Application of a Linear Regression Model to the Proactive Investment Strategy of a Pension Fund

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The consulting actuary is typically concerned with pension plan design and funding issues. For large pension plans, the preparation of asset/liability studies and fund projections will present many challenges and opportunities for the consultant, including the development of optimal asset allocation strategies and risk mitigation strategies. Investment performance measurement and attribution analysis are two other areas where actuaries can make useful contributions to effective asset/liability management. This paper advocates a broader role for actuaries in the proactive investment strategy of a pension fund by utilizing quantitative techniques and feedback analysis.

In Modern Portfolio Theory (MPT), a simple but effective tool for analyzing investment performance is the linear regression model where the actual performance of a fund or manager is compared against an appropriate benchmark or index. The model produces a set of seven basic statistical measures of investment return and risk; these are based on a number of sequential observations:

- Mean (rate of return)
- Standard Deviation (of rate of return)
- Alpha Coefficient
- Beta Coefficient
- R^2 Coefficient of Determination
- Coefficient of Variation (standard deviation ÷ mean)
- Reward-Risk Ratio (alpha ÷ standard deviation)

Alternative measures of reward-risk ratios may be utilized; for example, those developed by Sharpe and Treynor. Typically a measurement period of 36 or 60 months might be chosen with investment performance rates of return computed for each month; the mean and standard deviation are readily derived from the set of observations. The choice of an appropriate comparative benchmark will depend on the asset class or mix of asset classes in the pension fund being studied.

Typical choices might be:

- Standard & Poor's 500 Stock Index (S&P)
- Lehman Brothers Government /Corporate Bond Index (LBGC)
- Composite index such as: 60% (S&P) plus 40% (LBGC)

The corresponding inputs of mean and standard deviation of rates of return are required for the benchmark for the linear regression model; the model produces a "best fit" for the two data series for the pension fund portfolio and the benchmark, expressed by the linear relationship:

Portfolio rate of return equals alpha plus product of beta and benchmark return.

In a geometric analogy, beta is the "slope" and alpha is the "intercept" of a characteristic line in a simple two-dimensional chart. The R^2 coefficient of determination measures the extent of the relationship between the two variables and is often referred to as correlation or "fit".

When the set of seven basic MPT statistics is produced from the linear regression model, a wealth of information is available that provides a preliminary diagnostic analysis of the return and risk characteristics of the fund. An MPT analysis summary from an actual case study for a multi-asset and multi-manager pension fund is presented in Exhibit I. This exhibit covers 31 three-year periods comprising a sequence of successive rolling three-year periods measured over a 30-month time frame.

Exhibit II presents the mean, standard deviation and coefficient of variation for the stock and bond indices for the corresponding periods, together with a composite 60/40 index that was used as the benchmark for the study.

Analyzing and interpreting the results of such a study will reveal a great deal of information about the "health" of the pension fund and will indicate areas for potential corrective action. In reviewing a set of MPT results, the consultant will consider:

- Is the portfolio mean rate of return above or below the benchmark and by how much?
- Is the portfolio standard deviation above or below the benchmark and by how much?

- Is the alpha coefficient positive? Alpha is interpreted as a measure of independent “value-added” by the investment strategy unrelated to general market movements.

- Is the beta coefficient below or above par? Beta is interpreted as a measure of leverage or risk related to the general market movements.

- Is the coefficient of variation below the benchmark? This would indicate a situation of “more return for less risk”.

- Is the reward-risk ratio positive? And how large? This ratio is an important measure of the risk-adjusted “value-added” coefficient relative to the benchmark.

- How close is the coefficient of determination to par? It provides a measure of relative diversification in comparison with the benchmark.

As results of the study are analyzed and interpreted, they may be used to provide feedback and input to future strategies to achieve the desired objectives.

The results of the MPT linear regression analysis of investment performance can be used directly to formulate pro-active investment strategies to achieve superior performance. For example, targets could be set for each of the seven statistical measures and then the mix of asset classes, styles and managers could be “engineered” to achieve the objectives. Typical actions to enhance the risk and reward profile of a fund would be:

- Re-allocate assets by asset class, e.g. equities, fixed income, and real estate to reduce standard deviation and increase reward-risk ratio

- Eliminate an under-performing manager with low or negative alpha

- Add a superior-performing manager with a track record of positive alpha

- Extend diversification by asset class or manager style, e.g., international or small-cap equity to increase reward-risk ratio

- Re-allocate assets among existing managers within an asset class or style to achieve optimum allocation and improve reward-risk profile

- Discuss and implement modifications to strategies of existing managers.

In practice, the model and analysis may be applied at the portfolio level, asset class level or manager level and can be applied to any time period (rolling 3-year, 5-year or 10-year periods) and can be applied to the results derived from any measurement interval (monthly, quarterly or annual).

