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Sequential Credibility Evaluation via Stochastic Approximation

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Abstract

Stochastic approximation is a powerful tool for sequential estimation of zero points of a function. This methodology is defined and is shown to be related to a broad class of credibility formulae derived for the *Exponential Dispersion Family.* We further consider a Location Dispersion Family which is rich enough and for which no simple credibility formula exists. For this case, a Generalized Sequential Credibility Formula is suggested and an optimal stepwise gain sequence is derived.

Key words: Stochastic approximation, Sequential credeblity, Location Dispersion Family, Optimal stepwise gain sequence

1 Introduction

Stochastic approximationΓoriginally proposed by Robbins and Monroe (1951)Γis concerned with the problem of finding the root of a function which is neither known nor directly observable.

Let $g(t)$ be a trend of some stochastic process $X(t)$ with a unique root τ, i.e.Γ

$$g(t) = EX(t) = \int x f(x|t)dx,$$

$$g(\tau) = 0,$$

where $f(x \mid t)$ is a probability density function of $X(t)$. τ and $g(t)$, which can also be regarded as a regression function of $X(t)$ conditioned on tΓare assumed unknown andΓfurtherΓ$f(x|t)$ need not be known.

Stochastic approximation theory examines the random sequence $T_0, T_1, \ldots$, in which T_0 is an initial guess and $T_1, T_2, \ldots$Γare evaluated in terms of successive observations $X_1(t_1), X_2(t_2), \ldots$, by

$$T_{n+1} = T_n - a_n X_n(T_n), \tag{1}$$

where a_n is a gain sequence of positive numbersΓand for $i = 0, 1, \ldots, n$, $X_i(T_i)$ is random variableΓwhose distribution under condition $T_0 = t_0, T_1 = t_1, \ldots, T_n = t_n$ is the same as the distribution of $X(t_i)$ (see Hall and Heyde (1980)ΓSection 7.6.2).

Over the yearsΓstochastic approximation has been extensively studied and proved to be a powerful and useful tool. For a recent publication see Kushner and Yin (1997).

The main goal of this paper is to identify the relationship between the credibility formula and stochastic approximation. This relationship is then explored for cases where the traditional credibility formula fails in the sense that it does not provide the exact expression for the predicted mean and where stochastic approximation gives rise to some kind of *quasi-credibility*.

Let X denote the claim size and let θ denote the risk parameter. Given θ, the distribution of X is given by P_θ, a member of a family of distribution$\{P_\theta, \theta \in \Theta \subset R^1\}$. It is further assumed that θ has a prior distribution $\pi(\theta)$, often referred to as the *structure distribution.* At the center of experience rating is the problem of estimating the *fair premium* $\mu = \mu(\theta) = E(X \mid \theta)$, given n years individual experience $x_1, x_2, \ldots x_n$, and the collective fair premium $m = \int_\Theta \mu(\theta)\, \pi(\theta) d\theta$. This is traditionally done by means of a credibility formula

$$\hat{\mu}_n = (1 - \alpha_n) m + \alpha_n \, \bar{x}_n, \tag{2}$$

where α_n is the credibility factor which tends to 1 as n increasesΓthus giving less weight to the collective fair premium in favor of the individual experience.

It is straight forward to establish that with $\hat{\mu}_0 = m$, (2) can be written in a sequential form as follows:

$$\hat{\mu}_n = \hat{\mu}_{n-1} - \frac{\alpha_n}{n}(\hat{\mu}_{n-1} - x_n), \tag{3}$$

which is a stochastic recursion of the type defined in (1)Γwhere the gain sequence is the credibility factor divided by n. While (3) is identical to (2)Γthe latter is particularly suited for the sequential evaluation of the fair premium. The strength of stochastic approximation lies beyond this technical issue. As we shall show belowΓit offers means for the evaluation of credibility in cases where the traditional credibility formula fails. For a full version of the paper including proofs of theorems see Landsman and Makov (1999b).

