

Extreme Value Statistics,
Resampling,
and
Insolvency Testing

by

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Abstract: By the use of resampling and extreme value statistics we will develop a method to reduce the time and costs of testing insurance company insolvency. Most ruin models require assumptions about the surplus distribution and/or assumptions about the claims count and severity distributions. The actuary may not be comfortable in making these ruin assumptions. Instead, he or she may generate such distributions by running extensive computer simulations of corporate models. I introduce a method to reduce the number of simulations necessary to estimate the 1 and 0.1 percentiles of the surplus distribution. This approach 'cuts off' the tail at approximately the tenth percentile of the sample distribution and generates new tails for the distribution by resampling. Averages of the new order statistics approximate the 1 and 0.1 percentiles.

Keywords: Extreme Value distributions, Bootstrapping, Insolvency, Surplus analysis, Normalized distances, Resampling.

Introduction

In basic ruin analysis, Bowers et al. [2] set up a stochastic process with the following assumptions:

1. Claims count distribution,
2. Claims amount distribution,
3. No interest or asset performance,
4. Constant premiums,
5. Constant expense loadings.

Even with these simple assumptions, there is still no closed formula for the probability of ruin (i.e., when surplus drops below zero), except when the process is compound Poisson with an exponential claims amount distribution. In the industry, however, we are required by regulations

or professional standards to conduct computer simulation analysis upon different lines of business to observe when business performs poorly. We model our business as accurately as possible, allowing for interest and asset performance, changing premium and expense loadings. We do not make assumptions on the claims count or amount distributions; however, we do make many other assumptions such as the term structure of interest rates, relationship of our decrements to the level of interest rates, and asset default probabilities. Computer simulations reveal our business's behavior relative to our assumption of the term structure of interest rates. Optimally, we want to calculate the probability of ruin within the accuracy of these computer models.

There are three modeling techniques to consider: parametric, nonparametric, and semiparametric. Parametric methods like those in [11] require running n simulations and fitting the results to the best parametric distribution. Six problems with this method are:

1. Finding the proper size of n ,
2. Finding the proper distribution,
3. Finding the proper fitting algorithm,
4. Determining whether or not the fitting algorithm is data dependent,
5. Estimating the support of the fitted distribution (i.e., the range of possible values of surplus),
6. Measuring extraneous information which parametric models introduce relative to nonparametric models. Currently there are no diagnostics that can measure this information. See [14] for a brief discussion of this problem.

Nonparametric models [16] for fitting unknown distributions have these problems:

1. The determination of the proper algorithm,
2. The misestimation of the support,
3. The need of a stopping rule (i.e., when to stop augmenting the model),
4. The distortion in the extreme tails,
5. The increase in the size of the confidence intervals.

The increase in the size of the confidence intervals is most likely related to the inability to measure extraneous information.

Semiparametric methods force a structure upon the model, but usually do not require estimates of specific parameters. The best example of this concept in Actuarial Science is the Whittaker-Henderson graduation technique, where Whittaker chose his distribution of true values “by analogy to the normal frequency law.” However, he did not assume the distribution to be normal [10, 19]. Semiparametric methods have problems similar to parametric ones, such as:

1. Determining the proper size of n ,
2. Determining the choice of the best distributional structure for the model,
3. Determining the choices of the best fitting algorithm,
4. Determining whether the fitting algorithm is data dependent or not,
5. Determining the support,
6. Measuring extraneous information.

The resampling technique described in Section 3 is best described as semiparametric. However, it addresses the support misestimation problem.

In addition to the issue of choosing an appropriate modeling technique, there remains the challenge of properly selecting the level of confidence. That is, if estimating an extreme

percentile like 1% or 0.1%, should the confidence level be at 90% or 95%, or should it be comparable to the level of the percentile, like 99.5% or 99.95%?

In my company research, I have been restrained to a nonparametric setting with a high level of confidence in the left tail. Using the following formula from [7]¹:

$$Pr(Y_k < \xi_p) = \sum_{w=k}^n \frac{n!}{w!(n-w)!} p^w (1-p)^{n-w},$$

where Y_k is the k 'th ascending order statistic (i.e., Y_1, Y_2, \dots, Y_n , where $Y_1 \leq Y_2 \leq \dots \leq Y_n$), and ξ_p is the p 'th percentile, I constructed a Quick Basic program as shown in Appendix I. Setting the number of scenarios n (n must be even) and the percentile p , the program steps through the various Y_k until the desired level of confidence is obtained. We found that estimating ruin at the 0.1% level at a 99.95% left tail confidence level required $n = 10,000$. Using 10,000 interest rate scenarios for multiple lines of business, even though very accurate, is expensive, time consuming, hard to organize, and not as succinct as parametric ruin theory. In this paper, I introduce a method to analyze surplus distributions with fewer ($n=1,000$) simulations, giving rules of thumb that help determine whether the 1,000 trials are sufficient.

Klein [9] noted that changing the underlying distribution of the term structure produces a dramatically different surplus distribution. We will not address this complex issue in this paper, other than to note that the term structure which generates the surplus results in Section 5 is consistent between the different lines of business in a given year of study.

In Section 1, I briefly describe resampling and discuss extreme value statistics in detail. In

¹Please note that later editions have eliminated a lengthy discussion of distribution free confidence intervals.

Section 2, I describe visual and statistical tests which allow the analyst to estimate the specific extreme value distribution of the surplus distribution. Section 3 discusses the semiparametric bootstrap technique. In Section 4, we examine the bootstrap technique on some known parametric distributions. In Section 5, we study fifty-one different lines of business and compare the bootstrap estimates with the 10,000 trial empirical distributions for each of these lines of business. This section also develops the rules of thumb. In Section 6, we conclude with a discussion of results, other applications, and further research.

Section 1. Resampling and Extreme Value Statistics

The bootstrap resampling technique is very flexible and can be applied to a broad class of problems. See [4] for a good introduction to its use. The bootstrap allows you to acquire approximate samples of a random variable without specifying a parametric model for the distribution. One constraint on this form of resampling is that the original samples must be independent and identically distributed (denoted iid) samples.

Extreme value statistics had its founding as an actuarial problem. Nicolaus Bernoulli considered when n men of equal age die within t years, and he determined the mean duration of life of the last survivor. See [6].

All statistical analysis below is constructed to analyze the right tail of a distribution. In our insolvency studies, we desire to study the left tail. In Section 5, we convert the left tail problem into right tail by multiplying the surplus values by -1 .

Now, let $X_{1n} > X_{2n} > \dots > X_{nn}$ denote the descending order statistics of a sample of size n from a population with CDF $F(x)$. The CDF $F(x)$ is said to be in the domain of attraction of the

distribution $G(x)$, denoted $D(G)$, if the following theorem holds for $F(x)$

Theorem 1. (Central Limit Theorem for Extreme Value Statistics) If there exist real numbers

$a_n > 0, b_n, n = 1, 2, \dots$ such that for all real x

$$\lim_{n \rightarrow \infty} F^n((x - b_n)/a_n) = G(x)$$

and if the above limit is nondegenerate then $G(x)$ takes on one of three functional forms (location and scale parameters aside):

$$G_{1,\alpha}(x) = \begin{cases} 0, & x \leq 0 \\ \exp(-x^\alpha), & x > 0 \end{cases}$$

$$G_{2,\alpha}(x) = \begin{cases} \exp(-(-x)^\alpha), & x \leq 0 \\ 1, & x > 0 \end{cases}$$

$$G_3(x) = \exp(-\exp(-x)), \quad -\infty < x < \infty$$

where $\alpha > 0$.

Fréchet found $G_{1,\alpha}$ in 1927, and Fisher and Tippett found the other two in 1928.

Gnedenko proved the above Central Limit Theorem in 1943. After Gumbel used the three distributions [6], later literature tends to refer to them as the Gumbel Type II, III, I distributions, respectively. Our naming convention follows that of Falk [5]. See [3, 5, 6, 15, 21] for further discussion. Restated, the theorem says that as the number of samples approaches infinity, the distribution of the largest order statistic is either degenerate or one of the three above distributions. Gnedenko stated that $b_n = F^{-1}(1 - 1/n)$ and $a_n = F^{-1}(1 - e^{-1/n}) - b_n$ (See [15]).

Using the notation from Falk [5], below we use the relation $x \asymp y$ to indicate that the

symbol y is defined to be the expression x on the left hand side.

The generalized Pareto distribution is defined as

$$\begin{cases} 1-x^{-\alpha}=:W_{1,\alpha}(x), & x>1 \\ 1-(x)^{\alpha}=:W_{2,\alpha}(x), & -1\leq x\leq 0 \\ 1-\exp(-x)=:W_3(x), & x>0 \end{cases}$$

Note that $W_{1,\alpha}(x)$ is the standard Pareto distribution, $W_{2,1}(x)$ is the uniform distribution on $[-1,0]$ and $W_3(x)$ is the standard exponential distribution. Denote the three distributions collectively as $W(x)$.

Also, letting $\alpha > 0$, and if $G = G_{1,\alpha}, G_{2,\alpha}, G_3$, then note that $1+\ln(G(x))=W(x)$. Also note the following relationship:

$$\begin{aligned} 1+\ln(G(x)^{1/n}) &= \begin{cases} W_{1,\alpha}(n^{-1/\alpha}x), & x>n^{-1/\alpha}, \\ W_{2,\alpha}(n^{-1/\alpha}x), & -n^{-1/\alpha}\leq x\leq 0, \\ W_3(x+\ln(n)), & x>-\ln(n), \end{cases} \\ &=:W_{(n)}(x). \end{aligned}$$

We will call the $W_{(n)}(x)$ a shifted generalized Pareto distributions.

Theorem 2. (Distribution Tail Classification Theorem) $F \in \mathcal{D}(G)$ iff its upper tail can be approximated in an appropriate way by a shifted generalized Pareto distribution,

i.e., $F((x-\hat{b})/\hat{a}) \approx W_{(n)}(x)$ for some shift parameter \hat{b} and scale parameter \hat{a} , as n approaches infinity.

Proof: See Falk [5].

To summarize, we can classify the tails of all distributions that satisfy Theorem 1 (up to

scale and shift parameters) as either Pareto, exponential or $W_{2,\alpha}(x)$ with compact support².

With the above information, I attempted to approximate the extreme quantiles of $F(x)$ by fitting it to a shifted generalized Pareto distribution, using a maximum likelihood estimator technique by Hosking [8], as well as a moments method by Pickands [12]. However, these estimates were inconsistent with actual values from the parametric distributions in Section 4. When trying to fit the exponential distribution, Hosking's algorithm produced an extreme value distribution which was not exponential. This result be due the fact that Hosking's algorithm used all data observed; Castillo et al. [3] recommended against this approach. I confirmed this recommendation while examining the standard normal distribution. Here, when using only the positive samples, Hosking's algorithm would overestimate the higher percentiles. When all the data was used, the algorithm would underestimate the higher percentiles.

The following technical theorem guarantees that the joint distribution of the tail order statistics converge to one of three forms. This joint distribution will be simulated in Section 3 using resampling. Dwass generalized the Central Limit Theorem in 1966, and Weissman proved it in 1977.

Theorem 3. (Generalized Central Limit Theorem) $(X_{1n} - b_n)/a_n$ converges in distribution to an extreme value distribution G iff for any $k \in \mathbb{N}$ the ordered n -tuple,

$$((X_{1n} - b_n)/a_n, (X_{2n} - b_n)/a_n, \dots, (X_{kn} - b_n)/a_n),$$

converges in distribution to $G^{(k)}$. Here $G^{(k)}$ has the k -dimensional Lebesgue-density

²Here compact support is used to indicate that the random variable represented by the distribution, takes on finite values. That is, the tails don't extend to either $\pm\infty$. For example, the continuous uniform distribution has compact support.

$$g^{(k)}(x_1, x_2, \dots, x_k) = G(x_k) \prod_{i=1}^k G'(x_i)/G(x_i),$$

where $x_1 > x_2 > \dots > x_k$ and is zero elsewhere.

So $G^{(k)} \in \{G_{1,\alpha}^{(k)}, G_{2,\alpha}^{(k)}, G_{3,\alpha}^{(k)}\}$ is the only limit for the joint distribution of the k largest order statistics, standardized with the same location and scale parameters (the a_n and b_n don't have to be the same as given by Gnedenko). See [5] for a proof.

Weissman [18] proved the final theorem we will need:

Theorem 4. (Normalized Spacing Theorem) For F in the domain of distribution $G_j(x)$, and for fixed k , as $n \rightarrow \infty$, the normalized spacings $ia_n^{-1}(X_{in} - X_{(i-1)n})$ ($i = 1, 2, \dots, k$) are asymptotically jointly distributed as independent standard exponential random variables.

Reiss [13] discusses the appropriate value of k : “We see that in both cases there is a trade off between the following two requirements:

- (a) k has to be large to gain efficiency,
- (b) k has to be small enough to get asymptotic normality of the estimator.”

This search for the proper k , which Boos [1] characterizes as where the distributional tail begins, is critical to models in [1, 21, 15]. Several methods to determine k are outlined in [15, 17, 21]. Boos [1] takes a more empirical estimate. My attempts to estimate the 1 and 0.1 percentiles by using several techniques outlined in [15,21] produced inconsistent results. Therefore, I took a more empirical approach like Boos [1] and set $k = 0.1n + 2$ for all my simulations.

Once k is determined, Weissman provides two approximations for a_n and b_n . The two sets of estimators are:

$$\begin{aligned}
 \text{Minimum Variance Estimators:} \quad \hat{a} &= X_{k:n} - X_{(k-1):n} \\
 \hat{b} &= \hat{a}\psi(k+2) + X_{(k-1):n} \\
 \text{Maximum Likelihood Estimators:} \quad \tilde{a} &= \bar{X}_{k:n} - X_{k:n} \\
 \tilde{b} &= \tilde{a}\ln(k) + X_{k:n}
 \end{aligned}$$

where the digamma function ψ is defined by

$$\psi(1) = -\gamma, \quad \psi(n) = -\gamma + \sum_{k=1}^{n-1} \frac{1}{k} \quad (n > 2), \quad \gamma = .5772156649\dots$$

These formulas are used to estimate the $100(1-c/n)$ percentile $\hat{\eta}_c$ of $F(x)$ where c is small when compared to n . Here $\hat{\eta}_c := b - a\ln(c)$ where b is either location parameter above, and a is either scale parameter. See [15,18]. Boos [1] said “that Monte Carlo work showed that [the minimum variance estimators] are not an improvement over the maximum likelihood estimators...”

