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Interactive Instructional Software for Actuarial Mathematics

Arnold F. Shapiro and Donald A. Jones

Penn State University and Oregon State University

Abstract

In an earlier paper, Shapiro **et. al.** (1992) discussed an interactive instructional software for actuarial students. The purpose of that paper was to show that techniques of computer assisted instruction (CAI) can materially improve both classroom instruction and independent study in mathematics of finance courses.

This paper is a sequel to Shapiro **et. al.** in that it discusses a similar role for CAI in topics specific to the text **Actuarial Mathematics**, by Bowers **et. al** (1986). The topics covered are the previous topics of actuarial notation, basic concepts, and what-if analysis, along with some new ideas for the presentation of stochastic models.

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INTRODUCTION

In recent years, more and more educators are investigating the role of computer assisted instructional (CAI) techniques in the education of students and the presentation of research results. Among the applications have been the packages developed in chemistry by Smith and Jones (1987), in the health sciences by O'Neill (1990), in foreign languages by Patrikis **et. al.** (1990), and in insurance by Shapiro (1991).

While these applications have many positive attributes, one of their major limitations has been their implementation requirements since, among other things, workstations often include an audio-visual interface. In response to this concern, Shapiro **et. al.** (1992) discussed an interactive instructional software for mathematics of finance that could materially improve both classroom instruction and independent study at relatively low cost.

This paper is a sequel to Shapiro **et. al.** in that it discusses a similar role for CAI in topics specific to the text **Actuarial Mathematics**, by Bowers **et. al** (1986). The topics covered are the previous topics of actuarial notation, basic concepts, and what-if analysis, along with some new ideas for the presentation of stochastic models.

ACTUARIAL NOTATION

One of the first thing an actuarial student learns about actuarial mathematics is that it involves notation unique to actuarial science and that there is such a thing as International Actuarial Notation. Thus, these students need to be familiar with actuarial notation and instructional software for actuarial mathematics should help to accomplish this.

With this in mind, a module was developed which presents dynamically what is in Appendix 4 of **Actuarial Mathematics**. The basic screen from this module is shown in the Figure 1. As described in the information scroll[1], on the upper left, the actuarial symbol is made up of six parts, labelled A to F in the content area[2] of the screen. Here,

[1]The information scroll is the rectangle to the upper left of the page. It contains information about and a description of the topic on the page. Note that this rectangle has a scroll on its right side. If the text exceeds the space provided, you can use the scroll to view the rest of it.

[2]The content area is the large rectangle on the right of the page where most of the content is located.

the "A" is highlighted, indicating the basic symbol location, and two examples of basic symbols, "v" and "E", are provided. By moving the mouse to each of the locations, "B" to "F", the user can get a description of the modifier and an example of its use.

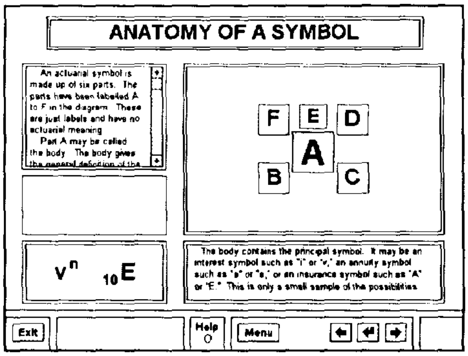

Figure 1

It is clear that this way of presenting actuarial notation is an improvement over traditional transparencies and slides. A primary advantage is that the highlighting provides an effective cuing device which helps students attend to important concepts and helps coordinate the relationships between the text and the figure.

BASIC CONCEPTS

Many of the basic concepts in Chapter 3 of **Actuarial Mathematics** can be readily presented using a dynamic format. This section shows how this can be done for such things as survivor probabilities and fractional age assumptions.

Survivor Probabilities

Table 3.1.2 summarizes ways to characterize the distribution of the time-until-death random variable. One approach to making that table dynamic is depicted in Figure 2, which gives four ways of showing this: distribution function, probability density function, survival function, or force of mortality. As indicated, any one of the four can be expressed in terms of any of the others.

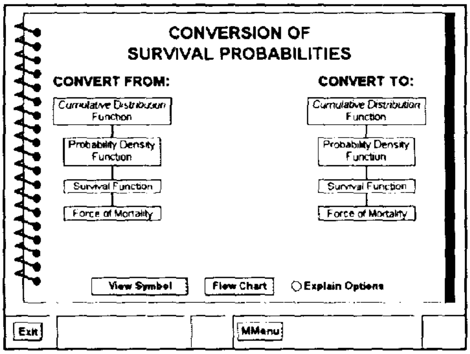

Figure 2

The student implements this screen by choosing an item from the "CONVERT FROM" column and item from the "CONVERT TO" column, and then clicking on the "View Symbol" button, as shown in Figure 2. Students who are unfamiliar with the functions can click the "Explain Options" button to get a description of each of them.

