

**A Retirement Plan Based On Fixed Accumulation
And Variable Accrual**

by

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Abstract

A new type of retirement plan is proposed, aimed at achieving a compromise between stability in cost for the plan sponsor and guaranteed benefits for the participants. The proposed plan is shown to smooth out the fluctuation in benefits that occurs in a money purchase plan.

MONEY PURCHASE PLAN

C = allocation per member (paid by employer)

i_t = interest earned between time $t-1$ and t

Benefit arising from allocation at time t for a member
 n years from retirement:

$$C \times (1 + i_{t+1}) \times (1 + i_{t+2}) \times \dots \times (1 + i_{t+n})$$

VARIABLE ACCRUAL PLAN

C = true employer cost per member

C_t = allocation per member at time t

In general: $C_t \neq C$

Benefit arising from each allocation:

$$= C_t \times g, \text{ where } g > 1$$

VARIABLE ACCRUAL BENEFIT

Fund arising from allocation at time t for a member
 n years from retirement:

$$C_t \times (1 + i_{t+1}) \times (1 + i_{t+2}) \times \dots \times (1 + i_{t+n})$$

At time t :

- forward contract between member and employer
- accumulation of C_t is sold by member to employer
- price of forward contract = $C_t \times g$
- settlement date = retirement date of member

**Settlement of forwards are added to / subtracted from
the allocation to the active members**

HOW IS C_t CALCULATED?

$$C_t = C \pm \frac{\text{Settlement of forwards at time } t}{\text{no. of active members at time } t}$$

HOW IS g CALCULATED?

g = expected average accumulation on C_t

Let:

i_0 = expected average interest rate

x_e = average entry age

x_b = average age of benefit payment

$$\text{Then } g = \frac{\ddot{S}^{i_0}}{x_b - x_e}$$

POTENTIAL PROBLEMS

1) Investment returns are very poor or very good.

$$C_t = C + \frac{\text{Settlement of forwards at time } t}{\text{no. of active members at time } t}$$

- Very poor investment returns $\rightarrow C_t < 0$
- Very good investment returns $\rightarrow C_t > \text{tax limit}$
- True employer cost may have to vary from C to keep C_t within acceptable limits.

2) Number of active members is declining.

- Settlements spread over fewer members
- C_t becomes more volatile
- True employer cost more likely to vary from C

PROJECTIONS FOR A MODEL PLAN

- $C = 1$ unit per member, paid annually in advance
- 1 active member at each age from 25 to 64
- All members enter at 25 and retire at 65

INITIAL CONDITIONS

- $i_0 = 2\%$
- Asset share of member aged $x = \ddot{S}_{x-25}^{2\%}$

VARIABLE ACCRUAL PLAN

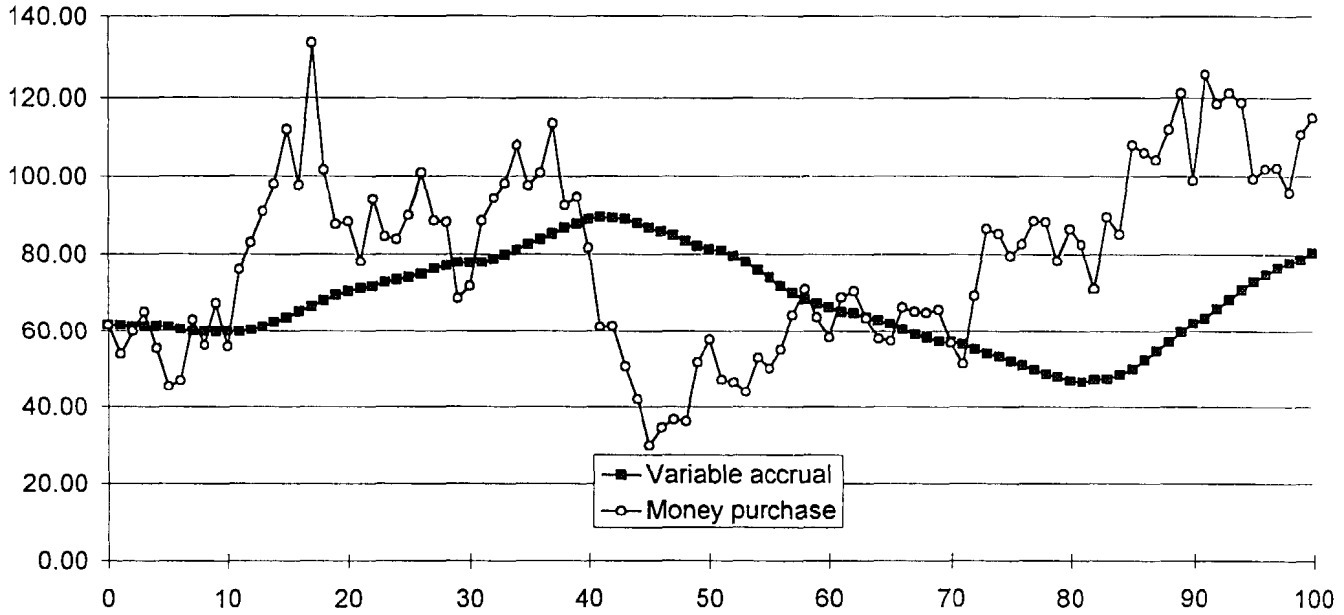
- $g = \frac{\ddot{S}_{40}^{i_0}}{40}$
- $0 \leq C_t \leq 2$

