Living to 100: Survival to Advanced Ages: Insurance Industry Implication on Retirement Planning and the Secondary Market in Insurance

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Abstract

The paper will focus on two broad areas:

1. Integration of insurance products with investment products to mitigate the risk of outliving one's assets in post-retirement financial planning

2. Modeling and pricing for the longevity risk in the secondary market in insurance.

For the first area, the author will discuss different designs of variable and fixed immediate annuities together with investment products in order to manage the longevity risk. In particular, the author will discuss some theoretical results from the doctoral research work of one of his PhD students on integrated post-retirement financial planning. The research describes asset allocation techniques between investment, annuity and insurance products in order to optimally manage post-retirement income and bequest needs subject to ruin probabilities being kept within a prescribed minimum level.

For the second area, the author will describe techniques on how to manage the longevity risk in the secondary market in insurance for impaired policyholders needing liquidity from their existing life insurance policies. The author will describe how he has adapted into the secondary market in insurance the doctoral research work of his PhD student in developing a provision for adverse deviation (PAD) model for the longevity risk of structured settlements. In particular, the author will discuss how life expectancy and qualitative information from external underwriters can be utilized to quantify the slope risk, underwriter misstatement risk and statistical volatility risk of impaired policyholders in order to develop a PAD for the longevity risk.

1. Survival to Advanced Ages: Implications on Retirement Planning

There are several risks facing retirees with limited assets to cover their financial needs: the mortality risk of living too long and outliving one's assets, the morbidity risk of a prolonged illness eroding one's assets and the investment risk of poor or volatile asset performance. There are other risks like interest rate fluctuations, inflation and taxes, which could have an impact on retirement planning, but they are relatively stable over time and can be modeled deterministically using reasonable assumptions.

For the longevity risk, the insurance products available in the marketplace to mitigate this risk are fixed and variable immediate annuities. In the PhD research work of Dr. Peng Zhou at the University of Connecticut, who was jointly supervised by Professor Vinsonhaler and myself, he developed an integrated financial planning model which optimally allocates a retiree's assets into immediate annuities, investment products and health care products, in order to maximize a set of defined financial objectives, subject to a minimum ruin probability. In this paper, we will focus on just the longevity and investment risk and demonstrate that an optimal allocation of assets between immediate annuities and investment products is a "better" strategy than allocating all of a retiree's assets into investment products.

The financial objectives of a retiree are twofold:

1. Maximize current spending levels at retirement; and
2. Maximize estate value at death.

The constraints are a fixed initial level of retirement assets and a ruin probability within a given tolerance level. Ruin is defined as failing to meet either or both of the two financial objectives anytime from today until the time of death.

In general, immediate annuities have the drawback that they do not create any estate value. So, immediate annuities provide longevity insurance at the expense of estate protection. A pure investment or self-annuitization strategy, while failing to cover the longevity risk, does create an estate of outstanding or unused assets which declines with the duration of death. However, a well crafted investment and immediate annuity strategy can create both longevity risk protection and greater estate value than a pure investment strategy.

The theorem in Dr. Zhou's thesis demonstrates this result. Due to the significance of this result, I have reproduced this theorem verbatim from Dr. Zhou's thesis:

A retiree who has initial assets A will consume a certain amount of money, $C(t)$, in the $t-th$ time period. The self-annuitization strategy is to put all assets into a pure investment portfolio and withdraw $C(t)$ at the end of the $t-th$ period. We called the sequence of cash values of the pure investment portfolio at the end of every period after withdrawal the estate process under self-annuitization strategy, $\{M(t)\}$.