This type of presentation can be helpful to a student involved in self-study, without the benefit of an instructor, or for an instructor in the classroom, because, unlike a textbook, it gives a dynamic representation of the concepts.

Fractional Age Assumptions

Another important area of basic concepts has to do with fractional age assumptions and their implications. Three dynamic ways of presenting this material are by basic representation, using either bullets or graphs, and by tables.

Figure 3 is used to describe the three assumptions that are common to actuaries: uniform distribution of deaths, Balducci, and constant force.

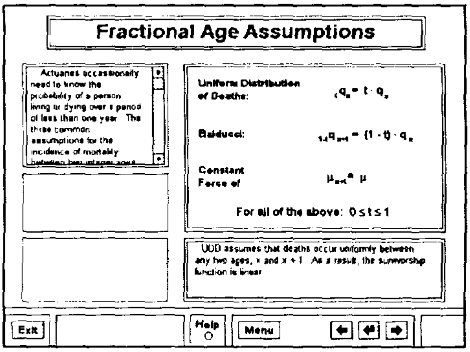

Figure 3

Passing the mouse over any of the laws highlights that item and gives a brief discussion of its attributes in the rectangle below the content rectangle. In this instance, the uniform distribution of deaths is being discussed.

This next screen shows the relative relationships of the force of mortality under the different assumptions. Here, the graph under constant force is highlighted and provided with an explanation.

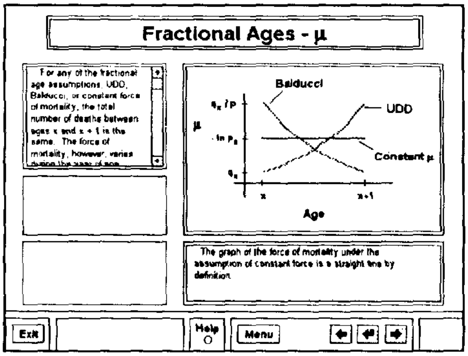

Figure 4

Finally, as shown in Figure 5, the impact of a change in assumptions on the fractional age calculations can be analyzed using a dynamic table.

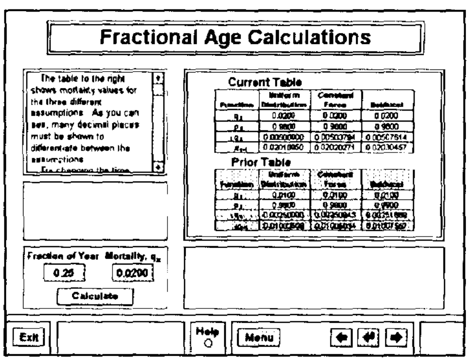

Figure 5

The user can vary the mortality rate to facilitate sensitivity study. Thus, as indicated, the user gets to see the before and after results of changing these parameters. Additionally, passing the mouse over a cell of the table activates a popup window which gives a definition of the cell, which helps conceptualize the relationships between the assumptions, the cell data, and the table.

LIFE INSURANCE CONCEPTS

Presentation screens and tables are also very helpful in presenting the insurance concepts of chapter 4 of **Actuarial Mathematics**. Figure 6, for example, shows a one-year term insurance representation.

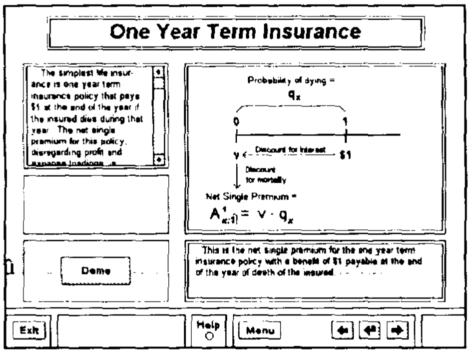

Figure 6

What makes this different than a standard textbook representation is its dynamics. As indicated, when the mouse is passed over the net single premium equation, an explanation of the equation appears at the bottom of the content area.

As shown in Figure 7, a complement to the foregoing representation would be a dynamic table based on the concept. As with other tables of this sort, the interest rate and/or the mortality table can be changed.

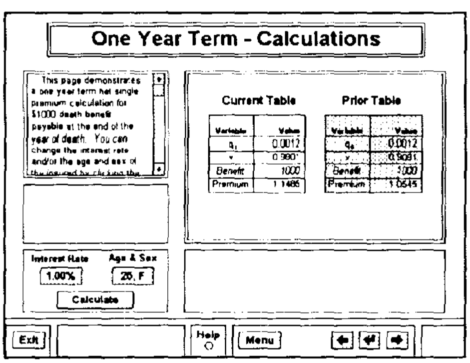

Figure 7

It is obvious that these types of analyses can be generalized to a term insurance of any number of years.

