Revisiting Pension Actuarial Science:
A Five-Part Series

Part 3
Fair Value of the Liability – A Market-Based Discount Rate

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Abstract for the Series

The current financial model put forth as the market value of public sector pension benefit liabilities is simply the expected cash flows of the accumulated benefit obligation, as defined for current private sector financial reporting, discounted using a risk-free yield curve. This model is in serious need of an overhaul. It fails to faithfully represent the fair value of a currently accrued public sector pension benefit liability in three important ways:

1. Its use of the accumulated benefit obligation cash flows fails to accurately represent the terms of the employment contract which gives rise to the obligation being valued – a violation of labor economics principles.

2. Its use of expected cash flows as if they were fixed fails to recognize the risk premium load, which a fair exit price would include for the potential for adverse cash flow experience – a violation of actuarial finance and pricing principles.

3. Its use of risk-free discount rates fails to adequately reflect the observable and not-so-observable inputs from market participants' behavior – a violation of financial engineering principles.

Parts 1 through 3 in this series propose solutions to these three flaws.

Part 4, "The Residual Benefit Liability," presents an alternate approach to obtaining the fair value of the public sector employer's pension benefit liability. It approaches the task by modeling the real world operation of the pension fund, rather than approaching the task from the perspective of a theoretical construct. This alternate approach dares to model the long-term agency operation of the plan rather than ignoring it in favor of a pass-through approach. The current model ignores the effectiveness (even the existence) of the pension fund itself, while the alternate approach attempts to model the plan's operation in practice over time in order to determine the employer's residual asset or liability.

In spite of these three improvements and the alternate model, we believe the fair value of public sector post-employment benefit liabilities has little to no usefulness in most venues. There are legitimate roles which the market or fair value might play in valuing an individual member's personal wealth, a minor role in the context of certain discussions concerning risk measurement and risk management, and a major role in the context of plan terminations and freezes.

However, for purposes of advance funding, taxpayers, financial reporting, lenders and rating agencies, comparability, and the major part of risk measurement and analysis, the decision-usefulness of market or fair value is negligible, possibly even misleading. Other existing models and methods are far more suitable for these purposes, including conventional actuarial approaches and others that are less conventional or popular, but which should be considered in the actuarial toolbox and have higher decision utility.

Part 5 in this series, "Consider the Measurement Purpose," addresses various purposes for measuring a public sector pension liability and which measures have the most practical usefulness.

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Fair Value of the Liability – A Market-Related Discount Rate

A. Modigliani-Miller Friction

Whether or not an entity's financing method affects its value is a key subject of modern finance research. Modigliani and Miller (1958), in a seminal work affecting the subject, showed that, under specific conditions, the value of a company is invariant with respect to the leverage policy, or the method of financing of the company, in general. This irrelevance proposition rests on the assumptions of no taxes, no bankruptcy costs, and no agency costs. Consequently, if we observe the method of financing affecting the value of a company, it must be so because of taxes, bankruptcy costs, or agency costs.

While a pension plan is not a company, the general frame of reference applies. Taxes should not matter, as a qualified plan is not subject to them. However, for our purposes, pension "taxes" include any required payments to the government or regulators – even if, not termed taxes. Such a situation exists if the plan surplus cannot be fully returned to the plan sponsor, while a deficit is covered by the sponsor.

Most importantly, for financial institutions, such costs of benefits are imposed by regulatory capital requirements. As pointed out by the Casualty Actuarial Society Task Force on Fair Value Liabilities (2000), financial intermediaries face capital requirements. The capital held by industrial firms is primarily fixed assets, such as plants and equipment, and working capital used for operations. Efficient firms that produce with less capital drive out less efficient firms. In a competitive industry, the capital used is typically at the level of the capital needed. But in insurance and banking, capital serves to produce the product, but also to protect consumers. Capital is determined by statutory requirements or by rating agency measures. Statutory risk-based capital (RBC) formulas are minimum capital standards, and insurers usually hold more than the RBC requirements to avoid regulatory interventions or compete for discerning consumers seeking financially strong firms. Additionally, insurers base their target capital on the rating agencies' standards. The cost of holding capital includes the rate of return that the capital must earn investment restrictions, double taxation, as well as competitive and other friction costs. The relative importance of each cost depends on the industry. For example, for property-casualty insurers, double taxation is a substantial cost, while for pension plans that cost is not relevant. On the other hand, if banks hold 10 percent of deposits as non-interest-bearing deposits with the central bank, and the opportunity cost of capital (i.e., the available rate of return on alternative uses of the funds) is 5 percent, the economic cost of capital is 0.5 percent of the deposits per annum.

