ASPECTS OF LOAN GUARANTEES PORTFOLIO DIVERSIFICATION

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ABSTRACT

This paper uses contingent claims analysis (CCA) to evaluate portfolios of vulnerable private loan guarantees and investigate their risk diversification properties. Results from Monte Carlo simulations are consistent with those in the previous literature on loan guarantees with respect to the asset value and the risk posture of the guarantor and the insured firms, as well as the correlation between them. In particular it is shown that, for plausible base line value of the parameters, the non-systematic diversifiable credit risk can be eliminated in a portfolio of ten insured firms. We also show how further diversification can be achieved by an appropriate choice of insured firms' risk postures and of correlations between them and the guarantor.

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INTRODUCTION

A financial guarantee is a promise from a guarantor to make good on payments to the funds providers in case of default of the debt borrowers. The guarantee can be private or public. Indeed, insurance companies and commercial banks as well as public agencies offer guarantees to a number of financial instruments such as a letters of credit, loans, deposits, swaps, mortgages and municipal bonds insurance (see e.g., Merton and Bodie 1992, Greenwald 1998, Lonkevich 2000). Guarantees can be implicit or explicit; for instance, subsidiaries often benefit from implicit guarantees by their parent companies with respect to their financial obligation, hence reducing their cost of capital.

Financial guarantees also play a major role in the development of international commerce. Financial guarantees improve access to international capital markets at lower costs, longer maturities and for larger amounts (*Euromoney Publications* 1997/1998, The World Bank 1995). Furthermore, exporting firms often benefit from governmental or private insurance guarantees to assist them expanding their activities overseas.[1] Multilateral development banks such as the World Bank endorse several private investment projects in emerging countries (The World Bank 1995, 1999).

Last but not least, financial guarantees have become a major tool for risk management and financial innovation in the context of credit enhancement and hedging. Recent financial crises in Asia and eastern Europe underscore the importance of appropriate immunization against default risk (*Euromoney Publications* 1997/1998, Pedroza and Roll 1998).

Options-based valuation models for financial guarantees stem from the seminal work of Merton (1977) who established an isomorphism between a financial guarantee and a put option. While most works have focused on public, or default-free guarantees (e.g., Jones and Mason 1980, Chen and Sears 1986), some authors have investigated private financial guarantees when the guarantor is subject to default risk (Johnson and Stulz 1987, Lai 1992, Lai and Gendron 1994).

Further, the extant literature on loan guarantees has focused on the evaluation of single loan guarantees. However, in practice, financial guarantors manage portfolios of several guarantees that might be correlated. As matter of fact, insured firms often operate in the same industry or are subject to the same structural factors exposing them to a common systemic risk, which could even affect the guarantor itself.

[1] The Insurance Bureau of Canada reports the increasing role of the Export Development Corporation, a public agency in the export related insurance market (*Journal de l'Assurance*, January 1999).

The purpose of this paper is to add to the literature on loan guarantees or portfolios of vulnerable complex put options by accounting explicitly for the non-zero correlation or non-independence between the debt borrowing firms themselves and between these firms and the guarantor.

Portfolio settings were considered in the analysis of bonds by McEnally and Boardman (1979) and Pedroza and Roll (1998), and of loans by Brealey, Hodges and Selby (1983), as well as by Evans and Archer (1968) and Statman (1987) for equity stocks. These authors have established possible risk reduction through diversification and estimated the level of systematic risk in their respective context.

This paper uses contingent claims analysis (CCA) to evaluate portfolios of vulnerable private loan guarantees and investigate their risk diversification properties. Results from Monte Carlo simulations are consistent with those in the previous literature on loan guarantees with respect to the value and the risk posture of the guarantor and the insured firms, and the correlation between them. We also show that depending on correlations, insuring the debts of five to fifteen firms produces a well-diversified portfolio of guarantees. These results are in line with those in Evans and Archer (1968) for stocks. We demonstrate how the level of systematic credit risk can be reduced through an appropriate combination of volatilities and correlations of all parties involved (e.g., Babbel 1989). Our results suggest that there is more to be gained by simple size portfolio diversification (i.e., increasing the number of insured firms in the portfolio) than by seeking cross-sector diversification (i.e., decreasing correlations between firms).

The rest of the paper is organized as follows. Section I sketches the model of a portfolio of loan guarantees under a vulnerable guarantor. Section II presents the simulation procedures and Section III analyzes the simulations results. Section IV concludes.

I. THE MODEL

We first present the dynamics of the debt borrowing firms and the financial guarantor. We then compute the value of the losses to the debtholders and finally the value of the guarantee, which is less than the value of the lenders' losses when the guarantor defaults.