What-if analysis

The same type of what-if analysis that was done for the theory of interest (Shapiro **et. al.**, p. 78) can be done for life contingency functions, as shown in Figure 8. Here, however, since a mortality table is used, computations are more involved and can be done more efficiently by linking to a spread sheet, Excel in this case.

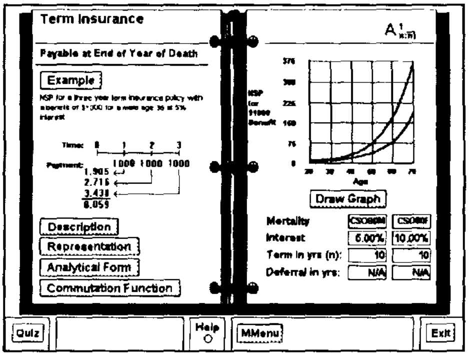

Figure 8

At the top left of the page is the general category of the page, Term Insurance, in this case. The subcategory tells us that the insurance is payable at the end of the year of death.

The next item is the simple representation of a term insurance. Here, the example shows the present value of 3-year term insurance, at a compound interest rate of 5% per year. Clicking on the "Example" button activates the example and shows in a step-wise progression how the time diagram is drawn.

The rest of the buttons on the left hand side behave just as they did in the simple interest example. So, there is a "Description" button, which provides a verbal description of the topic, a "Representation" button provides a general representation, an "Analytical Form" button gives an equation for the function based on the general representation, and the "Computational Form" button, which gives the commutation functions.

The right side of the page is used to investigate the sensitivity of the function. The first few items are the same as its theory of interest counterpart. Thus, at the top right of the page is the symbol which represents the topic, in this instance, a term insurance. The rest of the right side of the page is used to investigate the sensitivity of the symbol.

Again, there are two steps to this process: (1) set the values of the parameters, and (2) click the "Draw Graph" button. There are now 8 potential parameters, 4 for each curve; these are the interest rate, the mortality table, the term of the contract, and the deferred period, if any. In the figure, the first three are initially set at the CSO80 male and female table, 5% and 10%, and 10 years, respectively. Clicking on the "Draw Graph" button draws the new graph based on the settings of these parameters.

RANDOM VARIABLES

A basic innovation of **Actuarial Mathematics** over its predecessor is that it explicitly regards the time until a contingency as a random variable. One way to convey this idea to students is show in Figure 9, which gives an example where the random event is an accident, rather than death, and the time horizon is two years. The situation is that we have a group of 18 year olds, and we are interested in monitoring the distribution of the time until an accidents for this group.

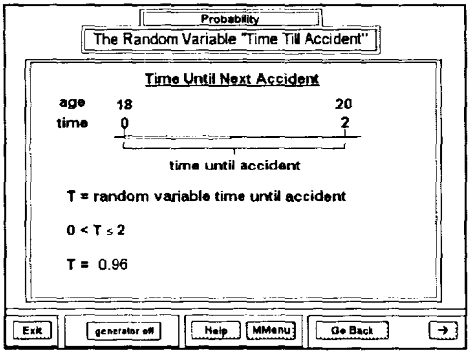

Figure 9:

The student obtains a visual image of the random variable by clicking the button labelled "generator on," which starts the random number generator. An example of this sort which deals with a situation a college student can easily relate to is ideal for getting across the notion of a random variable.

One can easily extrapolate from this to the case involving time until death. In this case, we would show, for example, the age at issue and the age at death of the insured, and if the time until death were integral, we would use "K" instead of "T." Extensions on this theme are the inclusion of variances, and the option for the student to choose a particular table as the basis for the simulation.

This next example expands the notion of a random variable one step further. Now, in addition to the issue of whether there is an accident or not, we have the probability distribution of the number of accidents, zero, one, two or three.