Alternatively, the annuitization strategy will use all initial assets A to buy a life annuity and open a side fund account. At the end of period t, the individual has income from this life annuity, $P(t)$. If $P(t)$ is more than $C(t)$, he will reinvest the surplus $P(t) - C(t)$ into the side fund account; otherwise, he will have to cash out the amount $C(t) - P(t)$ from the side fund account to cover living expenses. There is no cash value for a life annuity and we call the sequence of cash values in the side fund account the estate process under annuitization strategy, $\{A(t)\}$. Suppose the same type of fund is being used by the pure investment portfolio under the self-annuitization strategy, and the life annuity and side fund account under annuitization strategy. Denote the rate of return on this fund during the $t-th$ period $R(t)$. Then the estate process under self-annuitization strategy is

$$
\begin{aligned}
&M(0) = A,\\
&M(1) = M(0)(1 + R(1)) - C(1),\\
&\quad\cdots\\
&M(t) = M(t-1)(1 + R(t)) - C(t).
\end{aligned}
$$

For the annuitization strategy, suppose the assumed interest rate (AIR) of the life annuity is i. The initial payment $P(0)$ is determined based on guaranteed mortality and the AIR. It satisfies $A = P(0)a_x$. The actual payments from this life annuity would be adjusted periodically to reflect the investment earnings of the underlying asset portfolio. Subsequent payments will increase or decrease, depending on the actual investment performance of the annuity funds compared to the AIR. If the rate of return is higher (lower) than AIR, i.e. $R(t) > (<)i$, the following payment will increase (decrease). Atkinson and Dallas (2000) give the recursive formulas as follows,

$$
\begin{aligned}
P(1) &= P(0)\frac{1+R(1)}{1+i},\\
&\cdots\\
P(t) &= P(t-1)\frac{1+R(t)}{1+i}.
\end{aligned}
$$

The estate process under annuitization strategy is

$$
\begin{aligned}
A(0) &= A - P(0)\cdot a_x = 0,\\
A(1) &= A(0)(1+R(1)) + P(1) - C(1),\\
&\cdots\\
A(t) &= A(t-1)(1+R(t)) + P(t) - C(t).
\end{aligned}
$$

Theorem 1 There exists a duration n such that $a_x = a_{\overline{n}|}$ and $n \leq e_x$. Furthermore, for this duration n,

$$
\begin{aligned}
M(t) &> A(t) \quad \text{if} \quad t < n; \\
M(t) &= A(t) \quad \text{if} \quad t = n; \\
M(t) &< A(t) \quad \text{if} \quad t > n.
\end{aligned}
$$

Proof. We notice that the actuarial present value of a life annuity is no greater than that of an annuity-certain with duration e_x, i.e., $a_x \leq a_{\overline{e_x}|}$. The reason is simple.

$$
\begin{aligned}
\text{Let} \qquad h(t) &= a_{\overline{t}|} = (1 - v^t)/i, \\
\text{then} \qquad h''(t) &= -(\ln v)^2 v^t / i < 0.
\end{aligned}
$$

Jensen's Inequality states that for a random variable X and function $u(w)$,

$$
\begin{aligned}
&\text{if} \quad u''(w) < 0, \quad \text{then} \quad E[u(X)] \leq (E[X]), \\
&\text{if} \quad u''(w) > 0, \quad \text{then} \quad E[u(X)] \geq (E[X]).
\end{aligned}
$$

By Jensen's Inequality, we have

$$
a_x = E[a_{\overline{T(x)}|}] \leq a_{\overline{E[T(x)]}|} = a_{\overline{e_x}|}.
$$

It is obvious that the present value of an annuity certain $a_{\overline{t}|}$ as a continuous function of duration t is strictly increasing and $a_{\overline{0}|} = 0$. Recognizing $a_x \leq a_{\overline{e_x}|}$, we know there must exist a duration n, such that $a_x = a_{\overline{n}|}$ and $n \leq e_x$.

Next, we considered the relationship between estate processes under the annuitization and self-annuitization strategies.

$$
\begin{aligned}
M(t) &= M(t-1)(1+R(t)) - C(t) \\
&= M(0)\prod_{j=1}^{t}(1+R(j)) - \sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right] \\
&= A\prod_{j=1}^{t}(1+R(j)) - \sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right]. \\
A(0) &= 0 = A - P(0)\cdot a_x = A - P(0)\cdot a_{\overline{n}|}.
\end{aligned}
$$

$$
\begin{aligned}
A(t) &= A(t-1)(1+R(t)) + P(t) - C(t) \\
&= A(0)\prod_{j=1}^{t}(1+R(j)) + \sum_{j=1}^{t}\left[P(j)\prod_{k=j+1}^{t}(1+R(k))\right] \\
&\quad -\sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right]
\end{aligned}
$$