2 The Exponential Dispersion Family

The exponential dispersion family (EDF) was considered in Nelder and Wedderburn (1972)ΓTweedie (1984)Γand Jorgensen (1983Γ1986Γ1987Γ1992) and takes the following form:

$$dP_{\theta,\lambda} = f(x \mid \theta, \lambda) dx \ = e^{\lambda(x\theta - k(\theta))} q_\lambda(x) dx, \theta \in \Theta \subset R^1, \lambda \in \Lambda \in R^+, \tag{4}$$

$\Theta = \{\theta \mid \lambda k(\theta) = \ln \int e^{\lambda\theta x} q_\lambda(x) dx < \infty\}$.

The EDF has certain analogies with location and scale modelsΓwhere location is expressed by the population mean

$$E_{\theta,\lambda} X = \int x dP_{\theta,\lambda} = k'(\theta) = \mu(\theta) = \mu,$$

and the role of the scale parameter is played by the dispersion parameter $\sigma^2 = 1/\lambda$. It follows from (4) that the population variance is given by

$$V_{\theta,\lambda}X = k''/\lambda = V\left(\mu\left(\theta\right)\right)\sigma^2,$$

where $V(\mu)$ is called the variance function. The EDF is an extension of the Natural Exponential family (NEF). IndeedΓfor $\lambda = 1$Γthe EDF is reduced to the NEF. The EDF is characterized by the nature of its variance function. Morris (1982Γ1983) studied exponential families with quadratic variance functions. Cubic variance functions were discussed by Mora (1986) and Letac and Mora (1990). Power variance functions were investigated by Bar-Lev and Enis (1986) and Jorgensen (1987). See also Letac (1991)ΓBar-Lev *et al* (1992)Γ Kokonendji (1992) and Kokonendji and Seshadri (1994).

The study of the EDF in actuarial science is only now starting. It has been recently used for modelling compound Poisson claim data (see Jorgensen and Paes de Souza (1994)). Bayesian credibility formulae were derived in Landsman and Makov (1998). We now recall the main results of the latter paper and relate credibility to stochastic approximation.

Let the distribution of claims be given by (4) and let the conjugate prior distribution of θ be given by

$$\pi(\theta) \propto e^{n_0(x_0\theta - k(\theta))}, \tag{5}$$

where the choice of the hyperparameters n_0, x_0 renders $\pi(\theta)$ the properties of a density function (see Diaconis and YlvisakerΓ1979). Then for a given $\lambda \in \Lambda \in R^+$, the Bayesian credibility formula is as follows (see Landsman and Makov (1998))

$$\begin{aligned} E(x_{n+1} \quad | \quad x_1, x_2, ...x_n, \lambda) &= E(\mu \mid x_1, x_2, ...x_n, \lambda) \\ &= \frac{n_0}{n_0 + n\lambda} m + \frac{n\lambda}{n_0 + n\lambda}\ \bar{x}_n\ . \end{aligned} \tag{6}$$

Following (3)Γwe can write (6) in a stochastic approximation fashion:.

$$\hat{\mu}_n = \hat{\mu}_{n-1} - \frac{\lambda}{n_0 + n\lambda}(\hat{\mu}_{n-1} - x_n). \tag{7}$$

We shall now take this sequential credibility formula and discuss its role as a functional root finder of the trend of some process $X(t)$.

Let us reparametrize the EDFΓpassing from its *canonical* parameter θ to the parameter-expectation μ, usually called the *natural* parameter. We denote the family of densities (4) in the new parametrization by $\tilde{f}(x \mid \mu, \lambda)$ and the score functions of EDF with respect to μ,

$$\tilde{J}(x|\mu, \lambda) = \frac{\partial}{\partial\mu} \ln \tilde{f}(x|\mu, \lambda).$$

Theorem 1 *The stochastic approximation recursion (7) corresponds to the process*

$$X(t) = -V(t)\tilde{J}(X|t, \lambda) \tag{8}$$

and converges with probability 1 to μ, the root of function

$$g(t) = -V(t)E_{\mu,\lambda}\tilde{J}(X|t, \lambda). \tag{9}$$

So far we have considered credibility evaluation when λ is assumed known. When this is not the caseΓseveral alternatives are available (for details see Landsman and Makov (1998Γ 1999a)).

