Revisiting Pension Actuarial Science:
A Five-Part Series

Part 4
Fair Value of the Liability – The Residual Benefit Liability

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Abstract for the Series

The current financial model put forth as the market value of public sector pension benefit liabilities is simply the expected cash flows of the accumulated benefit obligation, as defined for current private sector financial reporting, discounted using a risk-free yield curve. This model is in serious need of an overhaul. It fails to faithfully represent the fair value of a currently accrued public sector pension benefit liability in three important ways:

1. Its use of the accumulated benefit obligation cash flows fails to accurately represent the terms of the employment contract which gives rise to the obligation being valued – a violation of labor economics principles.

2. Its use of expected cash flows as if they were fixed fails to recognize the risk premium load, which a fair exit price would include for the potential for adverse cash flow experience – a violation of actuarial finance and pricing principles.

3. Its use of risk-free discount rates fails to adequately reflect the observable and not-so-observable inputs from market participants' behavior – a violation of financial engineering principles.

Parts 1 through 3 in this series propose solutions to these three flaws.

Part 4, "The Residual Benefit Liability," presents an alternate approach to obtaining the fair value of the public sector employer's pension benefit liability. It approaches the task by modeling the real world operation of the pension fund, rather than approaching the task from the perspective of a theoretical construct. This alternate approach dares to model the long-term agency operation of the plan rather than ignoring it in favor of a pass-through approach. The current model ignores the effectiveness (even the existence) of the pension fund itself, while the alternate approach attempts to model the plan's operation in practice over time in order to determine the employer's residual asset or liability.

In spite of these three improvements and the alternate model, we believe the fair value of public sector post-employment benefit liabilities has little to no usefulness in most venues. There are legitimate roles which the market or fair value might play in valuing an individual member's personal wealth, a minor role in the context of certain discussions concerning risk measurement and risk management, and a major role in the context of plan terminations and freezes.

However, for purposes of advance funding, taxpayers, financial reporting, lenders and rating agencies, comparability, and the major part of risk measurement and analysis, the decision-usefulness of market or fair value is negligible, possibly even misleading. Other existing models and methods are far more suitable for these purposes, including conventional actuarial approaches and others that are less conventional or popular, but which should be considered in the actuarial toolbox and have higher decision utility.

Part 5 in this series, "Consider the Measurement Purpose," addresses various purposes for measuring a public sector pension liability and which measures have the most practical usefulness.

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Fair Value of the Liability – The Residual Benefit Liability

A. Modeling the Real World

Mathematical models, whether physics or financial models, are intended to simulate the operation of real world structures and, hopefully, present results that approximate real world outcomes. A fair value model for financial instruments should take care to simulate the operation of all moving parts in the structure and contract. Ignoring or glossing over major components can be disastrous.

Financial economists tend to group players into principals and agents. Among the qualities Warren Buffet looks for in a company, is solid management personnel whom he likes. Agents can add or deplete value for principals. As mentioned previously, one of the primary conditions for Modigliani-Miller's irrelevance proposition is that agency costs and benefits do not matter. That is a condition for the theoretical proposition—not a fact.

We emphasized in Part 1 of this series, "The Contractual Benefit Obligation," the importance of recognizing the employment contract terms between employer and employee when valuing the benefits for fair value purposes. Respect for this labor economics principle led us to revise the benefits valued from the accumulated benefits obligation (ABO) to the contractual benefits obligation (CBO).

Similarly, we must give the same respect to contract terms between the employer and the plan. The pension plan trustees and managers are agents standing between taxpayers and plan members, but the manner in which they discharge their duties directly affects the actual true cost to taxpayers of the benefits payable. Furthermore, the pension fund is not merely the employer's collateral for satisfying its direct contractual liability. That is not how it works.

Contractually speaking, the employer does not owe next year's pension benefit payments to plan members, nor the year after that, or the year after that. The pension plan owes them. If due to insolvency, the pension plan cannot pay the benefit, then the employer will step in to satisfy its original promise to plan members. While the public sector pension fund should not be deemed a mere pass through (more about this in the next section), the employer does retain a residual pension benefit liability in the event of the plan's default on its obligation to plan members.

In the spirit of modeling all moving parts carefully, we should consider an alternate model which simulates the operation of the pension fund over time, to see what liability or assets may remain for the employer to assume after the dust has settled. This alternate approach is truer to real world modeling.