$$
\begin{aligned}
&= A(0)\prod_{j=1}^{t}(1+R(j)) + P(0)\prod_{j=1}^{t}(1+R(j))\sum_{j=1}^{t}(1+i)^{-j} \\
&\quad -\sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right] \\
&= (A(0)+P(0)\cdot a_{\overline{t}|})\prod_{j=1}^{t}(1+R(j)) - \sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right] \\
&= (A(0)+P(0)\cdot a_{\overline{n}|} + P(0)\cdot a_{\overline{t}|} - P(0)\cdot a_{\overline{n}|})\prod_{j=1}^{t}(1+R(j)) \\
&\quad -\sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right] \\
&= A\prod_{j=1}^{t}(1+R(j)) - \sum_{j=1}^{t}\left[C(j)\prod_{k=j+1}^{t}(1+R(k))\right] \\
&\quad +P(0)(\cdot a_{\overline{t}|} - \cdot a_{\overline{n}|})\prod_{j=1}^{t}(1+R(j)) \\
&= M(t) + (a_{\overline{t}|} - a_{\overline{n}|})P(0)\prod_{j=1}^{t}(1+R(j)).
\end{aligned}
$$

Therefore, the difference of estates for these two strategies at the end of period t is

$$\Delta_{ES}(t) = A(t) - M(t) = (a_{\overline{t}|} - a_{\overline{n}|})P(0)\prod_{j=1}^{t}(1 + R(j)). \qquad (4)$$

Factor $P(0)\prod_{j=1}^{t}(1 + R(j))$ is always positive, so the sign of this difference depends on the sign of $a_{\overline{t}|} - a_{\overline{n}|}$. Again, $a_{\overline{t}|}$, as a function of t, is strictly increasing.

$$\begin{aligned} a_{\overline{t}|} &< a_{\overline{n}|} \quad \text{if} \quad t < n; \\ a_{\overline{t}|} &= a_{\overline{n}|} \quad \text{if} \quad t = n; \\ a_{\overline{t}|} &> a_{\overline{n}|} \quad \text{if} \quad t > n. \end{aligned}$$

The Proof is complete.

Note that this theorem holds under two key assumptions that typically do not hold in practice:

1. The variable immediate annuity earns the same return as the mutual fund in the pure investment strategy. In reality, spread charges are larger for variable annuities compared to mutual funds.

2. The mortality rates used to calculate the initial variable immediate annuity payment are the same as the experience mortality of the retirees. In practice, an insurer would use lower than expected mortality rates to provide a margin for contingencies.

The implications of this theorem that estate value preservation and longevity risk protection can both be accomplished using immediate annuities naturally led to the inclusion of additional insurance products in the retirement planning model to create a more robust optimal asset allocation strategy. Besides fixed and variable immediate annuities, mutual funds with varying risk/return characterizations and health care products for the morbidity risk, there are three additional insurance products that should be incorporated to create a complete and holistic retirement planning strategy:

1. Declining 1 year term insurance to cover the gap in estate value for death prior to life expectancy.

2. A deferred immediate annuity which starts making payments beyond the retiree's life expectancy. This is a significantly cheaper alternative to cover the longevity risk compared to an immediate annuity.

3. A layered immediate annuity strategy which purchases a layer of immediate annuity protection each year until a retiree's life expectancy.

Given the choice of insurance and investment products, the complexity of the objective function and the sensitivity of the optimal solution to the underlying actuarial and investment assumptions, the optimal asset allocation is reached using a stochastic simulation of all possible asset combinations.

There are a few observations about the optimal solution that are interesting:

1. For most levels of initial assets, the optimal mix incorporates a significant number 25 to 45 percent of immediate annuity products, which include some portion of deferred immediate annuities and the layered immediate annuity strategy. This is in contrast to current allocation levels of 2 to 4 percent of retiree assets to immediate annuities that most insurance companies are experiencing.

2. The declining 1 year term insurance asset allocation is relatively modest.

3. Purchasing catastrophic illness coverage upfront is almost always part of the optimization criteria versus a pay-as-you-go approach.