The pension plan's situation is quite different, as illustrated in comparison to an insurance company. Suppose an insurer has a liability to be paid in three years exactly, in the amount of $1,000,000.00. Assume that the risk-free interest rate is 3 percent per annum. The present value of the liability is $1,000,000.00 times 1.03^{-3}, or $915,141. 66. Suppose that the company is required by law and regulatory agencies to hold the amount of capital equal to 10 percent of the liabilities' market value, i.e., $91,514.17. As a consequence, the fair value of this liability is $915,141.66 + $91,514.17= $1,006,655.83, because any other entity assuming this liability must pay the liability, and simultaneously acquire appropriate capital to hold this liability. This

increase in the fair value of the liability, in relation to the equivalent market instrument paying the same amount at the same time, results from the fact that the equivalent market instrument cannot be legally represented to be an insurance policy. Only a firm organized as an insurance company and holding appropriate capital would be able to hold this liability -- then fair value would be established in the manner presented here.

In the case of a pension plan, the capital requirements are actually the exact opposite. Not only is a public pension plan not required to hold any form of risk-based capital, but it is specifically allowed, and encouraged by the regulation structure, to spread the funding of any surplus shortfall over an extended period of time. This means that the required regulatory capital for a public pension plan is, effectively, negative, and this results in a lower fair value of the liabilities. A financial economics approach calls for discounting of the pension plan liabilities cash flows at a risk-free rate appropriate for a given cash flow's maturity. The conventional actuarial approach uses the long-term expected rate of return on the asset portfolio. One can actually view the actuarial method of discounting as a substitute for the measurement of the reduction in the liability value due to delayed funding practice being encouraged in the reality of public pension plans.

Conventional actuarial practice is a reasonable approximation of the economic reality, while financial economics imposes complete prefunding, and inflexibility of funding, not applicable to the economic reality. The motivation of the financial economics approach is complete security of pension benefits, as it naturally and often imposes an investment strategy on a portfolio consisting entirely of risk-free bonds of appropriate maturities. In reality, pension plans are invested in a balanced portfolio of stocks and bonds.

To a degree, such a portfolio is imposed on pension plans by regulatory diversification requirements. Because of this situation, it is often argued (Gabriel, Roeder, Smith & Co., 2008) that financial economics imposes excessive funding requirements on present generations at the expense of future generations, because, historically, the actual realized rate of return has been consistently higher than the risk-free rate of return.

But we must also notice that the historical rate of return is subject to *survivorship bias:* only securities and securities markets that survived the period of measurement are included. Historical equity rates of return are higher than the risk-free rate, because there have been entire markets (e.g., the Russian stock market of 1900) or individual securities that did not survive the period of observation. If their rates of return are included in measurement, realized rates of return will be lower. Notably, for pension plans, historical averages that accurately represent experience should include what happened to pension plans that failed or required any form of emergency assistance from any form of government. We should also note survival of the pension plan itself, though the apparent goal of its funding and regulation is not assured. Thus the fair value of the pension liability that does not include a reduction allowing for the possibility of non-survival does not truly correspond to economic reality. Of course, such an adjustment is unlikely to be used in fair value accounting, as it would represent admission of funding regulatory failure. A survivorship bias adjustment applied to the asset side of the balance sheet without an equivalent adjustment to the liability side of the balance sheet represents a significant departure from the basic philosophy of fair value accounting principles -- and a rather pronounced inconsistency.

But one could argue that for a public pension plan, survival of the plan is assured by what one could call the *higher government backstop.* Imminent insolvency of a sole or agent plan and its government sponsor is very likely to bring about support from a higher level of government, or the Federal Government. This additional backstop can be represented in the balance sheet as an extra asset, or through a reduction in the actuarial liability. That reduction (or a part of it) can be—and in practice is—achieved by using a valuation rate which is higher than risk-free.