Under the standard contingent claims analysis (CCA) assumptions (e.g., perfect markets, no taxes, no transaction costs, symmetric information, etc.), we suppose no violation of the absolute priority rule (APR) and ignore potential agency problems inherent in financial contracting.[2] For each firm, we also assume a simple capital structure consisting of only one zero coupon bond and equity under a constant interest rate regime.[3] The

[2] Violation of APR requires modeling of bankruptcy trigger point and distribution of residual assets which brings us far beyond the scope of this paper. There exists a substantial literature on APR violation, for instance, Eberhart and Senbet (1993) demonstrated that departures from APR are effective in controlling the risk-shifting incentive of financially distressed firms.

[3] Incorporating time diversification issues under a stochastic interest rate regime in this analysis is left for further study.

insured portfolio consists of N firms' guarantees ($N \geq 2$). There are no cash outflows from the firms (e.g., dividend) before the debts mature. All the debts under study mature at the same time and we assume no other senior debts.

- **Firms dynamics :** The dynamics of the firms assets V ($V = [V_1, V_2, \ldots, V_N]'$) are described by the following system of stochastic differential equations :

$$dV / V = \mu_V \; dt + \Sigma \, dz_V + \sigma_{VW} \cdot \; dz_{VW}$$

  where $\mu_V = [\mu_{V1}, \mu_{V2}, \ldots, \mu_{VN}]'$ is the mean vector, $\Sigma^2 = \Sigma \; \Sigma'$ the covariance matrix, $\sigma_{VW} \cdot \; dz_{VW}$ the vector of covariances between the firms and the guarantor times their respective Wiener processes, and dz_V a vector of Wiener processes.

  These equations describe the dynamics of each borrowing firm interacting with the other firms and the guarantor. The relation between the firms themselves is captured by the matrix Σ while the one between the firms and the guarantor by σ_{VW}. This implies that a firm is influenced not only by its own characteristics but also by the dynamics of the other firms and the guarantor.

- **Guarantor dynamics :** The dynamics of the guarantor is described by the following process :

$$dW / W = \mu_W \, dt + \sigma_W \, dz_W + [\sigma_{WV}]' dz_{WV}$$

  with instantaneous mean μ_W and volatility σ_W. σ_{WV} is the vector of covariances between the firms and the guarantor and dz_W and dz_{WV} are Wiener processes.

  Note again that the dynamics of the guarantor depend on its covariance with the insured firms as well as its own characteristics.

We now proceed to the evaluation of the value of the losses to the lenders and then follow with the estimation of the value of the vulnerable guarantee.

Value of the loss per unit of face value to the debtholders

The value of the losses per unit of face value $P_{i,T}$ to the debtholders of firm i at maturity T is given by the following expression :

$$P_{i,T} = \frac{Max(0, F_i - V_{i,T})}{F_i} \quad \text{et} \quad \alpha_i = \frac{F_i}{\sum_{k=1}^{N} F_k} = \frac{F_i}{F}$$

where $V_{i,T}$ is the firm i value at maturity T, F_i the loan face value of firm i, F the sum of all the debts, and α_i the portfolio weight for firm i.

At maturity T, the value of the losses per unit of face value for the portfolio is the weighted average of $P_{i,T}$:

$$P_T = \sum_{i=1}^{N} a_i \times P_{i,T} .$$

By way of the equivalent martingale argument via the expectation E^* [.], the value of the portfolio losses per unit of face value P is the expected value of losses per unit at maturity discounted at the constant interest rate r , or :

$$P = E^*\left[e^{-rT} \sum_{i=1}^{N} \frac{F_i}{F} \times \frac{Max(0, F_i - V_{i,T})}{F_i} \right] = E^*\left[e^{-rT} \sum_{i=1}^{N} \frac{Max(0, F_i - V_{i,T})}{F} \right].$$

Analyzing this expression is tantamount to studying the normalized value with respect to the total face values of the debts of a portfolio of puts, each with exercise price F_i and underlying asset V_i.

Value of the guarantee per unit of face value to the debt lenders

Since our guarantor is not default-free, the expected losses are bounded by its insuring capacity. Hence P is greater than or equal to the value of the guarantee G, defined as :

$$G_T = Min\left[\sum_{i=1}^{N} \frac{Max(0, F_i - V_{i,T})}{F}, \frac{W_T}{F} \right] = \frac{1}{F} Min\left[\sum_{i=1}^{N} Max(0, F_i - V_{i,T}), W_T \right].$$

Using again the equivalent martingale argument, we obtain the following expression for the present value of the guarantee G :

$$G = E^*\left\{ e^{-rT} \frac{1}{F} Min\left[\sum_{i=1}^{N} Max(0, F_i - V_{i,T}) , W_T \right] \right\}.$$

This expression can be interpreted as a complex option on a portfolio of puts and on the value of the guarantor. Given the curse of dimensionality due to the large number of underlying assets involved in the problem, we resort to Monte Carlo simulation to evaluate these guarantees.