4. A well designed immediate annuity and investment strategy generally out-performs a pure investment strategy. This is the case even after recognizing larger investment spread charges for variable immediate annuities, compared to a pure investment strategy.

5. A combined investment and insurance strategy allows a retiree to adopt a more aggressive investment strategy since there is a layer of guaranteed income protection.

In conclusion, insurance risks are key risks to consider for any integrated financial planning. In particular, the longevity risk of outliving one's assets can be hedged by investing in immediate annuities. However, immediate annuities do not create any estate upon death and, as a result, do not constitute a significant portion (only 2 to 4 percent) of the assets of the retirement population. In an integrated retirement financial planning approach, which recognizes the longevity risk as one of several investment and insurance risks facing a retiree, immediate annuities (and other variations of immediate annuities), constitute a significant portion (25 to 40 percent) of the planning strategy for a retiree. The optimal strategy combining insurance and investment products outperforms a pure investment strategy in both maximizing spending income at retirement and ensuring a minimum estate value at death, for a given ruin probability.

2. Survival to Advanced Ages: Implications on the Secondary Market in Insurance

This section will address the longevity risk issue for the life settlements industry, which is one of the key components of the secondary market in insurance.

Unlike premium financed policies which are newly issued, the life settlements industry consists of the sale of in force insurance contracts where the policyholder has experienced deterioration in health status. The policies are reunderwritten by an external underwriter, and the settlement price of the policy is determined based on the life expectancy estimate provided by the external underwriter.

The risk of the institutional investors investing in life settlements continuing to make premium payments may be likened to the mortality risk faced by insurance companies involved in the structured settlements business or in issuing annuities to impaired policyholders. In the latter two instances, the policyholder is impaired and the risk is that the policyholder lives long enough that the value of the payments made by the insurance company exceeds the premium paid by the policyholder.

For life settlements, the risk is the same, but the transaction is reversed. The investor in a life settlements transaction makes the initial payment to the policyholder and continues to make the necessary premium payments to keep the policy in force. The payoff to the investor occurs upon the death of the policyholder from the death benefit proceeds. The risk is that the policyholder lives long enough that the value of the payoff is less than the combined value of the life settlements sale price and ongoing premium payments.

One of the complexities of the longevity risk for life settlements is that mortality experience data for impaired, older age annuitants is not available. The only information available to estimate the impaired mortality curve in a life settlements transaction is the external underwriter life expectancy (LE) estimate.

Most of the work described below is the research of Sudath Ranasinghe, a current PhD candidate at the University of Connecticut, who is jointly being supervised by Professor Vinsonhaler and me. In developing a provision for adverse deviation (PAD) methodology for the longevity risk, there are three issues to consider:

1) Assigning an appropriate mortality slope for the impaired policyholder such that the external underwriter LE is preserved. Different mortality slopes having the same LE can generate significantly different life settlement values.

2) Measuring and quantifying the PAD for the underwriter misstatement risk of understating the true LE of the policyholder.

3) Measuring and quantifying the PAD for the statistical volatility risk of the policyholder living beyond his LE.

In coming up with the appropriate substandard mortality slope, it is common practice to multiply the mortality rates of a base mortality table by a constant factor so as to reproduce the underwriting LE of the policyholder. Based on the underwriting LE and specific medical information on the impaired policyholder, we have generalized the substandard mortality slope to be of the form:

$$\mu^*(x+t) = a(t)\mu(x+t) + b(t)$$

where: $\mu^*(x+t)$ is the substandard force of mortality at age x+t

$\mu(x+t)$ is the standard force of mortality at age x+t

a(t), b(t) are parameters which vary by type of impairment and underwriting LE

The choice of mortality slope can have a significant impact on the value of the life settlement transaction. Consider a male, age 70, with an underwriting LE of 8 years, $1M Universal Life, Option 1, zero fund value policy issued at 65, with cost-of-insurance charges based on the 2001 VBT Basic Table. A constant mortality multiple substandard slope produces a life settlements value of approximately $246 per $1,000 of face amount. On the other hand, a constant additive substandard slope for 8 years produces a life settlements value of approximately $301 per $1,000 of face amount. This is about a 22 percent increase in value, which is significant.