Let us also note that while the conventional actuarial approach uses a discount rate in excess of the risk-free rate, it also effectively incorporates the flexibility of delayed funding granted to plan sponsors and, possibly a portion of the adjustment for survivorship bias on the liabilities side. It, therefore, represents a pragmatic and realistic alternative to a rigid risk-free discounting, which does not incorporate regulatory costs and benefits, and survivorship bias on the liability side.

The second friction cost identified in the Modigliani-Miller irrelevancy proposition was the cost of bankruptcy,. It should not matter in the case of a pension plan. The very existence of the plan is a design created for the purpose of lowering the probability of plan insolvency. Insurance against that calamity is also often provided by some form of regulatory agency, such as Pension Benefits Guaranty Corporation for private plans in the United States. Public pension plans are not subject to such insurance. Instead, it is the good faith and credit of the plan sponsor that provides the secondary or residual guarantee, in addition to plan pre-funding. In fact, expected continuous (in fact, infinite) existence of public plans sponsors ties the solvency of the plan to the sponsor's solvency, and given the taxing power of government sponsors, the cost of bankruptcy is significantly reduced, probably even to zero.

Additionally, given the state and federal governments' history of supporting municipal governments, one could conclude that the cost of bankruptcy is negligible. It frees management of the pension fund to take reasonable risks to generate greater potential for rewards than just investing entirely in Treasuries.

The final friction cost identified by the Modigliani-Miller irrelevancy proposition is the agency cost: the cost of the relationship of delegating control over property from its owner (principal) to agents. Generally in an insurance company case, agency problems raise the cost of holding capital. Imagine a situation where shareholders wish the insurer to write profitable but risky business, but managers avoid risk or buy reinsurance in order to protect their jobs. The managers' actions lower rates of return to shareholders. Shareholders may also incur the cost of incentives to induce managers to take more risk. These costs are no different for insurers than for other companies. They may become large, and are difficult to measure.

In pensions, these costs have complicated structures. Investing public pension plan assets in a diversified portfolio of stocks and bonds, as opposed to risk-free bonds, may result in lower plan cost, and the benefits of such actions will accrue to taxpayers, and not plan beneficiaries. This has been in fact put forth as one of the arguments for financial economics approach. However, if the cost of possible lower rates of return due to, for example, stock market declines, are also borne by the plan sponsor, this structure does not impose any agency costs on plan beneficiaries. That is a more realistic picture of the case of public pension plans. One could,

however, argue that agency costs in the case of public pension plans are only imposed on the liabilities side: as the plan participants are a small group with homogeneous interests, if they organize successfully, they can become a powerful political lobby that can raise benefits for plan participants by imposing small costs spread among a large number of taxpayers.

This situation of concentrated benefits and dispersed costs is a standard model in Public Choice Theory (Mueller, 1989). The case for it is especially powerful when the costs of benefits can be transferred to future taxpayers, not yet alive or resident, through borrowing. This again, reinforces the theme of postponement of funding: it is actually that phenomenon that lowers the fair value of pension liability in the case of public pension plans, as we presented it in the analysis of tax and regulatory costs.

It should be noted that in the case of private pension plans, overfunded plans may be subject to a form of "pension arbitrage"; liabilities may be settled by purchasing annuities from an insurance firm. This can only be done for an overfunded plan, as the same liabilities for pension payouts will have different values when held by a pension plan or an insurance company. An insurance firm can function as an insurance firm only if it holds appropriate capital, and it charges the cost of that capital to liabilities, causing the value of liabilities to increase. Financial economics may argue that this is yet another argument for a risk-free bond funding of pension plans. While an extreme viewpoint, some believe that this is a proof that an overfunded pension plan loses its reason for existence as a pension plan. A pension plan is created to spread the funding over an extended period of time, and in the case of public pension plans, such extension can be quite substantial, due to the infinite horizon of existence of plan sponsors. The financial economics approach fails to incorporate these considerations.

Finally, the Modigliani-Miller irrelevance proposition is not free to be invoked whenever it is convenient or supports one's position. It is a very theoretical construct. It requires numerous conditions, including no taxes, no bankruptcy, no agency costs and a competitive and complete capital market, a "perfect" market (Myers, 2001). It requires a frictionless, pristine environment. Modigliani-Miller is instructive. We appreciate its value for pedagogical reasons for the insight it brings to understanding the theoretical principles underlying capital structures. However, all its conditions never really exist in the real world.