However, when I tried to apply both Weissman estimators directly using methods to determine k from [1,15,21], I did not obtain consistent results. I believe the inconsistencies derive from the fact that the Weissman estimators were heavily dependent upon the worst case order statistic, which is known to have the largest variance of all the order statistics.

Section 2. Visual and Statistical Tests for Exponential Distributions

Boos [1] describes the following technique to determine the domain of attraction of a distribution's tail. Take the top $n/5$ sample order statistics, and plot them against $-\ln(i/(n+1))$ for $i=1,2,\dots,\left\lceil \frac{n}{5} \right\rceil$. If the graph appears to be a straight line, the distribution has an exponential tail attracted to $G_{(3)}^{(k)}(x)$. If the graph bends down (concave down) the distribution has compact

support similar to the uniform distribution and is attracted to $G_{(2,\alpha)}^{(k)}(x)$. The tails of these types of distribution have less area than the exponential distribution and are said to be lighter than exponential tails. If the graph bends up (concave up) the distribution has a Pareto like tail and is attracted to $G_{(1,\alpha)}^{(k)}(x)$. These tails are considered to be heavier than exponential and will have more area under them than exponential.

In addition to the visual test, the following statistical test (hereafter denoted CS test) by Castillo et al. [3] reduces confusion about the domain of attraction. Consider the following regions: $A_{12} = \{1, \dots, \lfloor \sqrt{n} \rfloor\}$ and $A_{34} = \{\lfloor \sqrt{n} \rfloor, \dots, \lfloor 2\sqrt{n} \rfloor\}$, where $\lfloor x \rfloor$ denotes the greatest integer less than x . Let S_{12} and S_{34} be the slopes of the least square lines between $-\ln(-\ln((n+1-i+0.5)/n))$ and the order statistics $\{X_{i,n}\}$ for i in A_{12} and A_{34} , respectively. The quotient S_{34}/S_{12} will have large values for distributions attracted to $G_{(2,\alpha)}^{(k)}(x)$, small values (near zero) for those attracted to $G_{(1,\alpha)}^{(k)}(x)$, and midrange values for those attracted to $G_{(1)}^{(k)}(x)$. See [3] for further explanation.

The results of the visual test are displayed below in Figures 1 through 3. The parameters and slope ratios of these three examples are contained in Section 4.

EXPON

4.0E+01
3.5E+01
3.0E+01
2.5E+01
2.0E+01
1.5E+01
1.0E+01
5.0E+00



Figure 1

BETA1

1.7E+00
9.0E-01
8.0E-01
7.0E-01
6.0E-01
5.0E-01
3.0E-01



Figure 2

PARETO

3.0E+03
2.5E+03
2.0E+03
1.5E+03
1.0E+03
5.0E+02
0.0E+00

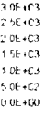


Figure 3

Section 3. Semiparametric Bootstrap Technique

This technique approximates by bootstrapping the joint distribution of the k largest order statistics $(X_{1:n}^{(k)})$ of Theorem 3. Zelterman developed this method, which is discussed in [15, 21, 22]

Zelterman [21] discusses the problem of limited support, where prior estimations of $X_{1:n}$ by bootstrapping did not have points larger than $X_{1:n}$. In fact, the bootstrap distribution had a point mass at the observed $X_{1:n}$. He overcame this limitation with the following approach.

Define the vector of normalized spaces $\mathbf{d} = \{d_i\}$ of the $k+1$ largest order statistics as

$$d_i = i(X_{1:n} - X_{i+1:n}) \quad i = 1, \dots, k$$

By Theorem 4, the $\{d_i\}$ are approximately iid exponential random variables when n is much larger than k . Please note realize that the normalized spacings above are different from Theorem 4.

Because a_n are unknown and constant across all the distances, we can remove a_n to perform the simulation. Let $\{d_1^*, \dots, d_k^*\}$ be a bootstrap resample of size k drawn with replacement from \mathbf{d} .

The bootstrap resample $X_{1:n}^* > X_{2:n}^* > \dots > X_{k:n}^*$ of the k largest order statistics is defined by

$$X_{jn}^* = X_{k-1n} + \sum_{i=j}^k i^{-1} d_i^* \quad (j = 1, \dots, k).$$

In the bootstrap, all that is needed is the fact that the d are approximately iid. We do not use the fact that they are also approximately exponentially distributed. Zelterman [21] goes on to show the theoretical underpinning of the above technique. Note also that the different X_{1n}^* are not limited by the original X_{1n} . The collective $\{X_{in}^*\}$ simulates a sample from the joint distribution discussed in Theorem 3.

In the analysis below, we resample each sample distribution's tail 500 times and collect the resampled X_{in}^* for specific i . The rule to estimate the 1% level is $0.01n + 1$. The rule to estimate the 0.1% level is $0.001n + 1$, if $n > 1000$. If $n < 1000$, we will linearly interpolate the first and second order statistics by $(1 - n/1000)X_{1n} + (n/1000)X_{2n}$. Formulas of this type are discussed in [1]. I did conduct an empirical experiment with the surplus data in Section 5 by resampling at 1000 times instead of 500. There were minor improvements, but they were not worth the extra time and computer memory in the simulations.

This technique essentially cuts the tail off and attaches additional tails with distances based on the original tail. When this technique is used to estimate heavier tails than exponential it should underestimate the quantiles. Obviously, for lighter than exponential tails, it will overestimate, which is the case with the distributions with compact support in Section 4.

At this point, I want to mention two other failed attempts at approximating the percentiles. First, I created a sample distribution for F by applying Theorem 1. In this attempt, I took $n=300$ and resampled 1000 times, only collecting values on the first order statistic, X_{1n}^* . I

sorted the 1,000 values into 100 bins and divided the size of each bin by 1000 to produce a sample density function. By summing the values in each bin, I created an empirical sample cumulative distribution \hat{G} . By Theorem 1, we can approximate the underlying distribution F by taking the $1/300$ root of \hat{G} . However, the estimated percentiles dramatically misestimated the actual parametric distributions percentiles. As mentioned above, I believe such a lack of fit results from the fact that $X_{1/n}^*$ has the largest variance among all the other order statistics. See [21] for a further discussion on the variance of $X_{1/n}^*$.

In my second attempt, I approximated the scale parameter a and the location parameter b from the known parametric distributions. From these samples, I resampled the a and the b and compared the averages of these resampled values to the Gnedenko theoretical values for a_n and b_n . They were not close. Note: I did not satisfy the iid requirement for this test.

The common thread to all the failed attempts mentioned here and in Section 2 is the attempt to approximate $G(x)$ using only $X_{1/n}$.

Instead of concentrating on $X_{1/n}$, Section 4 results are tied to resampling the extreme joint distribution of Theorem 3.

Section 4. Known Parametric Distributions

Initially, I studied the technique against fifteen parametric distributions to estimate the extreme quantiles. This section lists the distributions and their parameters, and the visual and statistical tests on each distribution. Table 1 lists the distribution, the parameters and the actual extreme quantiles. It also shows the observed extreme quantiles from a 10,000 sampling. (Note: Beta1 through Beta4 are from the Beta distribution with different parameters chosen to test

different compact support distributions)

Name	Parameters	Actual	Actual	10K	10K
		99.9%	99%	99.9%	99%
BETA1	$\alpha_1=0.2$ $\alpha_2=0.8$.984	.93	1.00	.98
BETA2	$\alpha_1=3$ $\alpha_2=3$.923	.86	.95	.89
BETA3	$\alpha_1=5$ $\alpha_2=1.5$.985	.96	1.00	.99
BETA4	$\alpha_1=2$ $\alpha_2=0.8$.992	.97	1.00	1.00
CAUCHY	$\alpha=1$ $\beta=1$	319.309	32.82	254.48	31.60
CHISQ	$\nu=5$	20.486	15.09	20.17	14.88
EXPON	$\beta=5$	34.539	23.03	37.67	23.28
GAMMA	$\alpha=5$ $\beta=1$	14.794	11.61	14.61	11.54
LOGISTIC	$\alpha=5$ $\beta=1$	11.907	9.60	12.14	9.50
LOGNORM	$\mu=1$ $\sigma=0.5$	12.745	8.70	12.71	8.64
NORM	$\mu=0$ $\sigma=1$	3.09	2.33	3.02	2.27

Name	Parameters	Actual	Actual	10K	10K
		99.9%	99%	99.9%	99%
PARETO	$\theta=1.1$ $a=5$	2668.350	328.97	2148.89	353.21
STUDENT	$v=5$	5.893	3.37	5.93	3.34
UNIFORM	$a=1$ $b=5$	4.996	4.96	5	4.96
WEIBULL	$\alpha=2$ $\beta=1$	2.63	2.15	2.68	2.16

Table 1.

Figures 4 through 15 are right tail graphs of the above parametric distributions, with the other three already displayed in Figures 1 through 3.

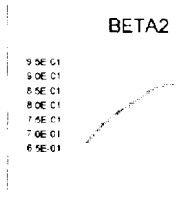


Figure 4

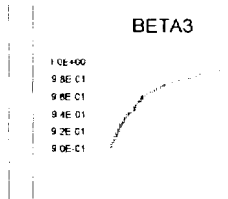


Figure 5

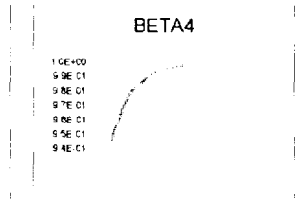


Figure 6

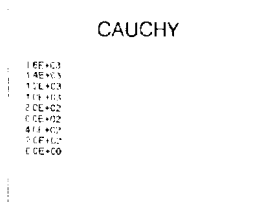


Figure 7

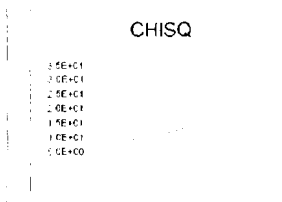


Figure 8

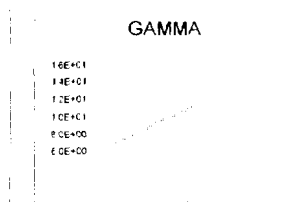


Figure 9

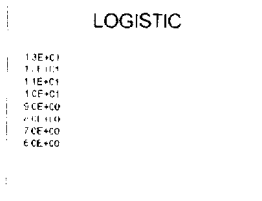


Figure 10

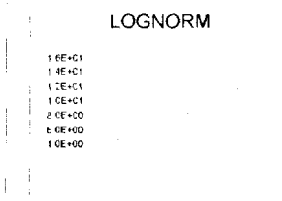


Figure 11

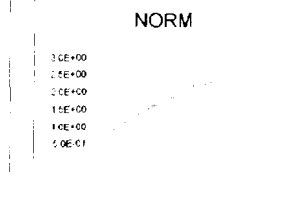


Figure 12

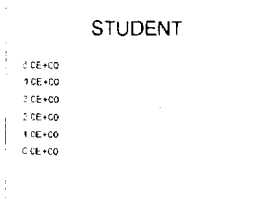


Figure 13

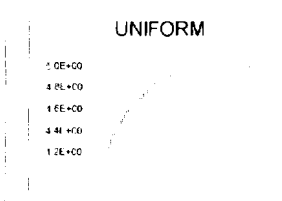


Figure 14

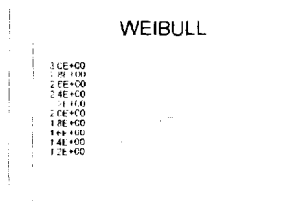


Figure 15

Table 2a compares the relative error of the 50% resampling estimation of the 99.9% and 99% quantiles of the above parametric distributions with the actual 99.9% and 99%, and the 99.9% and 99% 10,000-trial estimate. The high relative errors of the beta distributions were not surprising, since the visual and statistical tests indicate a lighter than exponential tail. The same is

true for the Cauchy and the Pareto, because the visual and statistical tests indicate that the tails are heavier than exponential. The only surprise is the Student distribution, where the visual and statistical tests state indicates that the tail is exponential (which is true), but the resampling technique dramatically underestimates the actual. This result may relate to the fact that the slope ratio is low (0.82) and the bootstrap estimator would tend to underestimate. This anomaly is also noted in some of the line of business graphs in Section 5.

Name	$\frac{S_{34}}{S_{12}}$	D(G)	99.9%	99.9%	99%	99%
			Actual	10K	Actual	10K
BETA1	10.40	2	-0.34	-0.32	-0.14	-0.08
BETA2	2.69	2	-0.10	-0.07	-0.05	-0.01
BETA3	2.64	2	-0.06	-0.04	-0.04	-0.01
BETA4	7.48	2	-0.03	-0.02	-0.03	-0.01
CAUCHY	0.03	1	0.77	0.71	-0.46	-0.52
CHISQ	0.62	3	0.07	0.06	0.04	0.03
EXPON	0.91	3	0.08	0.15	0.01	0.02
GAMMA	1.59	3	0.03	0.02	0.02	0.02
LOGISTIC	1.25	3	0.06	0.07	0.01	0.00
LOGNORM	1.00	3	0.10	0.10	0.01	0.00
NORM	1.82	3	0.06	0.03	0.06	0.03
PARETO	0.26	1	0.74	0.68	-0.27	-0.18
STUDENT	0.82	3	0.24	0.24	0.08	0.07
UNIFORM	6.44	2	-0.08	-0.08	-0.02	-0.02

Name	$\frac{S_{34}}{S_{12}}$	D(G)	99.9%	99.9%	99%	99%
			Actual	10K	Actual	10K
WEIBULL	1.34	3	-0.08	-0.06	-0.04	-0.04

Table 2a

Table 2b displays the mean, standard deviation, maximum and minimum values of the S_{34}/S_{12} ratio on 64 sets of 1,000 random samples for each parametric distribution. I arbitrarily set the S_{34}/S_{12} region of acceptance for Region 1 (heavier than exponential tails) for the ratio below .70, Region 3 (exponential tails) for the ratio between 0.7 and 1.99, and Region 2 (lighter than exponential tails) for the ratio greater than 1.99. The first probability column indicates the probability that the given distribution falls in Region 1. The second column indicates the probability that the given distribution falls in Region 2. For all the eight Region 3 distributions, the cumulative probability of Type I error is 0.1603. The majority of this is due to the large error (0.051) in the Student distribution. Two other surprises are how frequently the exponential and the lognormal distributions failed. The Type I error for the five Domain 2 distributions is 0.0814. The Type I error for the two Domain 1 distributions is 0.000.

