a+t)^2}{(a+t)(a+t+b+tm)}, \tag{4}$$

is the Bayesian premium for the variance principle.

So at the beginning of period $t+1$ we know the claim amounts $x_1, x_2, ..., x_t$ from the preceding periods which are conceived of being realizations of the random variables $X_1, X_2, ..., X_t$. The premium that the company charges could be given by $P^*_{\pi_0}(m) \times c$, where c denotes the assumed fixed (maybe, average) payment of a claim in the collective.

In the preceding section we have presented results about bounds for variance principle in a general situation, now the bounds for the variance principle in the no compound collective model Poisson-Gamma are given in the following corollaries, where we will name *indifference scene* to Γ^1_ε and *unimodality scene* to Γ^2_ε.

Corollary 1. In the setting of indifference, bounds of the Bayesian premium for the variance principle is given by

$$\inf_{\theta\in\Theta}\left[\sup_{\theta\in\Theta}\right]\frac{R_1 P^*_{\pi_0}(m)+R_2(\theta)}{R_1+R_3(\theta)},$$

where

$$R_1=(1-\varepsilon)a^b\Gamma(b+tm)(a+t+b+tm),$$

$$R_2(\theta)=\varepsilon\Gamma(b)\,(a+t)^{b+tm+1}\,(\theta+1)^2\,\theta^{tm}e^{-t\theta},$$

$R_3(\theta)=P_2(\theta)/(\theta+1)$ and $P^*_{\pi_0}(m)$ as in (3).

Proof. The proof follows from Theorem 1.□

Corollary 2. In the setting of unimodality, bounds of the Bayesian premium for the variance principle is given by $\inf_{z\geq 0}\left[\sup_{z\geq 0}\right] R(z)$, being

$$R(z)=\frac{R_1 P^*_{\pi_0}(m)+\frac{1}{z}\int_{\theta_0}^{\theta_0+z}R_2(\theta)d\theta}{R_1+\frac{1}{z}\int_{\theta_0}^{\theta_0+z}R_3(\theta)d\theta}\quad \text{if}\quad z>0,$$

and

$$R(0)=\frac{R_1 P^*_{\pi_0}(m)+R_2(\theta_0)}{R_1+R_3(\theta)},$$

with $R_1, R_2(\theta), R_3(\theta)$ and $P^*_{\pi_0}(m)$ as in Corollary 1.

Proof. The proof follows from Theorem 2.□

5. An illustration of the model

We find it clearer to describe our methodology in the context of a simple example. Assume the expected amount of a claim is fixed and its value is $c=100$ u.m.. Moreover, assume the actuary feels comfortable considering that the expected frequency is $E[\theta]=2.5$ (roughly speaking, the company can expect 5 claims every 2 years with this policy).

As Scollnik (1995) comments: "...prior information available for this parameter can be well modelled by a Ga(a,b) distribution, for some values of a and b. This is reasonable, since the shape of the Gamma density is very flexible. If we happen to have very little prior information concerning θ available, then we note that the selection ($a=2$, $b=1$) will result in a fairly satisfactory and relatively diffuse prior for θ". Hence, given our prior mean, it will be reasonable to assume that the base prior is Ga(2,5) (with this elicitation the actuary knows that the mode is around 2, i.e. $\theta_0=2$).

Table I. Number of claims during $t = 10$ years.

	Year	1	2	3	4	5	6	7	8	9	10	
Case 1		2	2	3	2	4	4	2	2	0	4	$m = \overline{x} = 2.5$
Case 2		2	2	7	7	8	6	6	4	2	6	$m = \overline{x} = 5$

The Bayesian robustness analysis developed in this article is illustrated by randomly generated data. Two situations are presented. In an standard Bayesian analysis, the Bayesian premium to be charged will be $P_{\pi_0}(m) \times c$. For instance, in the first case ($m = 2.5$) and under the variance premium principle we obtain 355.952 monetary units. Table I presents the number of claims observed grouped by year of exposure ($t = 10$ years).