Furthermore, Modigliani-Miller is about corporate finance and raising capital for corporate operations and investments. It is about the irrelevance of a corporate entity's capital structure (internal cash flow, debt and corporate equity) on its *corporate stock valuations*. Extending this theoretical concept to claim the irrelevance of a government or bidding entity's funding/investment policy on its *pension liability valuation* is a stretch.

One might argue that the value of the pension liability should be determined as its market or fair value, as tradable in the marketplace. Such a market-driven value would be independent of how that current employer/plan has, in the past, invested the assets set aside for that purpose. And one might argue that the public sector pension liability should be valued using a risk-free return in order to match the default-free nature of the obligation. These other arguments are more "on-point" and form a better foundation for advocating market or risk-free discount rates. One would not need, and should not, appeal to Modigliani-Miller to support that position.

In this section of Part 3, we have accommodated the professional discussion of the application of Modigliani-Miller irrelevance proposition, in part, by drawing upon how private sector insurance companies' capital structures affect reserving requirements. In reality, however, the Modigliani-Miller irrelevance proposition is virtually irrelevant to public sector pension liabilities in the real world.

Thus a fair value model may use discount rates higher than risk-free to reflect market-based observables.

B. Return to Fair Value Definitions

There is some logic in connecting the dots between a public sector pension benefit's lack of default risk and use of default-free discount rates. However, the markets do not necessarily conform to that notion.

There is an apparent inconsistency between current thinking about risk-free discount rates and discount rates that reflect the manner in which exchange prices would be developed in a market for pension liabilities. Fair value is not about assigning an economic value in a vacuum; it is about pricing; about how pricing would operate in a real market.

Market participants regularly demonstrate their willingness to pay higher prices for loans (even pledging personal collateral) for ventures in which they are more confident of profitable outcomes, whether short or long term. The market participant's intentions for investing the proceeds (regulated or not) and its capital structure will affect the price it is willing to quote for settling an employer's pension obligation. An exit price must be developed with consideration given to the settlement rates available in the market.

There are, indeed, markets (some active some imagined) where public sector pension liabilities might be settled. Exit price bids would likely be lower (higher discount rates) than risk-free prices.

C. Single Premium Group Annuity Market

Thanks to the U. S. Department of Labor's Interpretive Bulletin 95-1, there are only approximately 10 or 12 market participants who pass the threshold for the U. S. single premium group annuity market for settling corporate pension plan obligations. The single premium annuity market is highly regulated, with statutory, GAAP and tax reserves to consider. U. S. and European regulatory environments differ. As such, differences in prices quoted to settle pension obligations may occur.

Insurance companies setting the exit prices for employers who are settling their pension obligations do not discount the expected cash flows with risk-free rates. It is not because they know they can go out of business and renege on their "guarantee." It is because they take into consideration their own current general account investment portfolios, their plans for investing the proceeds and the fixed income market conditions at the time of the quote. They consider their capital structure, their own externally imposed reserving requirements, and a host of other factors.

The Modigliani-Miller irrelevance proposition does not apply so nicely and neatly to pricing in the real world of single premium group annuity contracts. The actual market for settling corporate pension obligations utilizes discount rates higher than risk-free rates. That is an observable market input for our fair value purposes at hand. According to financial engineering principles, as many components of our fair value pricing model as possible must be calibrated to real market data.

Very few state and local government defined benefit pension plans have ever been settled with insurance company single premium group annuity contracts. But there is nothing preventing them from doing just that. There is no reason insurance companies would price public sector plan settlements and different than private sector plan settlements, as long as the contract provisions are basically the same.

Thus, the single premium group annuity market can be thought of as the principal market for public sector pension plan exit transactions. The exit prices paid for such would be based on discount rates of high grade corporate bonds and other such investments expected to back up the promise.

