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Economic Scenario Generator for Insurance and Pension

Rational Decision Making Under Uncertainty

by
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ABSTRACT

We develop a stochastic generator for the generation of scenarios of the S & P 500 index, dividend yield, consumer price index, and U.S. Treasury yields. We first create a set of 'stylized facts' for these series. We estimate statistical models for these series. These in-sample statistical models are themselves not suitable for generation of scenarios for decision making, but instead are additional 'stylized facts' that assist in model development. The 'best' statistical model according to standard statistical model selection criteria can easily lead to a model that is highly unsuitable for generation of scenarios for decision making. We develop a stochastic generator that is suitable for decision making under uncertainty.

Some key words : Stylized facts; Double mean reverting process; ARIMA models; Transfer functions; Green's functions; Diffusion models; Rational decision making; Insurance; Pension; Uncertainty.

Economic Scenarios

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DMRP™ Model

- We based our Economic Model on Mark Tenney's DMRP™

DMRP™

- $du = \kappa_u(\theta - \lambda_u - u)dt + \sigma_u dz_u$
- $d\theta = \kappa_\theta(\bar{\theta} - \lambda_\theta - \theta)dt + \sigma_\theta dz_\theta$
- where z_u and z_θ follow a Wiener process with correlation ρ.

Series	Symbol	relationship	residuals
Short rate	r_t	$u_t = \ln(r_t)$	ε_{ut}
Intermediate rate	s_t	$\theta_t = \ln(s_t)$	$\varepsilon_{\theta t}$
Total Return Stock Index	T_t	$x_t = \Delta \ln(T_t)$	ε_{xt}
Price Return Stock Index	S_t	$y_t = \Delta \ln(S_t)$	ε_{yt}
Dividend Index	D_t		
Dividend Yield	$d_t = \frac{\sum D}{12 S_t}$		ε_{dt}
Inflation Index	P_t	$w_t = \Delta \ln(P_t)$	ε_{wt}

dY=(b+AY)dt+G'dZ

Regressions on u and θ

$x_t = .0168u_t - .0137\theta_t + .4349x_{t-1} + .0134 + \varepsilon_{xt}$

$d_t = .0001u_t + .9312d_{t-1} + .0006 + \varepsilon_{dt}$

Total Return Stock Index did not regress on u and θ.

Residual Formulae

$\varepsilon_{ut} = \Delta u_t - \kappa_u(\theta_t - u_t - \lambda_u)\,dt$

$\varepsilon_{\theta t} = \Delta\theta_t - \kappa_\theta(\bar{\theta} - \theta_t - \lambda_\theta)\,dt$

$\varepsilon_{wt} = w_t - (.0038u_t - .0037\theta_t + .4349w_{t-1} + .0034)$

$\varepsilon_{xt} = x_t - \bar{x}$

$\varepsilon_{dt} = d_t - (.0001u_t + .9312d_{t-1} + .0006)$

Residual Formulae

- $dt = 1/12$
- $\kappa_u = 0.5$
- $\lambda_u = 0.1982$
- $\kappa_\theta = 0.15$
- $\lambda_\theta = 0.4732$
- $\bar{\theta}$ = ln(0.106821)
- $\bar{x}$ = 0.00982 the average total return

Annualized Covariance Matrix

	u	θ	w	x	d
u	.097515	.018801	.000109	-.007519	.000015
θ	.018801	.042590	.000169	-.008271	.000025
w	.000109	.000169	.000080	-.000219	.000008
x	-.007519	-.008271	-.000219	.123506	-.000076
d	.000015	.000025	.000001	-.000076	.0000003

Choleski Decomposition G'

	u	θ	w	x	d
u	.312273	.000000	.000000	.000000	.000000
θ	.058927	.197781	.000000	.000000	.000000
w	.000130	.000075	.008927	.000000	.000000
x	-.020078	-.031646	-.320726	.145931	.000000
d	.000048	.000114	.000094	-.000474	.000194

b Vector

u	$-\kappa_u\lambda_u$
θ	$\bar{\theta}\kappa_\theta - \kappa_\theta\lambda_\theta$
w	0
x	$-0.5(\sigma_x)^2$
d	0

b Vector, Cont.