3 Location Dispersion Family

We now extend our discussion to *Location Dispersion Family (*LDF) (see Jorgensen (1983Γ 1987Γ1992))

$$dP_{\mu,\lambda} = f(x|\theta,\lambda)dx = a(\lambda)\exp(\lambda u(x-\theta))dx,\ x \in R, \tag{10}$$

$\theta \in R, \lambda \in R_{+}$.

The linear credibility formulaΓgiven in the right side of (6)Γis totally justified in the case of EDF since it provides the exact expression for $E(x_{n+1} \mid x_1, x_2, ... x_n, \lambda)$Γthe predicted mean of a future claim. Although in the case of LDFΓwhose membersΓwith the exception of the of normal distributionsΓare not EDFΓthe predicted mean is no longer linear with respect to the data (see Diaconis and Ylvisaker (1979)) and the equality between predicted mean and linear credibility formula now failsΓthe process (8) can still be considered and a stochastic approximation recursionΓcorresponding to this processΓcan be constructed. We consider such a recursion as a natural extension of the linear sequential credibility formula (7). CertainlyΓother alternativesΓusing some numerical methods can be suggested (see Young (1997Γ1998)). It should be emphasized that on the one hand the our approach preserves the optimal property of the predicted mean at any step n (stepwise optimal propertyΓsee Section 4)Γon the other hand it preserves a linear-fractional structure of the contribution of n in the prediction of a future claimΓwhich is typical for linear credibilityΓand it provides an improved m.s.e. (see Remark 2).

Theorem 2 *Let function $u(x)$ in (10) be concave (or convex), twice differentiable and $u'(x)^2$ be integrable with respect to density (10). Let $\mu_0(\lambda)$ be the expected value of (10) for $\theta = 0$, then the stochastic approximation recursion (1) with*

$$X(t) = \lambda u'(X - t + \mu_0(\lambda)) \tag{11}$$

(or $X(t) = -\lambda u'(X - t + \mu_0(\lambda))$ for a convex u) and a gain sequence meeting regularity conditions converges a.s. to the fair premium μ.

The recursion established in Theorem 2 takes the form

$$\hat{\mu}_n = \hat{\mu}_{n-1} - a_n \lambda u'(x_n - \hat{\mu}_{n-1} + \mu_0(\lambda)) \tag{12}$$

(or $\hat{\mu}_n = \hat{\mu}_{n-1} + a_n \lambda u'(x_n - \hat{\mu}_{n-1} + \mu_0(\lambda))$ for a convex $u(x)$), which we shall call *generalized Sequential Credibility formula.*

3.1 Examples

Let us show that for all the examples of LDF discussed in Jorgensen (1983)Γcondition C3 holds (in particularΓwe establish the concavity (or convexity) of $u(x)$) and therefore Theorem 2 applies.

- Log gamma is LDF with $u(x) = x - \exp(x), x \in R$, $u''(x) = -\exp(x) < 0$.
- The Barndorff -Nielesen (1977) hyperbolic distribution

$$f(x|\theta,\lambda) = \frac{1}{2\alpha K_1(\lambda)} \exp(-\lambda\{\alpha(1+(x-\theta)^2)^{1/2} - \beta(x-\theta)\}), x \in R,$$

where $\alpha^2 = 1+\beta^2, \beta \in R$Γand K_1 is a Bessel functionΓis a LDF with $u(x) = -\alpha\sqrt{(1+x^2)} + \beta x$. Then

$$u''(x) = -\alpha\ (1+x^2)^{-3/2} \begin{cases} \leq 0, \text{ if } \alpha \geq 0 \\ > 0, \text{ if } \alpha < 0 \end{cases}.$$

- Log generalized inverse Gaussian distribution

$$f(x|\theta,\lambda) = \frac{1}{2K_{\alpha\lambda}(\lambda)} \exp(-\lambda\{\alpha(x-\theta) - \beta\cosh(x-\theta)\}), x \in R,$$

$\alpha, \beta \in R$, is a LDF with $u(x) = \alpha x - \beta \cosh x$. It is clear that

$$u''(x) = -\beta \cosh x \begin{cases} \leq 0, \text{ if } \beta \geq 0 \\ > 0, \text{ if } \beta < 0 \end{cases}.$$