STOCHASTIC PROJECTIONS

- Annual investment returns are i.i.d.
- $\text{LN}(1+i_t) \sim N(0.02, 0.15)$

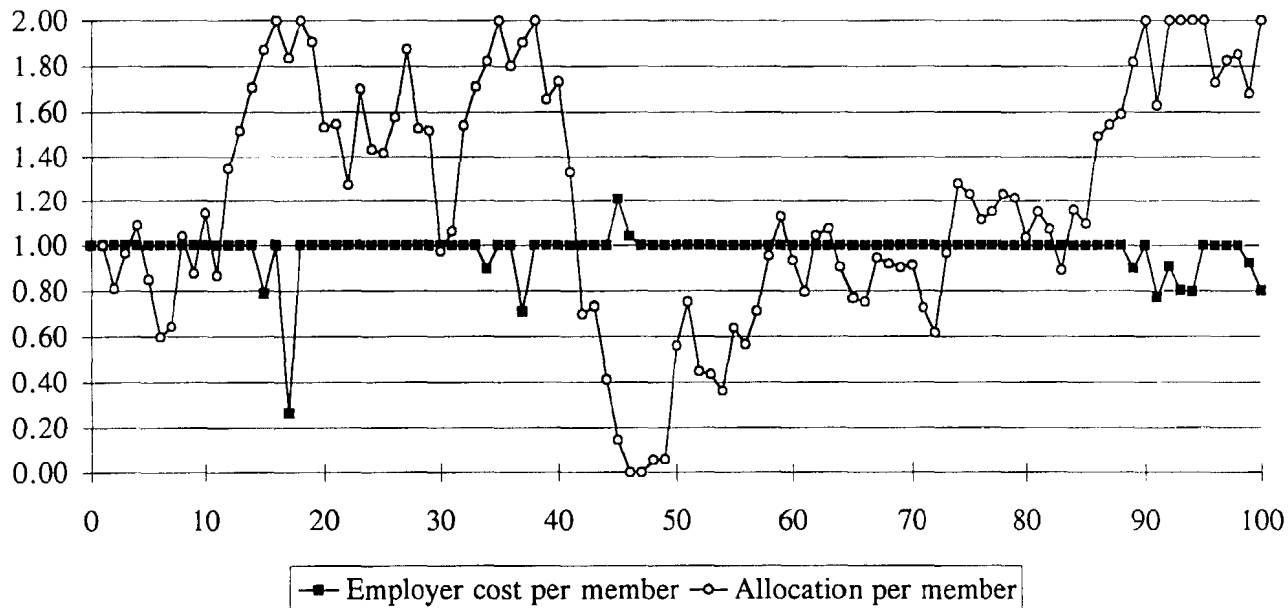
Comparison of benefit outgo

221



Variable accrual plan

zzz



COMPARISON OF PLANS

Money purchase benefit is based on:

- fixed allocation
- variable accumulation

Variable accrual benefit is based on:

- variable allocation
- fixed accumulation

Advantages of variable accrual plan

- inter-generational smoothing
- security of accrued benefits
- benefit is more predictable

