2018 Predictive Analytics Symposium

Session 45: Assessing Credibility of Predictive Model

SOA Antitrust Compliance Guidelines
SOA Presentation Disclaimer

Assessing credibility of predictive models

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Introduction

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Agenda

1. Pleased to meet you
2. Define key terms
3. Survey some specific predictive models
4. Focus on linear mixed effects models
5. Questions and discussion
6. Github repository:

   https://github.com/milliman/SOA_PAS_CrediblePredictiveModels

Definitions

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Credibility

ASOP 25:

- *A measure of the predictive value in a given application that the actuary attaches to a particular set of data (predictive is used here in the* ***statistical sense*** *and not in the sense of predicting the future).*
- *In [predictive models], credibility can be estimated based on the* ***statistical significance*** *of parameter estimates, model performance on a holdout data set, or the consistency of either of these measures over time.*

"Statistical sense" and "statistical significance" suggest a focus on data *quantity*

Credibility

ASOP 25:

- *A measure of the predictive value in a given application that the actuary attaches to a particular set of data (predictive is used here in the statistical sense and not in the sense of predicting the future).*
- *In [predictive models], credibility can be estimated based on the statistical significance of parameter estimates,* ***model performance on a holdout data set****, or the consistency of either of these measures over time.*

Other credibility considerations

Data integrity

- Large set of bad data worse than small set of bad data
- See: ASOP 23

Predictive model appropriateness

- Various methods will lead to various sets of “credible” predictions. Is one method most appropriate?

Bias-variance tradeoff

- Minimize this: $E\left[\left(y - \hat{f}(x)\right)^2\right]$

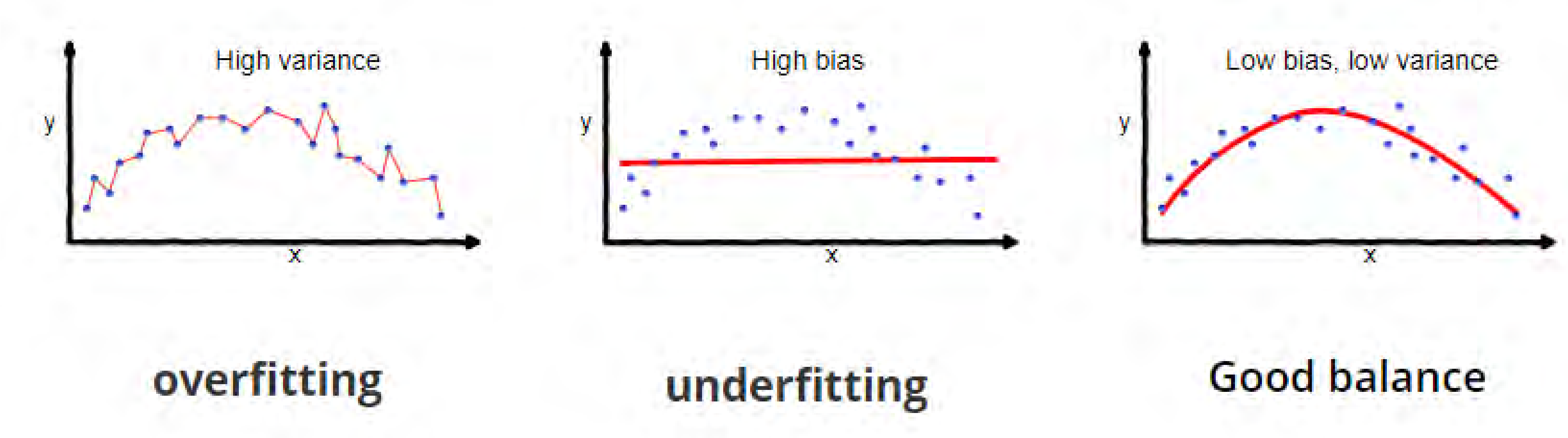
$$= \left(E\left[\hat{f}(x) - f(x)\right]\right)^2 + \left(E\left[\hat{f}(x)^2\right] - E\left[\hat{f}(x)\right]^2\right) + Var(y)$$

$$= bias^2 + (model)\ variance + random\ error$$

- A model complexity tradeoff between over- and under-fitting…between bias and variance