If a public sector pension fund or employer were to settle an accrued pension obligation in the single premium group annuity market (a genuine and likely market for relevant observables), there would be no more liability held in the books for the risk of the insurance company's insolvency or other default. There are also state insolvency guaranty funds to consider as a hedge against that possibility. That residual contingent liability is not appropriate for financial reporting.[1] More on the definition of liability for financial reporting can be found in Part 5 of this series, "Consider the Measurement Purpose." Furthermore, there is no need to recognize any residual contingent liability related to numerous other insurance transactions; none for long-term disability, health insurance, property/casualty insurance, general liability insurance. Such residual contingent liabilities are generally not measurable and do not qualify for reporting purposes.

Effectively, such a plan or employer which settled its pension obligation in the single premium group annuity market has settled the obligation at a market-related discount rate. Therefore, those plans and employers which have not yet actually settled their accrued obligation should not be held to a higher liability than those which have actually settled theirs at a fair market rate.

Thus, fair value should use discount rates higher than risk-free, as observable in the marketplace.

[1] The City of West Palm Beach settled an accrued pension obligation for general employees with an insurance company (1997). The negotiated price was based on market conditions and fixed plan cash flows. Since then, the city has not recorded any residual contingent pension obligation on its books.

D. High Quality Corporate Bond Settlement Rates

The FASB deliberately chose to require the use of settlement rates for discounting cash flows, which has evolved into the common use of high quality (AA or better) corporate bond yield curve observed at the measurement dates, which is much different from a risk-free yield curve.

Based on a valuation of our case study plan as of January 1, 2009, equivalent single discount rates were derived from risk-free (Ryan Labs) and high quality corporate (CitiGroup) spot yield curves, on the basis of the plan's expected ABO benefit cash flows. These equivalent single discount rates were obtained using this method for the spot yield curve for each December 31 from 1995 to 2008. Thus, the same duration and convexity were used to derive the equivalent single discount rate for each year's curve. Yields for spots above 30 years were assumed to be the same as the 30-year spots. Refer to the section on funding in Part 5 of this series, "Consider the Measurement Purpose," for more details on the methodology and sources. Figure 11 below presents these single equivalent rates for each of the last 14 years.

Figure 11

Single Equivalent Discount Rates			
Dec 31	Based on Treasury STRIPS	Based on High Quality Corporate Spots	Spread of Corporates over STRIPS
1995	6.32%	6.71%	0.39%
1996	6.68%	7.43%	0.75%
1997	6.00%	6.75%	0.75%
1998	5.38%	6.46%	1.08%
1999	6.72%	7.93%	1.21%
2000	5.43%	7.18%	1.75%
2001	5.59%	6.85%	1.26%
2002	4.91%	5.88%	0.97%
2003	5.12%	5.86%	0.74%
2004	4.88%	5.58%	0.70%
2005	4.58%	5.50%	0.92%
2006	4.84%	5.85%	1.01%
2007	4.45%	6.41%	1.96%
2008	2.82%	6.07%	3.25%
Average of 14 years			1.20%
Average of middle 12 years			1.09%

Thus, a fair-value model may use discount rates at least up to high quality corporate spot rates to represent settlement rates in the marketplace. This gives consideration to real market observables available for settling similar obligations, a fundamental principle of financial engineering and fair-value modeling.

E. Liquidity Risk

Small-denominated bond issues trade at higher yields (and lower prices), than do large ones with identical terms at the same time for the same issuer, because they are more difficult and expensive to sell. Private equity transactions have a higher return expectations (and lower initial prices) because they are not liquid. Investment economics principles require a higher return for illiquid instruments.

Public sector pension obligations are likely among the most illiquid financial instruments in existence. If we are to be true to the fair value measurement attribute, the fair value of public sector pension liabilities must recognize their illiquidity when setting the discount rates.

Again, fair value should use discount rates higher than risk-free.

F. Low Percentile in the Capital Asset Pricing Model

The single premium group annuity market is highly regulated and, thus, not a truly free market laboratory for adopting discount rates for fair value calculations.

A more advantageous market for transfer of public sector pension liabilities, if one needed to be imagined, might be one in which the market participants were other public sector pension funds. In such a market, these funds would buy and hold or sell pension liabilities for gain, just as they buy and hold or sell assets for gain. The plan exiting the liability would not likely be able to settle its obligation in that market at the expected long-term return of those pension funds.

