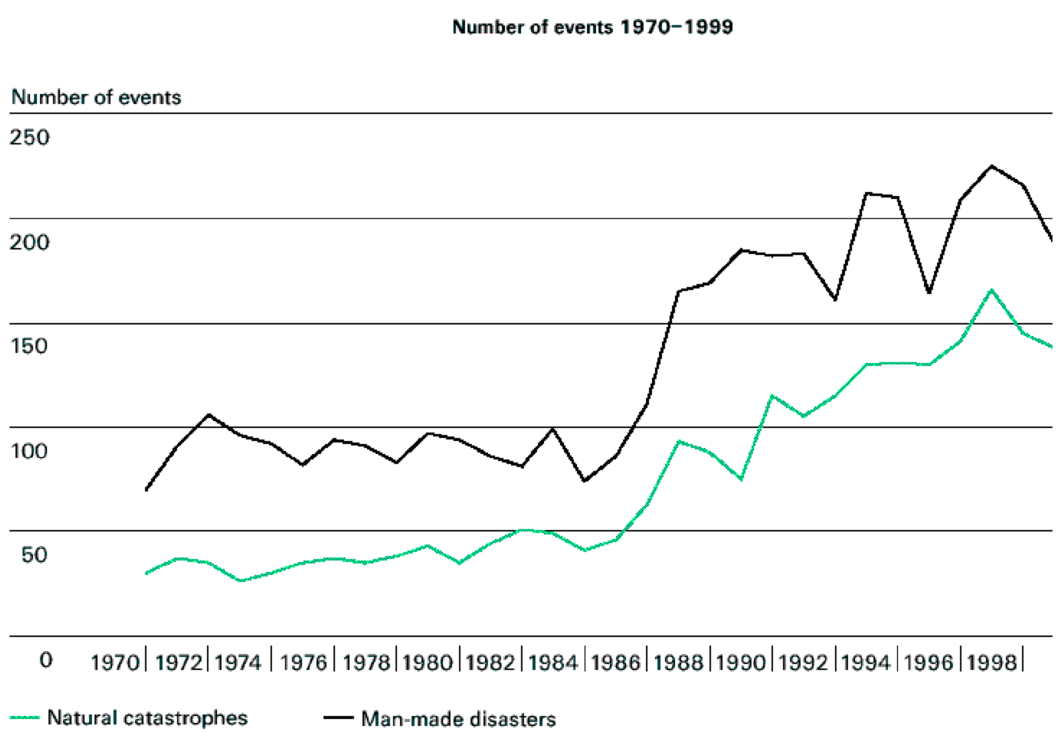
On the convergence of actuarial and financial methodologies

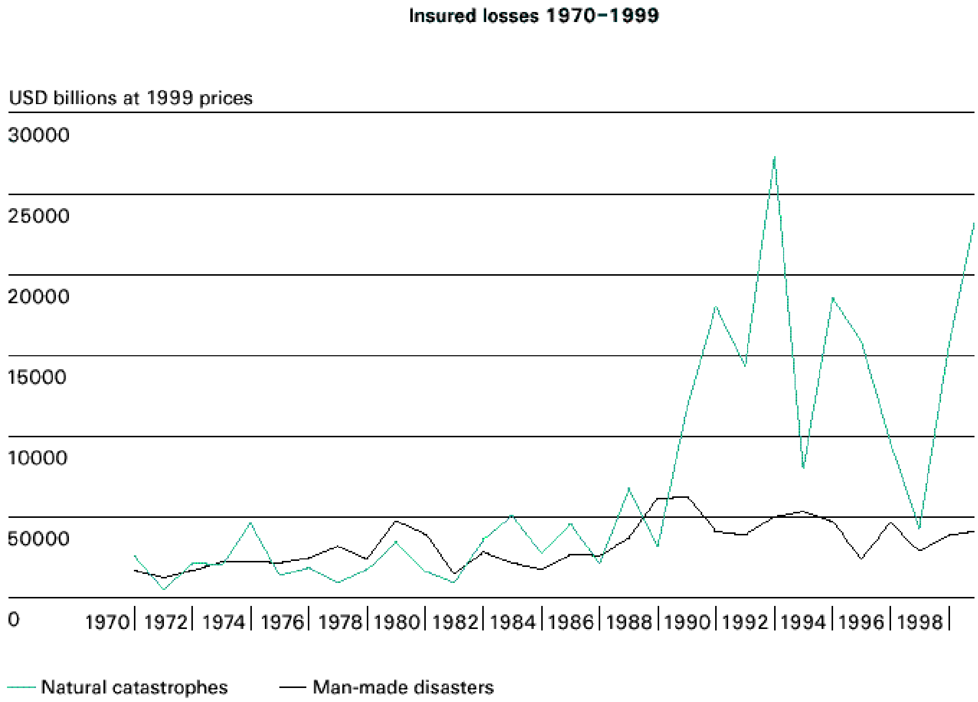
Diego Hernández
University of Waterloo
dhernand@uwaterloo.ca
August, 2000

The financial and insurance markets have always been interrelated. Nevertheless, since the late 1980s this interplay has become more intensive. Various elements have helped:

- Record number of natural catastrophes and their impact on insured losses
  source: Sigma Bulletin 2/2000



- Potential for losses >> insurance market capacity



source: Sigma Bulletin 2/2000

- Repositioning of the reinsurers → integrated financial services

- Finance related insurance products (CAT-bonds, insurance securities…)

3 stages:

I) Insurers use the readily available financial instruments to hedge the risks of assets and interest rate-sensitive liabilities, keeping reinsurance as the only mechanism for hedging underwriting risk.

