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Valuation of equity-indexed annuities

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In recent years, insurance companies have been introducing saving products whose returns are linked to equity market performance. Equity-indexed annuities (EIA's) are one of the products of this kind. In 1995, Keyport Life Insurance Co. introduced the first equity-indexed annuity in United States. Sales of equity-indexed annuities have doubled every year since and reached 3 billions in 1997. EIA's appeal to investors because they offer some of the benefits of fixed annuities and some of the potential of equities while limiting downside risk of the equity market. A typical EIA guarantees a minimum return (normally 3%) on a portion of the initial deposit amount, which is required by the so-called Nonforfeiture law. Its return is linked to an equity index. S&P 500 is the most commonly used index.

In Lin (1999), we consider an equity-indexed annuity which provides a minimum interest quarantee and a participation in the increase in an index. This index is calculated to the highest contract anniversary value of an equity index during the term of the contract. To focus on the application of our approach, we neglect surrender risk and mortality risk associated with the equity-indexed annuity.

Consider an equity-indexed annuity with the year of maturity T. Let r be the risk-free interest rate compounded continuously, and $S(t)$, $0 \leq t \leq T$, the value of an equity index. It is assumed that $S(t)$ follows a lognormal process under the risk-neutral probability measure, and is of the form

$$S(t) = S(0)e^{(r-\frac{1}{2}\sigma^2)t+\sigma W(t)}, \quad 0 \leq t \leq T, \tag{1.1}$$

where $W(t)$ is the standard Brownian motion and σ is the volatility of the index. Let P be the lump-sum premium received at the time of issue, f the guaranteed minimum interest rate compounded continuously, and p_f and p_S are the percentage of the premium to be accumulated at the guaranteed minimum interest rate and the participation rate in the increase in the index, respectively. Without the loss of generality, we assume that $P = S(0)$. The value of the annuity is thus calculated as follows:

- the minimum guarantee at time T is $p_f P e^{fT}$, where $p_f e^{fT} \geq 1$. This means the minimum guarantee will be no less than the initial premium;
- the value of the annuity at time T is

$$P + Pp_S \max_{k=0,1,\ldots,T} \left\{ \frac{S(k)}{S(0)} - 1 \right\}, \tag{1.2}$$

  if this value exceeds the value of the minimum guarantee.

To evaluate the total cost of the EIA, we employ the following investment strategy:

- invest a portion of the premium income in a risk-free bond maturing at time T to meet the minimum guarantee. The current value of the bond is $p_f P e^{(f-r)T}$;

- purchase T-year discrete lookback options with yearly sampling to meet the equity-option component. To determine the strike price of the lookback options and the number of units to purchase, express (1.2) as $(1-p_S)P + p_S \max_{k=0,1,\cdots,T} S(k)$. The payoff of the equity option-component in the EIA is

$$\left((1-p_S)P + p_S \max_{k=0,1,\cdots,T} S(k) - p_f P e^{fT}\right)_+,$$

which can be written as

$$p_S\left(\max_{k=0,1,\cdots,T} S(k) - [\frac{p_f P e^{fT} - (1-p_S)P}{p_S}]\right)_+. \tag{1.3}$$

Hence, we need to purchase p_S units of T-year discrete lookback options with strike price $K = \frac{p_f P e^{fT} - (1-p_S)P}{p_S}$ and $K \geq S(0)$.

Hence, the total cost of the EIA is the sum of the current value of the bond and the total price of the lookback options purchased. The portion of the total cost that exceeds the premium is thus considered the extra cost incurred by the guarantees of the EIA.

The estimated price of the T-year discrete lookback options with strike price K in (1.3) is given by the following formula.

$$S(0)\sum_{k=1}^{T} e^{-r(T-k)}\Phi(d_1(k), d_2(k); \rho_k) - e^{-rT}K\sum_{k=1}^{T}\Phi(d_3(k), d_4(k); \rho_k). \tag{1.4}$$

where $\Phi(x, y, \rho)$ is the standard bivariate normal distribution function with correlation coefficient ρ. The parameters $d_1(k), d_2(k), d_3(k), d_4(k)$ and ρ_k are the functions of the means and the variances of the anniversary values of the index, and the correlation coefficients between these index values.

These parameters are calculated recursively, using the Clark mehtod. For the detaled derivation of the formulae and numerical examples, see Lin (1999)

References

Lin, X.S (1999). Valuation of options on the maximum/minimum of multiple assets, discrete lookback options and equity-indexed annuities, preprint