B. The Pension Fund is Not a Pass Through

The public pension fund is the five-ton elephant in the room, which financial economics proponents ignore, in the name of "pass through."

There are inconsistencies in the arguments of those who advocate this pass through treatment. Pass through proponents never suggest that the pension fund assets be placed on the statement of net assets as a government asset in the name of pass through, like other collateral, such as cash with fiscal agents, or defeasance funds. Similarly, pass through proponents never suggest that the pension benefit liability itself be placed on the statement of assets as a long-term liability in the name of pass through. They insist that the *net* liability (market value of pension benefit liability minus market value of pension assets) be placed there. This feels much more like a residual liability, rather than pass through, but it is not modeled that way by financial economics proponents. Even the three improvements outlined above (CBO, risk-adjusted cash flows, and higher discount rates than risk-free) do not fix this flaw.

Let us turn our attention to the substantive ways in which the public sector pension fund is a material player (not to be ignored) to warrant specific treatment in our alternate fair model of pension benefit liabilities. This discussion applies to cost sharing employers as well as to sole and agent employers. While the arguments set forth in this section are stronger for cost sharing, multiple employer plans and their participating employers, they are equally applicable to sole and agent employers and their plans.

Multi-employer plans in the private sector have some of these same characteristics, which is why the pass-through concept is strained to the breaking point with private sector multi-employer plans. Private-sector, single-employer plans may have some of these characteristics, in theory only, which is why pass-through is not an unreasonable concept in that environment. However, these characteristics are much more apparent, exposed and exaggerated in the public sector.

The following qualities of public sector sole employer plans and agent multiple employer plans (in addition to cost-sharing, multiple-employer plans) drive this alternative model and our objection, in general, to the pass-through concept otherwise applicable to single-employer plans in the private sector environment. More on this topic and why public sector pension accounting is and should be different from the private sector can be found in the introduction to Part 5 of this series, "Consider the Measurement Purpose."

An Independent Entity. Typically, the public sector employer created the pension trust as a separate, independent entity. For example, Massachusetts' 106 public systems are independent from the municipalities by whom the members of the systems are employed[1], Michigan MERS is a statutory municipal employee plan that is a "public corporation" separate and apart from state government[2], and Missouri PSRS was created by state statute as a "body corporate." Often, the plan and its agent-staff are not even subject to the same administrative rules applicable to mere

[1] See Everett Retirement Bd. v. Board of Assessors of Everett, 19 Mass. App. Ct. 305, 473 N.E.2d 1162, Mass. App., 1985.

[2] Michigan Compiled Law 38.1536(1) and 38.1502c(3).

agencies of government. The pension fund is an independent trust and not taxpayers' money.[3] The financial managers of private sector companies are inextricably linked to the management of their single-employer pension plans. This is not true in the public sector.

Sue and be sued. A marker of independence is whether the plan is a jural entity that can sue and be sued on its own and in its own name. This is commonly a characteristic of public sector pension plans.[4]

Contract. There is a contract between the employer and the pension trust. In this contract, the employer has put the responsibility for pension *benefit* payments onto the pension plan in exchange for taking on a *funding* responsibility.

Enforcement of the exchange. There have been times when the employer has breached its agreement with the pension trust. Pension plans often have the authority to sue the employer for failure to fulfill its funding obligation under the contract, and as proof that this contract between the public sector employer and pension trust is a very real one, pension trusts have indeed exercised that authority and sued the employer for not contributing as scheduled in order to enforce the contract, for diverting funds and other breaches. Furthermore, employers have, at times, sued the plans.[5]. In fairness, there have been times in which the employer has reneged on its funding responsibility, and it was upheld in courts. However, the weight of common law is in favor of the employer's funding obligation enforced. This litigation seldom ever happens with single-employer plans in the private sector.

The Creditor. Clearly, the employer has a pension liability, which should be presented somewhere in its financial statements. But if the employer is a debtor for pensions, who is the debt owed to? If there is a debt to pay by the employer, who is the creditor? The transaction between the employer and the pension fund is a real one. The debt owed by the employer is not a benefit liability (benefits payable to plan members) but a funding liability (contributions payable to the pension trust). The employer owes payments to the pension fund, not to the employees. The pension fund is not a pass-through. It is the creditor.

The Payer. The pension trust is the benefit payer of first resort, while the employer is the benefit payer of last resort.