II) Use of the commodities futures market to hedge certain loss exposures.

III) Trading of insurance risk.

The transformation of insurance risk into an asset class allows:

a) Expanding the capital available to cover the insurance risks, with the advantages of increased liquidity and lower transaction costs and information asymmetries.

b) Providing the capacity to bring premiums to levels in which the transfer of risk is feasible.

c) Introduction of new participants acting as risk managers, not risk takers (reversibility).

d) Expanding the definition of insurability.

Actuarial vs. financial pricing

Actuarial Pricing

Data → Model → Premium Calculation Principle → Solvency → Reinsurance → Credibility

Notice that in general: $P = EX + \text{safety loading}$

i.e. Equivalence principle is "corrected" using

a) utility theory (risk aversion)
b) solvency considerations (ruin theory)
c) desirable properties of the premium calculation principle

Bowers(1989) p.16:

"In a competitive economy, market forces will encourage insurers to price short-term policies so that deviations of experience from expected value will behave as independent random variables. Deviations should exhibit no pattern that might be exploited by the insured or insurer to produce consistent gains. Such consistent deviations would indicate inefficiencies in the insurance market."

Financial Pricing (Harrison-Kreps (1979))

In the filtered probability space $(\Omega, F, (F_t)_{t\geq 0}, P)$,
with the underlying process denoted S_t in a finite horizon [0,T],
a contingent claim *X*, (*i.e.*, a r.v. F_T-measurable) has an arbitrage value at time *t*:

$$E^Q\left(\exp\left\{ -\int_t^T r(s)ds \right\} X \middle| F_t \right)$$

which at time *t=0* is:

$$E^Q\left(\exp\left\{ -\int_0^T r(s)ds \right\} X \right)$$

That is, a premium calculated using the equivalence principle, but with respect to a new probability measure Q. As Delbaen and Schachermayer (1994) notice:

> "The change of measure from P to Q can also be seen as a result of risk aversion. By changing the physical probability measure from P to Q, one can attribute more weight to unfavourable events and less weight to more favourable ones." (e.g. remember mortality tables).

Under the no-arbitrage assumption, we can take a preference-free version of the option price: the Q measure under which the discounted price process is a martingale, i.e. the representation of a market in which the investors' probability assignments are "risk-neutral". (Just as in Bowers' textbook!)

One more observation: in “nice” cases, this Q measure is the unique P-equivalent martingale measure, so the best predictor and unique price is the actuarial premium under Q.

This is the fundamental tool to price derivatives in the financial markets and its use in economic theory becomes popular after the M&M proposition.

Uncertainty in Economic Theory

General Equilibrium Theory (Arrow-Debreu-McKenzie)

“When there are markets and associated prices for all goods and services in the economy, no externalities or public goods and no information asymmetries or market power, then competitive markets allocate resources efficiently”

Key hypothesis: at some initial date, there is a market for each good produced or consumed in every possible future contingency, i.e., there is a complete set of contingent markets.

Problems:

a) Keynes(1936): Under uncertainty, agents will be reluctant to make more than limited contractual commitments into the future = some markets that should be active in matching future demand and supply are missing.

b) Simon(1947): There is a limited capacity of agents to foresee future events and to calculate the relative benefits of different courses of action, added to the fact that information is costly or even impossible to acquire. (Bounded rationality)

How to incorporate these in a general equilibrium model?

As Magill(1996) explains: “when agents have only limited knowledge and ability to cope with the uncertainties presented by the future, they trade sequentially and use a system of contracts which involve only limited commitments into the future.” (Rational expectations)

Completeness

Itô representation:

$$X = X_0 + \int_0^T x_t dS_t$$

Self replicating strategy: hold x_t in the risky asset S_t and

$$h_t = \left(X_0 + \int_0^t x_* dS_* \right) - x_t S_t$$

in the riskless asset, giving a value of the portfolio

$V_t = x_t S_t + h_t$ and therefore, $V_T = X$

$$\Rightarrow E^Q(X) = X_0$$

A market is complete if every contingent claim admits an Itô representation with respect to the process S_t.

Embrechts and Meister (1995)

"Nice" = Complete cases:

CRR (binomial)
Black-Scholes (geometric BM)
Multi-dimensional BM and some diffusions
Homogeneous Poisson process
Square integrable point process martingales

Incomplete cases:

Stochastic volatility
Stable processes
Compound Poisson processes
Jump diffusions

As Mark Davis notes: "In incomplete markets, exact replication is impossible and holding an option is a genuinely risky business, meaning that no preference independent pricing formula is possible. In technical terms, the problem is that no unique martingale measure exists.

What to do?

Gerber-Shiu (Esscher measure!)
Föllmer-Schweizer approach (minimizing expected squared hedge error)
Delbaen and Haezendonck (martingale approach to premium calculation principles)
M.H.A. Davis (Utility maximization)
Klüppelberg-Mikosch (approximating the distribution of the contingent claim under P + standard loading techniques)