Name	Mean	Stdev	Max	Min	Region 1	Region 2
					$Pr(S_{34}/S_{12} < 0.7)$	$Pr(S_{34}/S_{12} > 2.0)$
BETA1	6.470	1.814	12.104	3.058	0	1
BETA2	2.147	0.507	3.706	1.256	0	0.609
BETA3	3.543	0.937	6.197	1.929	0	0.984

Name	Mean	Stdev	Max	Min	Region 1	Region 2
					$Pr(S_{34}/S_{12} < 0.7)$	$Pr(S_{34}/S_{12} > 2.0)$
BETA4	7.518	2.055	16.271	3.734	0	1
CAUCHY	0.103	0.079	0.358	0.007	1	0
CHISQ	1.077	0.315	1.802	0.420	0.094	0
EXPON	1.037	0.332	1.775	0.493	0.188	0
GAMMA	1.273	0.402	2.194	0.615	0.047	0.047
LOGISTIC	1.062	0.325	2.318	0.595	0.094	0.016
LOGNORM	0.941	0.318	1.934	0.446	0.219	0
NORM	1.336	0.333	2.173	0.725	0	0.047
PARETO	0.119	0.089	0.361	0.0001	1	0
STUDENT	0.788	0.256	1.388	0.301	0.406	0
UNIFORM	5.494	1.521	10.279	3.031	0	1
WEIBULL	1.349	0.454	2.828	0.687	0.016	0.109

Table 2b

Table 3 contains a comparison of the 99.9% estimator using different sample sizes ($n = 100, 200, 400, 800, 1000$ and 2000). Here, instead of using the relative error, we compare the observed point estimate percentile in the 10,000 trial samples. The term “Over” is used if the bootstrap estimate fell outside the values in the 10,000 trials. The acceptance region for the 99.9% estimator was (99.85%, 99.94%).

99.9%						
Name	n=100	n=200	n=400	n=800	n=1000	n=2000
BETA1	Over	Over	Over	Over	Over	Over
BETA2	99.94	Over	Over	Over	Over	Over
BETA3	Over	Over	Over	Over	Over	Over
BETA4	Over	Over	Over	Over	Over	Over
CAUCHY	99.38	99.34	99.76	99.62	99.61	99.85
CHISQ	99.19	99.40	99.78	99.79	99.82	99.82
EXPON	99.38	99.62	99.73	99.73	99.77	99.79
GAMMA	96.96	99.76	99.84	99.85	99.85	99.85
LOGISTIC	99.15	98.95	99.39	99.62	99.69	99.85
LOGNORM	99.06	99.70	99.77	99.81	99.83	99.87
NORM	99.48	99.64	99.73	99.86	99.85	99.97
PARETO	99.03	99.35	99.27	99.44	99.52	99.47
STUDENT	98.05	98.53	99.37	99.52	99.64	99.81
UNIFORM	Over	Over	Over	Over	Over	Over
WEIBULL	98.61	99.11	99.85	99.92	99.95	99.95
Scorecard						
Over	4	5	5	5	6	7
Ok	1	0	0	3	2	4
Under	10	10	10	7	7	4

Table 3

Table 4 compares the 99% estimator with sample sizes ($n = 100, 200, 400, 800, 1000$). 0

The acceptance region for the 99% estimator is (98.50%, 99.49%). Table 4 has better results

than Table 3 because of a larger acceptance region.

99%					
Name	n=100	n=200	n=400	n=800	n=1000
BETA1	Over	Over	Over	Over	Over
BETA2	97.98	98.79	99.01	99.28	99.19
BETA3	99.33	98.41	99.19	99.72	99.84
BETA4	Over	Over	Over	Over	Over
CAUCHY	99.13	99.04	99.61	99.41	99.33
CHISQ	98.14	98.29	98.35	98.51	98.83
EXPON	98.75	98.76	98.73	98.75	98.94
GAMMA	95.26	98.83	98.68	98.83	98.81
LOGISTIC	98.18	97.28	98.01	98.73	98.98
LOGNORM	98.34	99.07	98.79	98.96	99.00
NORM	98.51	98.63	98.18	98.80	98.77
PARETO	98.80	99.07	98.71	99.07	99.18
STUDENT	96.45	96.69	97.90	98.49	98.65
UNIFORM	97.85	Over	Over	Over	Over
WEIBULL	96.96	97.43	98.62	99.24	99.24
Scorecard					
Over	2	3	4	4	4
Ok	5	7	7	10	11
Under	8	5	4	1	0

Table 4

Section 5. Empirical Surplus Distributions

Following is an application of the above techniques to the projected surplus of 51 different lines of business (LOBs), denoted "LOB m " for $m = 1, \dots, 51$. The analysis includes 3 year-end projections (1992-4) for each LOB. Appendix II shows the visual tests for 1992 and estimation tables for the 1992-4 LOBs.³ The following analysis examines the right tail of a distribution. To study the surplus values below zero, all the surplus values were multiplied by -1 to turn the left tail analysis into a right tail analysis. The visual and statistical tests have this adjustment as evident in the graphs. However, the estimation tables display discounted surplus values with the original signs. However, any references to high 10,000 trial quantiles refer to their complement. For example, 0.1% is displayed as 99.9%.

The given samples represent projected accumulated surplus at year 20, discounted to present. These values should give a good indication of long-term company solvency.⁴ Since we do not do a parametric fit, the following score card will compare the 10,000 trial quantiles with the resampled quantiles where $n=1,000$. The region of acceptance for the domain of convergence is the same as in Section 4. Table 5 shows scores for the 0.1% estimator.

99.9%	S_{34}/S_{12}	D(G)	10K	S_{14}/S_{12}	D(G)	10K	S_{34}/S_{12}	D(G)	10K
Name			%			%			%
	1992			1993			1994		
LOB01	1.22	3	99.89	0.97	3	99.91	NA	NA	NA

³Graphs for 1993 and 1994 are available upon request.

⁴These values are not perfect indications of solvency, since the LOBs could become insolvent after 20 years. However, we do not have access to a computer model that allows us to model at infinity. In addition, the business could become insolvent before 20 years, but recover and show positive surplus at year 20. To properly model these events, we would need a multivariate distribution, which is beyond the scope of this paper.

99.9% Name	S_{34}/S_{12} 1992	D(G)	10K %	S_{34}/S_{12} 1993	D(G)	10K %	S_{34}/S_{12} 1994	D(G)	10K %
LOB02	1.21	3	OVER	2.30	2	99.99	1.89	3	OVER
LOB03	1.83	3	99.91	0.98	3	99.91	1.29	3	99.99
LOB04	1.18	3	99.84	0.62	1	99.90	1.16	3	99.96
LOB05	NA	NA	NA	0.54	1	99.93	1.29	3	99.98
LOB06	1.75	3	99.91	0.55	1	99.93	0.81	3	99.99
LOB07	0.99	3	99.81	1.59	3	99.98	2.08	2	99.94
LOB08	NA	NA	NA	NA	NA	NA	1.26	3	99.79
LOB09	1.27	3	99.99	0.83	3	99.95	1.23	3	99.97
LOB10	2.11	2	99.90	1.35	3	99.98	0.90	3	99.99
LOB11	NA	NA	NA	NA	NA	NA	1.26	3	99.98
LOB12	NA	NA	NA	NA	NA	NA	1.27	3	99.98
LOB13	NA	NA	NA	1.53	3	99.96	2.07	2	99.96
LOB14	NA	NA	NA	0.77	3	99.88	NA	NA	NA
LOB15	1.37	3	99.94	1.06	3	99.95	NA	NA	NA
LOB16	1.35	3	99.88	1.41	3	99.94	NA	NA	NA
LOB17	0.81	3	99.75	0.79	3	99.80	0.94	3	99.95
LOB18	NA	NA	NA	NA	NA	NA	1.24	3	99.98
LOB19	2.53	2	99.99	1.63	3	99.94	NA	NA	NA
LOB20	NA	NA	NA	0.74	3	99.90	NA	NA	NA
LOB21	NA	NA	NA	1.43	3	OVER	NA	NA	NA
LOB22	NA	NA	NA	1.43	3	OVER	NA	NA	NA
LOB23	NA	NA	NA	0.76	3	99.92	NA	NA	NA
LOB24	NA	NA	NA	1.78	3	OVER	NA	NA	NA

99.9% Name	S_{34}/S_{12}	D(G)	10K	S_{34}/S_{12}	D(G)	10K	S_{34}/S_{12}	D(G)	10K
	1992		%	1993		%	1994		%
LOB25	NA	NA	NA	1.03	3	99.97	NA	NA	NA
LOB26	1.42	3	99.92	0.77	3	99.94	0.94	3	99.99
LOB27	1.78	3	99.90	0.82	3	99.93	1.47	3	99.99
LOB28	1.20	3	99.90	0.83	3	99.96	1.19	3	OVER
LOB29	2.30	2	99.98	0.83	3	99.98	1.05	3	OVER
LOB30	NA	NA	NA	1.35	3	99.99	NA	NA	NA
LOB31	1.23	3	99.76	1.65	3	99.97	NA	NA	NA
LOB32	1.33	3	99.85	1.52	3	99.92	NA	NA	NA
LOB33	0.85	3	99.80	2.21	2	99.98	1.25	3	99.71
LOB34	1.09	3	99.89	1.15	3	99.88	1.27	3	99.96
LOB35	0.87	3	99.63	2.04	2	99.98	1.63	3	99.85
LOB36	0.84	3	99.91	1.92	3	99.98	1.63	3	99.98
LOB37	NA	NA	NA	3.04	2	OVER	1.81	3	99.96
LOB38	NA	NA	NA	0.75	3	99.98	1.05	3	99.88
LOB39	NA	NA	NA	0.27	1	99.76	0.15	1	99.60
LOB40	NA	NA	NA	NA	NA	NA	0.57	1	99.95
LOB41	NA	NA	NA	NA	NA	NA	0.83	3	99.89
LOB42	0.74	3	99.88	0.73	3	99.96	NA	NA	NA
LOB43	NA	NA	NA	0.61	1	99.90	1.14	3	99.96
LOB44	1.80	3	99.87	0.80	3	99.95	1.11	3	OVER
LOB45	2.14	2	99.98	0.93	3	99.97	NA	NA	NA
LOB46	1.34	3	99.98	1.26	3	99.93	NA	NA	NA
LOB47	2.05	2	OVER	2.55	2	99.99	NA	NA	NA

99.9% Name	S_{14}/S_{12}	D(G)	10K	S_{14}/S_{12}	D(G)	10K	S_{14}/S_{12}	D(G)	10K
	1992		%	1993		%	1994		%
LOB48	1.82	3	99.98	1.55	3	99.96	3.23	2	OVER
LOB49	0.22	1	99.80	0.85	3	99.88	0.70	3	99.99
LOB50	2.14	2	OVER	0.83	3	99.94	NA	NA	NA
LOB51	1.67	3	99.98	1.34	3	99.96	NA	NA	NA
SCORE									
CARD									
1 OVER	NA	NA	0	NA	NA	0	NA	NA	1
1 IN	NA	NA	0	NA	NA	4	NA	NA	0
1 UNDER	NA	NA	1	NA	NA	1	NA	NA	1
2 OVER	NA	NA	5	NA	NA	5	NA	NA	2
2 IN	NA	NA	1	NA	NA	0	NA	NA	1
2 UNDER	NA	NA	0	NA	NA	0	NA	NA	0
3 OVER	NA	NA	5	NA	NA	20	NA	NA	21
3 IN	NA	NA	13	NA	NA	14	NA	NA	3
3 UNDER	NA	NA	6	NA	NA	1	NA	NA	2

Table 5

Table 6 is the scorecard for the 1% estimator. Notice that the estimator is very good at the 1% level, no matter what the domain of convergence. This result is fairly understandable because 1,000 trials will give a high confidence interval on the 1% estimator. (See Appendix 1)

SCORE CARD	1992	1993	1994
1 OVER	1	0	0
1 IN	0	5	2
1 UNDER	0	0	0
2 OVER	2	0	1
2 IN	4	5	2
2 UNDER	0	0	0
3 OVER	0	3	3
3 IN	24	32	23
3 UNDER	0	0	0

Table 6

Overestimation of the surplus is conservative, because the estimated surplus will indicate that the company is insolvent more frequently than in actuality. As you may see from Table 5, overestimation occurs at the 0.1% level, despite having exponential tails. This difficulty corresponds to the overestimation observed in the Student distribution mentioned in Section 4. This misestimation has led me to the following rules of thumb:

Rules of Thumb.

1. All business with tails heavier than exponential should use 10,000 trials.
2. All expensive business, where overestimation would be critical with tails exponential or lighter, use 10,000 trials.
3. If visual and statistical test behavior is maintained into subsequent years, the

additional runs may not be necessary.

Using Rule of Thumb 1, I determined that in 1992 LOB49 should be run 10,000 times due to heavier tails. In 1993, LOBs 4, 5, 6, 39 and 43 should be run. In 1994, LOBs 39 and 40 should be processed.

Section 6. Conclusions and Further Research

I tested my technique for robustness with another simulation not presented here. I set n equal to 400, and I examined all 1992-4 LOBs. Here, I compared the relative errors of the 0.1%, 1%, 5% and 10% estimators. There was no appreciable visual difference between the cumulative proportion of the first study and a collective set of twenty-five studies.