Recourse. To illustrate this exchange and its resultant benefit payment priorities, consider an employee who believes his pension was not calculated properly. In most situations, he will get nowhere appealing to or suing the employer. He must appeal to or sue the pension trust. The

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[3] Dadisman v. Moore, 384 S.E. 2d 816 (W.V. 1989); City of Miami v. Gates, 393 So. 2d 586 (Fla. 3rd DCA 1981).

[4] City of Houston HMEPS (Article 6243h, sec. 2(g), HMEPS v. Ferrel, Thayer v. HMEPS), City of St. Louis ERS (see State of Missouri, ex rel. Employees Retirement System of the City of St. Louis et al., v. Board of Estimate & Apportionment of the City of St. Louis, et al., 43 S.W.3d 887 Mo. App. E. D., 2001), Kentucky RS (KRS 61.645), Mississippi PERS (MCA Section 25-11-119(5)).

[5] City of St. Louis (refer to the previous citation and the related case of Neske v. City of St. Louis, 218 S.W.3d 417 (Mo. en banc. 2007), Illinois IMRF (given specific statutory authority to sue to enforce contributions, Section 7-172.1 of the Illinois Pension Code, 40 ILCS 5/1-101 *et seq*.); McDermott v. Regan, 624 N.E. 2d 985 (N.Y. 1993).

employer does not retain any responsibility for paying the benefits. If he loses with the pension trust, there is nowhere else to go.

Economic engine. Often the pension fund is larger than the employer itself, in terms of net assets. According to a recent study published by Boivie and Almeida (2009), in fiscal year 2005-06 expenditures from state and local pension benefits totaled $151.7 billion to 7.3 million pensioners and had a total economic impact of more than $358 billion, supported more than 2.5 million American jobs, and had a large multiplier effect with every taxpayer dollar invested in state and local pensions supporting $11.45 in total economic activity, while each dollar paid out in benefits supported $2.36 in economic activity.

Residual Assets and Liabilities. A clearer perspective of the employer's pension *benefit* liability would consider it a residual liability. After the pension fund has paid out all its assets on schedule with some remaining benefits yet to be paid, the residual obligation is an employer liability. On the other hand, if the last pensioner receives his last benefit and assets remains, some pension contracts say that such remainder may revert to the employer (after satisfaction of all liabilities), in which case the employer would reflect a residual asset. Other contracts may require that all assets be used for plan benefits of some sort or another, in which case there would be no residual asset. Nevertheless, it is difficult to imagine large, statewide, cost-sharing multiple-employer plans (as defined by GASB standards) running out of money with unpaid benefit obligations falling back onto each respective contributing employer ratably or otherwise. This model of the employer's residual benefit liability is more easily imagined for sole and agent employers.

The contract (including the enabling and operational documents) and the participating entity (the pension trust) are so very important in the delivery of public sector pension benefits, that their existence and operation must be considered when determining the fair value of the public sector pension benefit liability.

We propose an alternate approach to determine fair value of the employer *benefit* liability by modeling the operation of the public sector pension fund over time. This approach better represents the employer's risks and rewards, considering the true manner in which the pension obligation is defeased over time.

This alternate model is a fair value of the residual liability. The only *benefit* liability the employer has is a residual liability.

C. Modeling Residual Assets and Liabilities

If we modeled the operation of the plan's payment of the risk-adjusted CBO cash flows on schedule and the plan's investment return every year until the last pension benefit payment is due, we would have a model of the employer's residual asset or liability.

If there are benefits left to be paid after the pension fund comes to ruin, then the sum of their present values (discounted at a high quality corporate yield curve observed at the measurement date) would represent the employer's fair value of the residual pension benefit liability because the public sector employer would still be on the hook for their payment. On the other hand, there may be assets left after the last benefit payment is made. In that case, if the contract between the employer and the plan contains a reversion clause upon the satisfaction of all plan liabilities, then the present value of that remaining asset value (discounted at the same rate as the fund was assumed to have earned over the time period) would represent the employer's residual pension benefit asset.

Recall that there is a very real exchange agreement between the employer and plan in which the employer puts the pension *benefit* obligation to the pension fund, but accepts a substantial advance *funding* obligation pursuant to the pension fund's chosen actuarial cost method. The employer has no direct pension benefit liability until or unless the pension fund runs out of money before all pension benefits (contractually earned at the measurement date) are paid. Certainly, the employer may have a substantial *funding* obligation in accordance with conventional actuarial cost methods as of the measurement date, but its pension *benefit* obligation is the residual obligation described.

