

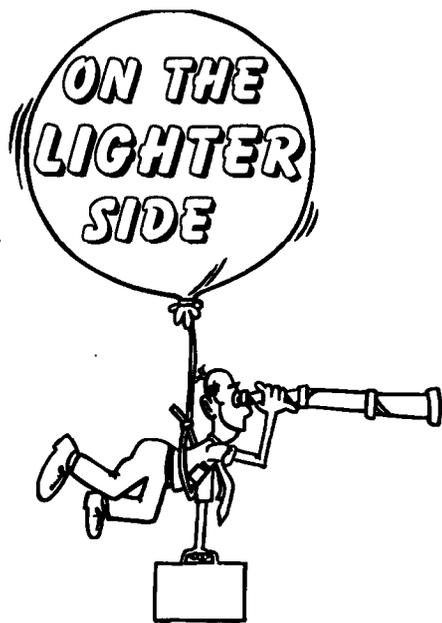


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Ping pong balls revisited

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Puzzle Editor

In the October 1992 issue, an Actuary reader sought comments on the relative probabilities for success in the National Basketball Association (NBA) lottery. The 11 teams who don't make the playoffs are thrown into a lottery to determine the order in which they will draft college players. The team with the worst record gets 11 ping pong balls; the next worst, 10 balls; and so on, down to one ball. The balls are identified by team and thrown into a hopper. One is randomly drawn, and that team gets first pick among available players. The probability for any team is simple: divide the number of balls it is allotted by 66 (total).

But what is the probability of Team A getting the second pick? This question was subdivided: (1) given that no balls have been drawn yet or (2) given that A is not the first team.

The reader commented that it's important to know that any remaining balls of the first team chosen are not removed from the hopper. If one of

NBA LOTTERY DRAFTING PROBABILITIES

Draft Pick	Rank from worst to 11th worst										
	Worst	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th
	# Balls										
	11	10	9	8	7	6	5	4	3	2	1
1	0.1667	0.1515	0.1364	0.1212	0.1061	0.0909	0.0758	0.0606	0.0455	0.0303	0.0152
2	0.1556	0.1447	0.1331	0.1207	0.1077	0.0940	0.0797	0.0648	0.0494	0.0334	0.0169
3	0.1435	0.1368	0.1289	0.1197	0.1092	0.0973	0.0842	0.0698	0.0542	0.0373	0.0192
4	0.5342	0.2992	0.1316	0.0350	---	---	---	---	---	---	---
5	---	0.2677	0.3463	0.2697	0.1163	---	---	---	---	---	---
6	---	---	0.1238	0.2821	0.3541	0.2399	---	---	---	---	---
7	---	---	---	0.0516	0.1881	0.3675	0.3929	---	---	---	---
8	---	---	---	---	0.0187	0.1049	0.3188	0.5577	---	---	---
9	---	---	---	---	---	0.0055	0.0475	0.2310	0.7159	---	---
10	---	---	---	---	---	---	0.0012	0.0159	0.1320	0.8508	---
11	---	---	---	---	---	---	---	0.0001	0.0030	0.0482	0.9487

Source: Kevin Larsen

these balls is chosen, it is discarded and the selection continues. Mark D. Evans, John Rutter, Steve Powell, Kevin Larsen, Richard Q. Wendt, Marc I. Whinston, Joe Nunes, and David Horrocks (son of SOA member Geoffrey Horrocks) all disagreed with the importance of such knowledge. As Evans wrote, "Actually, whether the balls are discarded immediately or as encountered is irrelevant. Removal of a redundant ball does not alter the relative probabilities of drawing the remaining eligible balls."

Question (2) was answered in two ways. The simpler way was to assume that the team receiving the first pick was known. Then the probability is $A/(66-N)$, where A is the number of balls that Team A started with, and N is the number of balls that the team drawn first started with. For example, assume that A starts with 10 balls and the team with 5 balls is drawn first. The probability of A being second is 10/61. The second way, used by Rutter and Larsen, only assumes that A is not the first team. Then, the required probability can be expressed, as suggested by Rutter, as:

$$\sum_{\substack{N=1 \\ N \neq A}}^{11} \left(\frac{A}{66-A} \right) \left(\frac{N}{66-N} \right)$$

the chances of each other team being chosen first from a universe excluding A, then multiplied by the probability of A being chosen second.

Of the seven who responded to Question (1), all agreed it is the sum of the conditional probabilities that

each team other than A is drawn first, times the probability given in Question (2):

$$\sum P(T \text{ is chosen first}) \cdot P(A \text{ is picked second} \mid T \text{ is chosen first})$$

which is:

$$\sum_{\substack{N=1 \\ N \neq A}}^{11} \left(\frac{N}{66} \right) \left(\frac{A}{66-N} \right) = \left(\frac{N}{66} \right) \sum_{\substack{N=1 \\ N \neq A}}^{11} \left(\frac{A}{66-N} \right)$$

Powell notes that for cases where A = 1 through 7, the probability of being second is greater than the probability of being first, and vice versa for A = 8 through 11.

Wendt related how "the local sports talk radio station was talking about the draft lottery probabilities one night and [he] was able to get on air with an 'expert' actuarial analysis. ... The host did not find this to be the most stimulating conversation of the evening."

Evans took his solution one step further, enclosing a formula for the probabilities that team T gets the nth pick, given that picks 1 through n-1 are known.

Larsen provided a summary of probabilities for all draft picks that incorporates the fact that only the first three result from a drawing. The remaining eight picks are given to the remaining eight teams ranked from worst to best record. (See Larsen's table on this page.)

Anyone who would like copies of calculations by those mentioned in this article should contact The Actuary staff editors at the Society of Actuaries office, 708-706-3500.