II. SIMULATION PROCEDURE

We use the Barraquand and Martineau (1995) procedure, first, to gauge the impact of the parameters which enter in the pricing of the portfolio of guarantees and second, to estimate the number of guarantees required to obtain a diversified portfolio? This procedure is a powerful technique with efficiency and speed for tackling complex options with multi-state variables. The technique combines the Monte Carlo simulation with the Stratified State Aggregation to price multivariate securities. The use of this technique to price our European complex put options is dictated by the large number of state variables (up to 100 variables) in our simulations.[4] This procedure consists of three major steps :

- the generation of the trajectory of the underlying assets,
- the aggregate state stratification along the payoff and the conditional probabilities, and
- the computation of the guarantee by an iterative backward procedure.

Details for these steps are provided in the appendix. We validated our simulation results for the case of three state variables with those obtained from the Boyle, Evnine and Gibbs (1989) algorithm which is based on the Cox, Ross and Rubinstein (1979) binomial tree. Results are similar in both techniques up to a factor 10^{-2}. In the next section, we discuss our results.

III. SIMULATION RESULTS

We first perform the comparative statics on the guarantees portfolio with respect to its value and risk determinants. For this purpose, we study the behaviour of a basic portfolio consisting of three firms and one guarantor. We then vary the number of firms in the portfolio to address the issue of size diversification.

Comparative statics for a portfolio of three firms and a guarantor

The base line values for the simulation parameters are presented below in Table 1.

Table 1 : Base line values for a portfolio of 3 firms and a guarantor

This table exhibits base line parameters for the simulations. It shows initial values of the 3 insured firms and the guarantor, the risk posture of the firms and the guarantor, the correlations between the firms and between the firms and the guarantor, and the face value of the borrowing firms debts.
V_i denotes the total assets of the firm i, W the total assets value of the guarantor, σ_i the volatility of the firm i measuring its risk, σ_w the volatility of the guarantor measuring its risk, ρ_{ij} the correlation between the firm i and the firm j, ρ_w the correlation between the firm i and the guarantor, F_i the face value of the debt of the firm i.

Firms and guarantor values	V_1	V_2	V_3	W
	30	40	50	80
Firms volatilities	σ_1	σ_2	σ_3	σ_W
	20 %	30 %	50 %	25 %
Correlations between firms	ρ_{12}	ρ_{13}	ρ_{23}	
	0.10	0.50	-0.30	
Correlations between firms and guarantor	ρ_{1W}	ρ_{2W}	ρ_{3W}	
	0	0	0	
Face values of firms debts	F_1	F_2	F_3	
	20	30	30	

[4] This technique is appropriate for handling the non-zero coupon situation and the investigation of time diversification under stochastic interest rates (see footnote 3).

The value of the guarantor is taken as two thirds of the total value of the insured firms. The firm's value varies in the range of 30 to 50. Firm volatilities vary between 20 % and 50 % per annum, Firm 1 being less risky than the guarantor (25 %). The correlations between the borrowing firms span from -0.30 to 0.50. The correlations between firms and the guarantor are initially set to zero and the firm leverages are around 65 %.

The simulation experiments were undertaken using a constant interest rate of 5 % (except for the comparative statics with respect to interest rates), while varying maturity from 1 to 10 years. Ceteris paribus, to study the individual effects of leverage, volatilities, correlations on P and G, we vary the key parameters under study such as interest rates, leverage, volatilities (Figures 1 and 2) and correlations (Figure 3 and 4).

We next present our comparative statics results with respect to the different parameters of the model. Recall that our two focal variables are the value of the losses per unit of face value to the lenders (P) and the value of the guarantee per unit of face value to the debtholders (G). In general, these two variables move in the same direction. When their results differ, we elaborate further on these cases.

The three following results are qualitatively the same as those in Lai (1992) and Lai and Gendron (1994).

1. *Comparative sensitivity to interest rate and debt maturity*

Both P and G decrease with interest rates and increase with maturities. These unsurprising results are the same as those in Merton (1974), Lai (1992), Lai and Gendron (1994).

2. *Comparative statics with respect to total assets value and nominal debt value of the borrowing firms*

P and G increase as firm value decreases or when the nominal amount of debt increases. This result follows from the fact that the probability of bankruptcy rises with increases in debt ratios in line with the classical results of Merton (1974).