This leaves an actuarially interesting question. What assumptions and methods as to the investment rates of return should be used for simulating the operation of the pension fund, so that we may know what are the residual expected (or rather, risk-adjusted) benefit cash flows that constitute the employer's obligation?

Deterministic Modeling

Certainly, for a deterministic forecast, the 50^{th} percentile of range of expected rates of return for the investment policy in place on the measurement date is a reasonable candidate. Alternatively, the 25^{th} percentile might be used to build in a margin for error. Additional guidance for answering this question can be found a in previous discussion in Part 3 of this series, "A Market-Related Discount Rate," about how public sector pension funds might select a discount rate for pricing their exit liability in a market where they buy and hold or sell pension liabilities for gain. Furthermore, if the portfolio were invested entirely in intermediate government securities, an expected return of 5.0 percent is reasonable (SunGard, 2009), and for a corporate bond portfolio, 5.75 percent (SunGard, 2009). Finally, a sequence of consecutive annual returns was reverse engineered from the STRIPS spot yield curve for Dec. 31, 2008. This sequence of returns themselves can be used to model the operation of a pension fund if it were invested entirely in Treasury STRIPS, matching the cash flow, as some financial economics advocates suggest.

These are scenarios of the future investment performance of the pension fund and are treated as deterministic assumptions in each future year. Stochastic approaches provide more information including likelihood ranges, and will be explored below. However, initially, for simplicity and illustration, we are limiting the forecast to deterministic views of the future. To summarize the candidates:

Figure 14

	Deterministic ROR Assumption
Balanced Portfolio 50th Percentile	7.60%
Balanced Portfolio 25th Percentile	6.59%
Equivalent Single Rate for Dec08 AA Corporate Spots (*CitiGroup*)	6.07%
Corporate Bond Expected Return	5.75%
Intermediate Govt Bond Expected Return	5.00%
Equivalent Annual Returns for Dec08 STRIPS (*Ryan Labs*)	Sequence[1]

The following figure presents risk-adjusted cash flows and plan asset values at five-year intervals until the end, using the expected return (50^{th} percentile) of the balanced portfolio in a deterministic forecast of the plan operation.

In the same manner as the full run of risk-adjusted CBO cash flows were discounted using the CitiGroup Pension Discount Curve observed as of Dec. 31, 2008, in Part 3 of this series, "A Market-Related Discount Rate," we are discounting the residual benefit payments using the same assumptions.

Figure 15

Deterministic Simulation (7.60% ROR; 50th Percentile) of Plan Operation under Alternate Fair Value Model							
$ In Thousands	MVA (BOY)	Risk-Adjusted CBO Cash Flows	Assumed Investment Earnings	MVA (EOY)	Residual Employer-Paid Benefits Due	CitiGroup High Quality Corp Spot Rates	PV at 1/1/2009 of Residual Benefits
2009	$ 380,717	$ 28,476	$ 28,935	$ 381,175	$ -	4.90%	$ -
2014	371,567	33,622	28,239	366,185	-	5.62%	-
2019	331,009	37,320	25,157	318,845	-	6.59%	-
2024	253,675	40,175	19,279	232,779	-	7.14%	-
2029	127,936	41,726	9,723	95,934	-	6.99%	-
2034	-	41,245	-	-	41,245	5.84%	9,441
2039	-	38,020	-	-	38,020	5.03%	8,304
2044	-	31,653	-	-	31,653	5.03%	5,409
2049	-	22,865	-	-	22,865	5.03%	3,057
2054	-	14,077	-	-	14,077	5.03%	1,473
2059	-	7,379	-	-	7,379	5.03%	604
2064	-	3,430	-	-	3,430	5.03%	220
2069	-	1,578	-	-	1,578	5.03%	79
2074	-	760	-	-	760	5.03%	30
2079	-	315	-	-	315	5.03%	10
2084	-	88	-	-	88	5.03%	2
2089	-	17	-	-	17	5.03%	0
2094	-	2	-	-	2	5.03%	0
2099	-	0	-	-	0	5.03%	0
2104	-	0	-	-	0	5.03%	0
2109	-	0	-	-	0	5.03%	0
Total Present Value at January 1, 2009 of the Residual Employer-Paid Benefits							$ 137,497,517

In this model of the operation of the pension fund, no future employer or employee contributions are made following the measurement date and no new contractual benefits accrue thereafter either. No administrative expenses are assumed and benefit payments are assumed to be made at year end.