Table II shows the range of variation of the experience rated premium for the variance premium principle and various ε, from 95 % ($\varepsilon = 0.05$) to 80 % ($\varepsilon = 0.20$) degree of confidence on π_0 by steps of 5 %. They also include a measure of which magnitudes do not depend on the units of measurement of the premium, the *relative sensitivity factor (R.S.)* introduced by Sivaganesan (1991), and which is given by

$$R.S.^i = \frac{1}{2P^*_{\pi_0}(m)} \left[\sup_{\pi \in \Gamma^i_\varepsilon} P^*_\pi(m) - \inf_{\pi \in \Gamma^i_\varepsilon} P^*_\pi(m) \right] \times 100\%, \qquad (i = 1,2)$$

that can be thought as the amount of percent variation of $P^*_\pi(m)$ around $P^*_{\pi_0}(m)$, as $\pi(\theta)$ varies in Γ^i_ε, ($i = 1,2$). It is also apparent the extreme robustness of the premium principles considered here. Thus, we conclude that the variance premium principles is little sensitive to departures from prior elicitations. In fact, $R.S.$ factor has not been particularly high in any of the cases considered here, that is, we have reasonably robust results. With respect to scenarios considered, $R.S.$ factor increases over ε and increase when Q_1 is used, however this is not exacerbated. Perhaps this Q_1 contains many unreasonable priors which artificially inflate the ranges of the Bayesian premium. In this case, for example, by $m = 2.5$ and $\varepsilon = 5\%$ the Bayesian premium can oscillate for the variance principle 1.06% around the Bayesian premium obtained by a base prior $\pi_0(\theta)$. This $R.S.$ decreases as far as 0.65% when we use *all unimodal distributions* and a considerable reduction (38.68 %) is obtained with Q_2, indicanting that the effect of the unimodality assumption is relatively larger.

It can be seen that unimodality effects are very important in modelling subjective beliefs on the risk parameter. Table II shows that a

Table II. Bounds for the Bayesian premium in the variance principle.

Indifference scene.

ε		0.05	0.10	0.15	0.20
Case 1	$\inf_{\pi\in\Gamma^1_\varepsilon} (P^*_\pi(m) \times c)$	352.512	349.226	346.061	342.987
$m = 2.5$	$\sup_{\pi\in\Gamma^1_\varepsilon} (P^*_\pi(m) \times c)$	360.086	364.060	367.916	371.689
$P^*_{\pi_0}(m) \times c = 355.952$	$R.S.^1$	1.06	2.08	3.06	4.03
Case 2	$\inf_{\pi\in\Gamma^1_\varepsilon} (P^*_\pi(m) \times c)$	554.454	546.502	540.046	534.509
$m = 5$	$\sup_{\pi\in\Gamma^1_\varepsilon} (P^*_\pi(m) \times c)$	600.966	622.153	637.374	649.447
$P^*_{\pi_0}(m) \times c = 565.174$	$R.S.^1$	4.11	6.69	8.61	10.16

Unimodality scene.

ε		0.05	0.10	0.15	0.20
Case 1	$\inf_{\pi\in\Gamma^2_\varepsilon} (P^*_\pi(m) \times c)$	352.546	349.270	346.100	343.013
$m = 2.5$	$\sup_{\pi\in\Gamma^2_\varepsilon} (P^*_\pi(m) \times c)$	357.208	358.405	359.551	360.651
$P^*_{\pi_0}(m) \times c = 355.952$	$R.S.^2$	0.65	1.28	1.88	2.47
Case 2	$\inf_{\pi\in\Gamma^2_\varepsilon} (P^*_\pi(m) \times c)$	561.197	557.495	553.992	550.630
$m = 5$	$\sup_{\pi\in\Gamma^2_\varepsilon} (P^*_\pi(m) \times c)$	575.536	583.009	588.686	593.164
$P^*_{\pi_0}(m) \times c = 565.174$	$R.S.^2$	1.26	2.25	3.06	3.76
Reduction in sensitivity (in %) $\left(\frac{R.S.^1 - R.S.^2}{R.S.^1} \times 100\%\right)$	Case 1	38.68	38.46	38.56	38.71
	Case 2	69.34	66.36	64.46	63.00

significant reduction in $R.S.$ factor is obtained if unimodality is considered in the prior elicitation process.