The stock return under risk-neutral pricing should be $r_f - 0.5(\sigma_x)^2$ where r_f is the risk free rate, and $(\sigma_x)^2$ is the variance of the stock residuals. Since the risk-free rate will be dynamic, the drift term will only contain the variance term. We will then build up the risk neutral stock returns from the yield curve process.

Generalized Method of Moments
Realistic Scenarios
&
Stylized Facts

Using GMM methods, you adjust the contents of the b vector and the A matrix to force the model to replicate certain stylized facts.

You leave the contents of G' alone.

A Matrix

	u	θ	w	x	d
u	$-\kappa_u$	κ_u	0	0	0
θ	0	κ_θ	0	0	0
w	0	0	0	0	0
x	0	0	0	0	0
d	0	0	0	0	0

Risk Neutral Parameter Values

κ_u 0.689

λ_u 0

κ_θ 0.0369

λ_θ 0

$\theta = 0.106821$

$0.5(\sigma_x)^2 = .011753$

Process Variable Names

u_t log of instantaneous rate

θ_t log of intermediate target rate

ε_{w_t} inflation residual

ε'_{x_t} Stock residual with variance drift

ε_{d_t} dividend yield residual

Inflation

$$w_t = .0038u_t - .0037\theta_t + .4349w_{t-1} + .0034 + \varepsilon_{w_t}$$

Dividend Yield

$$d_t = \max(\min(.0001u_t + .9312d_{t-1} + .0006 + \varepsilon_{d_t}, .01/12), .08/12)$$

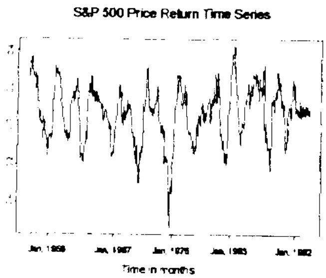
Total Return

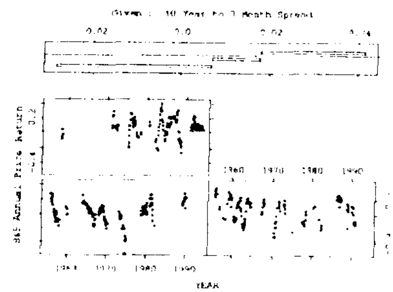
$$x_t = \exp(\Sigma\delta_t)\exp(\varepsilon_{x_t})$$

Price Return

$$y_t = x_t / (1 + d_t)$$

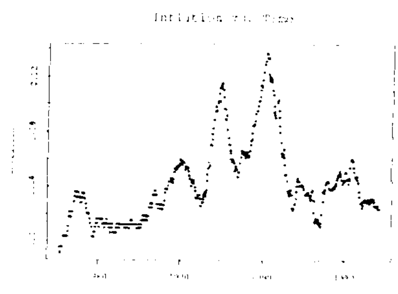
δ_t is the instanteous risk free force of interest

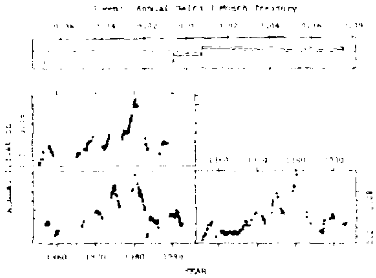




Realistic-Stock Market

- Model: 3 Observed Environments for the Stock Market, related to the spread between 10 year and 3 month treasuries
  - High
  - Middle
  - Low





Realistic-Inflation

- Model: 3 Observed Environments for the Stock Market, related to the delta between the current and last year's 3 month Treasury
  - High
  - Middle
  - Low