The fact that considerable enhancements to the return and risk profile can be achieved by a diligent application of this process may be demonstrated from this actual case study where the statistical measures were successfully engineered by a series of strategic initiatives over a 30-month period. Here are the "before" and "after" statistical profiles:

	Before	After
Alpha (monthly)	-0.0745%	+0.2816%
Beta	.9299	.8972
Mean Return (monthly)	0.9172%	1.2964%
Standard Deviation	2.7940%	2.0719%
R^2 Coefficient	.8690	.7826
Reward-Risk Ratio	-0.0267	+0.1359
Coefficient of Variation	3.0461	1.5982

The corresponding mean returns and standard deviations for the composite benchmark were 1.0664% and 2.8007% at the beginning of the study and 1.1311% and 2.0429% at the end of the 30-month period covered by the study. The alpha and beta coefficients for the benchmark are, ipso facto, zero and 1.0000 respectively.

Success was achieved by improving the fund performance from under-performing the benchmark to out-performing the benchmark and turning a negative alpha and reward-risk ratio into a positive position while at the same time reducing risk as measured by the standard deviation and coefficient of variation. In more practical terms, the relative performance of the fund, when ranked in a comparative universe of funds, was improved from a 70^{th} percentile ranking (third quartile) at the beginning of the study to a 12^{th} percentile ranking (top quartile) thirty months later.

The MPT regression model is a powerful and effective analytical tool that the consultant may use to diagnose and monitor a pension fund's investment performance and provide a basis for prescribing remedial actions to correct under-performance and achieve out-performance of a benchmark. Moreover, it provides tangible quantified evidence of the effect of the consultant's input to the strategic management of the pension fund's investment operations.

Exhibit 1

MPT STATISTICS FOR PENSION FUND

3 –Year Period Number	Alpha	Beta	Mean Return	Standard Deviation	R ^ 2	Reward/Risk	Coefficient of Variation
0	-0.0745%	0.9299	0.9172%	2.7940%	0.8690	-0.0267	3.0461
1	-0.0832%	0.9309	1.0069%	2.8063%	0.8703	-0.0296	2.7871
2	-0.1231%	0.9376	1.0845%	2.7851%	0.8698	-0.0442	2.5682
3	-0.0860%	0.9438	1.0353%	2.7834%	0.8729	-0.0309	2.6886
4	-0.0635%	0.9434	1.0252%	2.7796%	0.8699	-0.0228	2.7114
5	-0.0576%	0.9417	1.0111%	2.7924%	0.8710	-0.0206	2.7618
6	-0.0598%	0.9623	1.2214%	3.0611%	0.8903	-0.0195	2.5062
7	-0.0088%	0.9506	1.0820%	3.0157%	0.8818	-0.0029	2.7873
8	-0.0299%	0.9513	1.1349%	2.9831%	0.8777	-0.0100	2.6285
9	0.0085%	0.9428	1.0844%	2.9916%	0.8751	0.0029	2.7586
10	-0.0067%	0.9454	1.0211%	2.9723%	0.8712	-0.0023	2.9109
11	0.0103%	0.9519	0.9828%	2.9625%	0.8725	0.0035	3.0144
12	-0.0153%	0.9572	0.9314%	2.9905%	0.8706	-0.0051	3.2108
13	-0.0209%	0.9599	0.8540%	2.9043%	0.8625	-0.0072	3.4008
14	-0.0383%	0.9600	0.8023%	2.9128%	0.8651	-0.0132	3.6307
15	-0.0556%	0.9613	0.8197%	2.9121%	0.8656	-0.0191	3.5525
16	-0.0123%	0.9550	0.8579%	2.8841%	0.8708	-0.0042	3.3617
17	0.0272%	0.9603	0.9119%	2.9051%	0.8697	0.0094	3.1859
18	0.0375%	0.9610	0.9214%	2.9088%	0.8686	0.0129	3.1569
19	0.0119%	0.9708	1.0614%	2.7958%	0.8586	0.0042	2.6341
20	-0.0209%	0.9673	1.0450%	2.7999%	0.8509	-0.0075	2.6793
21	-0.0083%	0.9676	1.0533%	2.8032%	0.8490	-0.0030	2.6612
22	0.0281%	0.9586	1.0978%	2.7668%	0.8433	0.0101	2.5204
23	0.0456%	0.9581	0.9700%	2.5989%	0.8205	0.0175	2.6793
24	0.0015%	0.9585	0.9500%	2.6047%	0.8154	0.0006	2.7418
25	0.0079%	0.9790	0.9661%	2.5980%	0.8576	0.0030	2.6892
26	0.1485%	0.9286	1.2934%	2.2454%	0.8175	0.0661	1.7360
27	0.2298%	0.8941	1.3900%	2.0961%	0.8054	0.1096	1.5080
28	0.2564%	0.8941	1.4439%	2.0959%	0.8003	0.1223	1.4516
29	0.2475%	0.9051	1.3053%	2.0786%	0.7976	0.1191	1.5924
30	0.2816%	0.8972	1.2964%	2.0719%	0.7826	0.1359	1.5982