The analysis in this paper has concentrated on point estimates. As you can see in Tables 7-12 in Appendix II, percentiles other than the 50th percentile are displayed. To place a one sided confidence interval on the extreme quantile, just choose a lower percentile than the 50th. Further research is needed here, to see if the bootstrap estimator can give better interval estimators.

The resampling estimator appears to be a very effective estimator when coupled with the CS test for the type of tail. However, Strawderman and Zelterman [15] have developed a method that does not use resampling, but approximates the bootstrap CDF through a saddlepoint approximation found by Wood et. al. [20]. Further research could be conducted here.

Further analysis of the CS test should be conducted as well. One area is to test the sensitivity to changes in underlying parameters. Also, a more scientific method to choose the boundaries of the test should be developed.

Other areas of research would be to examine a better choice of k , instead of the simplified

rule used in this paper. Another promising method would be the technique developed in [17].

In close examination of the three years of surplus data, the 10,000 trials used were different for each year, however they were constant within the year. The models for each line of business may have been improved. For example, LOB49 was modeled the same in years 1992 and 1993, but was changed in 1994. However the domain of the distribution went from one to three between years 1992 and 1993, and remained at three in year 1994. It appears that term structure of interest rates affects the domain of convergence, and further research should be conducted on this effect.

Region 1 distributions, according to Gumbel [6] and Strawderman and Zelterman [15], can be transformed to Region 3 distributions using the natural logarithm. This transform may modify the rules of thumb.

In conclusion, we have developed a method to reduce the time and costs of testing insurance company insolvency. We have removed several assumptions about the surplus distribution and have replaced them with results from extreme value theory. We have introduced the CS test, which will allow actuaries to determine the riskiness of a line of business. We hope that these methods and tests will be adopted to reduce risk and develop better and safer products in our industry.

Acknowledgments: I would like to thank Steve Sedlak and Steve Hodges for their direction, insight and advice. Also, I would like to thank Russ Osborn and Aaron Bush for their skillful rewording and revision of the paper, and Gayle Williams for working on the preliminary graphics (which had to be changed after she left). Especially, I would like to thank the department wizard, Martha Rohlck, who managed the nightmare of generating the 10,000 trials for the last three years. All my love and gratitude go to my wife Patricia and my daughter Michelle and my sons, Samuel, Bradley, Evan and Carl, for their patience while researching and writing this paper.

Appendix I

```
CLS
PRINT "This program determines the confidence interval"
PRINT "of the k'th order statistic, of the p'th percentile"
INPUT "what is the total number of trials (n)?", N
INPUT "what is the percentile (in .xx)", p
SUM = 0

FOR K = 0 TO N - 1
  prod# = 1
  FOR I = 1 TO K
    prod# = prod# * (N - I + 1) / (K - I + 1) * p * (1 - p)
  PRINT prod#
  IF prod# < 1D-220 THEN prod# = 0!
  NEXT I
  IF K >= N / 2 THEN

    FOR I = 1 TO 2 * K - N
      prod# = prod# / (1 - p)
    NEXT I
  END IF
  IF K < N / 2 THEN
    FOR I = 1 TO -2 * K + N
      prod# = prod# * (1 - p)
    NEXT I
  END IF

  SUM = SUM + prod#
  COMPSUM = 1# - SUM
  IF COMPSUM <= .0000002# THEN GOTO ARB
  PRINT "prob. that the", K + 1, "th order stat is less than the", 100 * p, "% is ";
  COMPSUM
  IF (K + 1) / 20 = INT((K + 1) / 20) THEN
    INPUT "PRESS ENTER"; QRY$
    CLS
  END IF
NEXT K
ARB: END
```

Appendix II

LOB01

0.0E+00
1.0E+00
0.0E+00
1.0E+00
0.0E+00
1.0E+00

Figure 16

LOB04

0.0E+00
0.0E+00
1.0E+00
1.0E+00
0.0E+00
0.0E+00
1.0E+00
1.0E+00
1.0E+00

Figure 19

LOB09

3.1E+04
3.0E+04
3.2E+04
3.3E+04
3.4E+04
3.5E+04
3.6E+04

Figure 22

LOB16

0.0E+00
0.5E+00
0.1E+00
0.6E+00
0.2E+00
0.7E+00
0.3E+00

Figure 25

LOB02

1.0E+00
2.0E+00
0.0E+00
0.0E+00
4.0E+00
0.0E+00
2.0E+00

Figure 17

LOB06

4.0E+04
2.0E+04
0.0E+00
0.0E+04
4.0E+04
6.0E+04
8.0E+04

Figure 20

LOB10

4.0E+00
4.5E+00
1.0E+00
5.0E+00
6.0E+00
6.5E+00
7.0E+00

Figure 23

LOB17

1.0E+00
1.0E+00
0.0E+00
0.0E+00
1.0E+00
1.0E+00

Figure 26

LOB03

1.4E+00
1.2E+00
1.0E+00
0.0E+00
0.0E+00
4.0E+00
2.0E+00
0.0E+00
2.0E+00

Figure

LOB07

0.0E+04
0.0E+04
0.0E+04
1.0E+00
1.0E+00
1.0E+00
1.0E+00
1.0E+00

Figure 21

LOB15

3.0E+00
3.0E+00
3.0E+00
3.0E+00
3.0E+00
3.0E+00
3.0E+00

Figure 24

LOB19

0.0E+00
0.0E+00
0.0E+00
0.0E+00
0.0E+00
0.0E+00
0.0E+00

Figure 27

LOB26

4.0E+07
2.0E+07
0.0E+00
2.0E+07
5.0E+07
6.0E+05

Figure 28

LOB29

1.0E+06
4.0E+05
1.0E+07
1.0E+07
1.0E+05

Figure 31

LOB33

1.0E+04
1.5E+04
2.0E+04
2.5E+04
3.0E+04
2.7E+04
4.0E+04

Figure 34

LOB36

1.0E+05
4.0E+05
1.0E+05
6.0E+05
2.0E+05
8.0E+05

Figure 37

LOB27

1.0E+05
1.4E+05
1.0E+07
5.0E+04
0.0E+00
5.0E+04
1.0E+05

Figure 29

LOB31

0.0E+00
2.0E+05
7.0E+05
5.0E+05
4.0E+07
2.0E+05
8.0E+05
9.0E+05

Figure 32

LOB34

6.0E+05
7.0E+05
6.0E+07
9.0E+07
1.0E+04
1.1E+04
1.2E+04
1.3E+04
1.4E+04

Figure 35

LOB42

1.0E+04
1.0E+05
0.0E+00
1.0E+05
1.0E+06

Figure 38

LOB28

4.0E+04
2.0E+04
0.0E+00
2.0E+04
4.0E+04
4.0E+04
6.0E+04
1.0E+05

Figure 30

LOB32

6.0E+06
1.0E+06
1.0E+07
1.2E+07
1.4E+05

Figure 33

LOB35

1.7E+05
1.4E+05
1.6E+05
1.8E+05
2.0E+05
2.2E+05
2.4E+05
2.6E+05

Figure 36

LOB44

1.0E+06
5.0E+05
0.0E+00
1.0E+05
1.3E+06
1.5E+06

Figure 39

LOB45

6.0E+06
6.5E+06
7.0E+06
7.5E+06
8.0E+06
8.5E+06
9.0E+06
9.5E+06

Figure 40

LOB48

5.0E+05
6.0E+05
7.0E+05
8.0E+05
9.0E+05

Figure 43

LOB51

1.0E+06
2.0E+06
3.0E+06
4.0E+06
5.0E+06

Figure 46

LOB46

1.0E+06
1.5E+06
2.0E+06
2.5E+06
3.0E+06
3.5E+06
4.0E+06

Figure 41

LOB49

2.5E+05
3.0E+05
3.5E+05
4.0E+05
4.5E+05
5.0E+05
5.5E+05

Figure 44

LOB47

7.5E+07
8.0E+07
8.5E+07
9.0E+07
9.5E+07

Figure 42

LOB50

4.0E+06
4.4E+06
4.8E+06
5.2E+06
5.6E+06
6.0E+06

Figure 45

YEAR 1992 PERCENTILE 0.9990 K 102 N 1000

JOB	WORST	10K	1TH	\$	10K	RE	10TH	\$	10K	RE	25TH	\$	10K	RE	50TH	\$	10K	RE	MEAN	\$	10K	RE
	CASE	ACTUAL	EST	DELTA	%		EST	DELTA	%		EST	DELTA	%		EST	DELTA	%		EST	DELTA	%	
LOB01	-0.30	-0.15	-0.33	0.19	1.1000	-1.28	-0.23	0.08	0.9996	-0.54	-0.18	0.04	0.9995	-0.24	-0.14	-0.01	0.9989	0.07	-0.14	-0.00	0.9990	0.02
LOB02	19.48	19.63	18.91	0.73	1.1000	0.04	19.19	0.44	1.1000	0.02	19.30	0.33	1.1000	0.02	19.48	0.17	1.1000	0.01	19.43	0.20	1.1000	0.01
LOB03	-13.95	-11.87	-20.03	8.17	1.1000	-0.69	-15.90	4.04	1.1000	-0.34	-13.80	1.94	0.9998	-0.16	-11.99	0.12	0.9991	-0.01	-12.37	0.51	0.9994	-0.04
LOB04	-95.86	22.07	-101.36	123.43	1.1000	5.59	-19.10	41.17	0.9995	1.87	21.04	1.03	0.9991	0.05	55.94	-33.87	0.9984	-1.53	48.57	-26.50	0.9985	-1.20
LOB06	-0.07	-0.03	-0.08	0.04	1.1000	-1.30	-0.08	0.03	0.9996	-0.77	-0.05	0.02	0.9996	-0.44	-0.04	0.00	0.9991	-0.10	-0.04	0.00	0.9991	-0.13
LOB07	0.02	0.07	0.06	0.00	0.9992	0.05	0.07	-0.01	0.9989	-0.07	0.08	-0.01	0.9985	-0.16	0.08	-0.02	0.9981	-0.24	0.08	-0.02	0.9981	-0.24
LOB09	0.03	0.03	0.03	0.00	1.1000	0.04	0.03	0.00	1.1000	0.03	0.03	0.00	1.1000	0.02	0.03	0.00	0.9999	0.01	0.03	0.00	0.9999	0.02
LOB10	2.67	3.72	1.71	2.01	1.1000	0.54	3.08	0.65	0.9995	0.17	3.47	0.26	0.9993	0.07	3.79	-0.06	0.9990	-0.02	3.72	-0.00	0.9990	-0.00
LOB15	3.42	3.45	3.38	0.08	1.1000	0.02	3.41	0.04	1.1000	0.01	3.42	0.03	0.9998	0.01	3.44	0.01	0.9994	0.00	3.44	0.01	0.9996	0.00
LOB16	2.49	2.50	2.48	0.02	1.1000	0.01	2.49	0.01	0.9998	0.00	2.50	0.00	0.9992	0.00	2.51	-0.01	0.9988	-0.00	2.51	-0.01	0.9989	-0.00
LOB17	-17.29	-10.54	-15.69	5.15	0.9998	-0.49	-11.87	1.33	0.9994	-0.13	-9.90	-0.64	0.9987	0.06	-8.07	-2.48	0.9975	0.23	-8.38	-2.16	0.9978	0.20
LOB19	0.54	0.54	0.52	0.02	1.1000	0.04	0.53	0.01	1.1000	0.02	0.53	0.01	1.1000	0.02	0.54	0.01	0.9999	0.01	0.54	0.01	0.9999	0.01
LOB26	-67.79	-41.18	-93.68	52.50	1.1000	-1.27	-70.43	29.25	1.1000	-0.71	-57.80	16.62	0.9997	-0.40	-46.45	5.27	0.9992	-0.13	-48.28	7.10	0.9994	-0.17
LOB27	-0.22	-0.17	-0.30	0.13	1.1000	-0.73	-0.23	0.05	1.1000	-0.31	-0.19	0.02	0.9995	-0.11	-0.17	-0.01	0.9990	0.04	-0.17	-0.00	0.9990	0.01
LOB28	-0.08	-0.03	-0.08	0.04	1.1000	-1.30	-0.06	0.02	0.9998	-0.71	-0.05	0.01	0.9995	-0.36	-0.03	-0.00	0.9990	0.00	-0.04	0.00	0.9992	-0.04
LOB29	6.40	8.10	4.70	3.39	1.1000	0.42	6.02	2.08	1.1000	0.26	6.54	1.56	0.9999	0.19	7.00	1.10	0.9998	0.14	6.91	1.18	0.9998	0.15
LOB31	0.00	0.24	0.20	0.05	0.9994	0.19	0.29	-0.05	0.9988	-0.20	0.34	-0.10	0.9983	-0.40	0.39	-0.15	0.9976	-0.60	0.38	-0.14	0.9978	-0.57
LOB32	2.68	6.10	4.54	1.56	0.9997	0.26	5.58	0.53	0.9993	0.09	6.22	-0.12	0.9990	-0.02	6.80	-0.70	0.9985	-0.11	6.73	-0.63	0.9988	-0.10
LOB33	0.00	0.02	0.01	0.01	0.9996	0.41	0.01	0.00	0.9994	0.09	0.02	-0.00	0.9985	-0.07	0.02	-0.00	0.9980	-0.21	0.02	-0.00	0.9980	-0.18
LOB34	0.01	0.01	0.00	0.00	1.1000	0.45	0.01	0.00	0.9996	0.14	0.01	0.00	0.9994	0.05	0.01	-0.00	0.9989	-0.05	0.01	-0.00	0.9990	-0.02
LOB35	0.04	0.13	0.11	0.02	0.9995	0.18	0.14	-0.01	0.9985	-0.08	0.16	-0.03	0.9974	-0.20	0.17	-0.04	0.9963	-0.28	0.17	-0.03	0.9966	-0.26
LOB36	0.19	0.39	0.19	0.20	1.1000	0.51	0.31	0.07	0.9985	0.19	0.35	0.04	0.9993	0.10	0.39	0.00	0.9991	0.00	0.38	0.00	0.9991	0.01
LOB42	-0.98	-0.18	-0.66	0.50	0.9998	-3.11	-0.39	0.23	0.9995	-1.41	-0.27	0.11	0.9994	-0.67	-0.14	-0.02	0.9988	0.11	-0.16	0.00	0.9991	-0.02
LOB44	-1.65	-0.83	-1.99	1.15	1.1000	-1.39	-1.18	0.35	0.9995	-0.42	-0.91	0.08	0.9993	-0.10	-0.70	-0.14	0.9987	0.16	-0.75	-0.09	0.9988	0.10
LOB45	5.29	6.05	3.88	2.16	1.1000	0.36	4.70	1.35	1.1000	0.22	5.14	0.91	1.1000	0.15	5.52	0.52	0.9998	0.09	5.45	0.60	0.9999	0.10
LOB46	149.19	153.88	138.32	15.56	1.1000	0.10	143.65	10.23	1.1000	0.07	146.95	6.93	1.1000	0.05	150.44	3.44	0.9998	0.02	149.85	4.03	0.9999	0.03
LOB47	62.91	68.86	42.46	26.40	1.1000	0.38	53.45	15.42	1.1000	0.22	57.87	10.99	1.1000	0.16	61.87	6.90	1.1000	0.10	61.05	7.81	1.1000	0.11
LOB48	0.43	0.51	0.30	0.22	1.1000	0.42	0.38	0.14	1.1000	0.27	0.42	0.09	1.1000	0.18	0.47	0.04	0.9998	0.08	0.46	0.05	0.9998	0.10
LOB49	-9.30	-4.44	-0.63	-3.81	0.9983	0.86	0.29	-4.73	0.9981	1.07	0.41	-4.85	0.9980	1.09	0.48	-4.92	0.9980	1.11	0.43	-4.87	0.9980	1.10
LOB50	4.31	4.43	3.52	0.91	1.1000	0.20	3.84	0.59	1.1000	0.13	4.03	0.40	1.1000	0.09	4.17	0.26	1.1000	0.08	4.14	0.29	1.1000	0.07
LOB51	3.22	3.31	3.05	0.27	1.1000	0.08	3.16	0.15	1.1000	0.05	3.20	0.11	1.1000	0.03	3.25	0.05	0.9998	0.02	3.24	0.07	0.9999	0.02
TOTAL	55.85	234.65	-8.46	241.10			131.19	103.46			198.87	35.78			259.14	-24.50			247.22	-12.57		