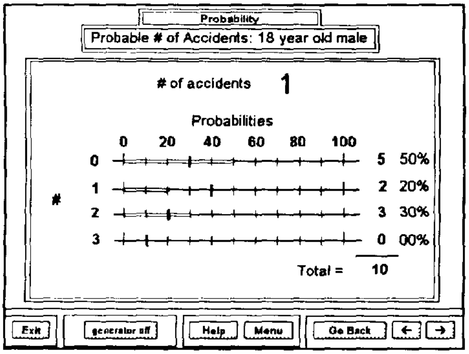

Figure 10

Turning the generator on long enough to generate 10 observations will provide an empirical distribution which is then compared to the underlying probability distribution in Figure 10. The heavy tic mark on each line shows the underlying probability for that result while the relative frequency in the sample is shown by the double line. For example, for zero accidents the underlying probability is 30% and there were 5 of the ten observations that had zero accidents for a relative frequency of 50%. (The frequencies and relative frequencies are also shown at the ends of the lines of the results.) At the result of one accident, the probability is 40% and the frequency for the sample was 2 so the relative frequency was 20%. Similarly, we had considerable disparity between the actual probabilities of two and three accidents, and the empirical probabilities based on the small sample. Thus, by controlling the size of the sample, the student gets a sense of how sample size impacts the credibility of empirical distributions.

THE ACTUARIAL CALCULATOR

One interactive facility that can be very helpful for analyzing the impact of various assumptions is the actuarial calculator shown in Figure 11.

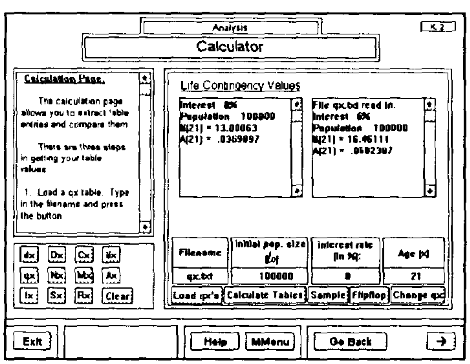

Figure 11

As indicated in the instructions on the information scroll, the student has to first load the q_x's, and calculate the tables. Alternatively, and this is an interesting application of computers, rather than use a deterministic table, the user can use a Monte Carlo process to generate a survivorship group, from which an experience table can be developed. The third step is to display the table.

In the example shown, the table is the Illustrative Life Table from **Insurance Mathematics**, the radix is 100,000, the attained age is 21, and the tables on the upper right compare the present value of a life annuity and whole life insurance at an interest rate of 8 percent (left table) with the similar contracts at 6 percent (right table).

The function keys on the lower left provide the actuarial functions shown in the table.

In this module, the computations of the actuarial functions are done in C++ to improve their speed. They could, of course, be done in a spread sheet, but this may reduce the computation speed considerably.

The next screen, shown in Figure 12, displays the calculator option for use in those instances when it is desirable to compare the entire table.

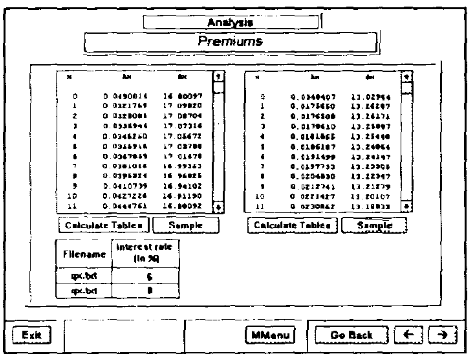

Figure 12

Once again, we have the whole life and annuity values for the same mortality table but two different interest rates.

Finally, as shown in Figure 13, the calculator has the option of displaying the graphs of the actuarial functions. In this instance, the graph shows the net single premium for a whole life insurance.

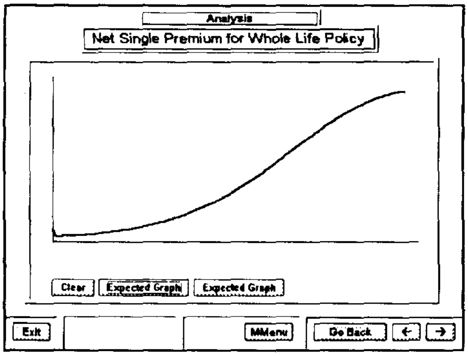

Figure 13

CONCLUSION

In some respects, the transition from the CAI for theory of interest to CAI for actuarial mathematics is relatively straightforward. This is the case, for example, with screens that do not explicitly involve decrement tables, such as those with bullets and simple animation. In these instances, all the computations can be done efficiently in a software construction set, such as ToolBook. However, where decrement tables are involved, the computations are more complicated so that speed becomes a consideration, and dynamic links must be used to incorporate other software, such as spreadsheets or C++. Nonetheless, since this other software can be implemented in a runtime version, they represent little or no extra cost to the user.

The purpose of this paper has been to show how relatively low cost CAI can materially improve communications for both classroom instruction and independent study of actuarial mathematics topics. It seems clear that this is the case. The CAI classroom environment can transform conventional actuarial lectures into a dynamic interaction between instructor and student and can be made self-instructional to accommodate individual learning styles and rates.

ACKNOWLEDGEMENT

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