3. *Comparative statics with respect to the insuring capacity of the insurer*

G is less valuable for a weak insurer (i.e., one with a small insuring capacity W) while P is not affected by the variability of the insurer total asset value. As matter of fact, P is a function of portfolio risk whereas G represents the average amount the guarantor will pay to all debtholders in case of bankruptcy. Obviously, a shaky guarantor will easily default and is unable to cover all of its obligations, hence a low guarantee value. This intuitive result concurs with those in Lai (1992) and Lai and Gendron (1994).

4. *Firm asset risk*

Both *P* and *G* increase with the firm volatility, i.e., risk. This is illustrated in Figure 1, which represents the evolution, with the debt maturity, of the risk premium as a function of Firm 1 volatility of 10 %, 30 % and 50 %.

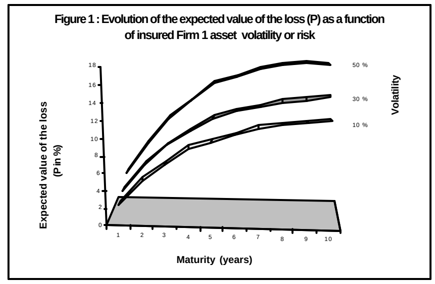


Figure 1 : Evolution of the expected value of the loss (P) as a function of insured Firm 1 asset volatility or risk

5. *Guarantor risk*

By construction of *P* and *G*, the guarantor insuring capacity affects only *G*. Indeed, the portfolio risk premium on a stand-alone basis, is only a function of the firms values. On the other hand, since a risky guarantor can default on its obligations (not a full faith and full credit guarantor), the guarantee depends not only on the firms dynamics, but also on the guarantor insuring capacity. Figure 2 plots the variations in the guarantee premiums for guarantor's risk of 5 %, 25 % and 45 % respectively.

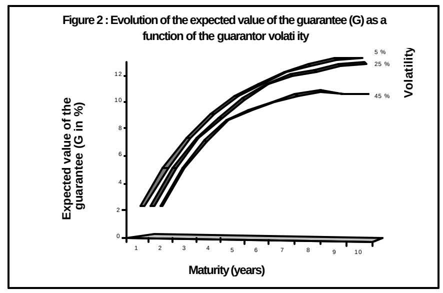


Figure 2 : Evolution of the expected value of the guarantee (G) as a function of the guarantor volati ity

6. *Correlations between firms*

Figure 3 shows how risk premium *P* is less valuable when there is negative correlations between the firms. However, in practice, the insured firms likely exhibit positive correlations among them due to sector specialization by the guarantor.

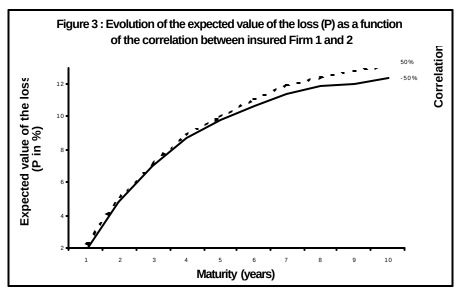


Figure 3 : Evolution of the expected value of the loss (P) as a function of the correlation between insured Firm 1 and 2

7. *Correlations between firms and guarantor*

Figure 4 shows that risk premium *P* is the lowest when correlations between the firms and the guarantor are zero. A bigger *P* is obtained for the same positive and negative correlations of 50 %. This stems from the dynamics of the generating process for V, $dV/V = m_V\ dt + S\ dz_V + s_{VW} \cdot dz_{VW}$ where $s_{VW} \cdot dz_{VW}$ is symmetric.

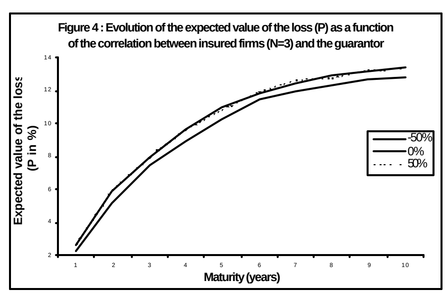


Figure 4 : Evolution of the expected value of the loss (P) as a function of the correlation between insured firms (N=3) and the guarantor

To sum up, in our portfolio setting :

- Results from Monte Carlo simulations are consistent with those in the previous literature on loan guarantees with respect to the value and the risk posture of the guarantor and the insured firms, and the correlations between them.

- In particular, the value of the guarantee depends on the covariance structure between the firms and between the firms and the guarantor. Positive correlations between the firms increase the risk premium of the portfolio whereas negative correlations decrease it. On the other hand, positive and negative correlations between the borrowing firms and the guarantor increase the risk of the portfolio of guarantees.