YEAR 1992 PERCENTILE 0.9900 K 102 N 1000

JOB	WORST	10K	11TH			RE	10TH			25TH			50TH			MEAN						
	CASE	ACTUAL	EST	\$	%		EST	\$	%	EST	\$	%	EST	\$	%	EST	\$	%				
LOB01	-0.30	0.07	-0.04	0.11	0.9972	1.53	-0.01	0.08	0.9960	1.11	0.01	0.07	0.9951	0.90	0.02	0.05	0.9942	0.68	0.02	0.05	0.9942	0.69
LOB02	19.49	19.91	19.69	0.22	0.9979	0.01	19.78	0.13	0.9965	0.01	19.82	0.09	0.9950	0.00	19.87	0.03	0.9920	0.00	19.87	0.04	0.9921	0.00
LOB03	-13.95	-6.57	-9.23	2.67	0.9971	-0.41	-8.07	1.50	0.9948	-0.23	-7.49	0.92	0.9929	-0.14	-6.88	0.29	0.9911	-0.04	-6.89	0.33	0.9912	-0.05
LOB04	95.86	168.22	110.37	57.85	0.9958	0.34	132.06	36.16	0.9944	0.21	143.70	24.52	0.9928	0.15	155.46	12.76	0.9915	0.08	154.11	14.11	0.9917	0.08
LOB06	-0.07	0.01	-0.01	0.02	0.9958	2.42	-0.00	0.01	0.9945	1.44	0.00	0.01	0.9933	0.96	0.00	0.01	0.9921	0.56	0.00	0.01	0.9923	0.60
LOB07	0.02	0.11	0.10	0.01	0.9949	0.07	0.10	0.00	0.9929	0.03	0.11	0.00	0.9915	0.01	0.11	-0.00	0.9894	-0.01	0.11	-0.00	0.9896	-0.01
LOB09	0.03	0.03	0.03	0.00	0.9980	0.01	0.03	0.00	0.9952	0.01	0.03	0.00	0.9945	0.01	0.03	0.00	0.9923	0.00	0.03	0.00	0.9924	0.00
LOB10	2.67	4.98	4.29	0.69	0.9975	0.14	4.60	0.38	0.9956	0.08	4.75	0.23	0.9942	0.05	4.89	0.09	0.9925	0.02	4.87	0.11	0.9926	0.02
LOB15	3.42	3.49	3.47	0.02	0.9969	0.01	3.48	0.01	0.9938	0.00	3.49	0.00	0.9916	0.00	3.50	-0.00	0.9867	-0.00	3.50	-0.00	0.9891	-0.00
LOB16	2.49	2.53	2.53	0.01	0.9952	0.00	2.53	0.00	0.9918	0.00	2.53	-0.00	0.9891	-0.00	2.54	-0.00	0.9857	-0.00	2.54	-0.00	0.9858	-0.00
LOB17	-17.29	-1.68	-4.43	2.74	0.9951	-1.63	-3.11	1.42	0.9933	-0.85	-2.49	0.81	0.9921	-0.48	-1.87	0.19	0.9908	-0.11	-1.89	0.21	0.9907	-0.12
LOB19	0.54	0.55	0.55	0.01	0.9975	0.01	0.55	0.00	0.9956	0.01	0.55	0.00	0.9936	0.00	0.55	0.00	0.9923	0.00	0.55	0.00	0.9925	0.00
LOB26	-57.79	-1.93	-22.99	21.06	0.9967	-10.91	-16.40	14.46	0.9951	-7.49	-12.49	10.56	0.9942	-5.47	-8.16	6.23	0.9926	-3.23	-8.35	6.42	0.9926	-3.33
LOB27	-0.22	-0.06	-0.11	0.05	0.9963	-0.80	-0.09	0.03	0.9940	-0.48	-0.08	0.02	0.9927	-0.30	-0.07	0.01	0.9917	-0.15	-0.07	0.01	0.9919	-0.17
LOB28	-0.08	0.02	-0.00	0.02	0.9957	1.23	0.00	0.01	0.9940	0.78	0.01	0.01	0.9934	0.52	0.01	0.00	0.9922	0.29	0.01	0.01	0.9922	0.30
LOB29	6.40	9.08	7.97	1.11	0.9991	0.12	8.36	0.72	0.9962	0.08	8.54	0.54	0.9975	0.06	8.70	0.38	0.9961	0.04	8.69	0.39	0.9964	0.04
LOB31	0.00	0.59	0.50	0.09	0.9950	0.15	0.54	0.05	0.9941	0.09	0.55	0.03	0.9935	0.06	0.57	0.01	0.9912	0.02	0.57	0.01	0.9914	0.03
LOB32	2.68	9.72	8.57	1.15	0.9959	0.12	8.87	0.85	0.9953	0.09	9.06	0.66	0.9941	0.07	9.26	0.46	0.9931	0.05	9.26	0.46	0.9931	0.05
LOB33	0.00	0.03	0.02	0.00	0.9945	0.12	0.03	0.00	0.9921	0.04	0.03	0.00	0.9910	0.01	0.03	-0.00	0.9881	-0.02	0.03	-0.00	0.9886	-0.02
LOB34	0.01	0.01	0.01	0.00	0.9955	0.08	0.01	0.00	0.9913	0.02	0.01	-0.00	0.9891	-0.01	0.01	-0.00	0.9877	-0.03	0.01	-0.00	0.9879	-0.03
LOB35	0.04	0.20	0.18	0.01	0.9930	0.06	0.20	0.00	0.9903	0.00	0.20	-0.00	0.9885	-0.02	0.20	-0.01	0.9853	-0.04	0.20	-0.01	0.9855	-0.04
LOB36	0.19	0.52	0.47	0.05	0.9956	0.10	0.50	0.03	0.9931	0.05	0.51	0.01	0.9918	0.02	0.52	-0.00	0.9898	-0.00	0.52	0.00	0.9901	0.00
LOB42	-0.98	0.23	0.06	0.16	0.9957	0.72	0.15	0.07	0.9929	0.32	0.20	0.03	0.9914	0.14	0.24	-0.01	0.9894	-0.06	0.24	-0.01	0.9894	-0.04
LOB44	-1.85	0.14	-0.26	0.40	0.9955	2.84	-0.10	0.24	0.9938	1.71	-0.00	0.15	0.9934	1.02	0.09	0.05	0.9916	0.39	0.07	0.07	0.9917	0.49
LOB45	5.29	7.08	6.29	0.78	0.9975	0.11	6.57	0.50	0.9961	0.07	6.69	0.37	0.9948	0.05	6.85	0.22	0.9931	0.03	6.84	0.23	0.9932	0.03
LOB46	149.19	183.08	157.58	5.50	0.9977	0.03	159.79	3.29	0.9954	0.02	160.78	2.30	0.9948	0.01	161.89	1.19	0.9927	0.01	161.88	1.20	0.9927	0.01
LOB47	82.91	76.75	68.18	8.56	0.9995	0.11	70.96	5.78	0.9979	0.08	72.29	4.46	0.9965	0.06	73.93	2.81	0.9958	0.04	73.78	2.86	0.9957	0.04
LOB48	0.43	0.63	0.56	0.06	0.9970	0.10	0.59	0.03	0.9948	0.06	0.61	0.02	0.9918	0.03	0.62	0.00	0.9904	0.00	0.62	0.00	0.9904	0.01
LOB49	-9.30	0.87	0.42	0.44	0.9980	0.51	0.57	0.30	0.9978	0.35	0.62	0.25	0.9975	0.28	0.67	0.20	0.9973	0.23	0.65	0.22	0.9975	0.25
LOB50	4.31	4.65	4.39	0.26	0.9996	0.06	4.50	0.15	0.9980	0.03	4.65	0.10	0.9962	0.02	4.61	0.04	0.9931	0.01	4.60	0.05	0.9937	0.01
LOB51	3.22	3.40	3.33	0.07	0.9988	0.02	3.36	0.04	0.9961	0.01	3.38	0.02	0.9939	0.01	3.40	0.01	0.9914	0.00	3.40	0.01	0.9914	0.00
TOTAL	55.85	486.64	362.48	104.15			400.35	66.29			420.45	46.16			441.62	25.02			439.75	26.89		