3.2 Symmetric Location Dispersion Family (SLDF)

The SLDF is defined by

$$f(x|\theta,\lambda) = a(\lambda)e^{\lambda u(|x-\theta|)},\ x,\theta \in R. \tag{13}$$

As a special caseΓwhen $u(x) = -|x|^\delta, \delta > 0$ we have *exponential power* family

$$f(x|\theta,\lambda) = \frac{\delta\lambda^{1/\delta}}{2\Gamma(1/\delta)} \exp(-\lambda|x-\theta|^\delta), \tag{14}$$

which offers a natural generalization of the Normal distributionΓfor which $\delta = 2$ (Jorgensen (1983)ΓBox and Tiao (1973)). We show below that in the case of SLDF the conditions of Theorem 2 can be relaxed.

Theorem 3 *Let $u(x)$ be differentiable monotone on $[0,\infty)$ whose integral of the square of the derivative with respect to density (13) exits. Then the stochastic approximation recursion (1) with*

$$X(t) = \lambda Sign(X-t)u'(X-t) \tag{15}$$

and a gain sequence meeting regularity conditions converges a.s. to the fair premium μ.

ClearlyΓexponential power family (14) satisfies conditions of the Theorem 3 for $\delta \geq 1$. The *Generalized Sequential Credibility formula for* SLDF takes the form

$$\hat{\mu}_n = \hat{\mu}_{n-1} - a_n \lambda \mathrm{Sign}(x_n - \hat{\mu}_{n-1}) u'(|x_n - \hat{\mu}_{n-1}|) \tag{16}$$

Remark 1 *For the truncated procedure the limits of the integrals in the Theorems 2 and 3 are bounded by the intervals $[\tau_1, \tau_2]$, and so the integrals exist automatically if the corresponding functions are, for example, continuous.*

4 Optimal stepwise gain sequence

In this section we discuss the choice of a gain sequence $\{a_n\}$ in the *Generalized Sequential Credibility formulae* (12)Γ(16).

Recall that $E_{\mu,\lambda}(\cdot)$ is expectation for given parameters μ, λ; $E_\lambda(\cdot)$ is the expectation with respect to measure $\pi(\mu) dP_{\mu,\lambda} d\mu$ with prior density $\pi(\mu)$ of parameter μ, given λ. (Instead of μ one can consider parameter θ and it's prior density $\pi(\theta)$).

We consider first the case of EDF. From (1) and (8) we can calculate $R_n = E_\lambda(\hat{\mu}_n - \mu)^2$Γ the Bayesian risk of $\hat{\mu}_n$Γas a function of a_n and further minimize such a function at fixed stage n.

Theorem 4 *Gain sequence a_n that minimizes Bayesian risk R_n at stage n is of the form*

$$a_n = \frac{R_{n-1}}{\lambda R_{n-1} + \lambda V(X)} \tag{17}$$

and a corresponding risk sequence is of the form

$$R_n = \frac{R_{n-1} V(X)}{R_{n-1} + V(X)} \tag{18}$$

Starting from $R_0 = E_\lambda(\mu - m)^2$, the variance of μ with respect to $\pi(\mu) d\mu$, and continuing the process of substituting we get

$$a_n = \frac{R_0}{n \lambda R_0 + \lambda V(X)}. \tag{19}$$

For the EDF family and conjugate prior (5)Γx_0 and n_0 have the following interpretation

$$x_0 = m, \;\; n_0 = \frac{\lambda V(X)}{R_0} \tag{20}$$

Comparing (19)Γ(20) with (7) we conclude that $\{a_n = 1/(n_0 + n\lambda)\}$ in the sequential credibility formula (7) is an *optimal stepwise gain sequence.*

We now adopt this approach for evaluating gain sequences for the LDF

Definition 1 *A gain sequence $\{a_n\}$ is called the first order optimal stepwise gain sequence if it minimizes (Bayes) risk $R_n = E_\lambda(\hat{\mu}_n - \mu)^2$ of sequential procedure (12) at step n up to the term $O(R^2_{n-1})$.*