In this deterministic forecast of the employer's residual benefit liability the pension fund comes to ruin in the year 2033. At the end of that year, there are no more funds left in the pension trust to fulfill its obligations under the contract it has with the employer. Hence, the employer becomes the payer of last resort and must pay the benefits promised. The present value of that residual benefit liability, discounted at market-related discount rates, is the employer's unfunded benefit obligation at the measurement date. Under a deterministic forecast, this represents the fair value of the employer's residual benefit liability.

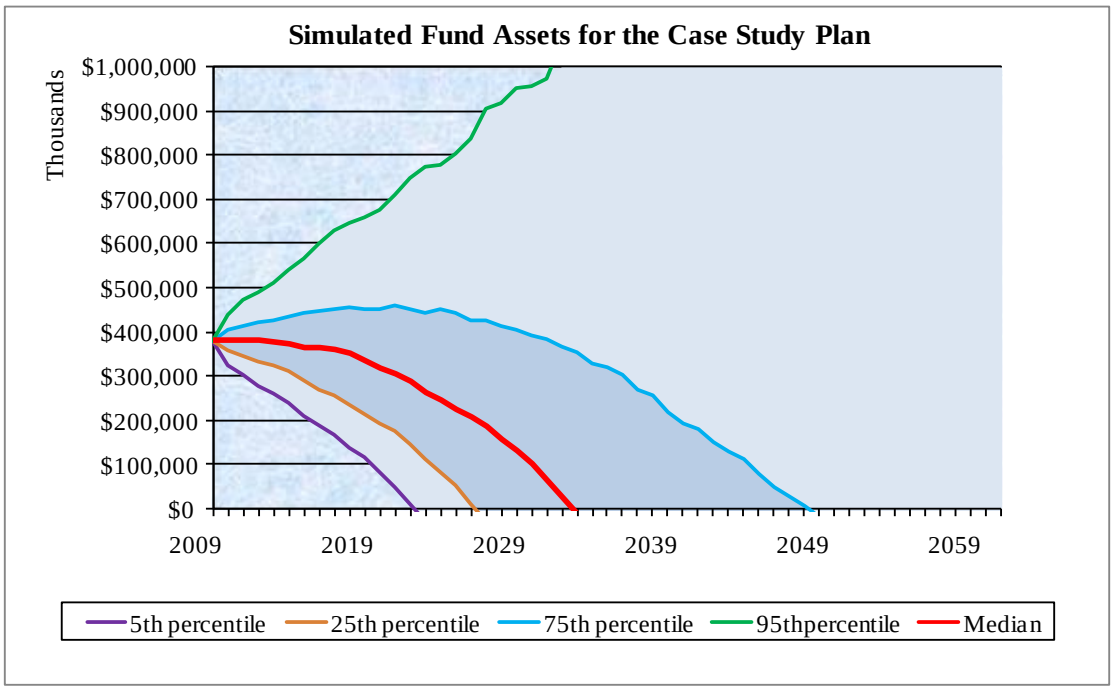
Stochastic Modeling

Rather than simply assume that the pension fund will earn exactly the expected return each year, more useful information can be extracted from the model by simulating, in a stochastic fashion, the pension fund's investment earnings in each future year.

We assumed that the future returns of the balanced portfolio will follow the normal/lognormal distribution curve. As mentioned previously, in Part 3 of this series, "A Market-Related Discount Rate," there is a renewed interest in fat-tail distributions such as Paretian and other log-stable distributions to more accurately reflect market swings in both tails of the distribution of returns. We seem to be having those once-in-a-century events every decade. But in the interest of simplicity and conventional practice, we will utilize the normal/lognormal distribution for modeling the pension fund returns, based on a mean of 7.97 percent and a standard deviation of 8.97 percent developed in Part 3 of this series. Again, these were developed using the capital asset pricing model and capital market assumptions from SunGard (2009).

A total of 500 trials were run (sufficient to stabilize the results), each producing returns for our case study plan for each of the next 100 years. The figure below tracks the value of plan assets over time.