We have investigated so far one form of diversification driven by correlations. However, there is another type of diversification, by number or size, which we study next.

Size diversification

With regard to size diversification, we address two research questions : 1) how many guarantees make a diversified portfolio for a financial guarantor? and 2) what is the undiversifiable systematic credit risk? For our simulation experiments, we use the base line parameters presented in Table 2 below.

Table 2 : Base line parameters for size diversification study of a portfolio of guarantees

This table exhibits the base line parameters used for the size diversification effect simulations. It presents the value of the firms and the guarantor, their volatilities, the debt ratio of firms and the correlations between firms and between firms and the guarantor. ***V_i*** *represents the value of the firm i and* ***s_i*** *his volatility with i varying from 1 to N. N is the number of firms in the portfolio,* ***W*** *represents the value of the guarantor and s_W his volatility,* ***d*** *the borrowing firms debt ratio or leverage and* ***r*** *the correlation coefficient.*

Firms and guarantor values	V_i = 40	W = 100
Volatilites of firms and guarantor	s_i Randomly selected between 10% and 35%	s_W = 15 %
Firms debt ratio (*d*)	d = 0.75	
Correlations between firms and between firms and guarantor	r = 0.20	

Consider *N* number of guarantees from 1 to 100. Each firm has a value of 40, with the guarantor having a value of 100. Following Brealey, Hodges and Selby (1983), firms' risks were drawn randomly in the interval [10%, 35%] from an uniform distribution. The guarantor risk is set at 15%. The base line correlations are 0.20 between the borrowing firms, and between the firms and the guarantor. The same leverage of 0.75 is applied for all the firms.

For each *N*, the reported standard deviation of the guarantees portfolio is obtained from averaging over 100 batches of 1000 replications for each batch (as done for instance in

Barraquand and Martineau 1995). Since we assume the firms to have the same value and debt ratio, the average losses are the same for all portfolios, and risks comparison is valid.

For each simulation, we first report the standard deviation (the absolute level of risk of the portfolio) then we normalize this standard deviation by the standard deviation of the portfolio consisting of one firm. The latter normalized risk can be seen as a relative risk measure which we use to gauge the "speed" of reduction of the non-systematic credit risk (diversification effect).

Next, we present our results related to diversification effects along two lines of inquiry :

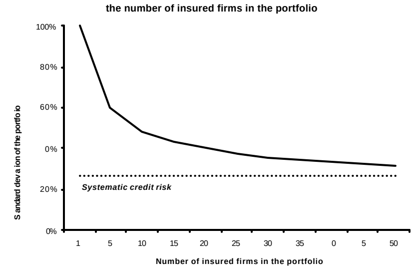
1) first, size diversification as a function of the number of firms composing the portfolio, and

2) second, the impact of correlations and leverage on the "speed" of diversification.

1. *Portfolio total risk as a function of the number of firms in the portfolio*

As shown in Figure 5, the higher the number of firms in the portfolio, the lower is the total risk. We find the familiar Evans and Archer (1968) type of curve, where the non-systematic risk (in our case credit risk) is drastically reduced with the number of firms in the portfolio. The undiversifiable portion is the systematic credit risk (e.g., Babbel 1989).

Figure 5 : Evolution of portfolio's standard deviation as a function of the number of insured firms in the portfolio



2. *The impact of correlations and leverage on the "speed" of diversification*

To gauge the impact of correlations and leverage on the speed of diversification, we compare three scenarios : the base case scenario and the other scenarios with high correlations or high leverage.

- ***Base case scenario*** : this corresponds to parameters values presented in Table 2. The firms values are set at 40, the guarantor at 100, the correlations at 0.20 and the leverages at 0.75.

- ***High correlations scenario*** : the parameters values correspond to the base case with correlations increased to 0.50.

- ***High leverage scenario*** : the parameters values correspond to the base case with leverage increased to 0.95.

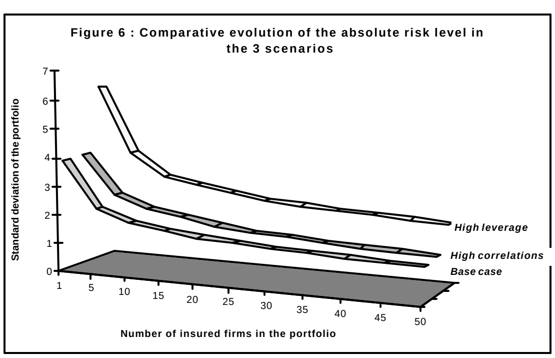
Table 3 below presents the risk of portfolios, as measured by its standard deviation for each scenario for maturities of 2, 6 and 10 years. The portfolio size varies from 1 to 50 firms. We show in the first column (*ABS*), the absolute value of the standard deviation of the portfolio, and in the second column (*REL*), the ratio of the standard deviation of the diversified portfolio on the one of the unit portfolio, a non-diversified portfolio consisting of only one guarantee.