Bias-variance breakdown

- The goal is to find the sweet spot; balance bias and variance so that overall error is low
  - Separate the signal from the noise effectively



*https://towardsdatascience.com/understanding-the-bias-variance-tradeoff-165e6942b229

Tabular model

- Data are segmented by a few dimensions, average outcomes are calculated in each segment
- Note that this *is* a predictive model
- High variance-low bias model on the variance bias tradeoff

Attained age	Sex	Smoker	q
…			
65	M	S	0.010
65	M	N	0.005
65	F	S	0.008
65	F	N	0.004
66	M	S	0.011
…			

Predictive model*

- A model with the ability to consolidate segments by identifying patterns between covariates and the outcome variable
- Can control model complexity more easily; can control bias-variance tradeoff;
- Including (but not limited to):
  - Linear regression
    - Regularization and mixed effects
  - Bayesian models
  - Decision trees (and ensembles thereof)
  - Support vector machines
  - Neural networks

A credible predictive model

1. The data are adequately cleaned and reviewed for reasonableness
2. The data are plentiful enough to generate confident predictions across a useful range of relevant dimensions
3. The predictive modeling method used is able to optimize the bias-variance tradeoff
   - Validation: use of holdout datasets to test goodness of fit
   - Consistency of parameter estimates and validation over time

Credibility methods

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Limited fluctuation credibility (LFC)

- More credible when the probability of proportionally small errors (k) is large (1 – α)
  - $P(|\bar{X} - \mu| < k\mu) > 1 - \alpha$
- For an observed cell, probability distribution typically comes from Central Limit Theorem
- We often look for at least 1,082 events in a cell for "full credibility." Where does 1,082 come from?

1,082 derivation (proportions)

- Recall: $\mu = q;\ \sigma^2 = \frac{q(1-q)}{n}$
- Work through some algebra and probability theory:

1. $P(|\hat{q} - q| < kq) > 1 - \alpha$
2. $P\left(\left|\frac{\hat{q}-q}{\sqrt{\frac{q(1-q)}{n}}}\right| < \frac{kq}{\sqrt{\frac{q(1-q)}{n}}}\right) > 1 - \alpha$
3. $\boldsymbol{P\left(|Z| < \frac{k\sqrt{nq}}{\sqrt{1-q}}\right) > 1 - \alpha}$
4. $\boldsymbol{nq > \left(\frac{Z_{\frac{\alpha}{2}}}{k}\right)^2 \cdot (1 - q)}$
5. *Choose $k = 5\%$ and $1 - \alpha = 90\%$*
6. $nq > \left(\frac{1.645}{0.05}\right)^2 (1 - q) = 1{,}082.4(1 - q)$ and 1 – q is typically very close to 1

LFC using confidence intervals

$$3.\ P\left(|Z| < \frac{k\sqrt{nq}}{\sqrt{1-q}}\right) > 1 - \alpha \ \Rightarrow\ 4.\ \boldsymbol{Z_{\frac{\alpha}{2}}} \cdot \sqrt{\frac{\boldsymbol{\hat{q}(1-\hat{q})}}{\boldsymbol{n}}} \le k \cdot \hat{q}$$

Width of confidence interval: $\mathbf{2} \cdot \boldsymbol{Z_{\frac{\alpha}{2}}} \cdot \sqrt{\frac{\boldsymbol{\hat{q}(1-\hat{q})}}{\boldsymbol{n}}}$

Width of error tolerance interval: $\mathbf{2} \cdot \boldsymbol{k} \cdot \boldsymbol{\hat{q}}$

1) LFC with a GLM

- GLM coefficients (and thus predictions) have approximate normal distributions, with variances derived from the variance-covariance matrix
- Compare GLM prediction confidence interval to an error tolerance interval to determine full credibility
- All can be done in base R

https://www.soa.org/Library/Newsletters/Predictive-Analytics-and-Futurism/2017/december/2017-predictive-analytics-newsletter-issue-16-kullowatz.aspx

