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A RESERVE DIFFERENCE FORMULA

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This note is motivated by Section 7.4 of *Actuarial Mathematics.* Consider two different fully-discrete policies issued to (x), each for a unit of life insurance during the first h years. Here, h is less than or equal to the shorter of the two premium-payment periods. For each positive integer m, $m \le h$, the retrospective formulas for the reserves are

$$_mV_{(1)} = P_{(1)}\, \ddot{s}_{x:\overline{m|}} - {}_mk_x$$

and

$$_mV_{(2)} = P_{(2)}\, \ddot{s}_{x:\overline{m|}} - {}_mk_x,$$

where the subscripts (1) and (2) are used to denote the two policies. The difference of the two reserves is

$$_mV_{(1)} - {}_mV_{(2)} = [P_{(1)} - P_{(2)}]\, \ddot{s}_{x:\overline{m|}}. \tag{1}$$

In particular, with m = h,

$$_hV_{(1)} - {}_hV_{(2)} = [P_{(1)} - P_{(2)}]\, \ddot{s}_{x:\overline{h|}}. \tag{2}$$

Dividing equation (1) by equation (2), we have

$$\frac{_mV_{(1)} - {}_mV_{(2)}}{_hV_{(1)} - {}_hV_{(2)}} = \frac{[P_{(1)} - P_{(2)}]\ddot{s}_{x:\overline{m|}}}{[P_{(1)} - P_{(2)}]\ddot{s}_{x:\overline{h|}}} = \frac{\ddot{s}_{x:\overline{m|}}}{\ddot{s}_{x:\overline{h|}}} = \frac{_hE_x}{\ddot{a}_{x:\overline{h|}}}\ddot{s}_{x:\overline{m|}} = P_{x:\overset{1}{\overline{h|}}}\, \ddot{s}_{x:\overline{m|}} = {}_mV_{x:\overset{1}{\overline{h|}}}.$$

Therefore we have the reserve difference formula

$$_mV_{(1)} - {}_mV_{(2)} = {}_mV_{x:\overset{1}{\overline{h|}}}\, [\,_hV_{(1)} - {}_hV_{(2)}], \qquad m \le h, \tag{3}$$

which can perhaps be inserted after (7.4.10) on page 218 of *Actuarial Mathematics*.

More generally, we consider two different policies issued to (x), with the same benefit structure (not necessarily a unit of insurance) during the first h years where h is less than or equal to the shorter of the two premium-payment periods. The premium payment methods of the two policies are level and of the same frequency. Then formula (3) can obviously be extended to this setting.

Three Applications of the Reserve Difference Formula

Exercise 7.17.b of *Actuarial Mathematics* :

$$
\begin{aligned}
{}_t\bar{V}(\bar{A}_{x:\overline{m+n}|}) - {}_t\bar{V}(\bar{A}^{1}_{x:\overline{m}|}) &= {}_t\bar{V}_{x:\overline{m}|}^{\;\;\;1}\,[{}_m\bar{V}(\bar{A}_{x:\overline{m+n}|}) - {}_m\bar{V}(\bar{A}^{1}_{x:\overline{m}|})] \\
&= {}_t\bar{V}_{x:\overline{m}|}^{\;\;\;1}\;{}_m\bar{V}(\bar{A}_{x:\overline{m+n}|}), \qquad 0 \le t \le m.
\end{aligned}
$$

Exercise 7.21 of *Actuarial Mathematics* :

$$
\begin{aligned}
{}_kV_{x:\overline{m+n}|} - {}_kV^{1}_{x:\overline{m}|} &= {}_kV_{x:\overline{m}|}^{\;\;\;1}\,({}_mV_{x:\overline{m+n}|} - {}_mV^{1}_{x:\overline{m}|}) \\
&= {}_kV_{x:\overline{m}|}^{\;\;\;1}\;{}_mV_{x:\overline{m+n}|}, \qquad 0 \le k \le m.
\end{aligned}
$$

Exercise 7.28.b of *Actuarial Mathematics* :

$$
\begin{aligned}
{}_kV^{(m)}(\bar{A}_{x:\overline{n}|}) - {}^n_kV^{(m)}(\bar{A}_x) &= {}_kV^{(m)}{}_{x:\overline{n}|}^{\;\;\;1}\,[{}_nV^{(m)}(\bar{A}_{x:\overline{n}|}) - {}^n_nV^{(m)}(\bar{A}_x)] \\
&= {}_kV^{(m)}{}_{x:\overline{n}|}^{\;\;\;1}\,(1 - \bar{A}_{x+n}), \qquad 0 \le k \le n.
\end{aligned}
$$

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REFERENCE

Bowers, N.L., Jr., Gerber, H.U., Hickman, J.C., Jones, D.A., and Nesbitt, C.J. 1997. *Actuarial Mathematics,* 2nd ed. Schaumburg, Ill.: Society of Actuaries.