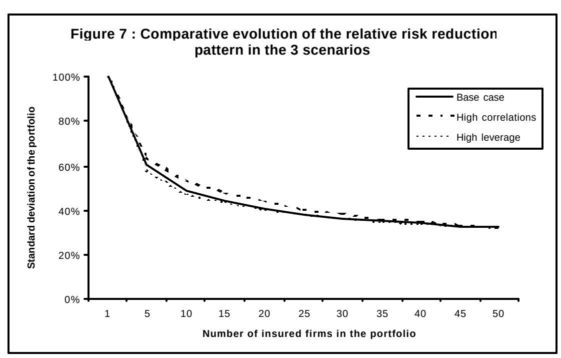
Table 3 : Standard deviation of the portfolio for different level of diversification and for the three scenarios

This table exhibits the absolute value and relative values of the standard deviation of the guarantees portfolio as a function of the number of firms and the maturities for the three scenarios. The three scenarios are described above, three maturities 2, 6, and 10 years and the number of firms varies from 1 to 50. The column ABS shows the absolute value of the standard deviation of the portfolio and the column REL exhibits the ratio of the standard deviation of the diversified portfolio on the one of the unit portfolio, a non-diversified portfolio consisting of only one guarantee.

Number of insured firms	Base case scenario						High correlations scenario						High leverage scenario					
	2 years		*6 years*		*10 years*		*2 years*		*6 years*		*10 years*		*2 years*		*6 years*		*10 years*	
	ABS	*REL*	*ABS*	*REL*	*ABS*	*REL*	*ABS*	*REL*	*ABS*	*REL*	*ABS*	*REL*	*ABS*	*REL*	*ABS*	*REL*	*ABS*	*REL*
1	1.88	1.00	3.86	1.00	4.80	1.00	2.12	1.00	3.92	1.00	5.11	1.00	4.71	1.00	6.23	1.00	7.79	1.00
5	1.24	0.66	2.26	0.59	2.72	0.57	1.36	0.64	2.55	0.65	3.18	0.62	2.64	0.56	3.93	0.63	4.30	0.55
10	0.97	0.52	1.88	0.49	2.25	0.47	1.17	0.55	2.13	0.54	2.56	0.50	2.21	0.47	3.10	0.50	3.52	0.45
15	0.87	0.47	1.67	0.43	1.99	0.42	1.04	0.49	1.93	0.49	2.32	0.45	1.99	0.42	2.93	0.47	3.21	0.41
20	0.79	0.42	1.51	0.39	1.90	0.40	1.00	0.47	1.73	0.44	2.12	0.41	1.88	0.40	2.68	0.43	2.96	0.38
25	0.75	0.40	1.46	0.38	1.72	0.36	0.88	0.42	1.59	0.41	1.94	0.38	1.74	0.37	2.48	0.40	2.88	0.37
30	0.70	0.37	1.38	0.36	1.68	0.35	0.83	0.39	1.56	0.40	1.87	0.37	1.69	0.36	2.40	0.39	2.72	0.35
35	0.68	0.36	1.33	0.35	1.64	0.34	0.78	0.37	1.44	0.37	1.73	0.34	1.62	0.34	2.29	0.37	2.57	0.33
40	0.69	0.37	1.28	0.33	1.54	0.32	0.78	0.37	1.38	0.35	1.67	0.33	1.59	0.34	2.27	0.36	2.52	0.32
45	0.64	0.34	1.23	0.32	1.53	0.32	0.73	0.34	1.34	0.34	1.59	0.31	1.54	0.33	2.18	0.35	2.42	0.31
50	0.64	0.34	1.22	0.32	1.49	0.31	0.73	0.34	1.27	0.32	1.54	0.30	1.52	0.32	2.09	0.34	2.41	0.31
100	0.55	0.29	0.99	0.26	1.22	0.26	0.55	0.26	0.98	0.25	1.18	0.23	1.22	0.26	1.76	0.28	1.96	0.25

Results from Table 3 demonstrate that risk increases with maturity and decreases with the number of firms in the portfolio. Contrary to Brealey, Hodges and Selby (1983), the risk reduction pattern remains the same regardless of maturity. This could be explained by the fact that we consider only zero coupon debts. The high correlations and high leverage scenarios exhibit the same risk reduction pattern as the one of the base case scenario. However, the risk posture for the high leverage scenario is substantially higher than the one for high correlations (roughly 200 % versus 10 % of the base case results). This reflects the fact that credit enhancement is mainly driven by financial leverage. This finding suggests that there is more to be gained by simple portfolio diversification (i.e., increasing the number of insured firms in the portfolio) than by seeking cross sector diversification (i.e., decreasing correlations between firms). Of course, marginal cost of diversification must also be accounted for.





