

Stability of Representative Pricing Scenarios

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Abstract

The primary focus of this paper is the adaptation of the representative scenario method such that the distribution of the primary rates used for product pricing, are consistent, regardless of the initial random number seed. A preliminary question is how many stochastically generated scenarios are required for consistency. This paper looks at the implications of: the use of antithetic random normal deviates upon the scenario process, ways of modifying the candidate list, and the choice function within the representative scenario process.

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SPDA PRICING

- ◆ Scenario consistency
- ◆ Impact of APL Seed



Why inconsistent results?

Is it the APL seed and/or the representative process?



Or are 1000 scenarios not enough?

Representative Process

- ◆ Create Scenarios
- ◆ Select Candidate List
- ◆ Identify Best Candidate
(Choice Function)

Weighing the options



- ◆ Identified 3 possible solutions
 - Eight combinations or methods
- ◆ Tested them
- ◆ Analyzed resulting data

Create Scenarios

- ◆ Random numbers
 - Key rates
 - Shape codes
- ◆ Additional Option
 - Antithetic normal (rates)

Candidate List Options

- ◆ Pricing “Adaptation”
 - $\mu \pm 2\sigma$
 - Percentiles:
 - » 5, 10, 15, 85, 90, 95
- ◆ Standard method
 - Matched extremes

Choice Function



- ◆ All Years Only
- ◆ All Years and First Year
 - Weights for First Year Statistics
 - 90% liability cash flows gone by end of 10 years



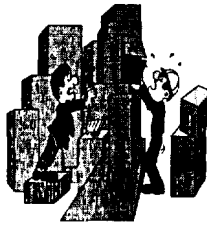
Poss.	Choice	List	Random
1	new	extremes	base
2	new	extremes	antithetic
3	new	2 std	base
4	new	2 std	antithetic
5	old	extremes	base
6	old	extremes	antithetic
7	old	2 std	base
8	old	2 std	antithetic

For each method:

- ◆Generated 10,000 scenarios
- ◆10 runs of 1000 scenarios
- ◆Determined Representative scenarios
- ◆Collected Data

DATA- Simple Statistics

- ◆12 maturities and shape
- ◆First 10 years and all years
- ◆ $5 \times 13 \times 2 = 130$ “variables”



Stability: 10000 Scenarios

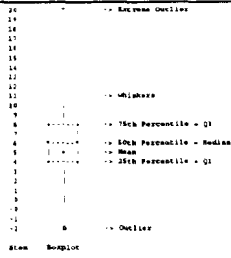
Significant differences: 5% level

- All times
 - »7 year median
- First 10 years
 - »1, 2 year means
 - »15, 20 ,30 year std.

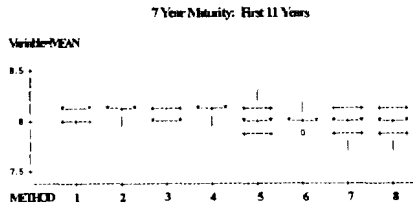
Representative Stability

- ◆ Compared methods
- ◆ Anova showed very significant differences
- ◆ Box-Whisker comparisons

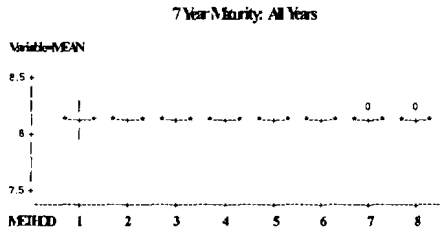
The Parts of a Box & Whisker Plot



Sample Results



Sample Results, Cont.



Analysis -First Years

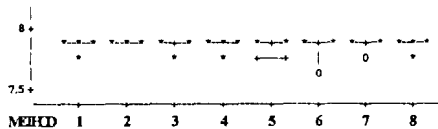
◆Methods 5-8 (Old choice)

- Lower mean
- Greater interquartile range
- Greater variance

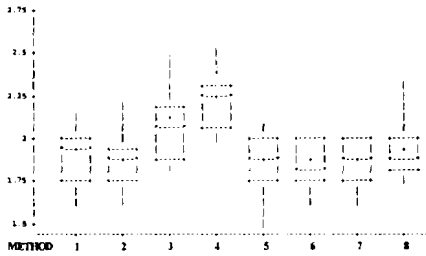
◆Methods 2 and 4 (Antithetic)