JOB	WORST CASE	10K ACTUAL	1TH			10TH			25TH			50TH			MEAN							
			EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%					
OB01	0.50	0.72	0.84	0.17	0.9999	0.24	0.82	0.06	0.9997	0.13	0.87	0.05	0.9994	0.07	0.71	0.01	0.9991	0.01	0.70	0.01	0.9991	0.02
OB02	-0.57	-0.40	-0.81	0.41	1.0000	-1.04	-0.89	0.29	1.0000	-0.74	-0.83	0.23	1.0000	-0.59	-0.55	0.16	0.9999	-0.40	-0.56	0.17	0.9999	-0.42
OB03	-12.30	-7.40	-14.78	7.37	1.0000	-1.00	-10.36	2.98	0.9997	-0.40	-8.60	1.40	0.9997	-0.19	-7.48	0.05	0.9991	-0.21	-7.72	0.31	0.9991	-0.04
OB04	-1.47	84.82	8.08	79.74	0.9999	0.84	48.78	35.08	0.9996	0.41	77.25	7.50	0.9993	0.08	102.40	-17.58	0.9990	-0.01	99.83	-15.01	0.9990	-0.18
OB05	-35.89	-18.05	-38.42	20.37	1.0000	-1.13	-29.38	11.34	0.9997	-0.83	-24.89	8.84	0.9997	-0.37	-19.83	1.78	0.9993	-0.10	-20.21	2.18	0.9994	-0.12
OB06	-0.09	-0.04	-0.10	0.06	1.0000	-1.55	-0.07	0.03	0.9999	-0.87	-0.08	0.02	0.9997	-0.50	-0.04	0.00	0.9993	-0.09	-0.05	0.01	0.9998	-0.18
OB07	-0.11	-0.09	-0.13	0.04	1.0000	-0.43	-0.11	0.02	0.9999	-0.25	-0.10	0.02	1.0000	-0.17	-0.10	0.01	0.9998	-0.08	-0.10	0.01	0.9998	-0.10
OB09	0.02	0.02	0.01	0.01	1.0000	0.31	0.02	0.00	0.9998	0.18	0.02	0.00	0.9997	0.12	0.02	0.00	0.9995	0.05	0.02	0.00	0.9995	0.06
OB10	-0.31	0.85	-2.27	3.12	1.0000	3.86	-0.85	1.70	1.0000	1.99	-0.21	1.06	0.9999	1.25	0.31	0.54	0.9998	0.84	0.19	0.88	0.9999	0.78
OB13	-0.06	-0.04	-0.08	0.04	1.0000	-0.87	-0.08	0.02	0.9999	-0.59	-0.06	0.02	0.9999	-0.39	-0.05	0.01	0.9998	-0.19	-0.05	0.01	0.9998	-0.22
OB14	-3.85	-2.52	-3.82	1.40	1.0000	-0.56	-3.13	0.81	0.9997	-0.24	-2.83	0.31	0.9998	-0.12	-2.45	-0.07	0.9998	0.03	-2.47	-0.05	0.9990	0.02
OB15	-5.71	-5.11	-8.24	1.13	1.0000	-0.22	-5.88	0.55	0.9999	-0.11	-5.44	0.32	0.9999	-0.08	-5.23	0.12	0.9995	-0.02	-5.28	0.18	0.9997	-0.03
OB16	0.00	1.45	1.00	0.45	0.9999	0.31	1.18	0.27	0.9998	0.18	1.22	0.18	0.9997	0.11	1.37	0.08	0.9994	0.06	1.39	0.10	0.9994	0.07
OB17	-8.23	-1.70	-3.87	1.97	0.9997	-1.16	-1.58	-0.12	0.9990	0.07	-0.59	-1.15	0.9985	0.87	0.28	-1.88	0.9880	1.15	0.07	-1.77	0.9982	1.04
OB19	0.27	0.38	0.24	0.14	1.0000	0.38	0.30	0.08	0.9999	0.20	0.33	0.05	0.9998	0.12	0.38	0.02	0.9994	0.05	0.35	0.02	0.9995	0.08
OB20	-0.06	-0.01	-0.17	0.18	1.0000	-13.59	-0.07	0.06	0.9998	-5.49	-0.04	0.02	0.9993	-2.11	-0.01	-0.00	0.9990	0.33	-0.01	0.00	0.9991	-0.28
OB21	128.42	145.00	72.95	72.05	1.0000	0.50	92.71	52.28	1.0000	0.36	104.87	40.33	1.0000	0.28	118.85	28.15	1.0000	0.19	114.75	30.25	1.0000	0.21
OB24	114.07	117.27	58.58	58.69	1.0000	0.50	79.07	38.21	1.0000	0.33	88.01	29.28	1.0000	0.25	97.40	19.87	1.0000	0.17	95.68	21.50	1.0000	0.18
OB22	122.82	138.92	59.83	79.10	1.0000	0.57	84.73	54.20	1.0000	0.39	97.12	41.80	1.0000	0.30	110.88	28.24	1.0000	0.20	107.63	31.29	1.0000	0.23
OB23	5.38	8.17	5.15	1.02	1.0000	0.17	5.80	0.37	0.9997	0.08	5.95	0.22	0.9994	0.04	8.12	0.05	0.9992	0.01	8.08	0.08	0.9992	0.01
OB25	118.11	259.13	203.14	55.99	0.9999	0.22	230.28	28.85	0.9999	0.11	241.18	17.95	0.9998	0.07	251.23	7.90	0.9997	0.03	248.88	10.25	0.9996	0.04
OB28	-88.18	-50.46	-98.41	47.95	1.0000	-0.95	-79.82	29.38	0.9999	-0.58	-68.25	17.79	0.9997	-0.35	-57.92	7.48	0.9994	-0.15	-59.40	8.94	0.9996	-0.18
OB27	-0.20	-0.11	-0.20	0.08	0.9999	0.72	-0.18	0.04	0.9997	-0.37	-0.14	0.03	0.9998	-0.23	-0.12	0.01	0.9993	-0.06	-0.13	0.01	0.9993	-0.10
OB28	-0.09	-0.05	-0.11	0.07	1.0000	-1.42	-0.09	0.04	1.0000	-0.91	-0.07	0.03	0.9998	-0.55	-0.06	0.01	0.9998	-0.23	-0.06	0.01	0.9996	-0.28
OB29	-1.81	0.33	-3.53	3.88	1.0000	11.59	-2.19	2.52	1.0000	7.58	-1.40	1.74	0.9998	5.21	-0.71	1.05	0.9998	3.15	-0.77	1.10	0.9998	3.30
OB30	9.85	13.87	1.77	11.90	1.0000	0.87	5.51	8.15	1.0000	0.80	7.84	8.02	1.0000	0.44	9.89	3.78	0.9999	0.28	9.53	4.14	1.0000	0.30
OB31	-0.18	-0.04	-0.19	0.15	1.0000	-4.27	-0.14	0.11	0.9999	-2.84	-0.11	0.07	0.9999	-2.10	-0.07	0.03	0.9997	-0.83	-0.07	0.03	0.9997	-0.90
OB32	3.40	5.94	1.28	4.66	1.0000	0.78	3.85	1.99	0.9999	0.33	4.80	1.14	0.9997	0.19	5.88	0.28	0.9992	0.05	5.48	0.48	0.9994	0.08
OB33	-0.03	-0.01	-0.04	0.03	1.0000	-1.84	-0.03	0.02	0.9999	-1.15	-0.03	0.01	0.9999	-0.78	-0.02	0.01	0.9998	-0.42	-0.02	0.01	0.9998	-0.48
OB34	-0.03	-0.02	-0.03	0.00	0.9997	-0.11	-0.02	0.00	0.9997	-0.08	-0.02	0.00	0.9992	-0.01	-0.02	-0.00	0.9988	0.03	-0.02	-0.00	0.9988	0.03
OB35	-0.22	-0.15	-0.32	0.17	1.0000	-1.11	-0.22	0.07	0.9999	-0.49	-0.19	0.04	0.9999	-0.30	-0.17	0.02	0.9998	-0.13	-0.17	0.02	0.9998	-0.17
OB36	-0.49	-0.38	-0.85	0.28	1.0000	-0.73	-0.58	0.18	1.0000	-0.48	-0.50	0.12	1.0000	-0.32	-0.44	0.08	0.9998	-0.18	-0.45	0.07	0.9999	-0.19
OB37	-0.78	-0.84	-1.41	0.77	1.0000	-1.21	-1.08	0.44	1.0000	-0.69	-0.88	0.32	1.0000	-0.50	-0.86	0.22	1.0000	-0.34	-0.88	0.24	1.0000	-0.38
OB38	-0.00	0.12	-0.10	0.22	1.0000	1.82	0.01	0.11	0.9999	0.03	0.05	0.07	0.9998	0.59	0.08	0.04	0.9998	0.33	0.07	0.05	0.9998	0.40
OB39	-14.12	-8.73	-7.81	-0.82	0.9987	0.09	-7.34	-1.39	0.9982	0.18	-7.09	-1.64	0.9978	0.19	-8.90	-1.83	0.9976	0.21	-8.94	-1.80	0.9976	0.21
OB42	-1.87	-0.78	-1.35	0.57	0.9999	-0.73	-1.08	0.30	0.9999	-0.38	-0.97	0.19	0.9998	-0.25	-0.87	0.08	0.9998	-0.11	-0.88	0.10	0.9998	-0.12
OB43	-8.83	80.08	-7.15	87.23	0.9999	1.09	41.20	38.88	0.9999	0.49	89.88	10.40	0.9993	0.13	93.93	-13.84	0.9990	-0.17	93.06	-12.88	0.9990	-0.18
OB44	-1.50	-0.98	-1.98	0.98	1.0000	-1.01	-1.48	0.48	0.9999	-0.49	-1.27	0.29	0.9997	-0.30	-1.09	0.11	0.9995	-0.11	-1.12	0.14	0.9997	-0.14
OB45	8.89	7.40	8.71	1.89	1.0000	0.23	8.41	0.99	1.0000	0.13	8.80	0.86	1.0000	0.08	7.16	0.24	0.9997	0.03	7.11	0.29	0.9997	0.04
OB48	0.00	178.28	183.98	14.30	0.9998	0.08	189.95	8.33	0.9998	0.05	173.42	4.88	0.9998	0.03	178.28	2.02	0.9993	0.01	178.08	2.20	0.9993	0.01
OB47	55.99	80.38	32.93	27.45	1.0000	0.45	48.28	14.10	1.0000	0.23	50.51	9.87	1.0000	0.18	56.01	4.37	0.9999	0.07	54.89	5.49	1.0000	0.08
OB48	0.40	0.45	0.29	0.18	1.0000	0.39	0.34	0.11	1.0000	0.24	0.38	0.07	1.0000	0.15	0.41	0.04	0.9999	0.08	0.41	0.04	0.9997	0.09
OB49	-29.80	-17.59	-29.46	11.87	0.9998	-0.87	-21.84	4.30	0.9995	-0.24	-18.27	0.68	0.9992	-0.04	-15.14	-2.45	0.9988	0.14	-15.48	-2.11	0.9988	0.12
OB50	-0.03	0.23	-0.03	0.28	0.9999	1.11	0.08	0.15	0.9998	0.86	0.14	0.09	0.9997	0.40	0.20	0.04	0.9994	0.15	0.19	0.04	0.9998	0.19
OB51	3.30	3.38	3.25	0.13	1.0000	0.04	3.30	0.08	0.9999	0.02	3.33	0.05	0.9999	0.02	3.38	0.02	0.9998	0.01	3.38	0.02	0.9997	0.01
TOTALS	358.80	989.70	392.32	587.38			653.45	330.25			790.58	199.15			920.58	89.15			932.89	88.82		