Figures 6 and 7 above present the risk reduction pattern for each scenario in absolute and relative term respectively. Figure 6 demonstrates that while the non-systematic risk reduction pattern is the same for the three scenarios, the systematic risk level is much higher for the high leverage scenario. Figure 7 confirms that the risk reduction trend is indeed very similar. Note that five firms are sufficient to reduce the risk by 40 % and 50 % of the risk is eliminated by only ten firms.

IV. CONCLUSION

The extant literature on loan guarantees has focused exclusively on the evaluation of single loan guarantees. In practice, however, financial guarantors manage portfolios of guarantees that are likely highly correlated. Indeed, insured firms often operate in the same industry or are subject to the same structural factors which expose them to a common systemic risk, which could even affect the guarantor itself.

The purpose of this paper is to extend the financial guarantees literature to a portfolio context by accounting explicitly for the non zero correlation between the debt borrowing firms themselves and between these firms and the guarantor.

We use contingent claims analysis (CCA) to evaluate portfolios of private and vulnerable (the guarantor can default) loan guarantees and investigate their risk diversification properties. Results from Monte Carlo simulations are consistent with those in the previous literature of loan guarantees with respect to the value of the guarantees and the risk posture of the guarantor and the insured firms, and the correlation between them. We also show that depending on correlations, insuring the debts of five to fifteen firms produces a well-diversified portfolio of guarantees. These results are in line with those of Evans and Archer (1968) for stocks, and McEnally and Boardman (1979) for bonds. We finally show how the level of the systematic credit risk can be reduce via appropriate combinations of firms' volatilities/risk and correlations (e.g., Babbel 1989, Pedrosa and Roll, 1998). Specifically, the value of the guarantee depends on the covariance structure between the firms and between the firms and the guarantor. Positive correlations between the firms increase the risk premium of the portfolio whereas the negative correlations decrease it. On the other hand, positive and negative correlations between the firms and the guarantor increase the risk of the portfolio of guarantees.

The high correlations and high leverage scenarios exhibit the same risk reduction pattern as the one of the base case scenario. However, the risk posture for the high leverage scenario is substantially higher than the one of high correlations (roughly 200 % versus 10 % of the base case results). Reflecting the fact that credit enhancement is mainly driven by financial leverage and credit ratings, our results suggest that there is more to be gained by simple size portfolio diversification (i.e., increasing the number of insured firms in the portfolio) than by seeking cross-sector diversification (i.e., decreasing correlations between firms).

APPENDIX

Barraquand and Martineau (1995) simulation procedure

This appendix summarizes the Barraquand and Martineau (1995) simulation procedure for tackling multivariable complex securities. The three steps of the simulation are as follows.

1. Generation of sample paths

The generation of sample paths for the underlyings and the calculation of the payoff (the values of the losses (*P*) and the value of the guarantee (*G*)). This step consists on generating a given number *M* of sample paths for the insured firms and the guarantor assets up to maturity. Let's consider a vector $X = (V_1,\dots,V_N, W)'$ representing the values of *N* firms and the guarantor. The values of the firms and the guarantor follow a lognormal process (Merton 1974). A simple explicit Euler scheme on *X* (X_i, *i = 1, …, N+1*) is given by :

" i Î {1, …, N+1}, $$X_i(t+\Delta t) = X_i(t)\exp\left((r - \tfrac{1}{2}k_{ii})\Delta t + \sum_{j=1}^{N+1} v_{ij}\sqrt{\Delta t}\, z_j^t\right)$$

where *Dt = T/d* with *d* the number of time steps in the interval *[0,T]*,

z_j^t follows independent standard normal distributions *N(0,1) for all j* and *t*;

r is the short-term riskless interest rate (risk free),

$K=[k_{ij}$ *; (i,j) Î {1,2, … N+1}*$^2]$ is the covariance matrix of the insured firms and the guarantor (the vector X),

$V = [v_{ij}$ *; (i,j) Î {1,2, … N+1}*$^2]$ is the volatility matrix and it's obtain from the Cholesky decomposition of *K* ($K = VV^T$).