Exhibit II

INDEX AND BENCHMARK STATISTICS

	S & P 500 Index			LB G/C Index			60/40 Index		
3 –Year Period Number	Mean Return	Standard Deviation	Coefficient of Variation	Mean Return	Standard Deviation	Coefficient of Variation	Mean Return	Standard Deviation	Coefficient of Variation
0	1.2306%	4.1508%	3.3731	0.8203%	1.2282%	1.4973	1.0664%	2.8007%	2.6262
1	1.3717%	4.1793%	3.0469	0.8700%	1.2086%	1.3892	1.1710%	2.8123%	2.4017
2	1.5289%	4.1060%	2.6856	0.9264%	1.2265%	1.3239	1.2879%	2.7703%	2.1511
3	1.3656%	4.1119%	3.0111	0.9219%	1.2220%	1.3254	1.1881%	2.7555%	2.3192
4	1.3269%	4.1054%	3.0939	0.8944%	1.2112%	1.3541	1.1539%	2.7480%	2.3814
5	1.2542%	4.1761%	3.3297	0.9561%	1.1573%	1.2104	1.1349%	2.7675%	2.4384
6	1.5214%	4.4984%	2.9568	1.0464%	1.2139%	1.1601	1.3314%	3.0015%	2.2544
7	1.2689%	4.4254%	3.4876	0.9653%	1.2806%	1.3267	1.1474%	2.9792%	2.5964
8	1.3736%	4.3796%	3.1884	1.0006%	1.2503%	1.2496	1.2244%	2.9378%	2.3994
9	1.2536%	4.4100%	3.5178	0.9725%	1.2732%	1.3092	1.1412%	2.9683%	2.6011
10	1.1911%	4.3700%	3.6688	0.9314%	1.2606%	1.3535	1.0872%	2.9346%	2.6992
11	1.0944%	4.3447%	3.9698	0.9125%	1.2401%	1.3590	1.0217%	2.9071%	2.8454
12	1.0692%	4.3568%	4.0750	0.8686%	1.1910%	1.3712	0.9889%	2.9150%	2.9476
13	0.9317%	4.1795%	4.4860	0.8811%	1.2063%	1.3690	0.9114%	2.8097%	3.0827
14	0.8219%	4.2036%	5.1142	0.9561%	1.1399%	1.1922	0.8756%	2.8221%	3.2230
15	0.8647%	4.1989%	4.8558	0.9794%	1.1393%	1.1632	0.9106%	2.8185%	3.0951
16	0.9394%	4.1652%	4.4336	0.8689%	1.1835%	1.3621	0.9112%	2.8182%	3.0928
17	0.9753%	4.1803%	4.2863	0.8403%	1.1938%	1.4207	0.9213%	2.8214%	3.0624
18	0.9461%	4.1742%	4.4120	0.8803%	1.1985%	1.3615	0.9198%	2.8210%	3.0671
19	1.1528%	3.9692%	3.4432	0.9736%	1.1648%	1.1963	1.0811%	2.6685%	2.4683
20	1.1544%	3.9693%	3.4383	1.0231%	1.1729%	1.1464	1.1019%	2.6699%	2.4230
21	1.1417%	3.9653%	3.4732	1.0303%	1.1678%	1.1335	1.0971%	2.6692%	2.4329
22	1.1419%	3.9650%	3.4721	1.0769%	1.1220%	1.0419	1.1159%	2.6507%	2.3753
23	0.9458%	3.7001%	3.9120	0.9933%	1.0903%	1.0976	0.9648%	2.4570%	2.5465
24	0.9744%	3.6913%	3.7881	1.0122%	1.1062%	1.0929	0.9896%	2.4538%	2.4797
25	0.9703%	3.6928%	3.8058	0.9914%	1.1060%	1.1156	0.9787%	2.4574%	2.5109
26	1.3269%	3.3099%	2.4945	1.0919%	1.0546%	0.9658	1.2329%	2.1863%	1.7733
27	1.4433%	3.1584%	2.1883	1.0792%	1.0608%	0.9830	1.2977%	2.1040%	1.6213
28	1.5103%	3.1448%	2.0822	1.0550%	1.0658%	1.0102	1.3282%	2.0971%	1.5789
29	1.3050%	3.0563%	2.3420	0.9644%	1.1083%	1.1492	1.1688%	2.0511%	1.7549
30	1.2631%	3.0466%	2.4120	0.9331%	1.1067%	1.1860	1.1311%	2.0429%	1.8061