LOB	WORST		10K		1TH		\$ 10K		RE		10TH		\$ 10K		RE		25TH		\$ 10K		RE		50TH		\$ 10K		RE		MEAN		\$ 10K		RE	
	CASE	ACTUAL	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA	%	EST	DELTA
OB01	0.50	0.86	0.80	0.06	0.0000	0.07	0.83	0.03	0.0030	0.04	0.84	0.02	0.0024	0.02	0.85	0.01	0.0008	0.01	0.85	0.01	0.0008	0.01	0.85	0.01	0.0008	0.01	0.85	0.01	0.0008	0.01	0.85	0.01	0.0008	0.01
OB02	-0.57	-0.26	-0.40	0.12	0.0001	-0.43	-0.35	0.08	0.0071	-0.27	-0.33	0.05	0.0050	-0.19	-0.30	0.03	0.0035	-0.10	-0.31	0.03	0.0036	-0.10	-0.31	0.03	0.0036	-0.10	-0.31	0.03	0.0036	-0.10	-0.31	0.03	0.0036	-0.10
OB03	-12.30	-1.85	-4.14	2.49	0.0057	-1.51	-3.11	1.46	0.0040	-0.58	-2.55	0.90	0.0028	-0.54	-1.97	0.32	0.0009	-0.19	-2.01	0.36	0.0011	-0.22	-2.01	0.36	0.0011	-0.22	-2.01	0.36	0.0011	-0.22	-2.01	0.36	0.0011	-0.22
OB04	-1.47	184.15	154.87	39.28	0.0057	0.50	189.87	24.48	0.0039	0.13	178.81	15.34	0.0028	0.08	188.49	5.66	0.0014	0.03	188.18	5.99	0.0015	0.03	188.18	5.99	0.0015	0.03	188.18	5.99	0.0015	0.03	188.18	5.99	0.0015	0.03
OB05	-35.90	-4.17	-10.50	8.33	0.0002	-1.52	-7.82	3.45	0.0042	-0.83	-6.17	2.00	0.0028	-0.48	-4.48	0.29	0.0005	-0.37	-4.64	0.47	0.0007	-0.11	-4.64	0.47	0.0007	-0.11	-4.64	0.47	0.0007	-0.11	-4.64	0.47	0.0007	-0.11
OB06	-0.09	0.01	-0.01	0.02	0.0006	2.52	-0.00	0.01	0.0045	1.47	0.00	0.01	0.0034	0.95	0.01	0.00	0.0000	0.38	0.00	0.00	0.0000	0.38	0.00	0.00	0.0000	0.38	0.00	0.00	0.0000	0.38	0.00	0.00	0.0000	0.38
OB07	-0.11	-0.07	-0.08	0.01	0.0073	-0.19	-0.07	0.01	0.0063	-0.12	-0.07	0.01	0.0033	-0.08	-0.07	0.00	0.0023	-0.05	-0.07	0.00	0.0023	-0.05	-0.07	0.00	0.0023	-0.05	-0.07	0.00	0.0023	-0.05	-0.07	0.00	0.0023	-0.05
OB09	0.02	0.03	0.02	0.00	0.0053	0.06	0.03	0.00	0.0038	0.04	0.03	0.00	0.0024	0.02	0.03	0.00	0.0004	0.00	0.03	0.00	0.0004	0.00	0.03	0.00	0.0004	0.00	0.03	0.00	0.0004	0.00	0.03	0.00	0.0004	0.00
OB10	-0.31	2.21	1.21	1.00	0.0004	0.45	1.82	0.59	0.0060	0.27	1.82	0.40	0.0048	0.18	2.06	0.15	0.0027	0.07	2.03	0.18	0.0028	0.06	2.03	0.18	0.0028	0.06	2.03	0.18	0.0028	0.06	2.03	0.18	0.0028	0.06
OB13	-0.06	-0.02	-0.03	0.01	0.0078	-0.74	-0.03	0.01	0.0062	-0.49	-0.03	0.01	0.0043	-0.38	-0.02	0.00	0.0024	-0.19	-0.02	0.00	0.0022	-0.12	-0.02	0.00	0.0022	-0.12	-0.02	0.00	0.0022	-0.12	-0.02	0.00	0.0022	-0.12
OB14	-3.85	-1.00	-1.48	0.47	0.0055	-0.47	-1.25	0.24	0.0037	-0.24	-1.09	0.08	0.0015	-0.08	-0.98	-0.04	0.0003	0.04	-0.98	-0.03	0.0005	0.03	-0.98	-0.03	0.0005	0.03	-0.98	-0.03	0.0005	0.03	-0.98	-0.03	0.0005	0.03
OB15	-5.71	-4.52	-4.83	0.32	0.0073	-0.07	-4.74	0.22	0.0066	-0.05	-4.67	0.18	0.0052	-0.03	-4.80	0.08	0.0031	-0.02	-4.60	0.09	0.0031	-0.02	-4.60	0.09	0.0031	-0.02	-4.60	0.09	0.0031	-0.02	-4.60	0.09	0.0031	-0.02
OB18	0.00	1.89	1.55	0.14	0.0081	0.08	1.60	0.09	0.0063	0.05	1.63	0.08	0.0053	0.04	1.66	0.03	0.0029	0.02	1.66	0.03	0.0034	0.02	1.66	0.03	0.0034	0.02	1.66	0.03	0.0034	0.02	1.66	0.03	0.0034	0.02
OB17	-6.23	2.42	1.60	0.82	0.0046	0.34	2.12	0.30	0.0018	0.12	2.43	-0.01	0.0009	-0.01	2.78	-0.34	0.0078	-0.14	2.70	-0.28	0.0080	-0.12	2.70	-0.28	0.0080	-0.12	2.70	-0.28	0.0080	-0.12	2.70	-0.28	0.0080	-0.12
OB19	0.27	0.45	0.41	0.04	0.0070	0.10	0.42	0.03	0.0049	0.06	0.43	0.02	0.0043	0.04	0.44	0.01	0.0022	0.02	0.44	0.01	0.0022	0.02	0.44	0.01	0.0022	0.02	0.44	0.01	0.0022	0.02	0.44	0.01	0.0022	0.02
OB20	-0.06	0.09	0.04	0.04	0.0007	0.51	0.06	0.02	0.0037	0.26	0.08	0.01	0.0017	0.11	0.09	-0.00	0.0002	-0.05	0.09	-0.00	0.0004	-0.05	0.09	-0.00	0.0004	-0.05	0.09	-0.00	0.0004	-0.05	0.09	-0.00	0.0004	-0.05
OB21	128.42	165.48	141.64	23.82	0.0004	0.14	149.15	16.31	0.0002	0.10	152.92	12.54	0.0005	0.08	158.99	8.47	0.0004	0.05	158.94	8.52	0.0004	0.05	158.94	8.52	0.0004	0.05	158.94	8.52	0.0004	0.05	158.94	8.52	0.0004	0.05
OB24	114.07	127.79	109.97	17.82	1.0000	0.14	118.86	10.93	0.0004	0.09	119.85	7.84	0.0005	0.08	123.43	4.36	0.0008	0.03	122.77	5.92	0.0070	0.04	122.77	5.92	0.0070	0.04	122.77	5.92	0.0070	0.04	122.77	5.92	0.0070	0.04
OB22	122.82	156.82	135.30	23.32	0.0005	0.15	142.51	16.11	0.0002	0.10	145.73	12.89	0.0008	0.08	149.74	6.88	0.0007	0.06	149.72	8.00	0.0007	0.06	149.72	8.00	0.0007	0.06	149.72	8.00	0.0007	0.06	149.72	8.00	0.0007	0.06
OB23	5.38	6.76	6.42	0.34	0.0072	0.05	6.55	0.21	0.0054	0.03	6.63	0.13	0.0037	0.02	6.70	0.07	0.0025	0.01	6.69	0.07	0.0026	0.01	6.69	0.07	0.0026	0.01	6.69	0.07	0.0026	0.01	6.69	0.07	0.0026	0.01
OB25	118.11	275.91	264.10	11.81	0.0000	0.04	269.49	8.45	0.0003	0.02	272.98	2.65	0.0002	0.01	278.54	-0.83	0.0002	-0.00	278.12	-0.21	0.0006	-0.00	278.12	-0.21	0.0006	-0.00	278.12	-0.21	0.0006	-0.00	278.12	-0.21	0.0006	-0.00
OB26	-66.18	-19.26	-37.43	18.17	0.0087	-0.94	-30.08	10.82	0.0044	-0.58	-28.43	7.18	0.0036	-0.37	-22.97	3.72	0.0019	-0.19	-23.06	3.80	0.0019	-0.20	-23.06	3.80	0.0019	-0.20	-23.06	3.80	0.0019	-0.20	-23.06	3.80	0.0019	-0.20
OB27	-0.20	-0.05	-0.07	0.02	0.0054	-0.50	-0.06	0.01	0.0031	-0.30	-0.06	0.01	0.0019	-0.17	-0.05	0.00	0.0008	-0.05	-0.05	0.00	0.0008	-0.05	-0.05	0.00	0.0008	-0.05	-0.05	0.00	0.0008	-0.05	-0.05	0.00	0.0008	-0.05
OB28	-0.09	-0.00	-0.03	0.02	0.0070	-7.61	-0.02	0.01	0.0042	-4.05	-0.01	0.01	0.0027	-2.58	-0.01	0.00	0.0010	-0.89	-0.01	0.00	0.0011	-0.89	-0.01	0.00	0.0011	-0.89	-0.01	0.00	0.0011	-0.89	-0.01	0.00	0.0011	-0.89
OB29	-1.81	2.53	0.89	1.84	0.0033	0.65	1.58	0.95	0.0006	0.36	1.67	0.68	0.0047	0.26	2.16	0.38	0.0034	0.15	2.15	0.38	0.0034	0.15	2.15	0.38	0.0034	0.15	2.15	0.38	0.0034	0.15	2.15	0.38	0.0034	0.15
OB30	9.86	17.88	14.10	3.58	0.0008	0.20	15.23	2.48	0.0008	0.14	15.89	1.79	0.0007	0.10	16.72	0.95	0.0000	0.05	16.68	1.01	0.0000	0.04	16.68	1.01	0.0000	0.04	16.68	1.01	0.0000	0.04	16.68	1.01	0.0000	0.04
OB31	-0.18	0.10	0.04	0.07	0.0063	0.85	0.06	0.04	0.0049	0.37	0.08	0.03	0.0033	0.27	0.09	0.01	0.0020	0.14	0.09	0.01	0.0020	0.14	0.09	0.01	0.0020	0.14	0.09	0.01	0.0020	0.14	0.09	0.01	0.0020	0.14
OB32	3.40	8.01	6.88	1.35	0.0075	0.17	7.31	0.71	0.0052	0.09	7.62	0.39	0.0031	0.05	7.82	0.09	0.0008	0.01	7.89	0.13	0.0008	0.02	7.89	0.13	0.0008	0.02	7.89	0.13	0.0008	0.02	7.89	0.13	0.0008	0.02
OB33	-0.03	0.00	-0.01	0.01	0.0070	16.30	-0.01	0.01	0.0051	11.00	-0.00	0.00	0.0037	7.30	-0.00	0.00	0.0004	3.84	-0.00	0.00	0.0004	4.20	-0.00	0.00	0.0004	4.20	-0.00	0.00	0.0004	4.20	-0.00	0.00	0.0004	4.20
OB34	-0.03	-0.02	-0.02	0.00	0.0073	-0.10	-0.02	0.00	0.0000	-0.08	-0.02	0.00	0.0003	-0.06	-0.02	0.00	0.0002	-0.04	-0.02	0.00	0.0002	-0.04	-0.02	0.00	0.0002	-0.04	-0.02	0.00	0.0002	-0.04	-0.02	0.00	0.0002	-0.04
OB35	-0.22	-0.08	-0.12	0.04	0.0071	-0.58	-0.10	0.03	0.0046	-0.37	-0.09	0.02	0.0034	-0.23	-0.08	0.01	0.0011	-0.10	-0.08	0.01	0.0011	-0.10	-0.08	0.01	0.0011	-0.10	-0.08	0.01	0.0011	-0.10	-0.08	0.01	0.0011	-0.10
OB36	-0.49	-0.22	-0.31	0.06	0.0072	-0.40	-0.27	0.05	0.0047	-0.22	-0.25	0.03	0.0035	-0.13	-0.23	0.01	0.0008	-0.04	-0.23	0.01	0.0010	-0.05	-0.23	0.01	0.0010	-0.05	-0.23	0.01	0.0010	-0.05	-0.23	0.01	0.0010	-0.05
OB37	-0.76	-0.48	-0.72	0.23	0.0006	-0.48	-0.62	0.14	0.0067	-0.26	-0.58	0.10	0.0071	-0.20	-0.54	0.08	0.0049	-0.12	-0.55	0.06	0.0049	-0.13	-0.55	0.06	0.0049	-0.13	-0.55	0.06	0.0049	-0.13	-0.55	0.06	0.0049	-0.13
OB38	-0.00	0.18	0.13	0.05	0.0007	0.29	0.15	0.03	0.0067	0.19	0.15	0.02	0.0051	0.13	0.17	0.01	0.0028	0.07	0.17	0.01	0.0032	0.08	0.17	0.01	0.0032	0.08	0.17	0.01	0.0032	0.08	0.17	0.01	0.0032	0.08
OB39	-14.12	-6.25	-8.57	0.33	0.0059	-0.05	-8.48	0.21	0.0048	-0.03	-8.38	0.13	0.0029	-0.02	-8.29	0.05	0.0014	-0.01	-8.30	0.05	0.0018	-0.01	-8.30	0.05	0.0018	-0.01	-8.30	0.05	0.00					

YEAR 1994 PERCENTILE 0.9990 K 102 N 1000

LOB	WORST			10K			1TH			\$			10K			RE			10TH			\$			10K			RE			50TH			\$			10K			RE			MEAN		
	CASE	ACTUAL	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE	EST	DELTA	%	RE							
LOB02	0.96	1.10	-0.18	1.29	1.1000	-1.17	0.27	0.83	1.1000	0.75	0.46	0.65	1.1000	0.58	0.68	0.43	1.1000	0.39	0.63	0.47	1.1000	0.43	-21.02	-16.69	-31.32	14.63	1.1000	-0.88	-25.70	9.01	1.1000	-0.54	-22.71	6.02	1.1000	-0.36	-20.40	3.71	0.9999	-0.22	-20.72	4.02	0.9999	-0.24	
LOB04	-203.03	-98.00	-385.88	287.88	1.1000	-2.94	-256.17	158.17	1.1000	-1.61	-206.36	108.36	1.1000	-1.11	-148.48	50.48	0.9996	-0.52	-158.40	60.40	0.9997	-0.62	-59.47	-51.00	-83.44	32.44	1.1000	-0.64	-72.32	21.32	1.1000	-0.42	-66.03	15.02	1.1000	-0.29	-58.14	7.14	0.9998	-0.14	-59.13	8.13	0.9999	-0.16	
LOB06	-0.15	-0.12	-0.22	0.09	1.1000	-0.76	-0.18	0.06	1.1000	-0.50	-0.17	0.04	1.1000	-0.36	-0.15	0.03	0.9999	-0.21	-0.15	0.03	0.9999	-0.23	-0.21	-0.14	-0.25	0.11	1.1000	-0.78	-0.20	0.06	0.9999	-0.42	-0.17	0.03	0.9999	-0.22	-0.15	0.01	0.9994	-0.08	-0.15	0.01	0.9995	-0.11	
LOB08	-0.07	-0.05	-0.06	0.01	0.9998	-0.18	-0.06	0.00	0.9995	-0.04	-0.05	-0.00	0.9989	0.01	-0.05	0.00	0.9979	0.05	-0.05	0.00	0.9979	0.05	-0.05	-0.05	-0.05	-0.06	0.01	1.1000	-0.23	-0.05	0.01	1.1000	-0.14	-0.05	0.00	0.9996	-0.09	-0.05	0.00	0.9997	-0.05	-0.05	0.00	0.9998	-0.06
LOB10	-6.50	-4.94	-11.61	6.67	1.1000	-1.35	-8.81	3.86	1.1000	-0.78	-7.53	2.58	1.1000	-0.52	-6.35	1.41	0.9999	-0.29	-6.56	1.71	1.1000	-0.35	-233.36	-197.04	-347.10	150.06	1.1000	-0.78	-281.25	84.21	1.1000	-0.43	-255.92	58.88	1.1000	-0.30	-229.90	32.86	0.9998	-0.17	-234.64	37.60	1.1000	-0.19	
LOB12	-233.35	-197.02	-353.94	156.92	1.1000	-0.80	-283.01	85.99	1.1000	-0.44	-256.78	59.76	1.1000	-0.30	-230.46	33.44	0.9998	-0.17	-235.03	38.01	1.1000	-0.19	-0.23	-0.16	-0.24	0.08	1.1000	-0.54	-0.21	0.05	0.9999	-0.34	-0.19	0.04	0.9999	-0.23	-0.17	0.02	0.9996	-0.10	-0.17	0.02	0.9997	-0.11	
LOB13	-22.73	-16.50	-32.56	16.05	1.1000	-0.97	-25.60	9.09	1.1000	-0.55	-22.20	5.70	0.9999	-0.35	-18.29	1.79	0.9995	-0.11	-18.95	2.44	0.9997	-0.15	0.00	0.02	-0.01	0.02	1.1000	1.01	-0.00	0.02	1.1000	1.06	0.00	0.01	0.9995	0.74	0.01	0.01	0.9998	0.48					
LOB18	-147.33	-124.55	-206.15	81.60	1.1000	-0.66	-168.88	45.43	1.1000	-0.36	-155.21	30.66	1.1000	-0.25	-140.55	16.00	0.9999	-0.13	-142.57	18.02	0.9999	-0.14	-0.29	-0.24	-0.42	0.18	1.1000	-0.78	-0.34	0.10	1.1000	-0.44	-0.31	0.07	1.1000	-0.29	-0.28	0.04	0.9999	-0.16	-0.28	0.04	0.9999	-0.18	
LOB27	-0.29	-0.24	-0.22	0.10	1.1000	-0.77	-0.18	0.06	1.1000	-0.46	-0.17	0.04	1.1000	-0.33	-0.15	0.02	1.1000	-0.20	-0.15	0.03	1.1000	-0.20	-13.45	-11.00	-20.38	9.38	1.1000	-0.85	-18.93	5.93	1.1000	-0.54	-15.26	4.25	1.1000	-0.39	-13.78	2.78	1.1000	-0.25	-14.02	3.02	1.1000	-0.27	
LOB29	-0.03	0.01	0.00	0.00	0.9995	0.38	0.01	-0.00	0.9987	-0.46	0.01	-0.01	0.9983	-1.02	0.02	-0.01	0.9971	-1.66	0.02	-0.01	0.9978	-1.58	-0.03	-0.01	-0.02	0.01	1.1000	-0.82	-0.02	0.01	1.1000	-0.48	-0.02	0.00	0.9998	-0.31	-0.02	0.00	0.9996	-0.19					
LOB34	-0.02	-0.01	-0.02	0.01	1.1000	-0.82	0.00	0.01	0.9996	0.79	0.01	0.00	0.9993	0.14	0.02	-0.01	0.9985	-0.64	0.02	-0.01	0.9985	-0.60	-0.08	-0.01	-0.02	0.03	0.9998	2.14	0.00	0.01	0.9996	-2.59	-0.15	0.09	0.9999	-1.60	-0.10	0.05	0.9998	-0.94					
LOB36	-0.27	-0.06	-0.29	0.23	1.1000	-4.04	-0.20	0.15	0.9999	-2.59	-0.55	0.18	0.9999	-0.47	-0.47	0.10	0.9996	-0.26	-0.48	0.11	0.9997	-0.29	-1.22	-0.86	-1.46	0.60	1.1000	-0.70	-1.13	0.27	0.9998	-0.32	-0.98	0.11	0.9995	-0.13	-0.84	-0.02	0.9988	0.02					
LOB37	-0.56	-0.37	-0.83	0.46	1.1000	-1.23	-0.66	0.28	1.1000	-0.76	-0.55	0.18	0.9999	-0.47	-0.47	0.10	0.9996	-0.26	-0.48	0.11	0.9997	-0.29	-1.04	-0.86	-1.46	0.60	1.1000	-0.70	-1.13	0.27	0.9998	-0.32	-0.98	0.11	0.9995	-0.13	-0.84	-0.02	0.9988	0.02					
LOB38	-1.22	-0.86	-1.46	0.60	1.1000	-0.70	-1.13	0.27	0.9998	-0.32	-0.98	0.11	0.9995	-0.13	-0.84	-0.02	0.9988	0.02	-0.48	0.11	0.9997	-0.29	-17.04	-10.85	-12.50	1.64	0.9997	-0.15	-9.08	-1.78	0.9982	0.16	-7.51	-3.34	0.9969	0.31	-8.18	-4.87	0.9960	0.43	-6.62	-4.23	0.9960	0.39	
LOB39	-110.74	-89.05	-144.65	55.60	1.1000	-0.62	-123.67	34.62	1.1000	-0.39	-108.13	19.08	0.9998	-0.21	-95.58	6.53	0.9995	-0.07	-97.80	8.75	0.9996	-0.10	-37.78	-22.89	-33.81	10.73	0.9999	-0.47	-26.78	3.67	0.9996	-0.17	-23.44	0.55	0.9994	-0.02	-20.73	-2.16	0.9989	0.09	-21.31	-1.58	0.9989	0.07	
LOB41	-37.78	-22.89	-33.81	10.73	0.9999	-0.47	-26.78	3.67	0.9996	-0.17	-23.44	0.55	0.9994	-0.02	-20.73	-2.16	0.9989	0.09	-21.31	-1.58	0.9989	0.07	-204.66	-100.67	-384.27	283.60	1.1000	-2.82	-273.04	172.37	1.1000	-1.71	-213.11	112.44	1.1000	-1.12	-153.68	53.01	0.9998	-0.53	-165.50	64.83	0.9997	-0.64	
LOB43	-2.85	-2.55	-4.27	1.72	1.1000	-0.67	-3.53	0.98	1.1000	-0.38	-3.23	0.66	1.1000	-0.26	-2.88	0.33	1.1000	-0.13	-2.92	0.37	1.1000	-0.15	-18.32	-14.10	-35.01	20.92	1.1000	-1.48	-26.88	12.80	1.1000	-0.91	-23.76	9.67	1.1000	-0.69	-20.97	6.88	1.1000	-0.49	-21.56	7.46	1.1000	-0.53	
LOB45	-18.32	-14.10	-35.01	20.92	1.1000	-1.48	-26.88	12.80	1.1000	-0.91	-23.76	9.67	1.1000	-0.69	-20.97	6.88	1.1000	-0.49	-21.56	7.46	1.1000	-0.53	-78.47	-63.48	-118.10	54.62	1.1000	-0.86	-99.74	36.26	1.1000	-0.57	-87.76	24.26	1.1000	-0.36	-77.11	13.63	0.9999	-0.21	-78.52	15.04	1.1000	-0.24	
LOB49	-78.47	-63.48	-118.10	54.62	1.1000	-0.86	-99.74	36.26	1.1000	-0.57	-87.76	24.26	1.1000	-0.36	-77.11	13.63	0.9999	-0.21	-78.52	15.04	1.1000	-0.24	TOTAL	-1410.47	-1021.40	-2209.08	1187.88		-1705.45	684.05		-1477.24	455.85		-1245.21	223.81		-1286.16	264.76						