We simulate the sample paths of the each element of *X* in the interval *[0,T]* by dividing this interval in *d* sub intervals $[t_j,t_{j+1}]$ with length *Dt* ($t_{j+1} - t_j$ *= Dt*). We repeat this generation procedure *M* times, then we obtain *M* batches. For each batch and at each instant *t* with *t Î* $\{t_0, t_1, \dots, t_d\}$, we calculate the values of the payoff (the value of the losses and the value of the guarantee) $f(X(t)) = (f^1(X_1(t),\dots,X_N(t)), f^2(X_1(t),\dots,X_{N+1}(t)))$ as follows :

$$f^1(V_{1,t},\dots,V_{N,t}) = \sum_{i=1}^{N} \frac{Max(0, F_i - V_{i,t})}{F} \quad ,$$

$$f^2(V_{1,t},\dots,V_{N,t},W_t) = Min\left[\sum_{i=1}^{N}\frac{Max(0,F_i - V_{i,t})}{F}, \frac{W_t}{F}\right] = Min\left[f^1(V_{1,t},\dots,V_{N,t}), \frac{W_t}{F}\right].$$

2. Stratified State Aggregation (SSA) and calculation of conditional probabilities and payoff expectations

Once the M paths $X^1(t), \ldots, X^M(t)$ are computed, we proceed to the Stratified State Aggregation Along the Payoff (SSAP) of the expected values (for more details, see Barraquand et Martineau, 1995). This consists in dividing the state space in cells on which the payoff value is equal. Then we calculate the conditional probabilities. The division of the state space in k sub-cells is as follows :

$$\pi_i(t) = \{X \in \mathbb{R}^n{}_+, A(t)\, e^{B(t)(i-2)} < f(X) \leq A(t)\, e^{B(t)(i-1)}\}, \ \forall i \in \{2,\ldots, k-1\}$$

$$\pi_1(t) = \{X \in \mathbb{R}^n{}_+, f(X) \leq A(t)\}$$

$$\pi_k(t) = \{X \in \mathbb{R}^n{}_+, A(t)\, e^{B(t)(k-2)} < f(X)\}$$

where k corresponds to the number of desegregated cells and $A(t)$ and $B(t)$ are adjusted automatically to assure $Prob\{X(t) \in \pi_1(t)\} \approx Prob\{X(t) \in \pi_k(t)\} \approx 0.1\ \%$. The set of $\{\Pi_i, i \in \{1, 2, \ldots, k\}\}$ constitute a finite partition of space of the payoff values.

Next the number $a_i(t)$ of the sample crossing $\pi_i(t)$ and the number $b_{ij}(t)$ of the samples moving from $\pi_i(t)$ to $\pi_j(t+\Delta t)$ are easily computed,

$$a_i(t) = Cardinal\{p \in [1,M], X^p(t) \in \pi_i(t)\}, \ \forall i \in [1,k]$$

$$b_{ij}(t) = \Sigma_{\{p \in [1,M], X^p(t) \in \pi_i(t), X^p(t+\Delta t) \in \pi_j(t+\Delta t)\}}\, e^{-r\Delta t}, \ \forall i,j \in [1,k]^2.$$

Similarly, the sum $c_i(t)$ (payoff expectations) over the samples X^k of the payoff values $f(X^k(t))$ is computed from :

$$c_i(t) = \Sigma_{\{p \in [1,M], X^p(t) \in \pi_i(t)\}}\, e^{-r\Delta t} f(X^p(t)), \ \forall i \in [1,k].$$

By the law of large numbers, we have the following identities for the value of conditional probabilities p_{ij} (for the passage from state i to state j, assuming that the process is Markovian) :

$$p_{ij} = \lim_{M \to +\infty} b_{ij}(t) / a_i(t) \ \text{ et } \varphi_i(t) = \lim_{M \to +\infty} c_i(t) / a_i(t),$$

where lim is the symbol of limit.

3. Backward integration algorithm

Using the above Monte Carlo estimates of the conditional probabilities and payoff expectations, an approximate of the values of losses and guarantee can be computed backward in time using the simple algorithm described below :

i) At the maturity date T, the approximate SSAP price is initialized at $C(i,T) = c_i(T) / a_i(T)$.

ii) At time $T\text{-}\Delta t$, we computed for all $i \in [1, k]$,

$$C(i, T - \Delta t) = \max\left(\frac{c_i\,(T-\Delta t\,)}{a_i\,(T-\Delta t\,)}, \sum_{j=1}^{k} C(\,j\,, T\,) \frac{b_{ij}\,(T-\Delta t\,)}{a_i\,(T-\Delta t\,)} \right).$$

iii) The above procedure is then applied recursively, backward in time to compute all the prices $C(i,T\text{-}2\Delta t)$, $C(i,T\text{-}3\Delta t)$, … , $C(1,0) = C_{SSAP}$ representing the values to be computed, which are the value of losses to the debtholders (P) and the value of the guarantee (G) analyzed in the main text.

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