6. Concluding remarks

In actuarial practice one is interested in using all available source of information. Some prior beliefs could be established by practitioner and a base prior $\pi_0(\theta)$ matching those requirements is elicited. In this paper we combine standard and robust Bayesian tools to study how the choice of prior can critically impact the premium to be charged. Robust

Bayesian analysis assumes uncertainty on the structure function $\pi_0(\theta)$, modelling such uncertainty by considering classes of priors for which robustness analysis is carried out. The robustness Bayesian analysis must be interpreted as follows. If the model is not sensitivity the investigator (the actuary) can be calm with your conclusions; the insurance company can charge the Bayesian premium. If the model is sensitivity, not robust, the investigator must be careful with your conclusions. In this case, the results can differ markedly of your estimations.

In this paper we show the advantages of bringing together the more commonly used methods of robust Bayesian methodology along with a practical situation in credibility theory using Poisson no compound model. This approach allows for slightly more flexibility than standard Bayesian methods in credibility theory allowing competitive situations with the same prior information. If only mean and/or unimodality prior information is available and a base prior is elicited, ε-contamination classes here considered to yield robust range of Bayes premium to be charged.

Even though the model is very robust, the consideration of unimodality does significantly reduce the sensitivity of the Bayesian premium arising from a base prior π_0. Therefore, unimodality turns out to be very convenient for modelling subjective beliefs about the risk parameter.

Acknowledgements

Authors want to express their appreciation to the Dirección General de Investigación Científica y Técnica, Spain, for its support under grant PB95-1194.

References

Berger, J.O. (1985). *Statistical Decision Theory and Bayesian Analysis*. Springer, New York.

Berger, J.O. (1994). An Overview of Robust Bayesian Analysis. *Test*, **3**, 5–124. (with discussion)

Berger, J.O. and O'Hagan, A. (1988). Range of Posterior Probabilities for the Class of Unimodal Priors with Specified Quantiles. In *Bayesian Statistics 3* (J.M. Bernardo, M.H. DeGroot, D.V. Lindley and A.F.M. Smith, eds.). Oxford University Press, New York.

Eichenauer, J; Lehn, J. and Rettig, S. (1988). A gamma-minimax result in credibility theory. *Insurance: Mathematics & Economics*, **7**, 49–57.

Gerber, H. (1979). *An Introduction to Mathematical Risk Theory*. S.S. Huebner Foundation for Insurance Education, University of Pennsylvania, Philadelphi.

Heilmann, W. (1989). Decision Theoretic Foundations of Credibility Theory. *Insurance: Mathematics & Economics*, **8**, 77–95.

Klugman, S.A. (1992). *Bayesian Statistics in Actuaril Science: with Emphasis on Credibility*. Kluwer Academic Publishers, Boston.

Lavine, M. (1991). Sensitivity in Bayesian Statistics: the Prior and the Likelihood. *Journal of the American Statistical Society*, **86**, 400 403.

Makov, U.E., Smith, A.F.M. and Liu, Y.H. (1996). Bayesian Methods in Actuarial Science. *The Statistician*, **45**, 4, 503–515.

Scollnik, D.P.M. (1995). The Bayesian Analysis of Generalized Poisson Models for Claim Frequency Data Utilising Markov Chain Monte Carlo Methods. *Actuarial Research Clearing House*, **1**, 339–356.

Sivaganesan, S. (1988). Range of Posterior Measures for Priors with Arbitrary Contaminations. *Communications in Statistics and Theory Methods*,**17**,5, 1591-1612.

Sivaganesan, S. (1991). Sensitivity of Some Posterior Summaries when the Prior is Unimodal with Specified Quantiles. *The Canadian Journal of Statistics*,**19**, 1, 57 65.

Sivaganesan, S. and Berger, J.O. (1987). Ranges of Posterior Measures for Priors with Unimodal Contaminations. *Technical Report* ♯ *86-41*. Department of Statistics. Purdue University.

Sivaganesan, S. and Berger, J.O. (1989). Ranges of Posterior Measures for Priors with Unimodal Contaminations. *The Annals of Statistics*, **17**, 2, 868 889.