YEAR 1994 PERCENTILE 0.9900 K 102 N 1000

LOB	WORST		1TH	10K		RE	10TH	5		10K	RE	25TH	5		10K	RE	50TH	5		10K	RE	MEAN		5	10K	RE
	CASE	ACTUAL		EST	DELTA			%	EST				DELTA	%				EST	DELTA			%	EST			
LOB02	0.96	1.32	0.98	0.34	0.9998	0.26	1.11	0.21	0.9990	0.16	1.19	0.14	0.9999	0.10	1.24	0.08	0.9952	0.06	1.24	0.08	0.9952	0.06	1.24	0.08	0.9952	0.06
LOB03	-21.02	-10.47	-15.48	5.01	0.9981	-0.48	-13.59	3.12	0.9965	-0.30	-12.72	2.25	0.9951	-0.21	-11.80	1.33	0.9931	-0.13	-11.86	1.39	0.9931	-0.13	-11.86	1.39	0.9931	-0.13
LOB04	-203.03	39.28	-41.12	80.40	0.9971	2.05	-6.43	45.70	0.9950	1.16	11.50	27.77	0.9937	0.71	29.75	9.53	0.9916	0.24	28.16	11.12	0.9918	0.28	28.16	11.12	0.9918	0.28
LOB05	-59.47	-31.37	-42.94	11.57	0.9970	-0.37	-39.54	8.17	0.9964	-0.26	-36.66	5.29	0.9944	-0.17	-34.04	2.67	0.9926	-0.09	-34.14	2.77	0.9926	-0.09	-34.14	2.77	0.9926	-0.09
LOB06	-0.15	-0.07	-0.11	0.03	0.9983	-0.47	-0.09	0.02	0.9955	-0.27	-0.09	0.01	0.9946	-0.20	-0.08	0.01	0.9926	-0.11	-0.08	0.01	0.9926	-0.11	-0.08	0.01	0.9926	-0.11
LOB07	-0.21	-0.08	-0.11	0.03	0.9963	-0.42	-0.10	0.02	0.9945	-0.27	-0.09	0.01	0.9937	-0.18	-0.08	0.01	0.9919	-0.07	-0.08	0.01	0.9919	-0.07	-0.08	0.01	0.9919	-0.07
LOB08	-0.07	-0.04	-0.05	0.00	0.9963	-0.11	-0.05	0.00	0.9944	-0.06	-0.04	0.00	0.9936	-0.04	-0.04	0.00	0.9917	-0.02	-0.04	0.00	0.9918	-0.02	-0.04	0.00	0.9918	-0.02
LOB09	-0.05	-0.04	-0.04	0.00	0.9984	-0.11	-0.04	0.00	0.9973	-0.07	-0.04	0.00	0.9958	-0.05	-0.04	0.00	0.9944	-0.03	-0.04	0.00	0.9944	-0.03	-0.04	0.00	0.9944	-0.03
LOB10	-6.50	-2.95	-5.23	2.26	0.9995	-0.77	-4.32	1.37	0.9976	-0.48	-3.97	1.02	0.9964	-0.34	-3.57	0.62	0.9947	-0.21	-3.60	0.65	0.9948	-0.22	-3.60	0.65	0.9948	-0.22
LOB11	-233.38	-126.35	-174.38	48.03	0.9974	-0.38	-156.97	30.82	0.9961	-0.24	-148.36	22.01	0.9948	-0.17	-137.58	11.23	0.9930	-0.09	-138.22	11.67	0.9930	-0.09	-138.22	11.67	0.9930	-0.09
LOB12	-233.35	-126.38	-174.94	48.58	0.9975	-0.38	-157.63	31.27	0.9961	-0.25	-148.57	22.21	0.9948	-0.18	-137.23	10.87	0.9929	-0.09	-138.33	11.97	0.9930	-0.09	-138.33	11.97	0.9930	-0.09
LOB13	-0.23	-0.09	-0.13	0.03	0.9965	-0.36	-0.12	0.02	0.9951	-0.23	-0.11	0.02	0.9940	-0.16	-0.10	0.01	0.9924	-0.06	-0.10	0.01	0.9924	-0.06	-0.10	0.01	0.9924	-0.06
LOB17	-22.73	-4.81	-10.81	6.00	0.9968	-1.25	-8.31	3.50	0.9954	-0.73	-6.87	2.06	0.9930	-0.43	-5.53	0.72	0.9916	-0.15	-5.72	0.90	0.9917	-0.19	-5.72	0.90	0.9917	-0.19
LOB18	0.00	0.02	0.02	0.01	0.9989	0.31	0.02	0.01	0.9973	0.21	0.02	0.00	0.9961	0.14	0.02	0.00	0.9950	0.08	0.02	0.00	0.9951	0.09	0.02	0.00	0.9951	0.09
LOB26	-147.33	-79.73	-110.09	30.36	0.9980	-0.38	-97.14	17.42	0.9960	-0.22	-91.61	11.88	0.9945	-0.15	-85.88	5.96	0.9925	-0.07	-86.35	6.82	0.9928	-0.08	-86.35	6.82	0.9928	-0.08
LOB27	-0.29	-0.16	-0.22	0.06	0.9977	-0.38	-0.19	0.04	0.9966	-0.24	-0.18	0.02	0.9945	-0.16	-0.17	0.01	0.9924	-0.07	-0.17	0.01	0.9927	-0.09	-0.17	0.01	0.9927	-0.09
LOB28	-0.15	-0.08	-0.11	0.03	0.9976	-0.41	-0.10	0.02	0.9968	-0.26	-0.10	0.02	0.9955	-0.19	-0.09	0.01	0.9932	-0.11	-0.09	0.01	0.9935	-0.11	-0.09	0.01	0.9935	-0.11
LOB29	-13.45	-7.13	-10.48	3.35	0.9983	-0.47	-9.26	2.14	0.9971	-0.30	-8.64	1.51	0.9959	-0.21	-7.96	0.85	0.9941	-0.12	-8.07	0.94	0.9944	-0.13	-8.07	0.94	0.9944	-0.13
LOB33	-0.03	0.03	0.03	0.00	0.9932	0.14	0.03	0.00	0.9912	0.05	0.03	0.00	0.9902	0.00	0.03	-0.00	0.9884	-0.04	0.03	-0.00	0.9887	-0.04	0.03	-0.00	0.9887	-0.04
LOB34	-0.02	-0.01	-0.01	0.00	0.9988	-0.37	-0.01	0.00	0.9977	-0.22	-0.01	0.00	0.9966	-0.17	-0.01	0.00	0.9950	-0.10	-0.01	0.00	0.9952	-0.11	-0.01	0.00	0.9952	-0.11
LOB35	-0.08	0.08	0.05	0.02	0.9947	0.24	0.06	0.01	0.9932	0.12	0.06	0.00	0.9920	0.06	0.06	0.00	0.9901	0.01	0.06	0.00	0.9902	0.01	0.06	0.00	0.9902	0.01
LOB38	-0.27	0.08	-0.00	0.08	0.9979	1.03	0.03	0.05	0.9953	0.67	0.04	0.03	0.9932	0.42	0.06	0.02	0.9915	0.21	0.06	0.02	0.9916	0.22	0.06	0.02	0.9916	0.22
LOB37	-0.56	-0.17	-0.29	0.12	0.9967	-0.68	-0.23	0.06	0.9943	-0.37	-0.21	0.04	0.9932	-0.22	-0.17	0.00	0.9901	-0.01	-0.17	0.00	0.9905	-0.02	-0.17	0.00	0.9905	-0.02
LOB38	-1.22	-0.34	-0.58	0.25	0.9969	-0.73	-0.48	0.15	0.9950	-0.43	-0.43	0.10	0.9937	-0.28	-0.36	0.04	0.9922	-0.13	-0.39	0.05	0.9923	-0.15	-0.39	0.05	0.9923	-0.15
LOB39	-1.704	-3.58	-6.32	2.76	0.9960	-0.78	-5.23	1.68	0.9949	-0.47	-4.59	1.03	0.9932	-0.29	-4.12	0.57	0.9918	-0.16	-4.20	0.64	0.9922	-0.18	-4.20	0.64	0.9922	-0.18
LOB40	-110.74	-43.59	-65.25	21.66	0.9962	-0.50	-58.47	14.88	0.9947	-0.34	-54.07	10.48	0.9943	-0.24	-49.07	5.48	0.9921	-0.13	-49.48	5.89	0.9922	-0.14	-49.48	5.89	0.9922	-0.14
LOB41	-37.78	-10.59	-15.60	5.01	0.9967	-0.47	-14.20	3.60	0.9955	-0.34	-13.17	2.58	0.9943	-0.24	-12.09	1.49	0.9931	-0.14	-12.19	1.59	0.9932	-0.15	-12.19	1.59	0.9932	-0.15
LOB43	-204.66	35.92	-54.40	90.32	0.9973	2.51	-14.21	50.13	0.9951	1.40	6.46	29.46	0.9938	0.82	25.38	10.55	0.9914	0.29	23.88	12.05	0.9915	0.34	23.88	12.05	0.9915	0.34
LOB44	-2.85	-1.58	-2.13	0.55	0.9972	-0.35	-1.98	0.38	0.9961	-0.24	-1.84	0.26	0.9943	-0.16	-1.72	0.14	0.9926	-0.09	-1.72	0.14	0.9926	-0.09	-1.72	0.14	0.9926	-0.09
LOB48	-18.32	-11.83	-17.74	5.91	1.1000	-0.50	-15.37	3.55	0.9969	-0.30	-14.37	2.54	0.9993	-0.21	-13.30	1.48	0.9969	-0.12	-13.48	1.68	0.9977	-0.14	-13.48	1.68	0.9977	-0.14
LOB49	-78.47	-41.19	-61.07	19.89	0.9986	-0.48	-53.20	12.01	0.9970	-0.29	-48.60	7.41	0.9950	-0.18	-44.81	3.62	0.9933	-0.09	-45.32	4.13	0.9933	-0.10	-45.32	4.13	0.9933	-0.10
TOTAL	-1410.47	-425.87	-806.56	382.70			-856.02	230.15			-576.03	150.17			-493.15	67.29			-500.39	74.53			-500.39	74.53		

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