- Tighter Distribution

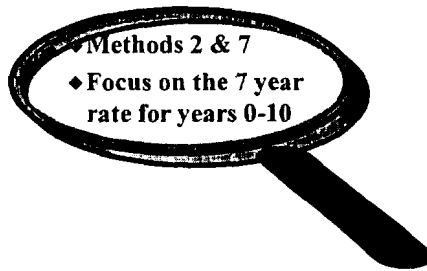
7 Year Median- First Years



7 Year Maturity -Std. First



A Closer Look At:



Method 2

7 Year Mean -Percentiles

Quantiles (Def=5)

100% Max	8.084	99%	8.084
75% Q3	8.081	95%	8.084
50% Med	8.0725	90%	8.083
25% Q1	8.07	10%	8.062
0% Min	8.06	5%	8.06
		1%	8.06
Range	0.024		
Q3-Q1	0.011		
Mode	8.071		

Method 7

7 Year Mean -Percentiles

Quantiles (Def=5)

100% Max	8.183	99%	8.183
75% Q3	8.084	95%	8.183
50% Med	8.0205	90%	8.16
25% Q1	7.882	10%	7.794
0% Min	7.764	5%	7.764
		1%	7.764
Range	0.419		
Q3-Q1	0.202		
Mode	7.764		

Method 2

7 Year Median

Quantiles (Def=5)

100% Max	7.924	99%	7.924
75% Q3	7.924	95%	7.924
50% Med	7.924	90%	7.924
25% Q1	7.904	10%	7.8495
0% Min	7.823	5%	7.823
		1%	7.823
Range	0.101		
Q3-Q1	0.02		
Mode	7.924		

Method 7

7 Year Median

Quantiles (Def=5)

100% Max	7.924	99%	7.924
75% Q3	7.924	95%	7.924
50% Med	7.924	90%	7.924
25% Q1	7.883	10%	7.8115
0% Min	7.765	5%	7.765
		1%	7.765
Range	0.159		
Q3-Q1	0.041		
Mode	7.924		

Method 2

7 Year Standard Deviation

Quantiles (Def=5)

100% Max	2.195	99%	2.195
75% Q3	1.964	95%	2.195
50% Med	1.8925	90%	2.131
25% Q1	1.764	10%	1.686
0% Min	1.646	5%	1.646
		1%	1.646
Range	0.549		
Q3-Q1	0.2		
Mode	1.646		

Method 7

7 Year Standard Deviation

Quantiles (Def=5)

100% Max	2.031	99%	2.031
75% Q3	1.987	95%	2.031
50% Med	1.9015	90%	2.0115
25% Q1	1.729	10%	1.675
0% Min	1.649	5%	1.649
		1%	1.649
Range	0.382		
Q3-Q1	0.258		
Mode	1.649		

Method 2

7 Year Minimum

Quantiles (Def=5)

100% Max	4.509	99%	4.509
75% Q3	4.136	95%	4.509
50% Med	4.045	90%	4.4895
25% Q1	3.669	10%	3.553
0% Min	3.5	5%	3.5
		1%	3.5
Range	1.009		
Q3-Q1	0.467		
Mode	3.5		

Method 7

7 Year Minimum

Quantiles (Def=5)

100% Max	4.014	99%	4.014
75% Q3	3.754	95%	4.014
50% Med	3.6435	90%	3.943
25% Q1	3.5	10%	3.5
0% Min	3.5	5%	3.5
		1%	3.5
Range	0.514		
Q3-Q1	0.254		
Mode	3.5		

Method 2

7 Year Maximum

Quantiles (Def=5)

100% Max	22.919	99%	22.919
75% Q3	18.492	95%	22.919
50% Med	17.511	90%	22.4975
25% Q1	16.329	10%	14.212
0% Min	14.033	5%	14.033
		1%	14.033
Range	8.886		
Q3-Q1	2.163		
Mode	14.033		

Method 7

7 Year Maximum

Quantiles (Def=5)

100% Max	16.636	99%	16.636
75% Q3	15.194	95%	16.636
50% Med	14.949	90%	16.1655
25% Q1	14.744	10%	14.057
0% Min	13.961	5%	13.961
		1%	13.961
Range	2.675		
Q3-Q1	0.45		
Mode	13.961		

Conclusions

- ◆ **10,000 scenarios are very stable**
- ◆ **1,000 scenarios are ok**
- ◆ **Found method with promise of cutting variation in 50 scenarios due to seed from 41 bp to 2.5 bp**