Figure 16



Many plans had worse funded ratios than our case study plan as of Jan. 1, 2009. On the other hand, at times in the past (and hopefully in the future again), when the equity markets were not so depressed, many plans had much better funded ratios. In a not-so-depressed equity market, this model of the employer's residual liability would show a much more favorable picture. The distribution of the year of ruin (and resultant residual liability) is very sensitive to the beginning

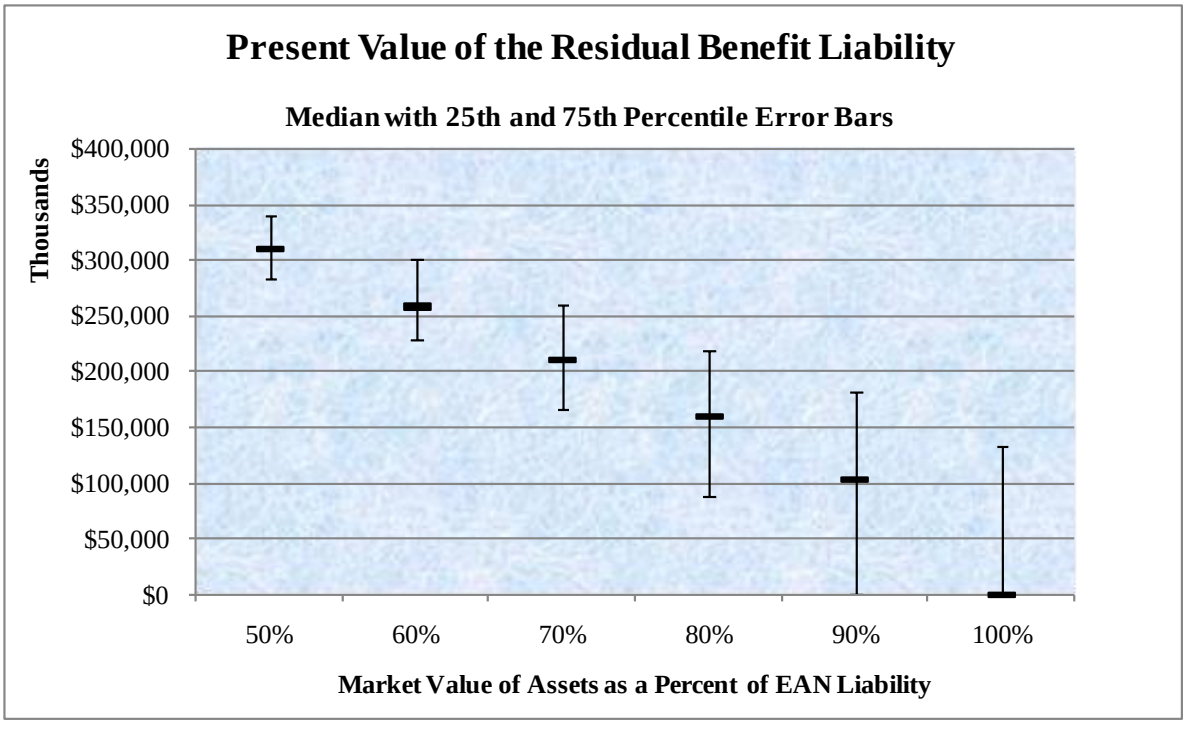
value of plan assets. In a better market, this model might easily show no ruin even at the 5th percentile.

With such volatility in the investment market and such volatility in the bond yields, the resultant volatility in the residual benefit liability is partly why the fair value of a public sector pension liability is of such limited utility for most purposes in a practical world where public sector plans seldom terminate or freeze.

Our case study plan had a beginning market value of assets equal to approximately 85 percent of the entry age actuarial accrued liability. We adjusted the beginning market value of assets up and down to illustrate the sensitivity of the resultant present value of the employer's residual benefit liability.

The figure below presents the median (50th percentile) value of the residual liability with error bars at the 25th and 75th percentiles, based on 500 trials of the same normally distributed returns, the same risk-adjusted CBO cash flows, and the same discounting of the residual benefit payments due using the CitiGroup Pension Discount Curve observed at Dec. 31, 2008.

Figure 17



Summary of Unfunded Obligations

Part 1 in this series, "The Contractual Benefit Obligation," proposed an improvement to the current model of the market value of liability, by revising the benefits valued to be more consistent with labor economics reflected in the contract between the employer and employee. We presented a bar chart comparing the present values of the ABO and CBO.