Let

$$I_\lambda = \lambda^2 a(\lambda)\int_{-\infty}^{\infty} u'(x)^2 \exp(\lambda u(x))dx \tag{21}$$
$$= -\lambda a(\lambda)\int_{-\infty}^{\infty} u''(x)\exp(\lambda u(x))dx$$

be a Fisher Information about parameter μ for the LDFΓand define

$$B_\lambda = \lambda^2 a(\lambda)\int_{-\infty}^{\infty} u''(x)^2 \exp(\lambda u(x))dx \tag{22}$$

Theorem 5 *Let in addition to the conditions of Theorem 2 $u''(x)^2$ be integrable with respect to density (10). Then the first order optimal stepwise gain sequence takes the form*

$$a_n = \frac{I_\lambda R_{n-1}}{I_\lambda + B_\lambda R_{n-1}} \tag{23}$$

and the corresponding Bayes risk

$$R_n = \frac{I_\lambda R_{n-1}}{I_\lambda + B_\lambda R_{n-1}} + O(R_{n-1}^2). \tag{24}$$

Let us notice that LDF is EDF if X is Gaussian $\mathcal{N}(\mu, \sigma^2)$. Then $\lambda = \sigma^2$, $u(x) = -\frac{1}{2}x^2, I_\lambda = \lambda, B_\lambda = \lambda^2$. For this distribution we have

$$a_n = \frac{R_{n-1}}{1+\lambda R_{n-1}} = \frac{R_{n-1}\sigma^2}{R_{n-1}+\sigma^2}$$

which conforms with (17).

We now offer a modification for the risk and gain sequences ((23) - (24)) by dropping the last term on the right hand side of (24).

Theorem 6 *Under conditions of Theorem 5, the modified first order optimal stepwise risk and gain sequences given by*

$$a_n^* = R_n^* = \frac{R_0}{n\kappa R_0 + 1}, n = 1, 2, ..., \tag{25}$$

where $\kappa = B_\lambda / I_\lambda$. Then the generalized Sequential Credibility formula is of the form

$$\hat{\mu}_n = \hat{\mu}_{n-1} - \frac{\lambda}{n\kappa + 1/R_0} u'(x_n - \hat{\mu}_{n-1} + \mu_0(\lambda)) \tag{26}$$

(or $\hat{\mu}_n = \hat{\mu}_{n-1} + \frac{\lambda}{n\kappa+1/R_0} u'(x_n - \hat{\mu}_{n-1} + \mu_0(\lambda))$ for a convex $u(x)$).

It should be emphasized that on the one hand the our approach preserves the optimal property of the predicted mean at any step n (up to the term $O(R_{n-1}^2)$)Γon the other hand it preserves a linear-fractional structure of the contribution of n in the prediction of a future claimΓwhich is typical for linear credibility.

The attention should be paid to the relevance of coefficient κ, which monitors the decreasing rate of a_n to 0, that in some sense defines the score of the next observation in the estimation of the fair premium. The next theorem provides a lower bound for κ.

Theorem 7 *Under conditions of Theorem 2*

$$\kappa \geq I_\lambda \geq V_{\lambda,\mu}(X)^{-1} \tag{27}$$

and $\kappa = I_\lambda = V_{\lambda,\mu}(X)^{-1} = c\lambda$ *($c > 0$ - some constant) iff X is Gaussian.*

Remark 2 *Theorem 7 is important for comparing the m.s.e. of stepwise optimal generalized sequential credibility procedure and linear sequential procedure (7) (which is the simple linear credibility formula). In fact, let R_n be the Bayes risk for the sequential linear credibility formula (7) and R_n^* be the modified optimal stepwise Bayes risk for generalized sequential credibility procedure (26). From Theorem 7 and (18) it follows that*

$$R_n^* = \frac{1}{n\kappa + \frac{1}{R_0}} \leq R_n.$$

and the equality holds iff $\kappa = I_\lambda = V_{\lambda,\mu}(X)^{-1}$*,i.e. iff X is Gaussian.*

Let us notice that in the Gaussian case (LDF is EDF (4) with $c = 1$)Γ$\kappa = \lambda$ and therefore (25) coincides with (19).

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