Although if the terms of the exchange transaction were to guarantee that the exiting plan would never have to retain any residual liability, there might actually be bidders who would quote the low price, based on their own expected returns. Usually, in other commercial markets, such low bidders are dismissed as not having the backing or solvency necessary to make good on the transfer. However, if the bidders are all viable public sector pension funds, with the ability to go back to their own respective employers to make good on the liability for the benefits of its own members or of those in the acquired block, then the exiting employer may not have the same concern about solvency as in other commercial markets.

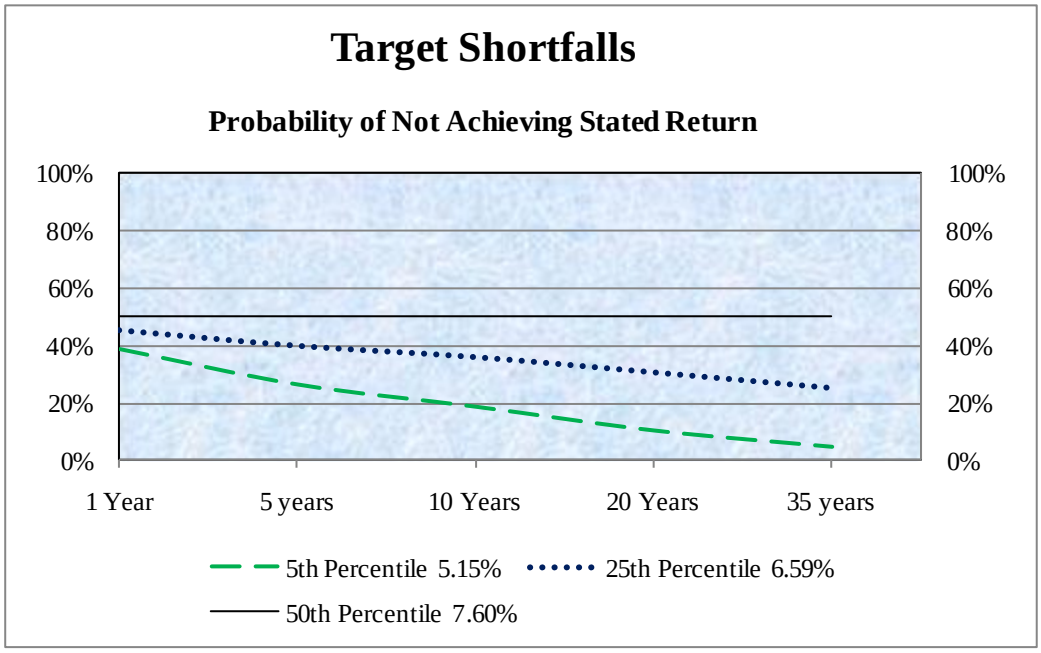
Nevertheless, the exit prices will more likely be set using discount rates below the expected long-term return on the underlying portfolio, based on fairly sophisticated assessments of risk. We imagine the pension funds would consult with their chief investment officer, investment consultants and actuarial consultants to assist in pricing the liability. Such analysis would surely include Monte Carlo simulations involving risk tolerance and stress-testing.

To simplify the analyses for illustration purposes, consider a pension fund whose investment asset allocation is 50 percent in domestic, large-cap stocks (split evenly between value and growth), 10 percent in international stocks, 35 percent in fixed income (split evenly between intermediate term government bonds and corporate bonds) and 5 percent in cash equivalents.

Based on current capital market assumptions for each asset class (SunGard, 2009) and assuming the alpha achieved equals the investment-related expenses, under a conventional capital asset pricing model, the mean return is 7.97 percent with standard deviation of 8.97 percent under a normal/lognormal distribution model. While interest in fat-tail distributions such as Paretian and other log-stable distributions is returning of late, we will use the currently accepted conventional analysis.

Because of the volatility drag, such a portfolio has a 50th percentile return of 7.60 percent, a 25th percentile return of 6.59 percent and a 5th percentile return of 5.15 percent over a 35-year period.