1) LFC with a GLM pros and cons

Pros

- Follows from commonly used credibility method
- GLMs are interpretable and commonly used

Cons

- No obvious weighting scheme for predictions with less than full credibility
  - $Weight = Z = \sqrt{\frac{n}{n^*}}$

2) Bayesian model (MCMC)

- Modeler declares model parameters, their prior distributions, and their likelihood functions
- "Coefficients" are represented by posterior distributions, derived from sampling the data and Bayes' theorem
- "rstan" package in R

https://www.soa.org/Library/Newsletters/Predictive-Analytics-and-Futurism/2017/june/2017-predictive-analytics-newsletter-issue-15.pdf

2) Bayesian model pros and cons

Pros

- Flexible model specification
- Priors are an intuitive form of "credible targets"
- Produces full distributions of parameters (coefficients)

Cons

- Computationally intensive
- Steeper learning curve: specifying the model, priors, and likelihood functions can be an involved process

3) Using offset as “credibility target”

What is an offset?

- A covariate that is constrained to have a coefficient of 1
- Like a null hypothesis

How is it implemented?

- Input as a vector in most R model functions
- **Should be in units of the prediction**

What can it do for us?

- Input some known credible assumptions as an offset to serve as a credibility target

3) Logistic regression offset

- $\ln\left(\frac{\hat{q}}{1-\hat{q}}\right) = \widehat{\beta_0} + \widehat{\beta_1} x_1 + \cdots + \widehat{\beta_p} x_p + 1 \cdot offset$
- Observation *i* has prescribed mortality of $q_i = 0.01$
- $\ln\left(\frac{q_i}{1-q_i}\right) = \ln\left(\frac{0.01}{0.99}\right) = -4.595$

3) Regularization with offset

- Additional constraints (“penalties”) on the model coefficients
- Set the offset to some “credibility target”
  - The offset effectively forces the model to identify regions where the data stray from the null hypothesis
- The coefficient constraints serve as a credibility weighting method
- Where data are scarce, the model will be weighted heavily toward the offset

3) Regularization details

- Penalties:
  - GLM: maximize(log-likelihood)
  - Lasso: maximize(log-likelihood - $\boldsymbol{\lambda} \sum |\boldsymbol{\beta_j}|$)
  - Ridge: maximize(log-likelihood - $\boldsymbol{\lambda} \sum \boldsymbol{\beta_j^2}$)
- Requires standardized covariates because the value of the coefficient is part of the penalty
- “glmnet” package in R

3) Regularization pros and cons

Pros

- Allows the regularization penalty to weight your data against the credibility target (offset)

Cons

- The “weighting” is less interpretable

4) Linear mixed effects models

- Model parameters can be fit as a random effects, which allows for some family-wise shrinkage
- Similar to the regularization method in concept
- “lme4” package in R has a relatively efficient algorithm for such fits

Linear mixed effects models: R example (lme4 package)

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Linear mixed effects model (LMM)

Case study:

- A company uses many distinct distributors, but not a lot of exposure for many of the distributors. It would like to generate "credible" surrender predictions for policyholders, ***taking distributor into account***.
- Many distributors will have limited exposure in the datasets
- GLMs struggle with fixed effects where either 0% or 100% of the observations recorded an event

LMM data and model

Data:

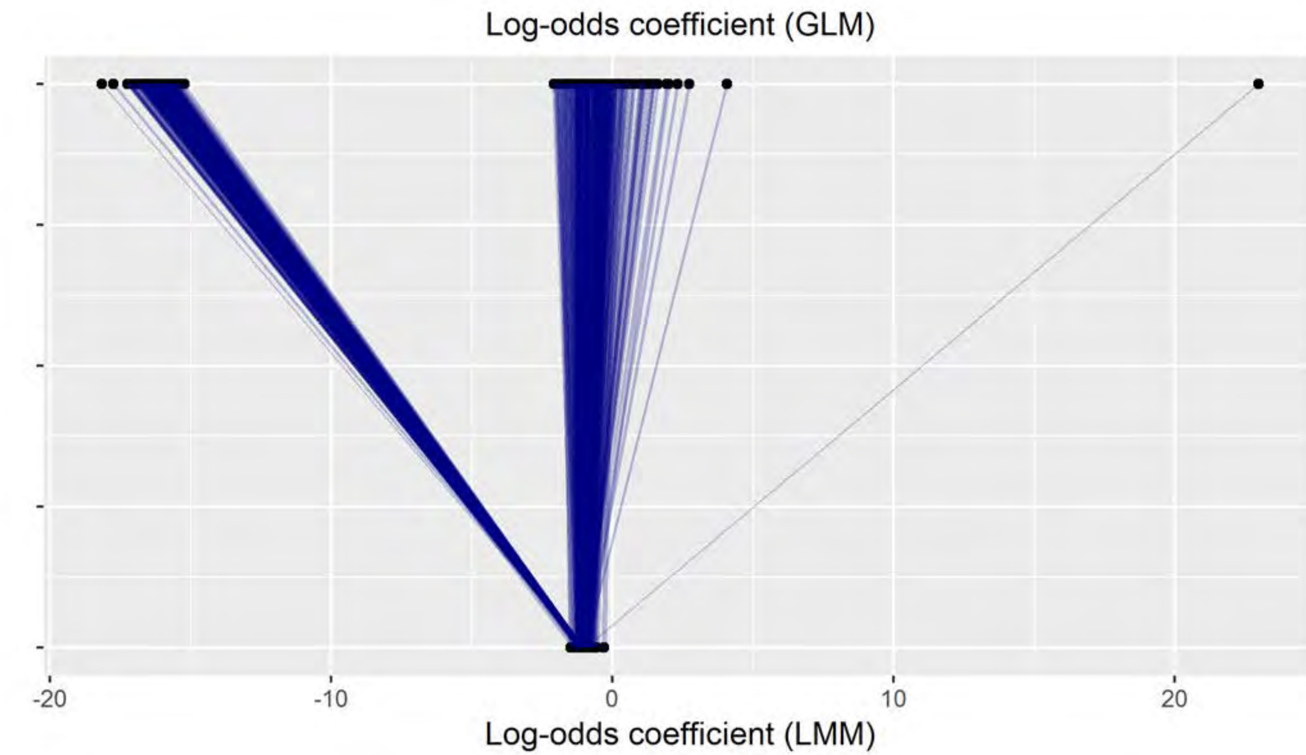
- 50,000 observations from 10,000 policyholders, undersampled to produce a 2% quarterly surrender rate.

Model:

- Probability of surrender (Surr) is a function of **moneyness** of the guarantee (ITM), **duration** (q), the **surrender charge phase** (IN, OUT), and **distributor** (DistCode)
- GLM
  - Surr ~ IN + Dur_IN + ITM:Dur_IN + Dur_OUT + ITM:Dur_OUT + DistCode
- LMM
  - Surr ~ IN + Dur_IN + ITM:Dur_IN + Dur_OUT + ITM:Dur_OUT + **(1 | DistCode)**

LMM continued

Fixed distributor effects from the GLM are shrunk to more credible random effects in the LMM:



LMM interpretation

- Coefficient comparison: Even with shrinkage, we get differentiation of distributors
  - More than 3x surrender rates from maximum effect to minimum
  - More than 1.5x surrender rates across middle 95%
- Model comparison:
  - LMM fits much better than GLM on holdout dataset (not including DistCode)

LMM credibility

- We were able to arrive at more credible surrender predictions by distributor…
- …and more credible estimates of the effects of each distributor
- What makes the predictions credible overall?
  - The random effects process
  - Performance on a holdout dataset, and the consistency thereof over time (ASOP 25)
- Other ideas:
  - Could have used an offset as credibility target
  - Could have fit other covariates as random effects

Lightning round: things to consider

Things to consider

1. If underlying distribution changes over time, are you relying on a consistency that doesn't actually exist?
   - E.g. rising interest rate scenario
   - Mortality: improvement, cure shocks, etc.
2. Weighting recent data vs. distant past data
3. Thresholds for data cleanliness
4. What proportion of deriving credible estimates should be qualitative?

Questions

Thank you!

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