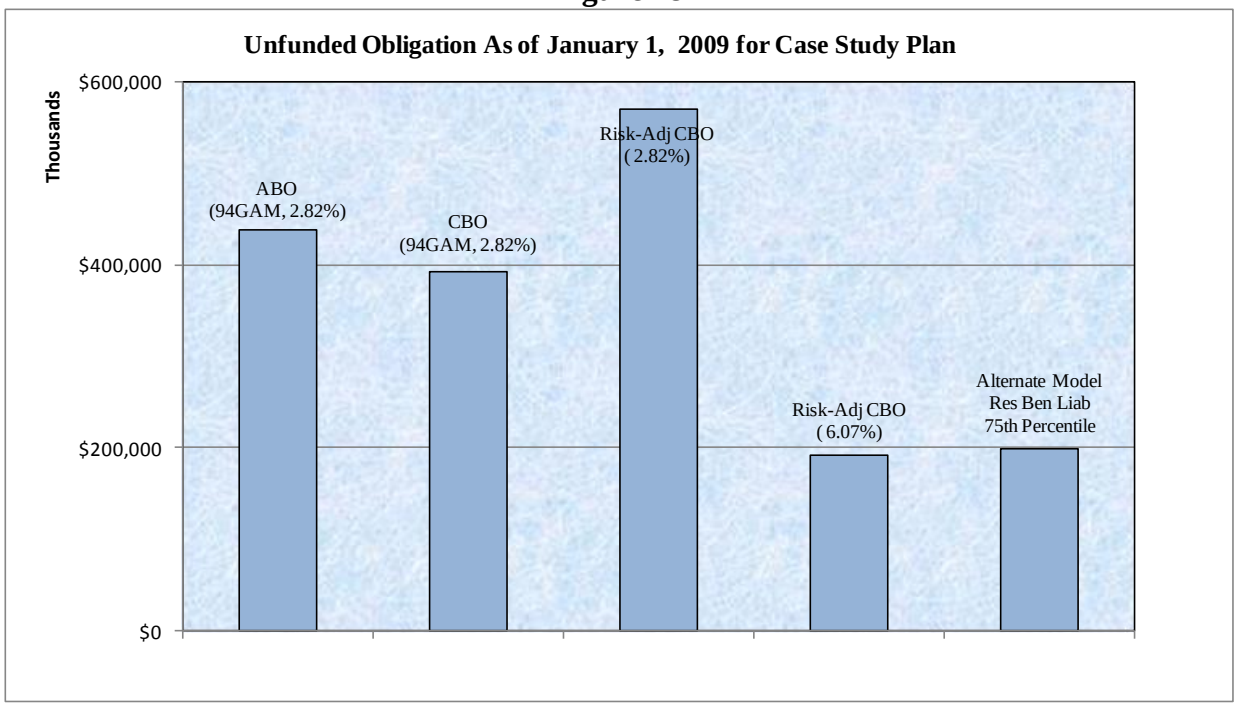
Part 2 in this series, "Risk-Adjusted CBO Cash Flows" proposed utilizing risk-adjusted cash flows instead of treating the expected cash flows as if they were fixed, to be more consistent with actuarial finance and pricing of a fair value. That part added the value of the risk-adjusted CBO cash flows to the bar chart for comparison.

Part 3 in this series, "A Market-Related Discount Rate," proposed the third improvement to the current model by suggesting a higher discount rate would be observable in the market, in order to be more consistent with financial engineering principles of fair value. That part added the fair value (using all three improvements) to the comparative bar chart.

Finally, this Part 4 of the series, "The Residual Benefit Liability," proposed an alternate model for measuring the fair value of the employer's pension benefit obligation, to reflect more consistently all the moving parts and the inherent contract between the employer and the pension plan.

For comparison purposes, the figure below adds the case study plan's employer residual liability expected at the 75th percentile (where percentiles above 50 are worse scenarios) to the bar chart of other unfunded pension obligations.

Figure 18



A great deal of effort has been expended in this series of papers proposing improvements and an alternate to the current model of market value of liability. As discussed previously, these models of employer liability have little usefulness in the real world.

But in the event that public sector actuaries, employers and plans are required to calculate and publish a fair value of the pension benefit liability, the current model needed a major overhaul. We wished to contribute to the body of knowledge for improving our methods to have more consistent integrity with the fair-value measurement attribute.

Part 5 of this series, "Consider the Measurement Purpose," explores eight common purposes for which pension costs and liabilities must be calculated, demonstrates the lack of decision utility inherent in a fair value model, and presents more suitable models for these common purposes.

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Special Thanks

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