Figure 12

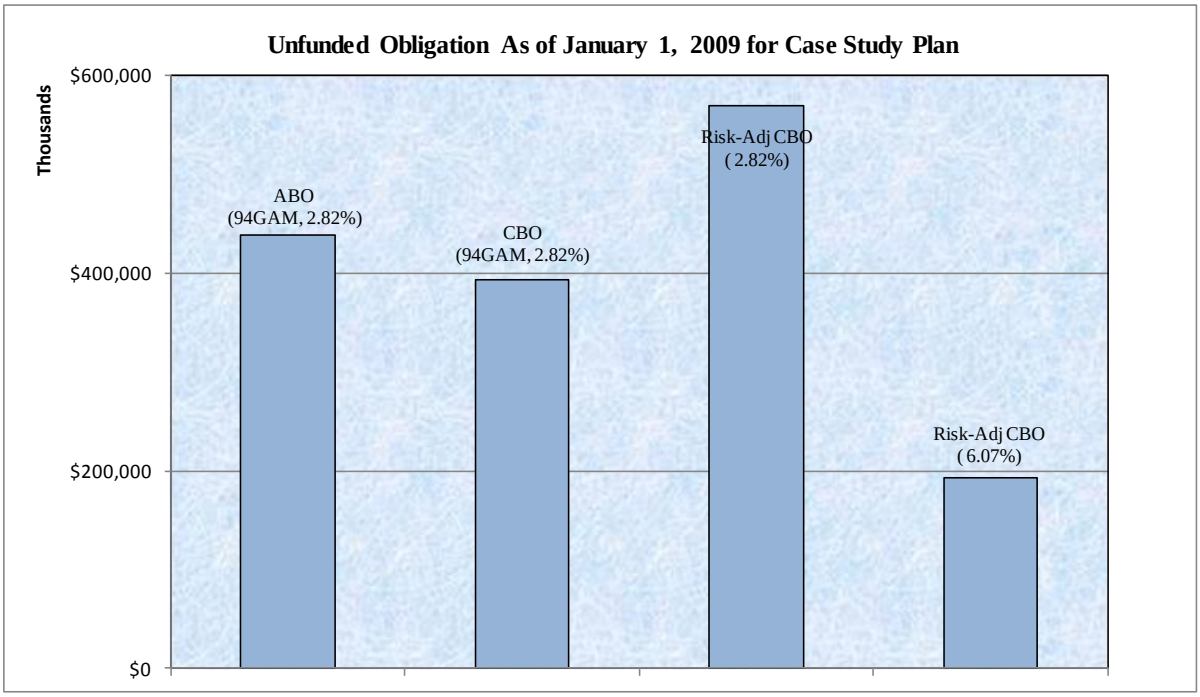


For a market in which the participants are other public sector pension funds, which do not have the reserving constraints that insurance companies in the single premium group annuity market have, it is reasonable to expect that these market participants can and would guarantee the payment of the transferred pension benefit obligation while quoting a price for risk-adjusted cash flows discounted at rates higher than risk-free but below the expected long-term return of their balanced portfolio. The rate might be at or near the 25th percentile (6.59 percent), but possibly down as low as the 5th percentile (5.15 percent).

In any event, the fair value of the exiting employer's pension benefit liability may be discounted using rates significantly higher than risk-free rates. The above analysis does not arrive at a single bright-line discount rate to use. Judgment is needed to set the discount rate in response to market inputs.

In Figure 13, the risk-adjusted CBO is discounted at a higher rate and added to the comparison chart. For illustration, we have used 6.07 percent, the single equivalent rate for the high quality corporate yield curve observed at Dec. 31, 2008.

Figure 13



G. Possible Resolution

Pension liabilities are another form of financial instrument. They are private issues, not marketable and not traded. This makes establishing their exact market value impossible. But financial theory does provide us with methods of calculating or approximating the values of privately traded and illiquid financial assets. This emerging methodology calls for stochastic modeling of future cash flows, and valuation based on either the risk-neutral model with risk-free discounting, or risk-adjusted model with real world probabilities and real world interest rates scenarios.

These approaches are represented by emerging regulatory methodologies of principles-based reserving, required cash flow and stress testing, and risk-based capital modeling using value-at-risk, or conditional tail expectation. We believe that a realistic pension liability valuation methodology may incorporate fully stochastic modeling of liability cash flows, using investment-risk-adjusted methodologies, essentially producing fair values of the public sector pension liability.

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