Once the appropriate substandard mortality slope is determined, the PAD for the underwriter misstatement risk and statistical volatility risk is calculated and added to the underwriter LE. The PAD for the underwriter misstatement risk requires three inputs:

- Level of confidence or reliability of the underwriter
- Level of tolerance of the cost of the LE misstatement
- Complexity and level of control of the policyholder impairments.

The following is a simplified example on how the PAD for the underwriter misstatement risk is calculated. Assume the following:

- Impaired underwriting LE = 5 years
- LE of corresponding healthy lives at same issue age = 10 years
- i = 0
- Underwriter reliability = 85 percent

- Level of tolerance = 5 percent

- If “true” LE is 6 years, then normalized cost is (6-5)/5 = 1/5
- Using a “sum-of-digits” technique to derive the probability distribution of the normalized cost, we get the following:

"True" LE	Cost	Probability	Cost * Probability
<=5	-	0.85	-
6	0.20	0.05	0.010
7	0.40	0.04	0.016
8	0.60	0.03	0.018
9	0.80	0.02	0.016
10	1.00	0.01	0.010
	TOTAL	**1.00**	**0.07**

- Suppose the PAD of 1 year increase in LE is used

  i.e., pricing LE = 6 years

- Then, normalized cost distribution is as follows:

	NO PAD			**PAD OF 1 YEAR INCREASE IN LE**			
"True" LE	**Cost**	**Prob.**	**Cost * Prob.**	**"True" LE**	**Cost**	**Prob.**	**Cost * Prob.**
<=5	-	0.85	-	<=6	-	**0.90**	-
6	0.20	0.05	0.010				
7	0.40	0.04	0.016	7	0.17	0.04	0.007
8	0.60	0.03	0.018	8	0.33	0.03	0.010
9	0.80	0.02	0.016	9	0.50	0.02	0.010
10	1.00	0.01	0.010	10	0.67	0.01	0.007
	TOTAL	**1.00**	**0.07**		**TOTAL**	**1.00**	**0.03**

= (6 - 5) / 5

= (7 - 6) / 6

- Since E(normalized cost) <= 5%, PAD for misstatement risk equals 1 year increase in LE

While many of these input parameters are subjective, some general observations can be made:

- The greater the level of underwriter confidence, the smaller the PAD.
- The smaller the level of tolerance for the misstatement cost, the greater the PAD.
- The more complex the impairments or the greater the level of individual control over these impairments, the greater the PAD.

- The underwriter misstatement risk is a non-diversifiable risk.

The second PAD component for the statistical volatility risk recognizes the fact that the LE estimate is simply the mean of the future lifetime random variable of the policyholder. The volatility of the future lifetime random variable can be calculated for each policy in a pool of life settlement polices. The PAD for the statistical volatility risk is based on the standard deviation of the average future lifetime random variable of the pool. Unlike the underwriter misstatement risk, the statistical volatility risk is a diversifiable risk which reduces as the size of the pool increases.

The techniques described above to mitigate the longevity risk in the life settlements industry do not directly capture or attempt to predict medical breakthroughs in the future which could significantly extend life expectancies for older lives. It is important that institutional investors who invest in life settlement pools ensure that the pools are sufficiently large and diverse by LE, age distribution and policy face amounts, as well as types of impairment. A lesson should be drawn from the collapse of several viatical companies investing in predominantly AIDS impaired policies, when LEs were significantly lengthened with the introduction on anti-retroviral drugs.

In conclusion, the life settlements industry is growing by leaps and bounds, and many investors are being drawn in by promises of high returns. However, a disciplined approach to measuring and quantifying the risks associated with this asset class is not available. This is because the single most critical risk impacting investor returns is the longevity or extension risk of older, impaired policyholders, which is an actuarial risk not fully understood by the investment community. Besides pricing for this risk through an explicit provision for adverse deviation in the estimated life expectancy, and determining the most appropriate impaired mortality slope for the policy, ongoing monitoring of actual to expected mortality experience for a portfolio of life settlement assets is critical as well. Like any other investable asset class which is regularly repriced and traded, life settlement assets have to be managed similarly using sound actuarial techniques to reflect the embedded longevity risk.

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