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TERM CONVERSION OPTION

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The option of conversion to permanent insurance without evidence of insurability began to be common in term policies on this continent during the 1890's. Term policies limited to term insurance had previously been issued for many years.

So far as I have been able to find, no United States company included the conversion option in its policies before 1890. However, there were two Canadian companies that did—the Confederation Life which commenced offering the attained age benefit in 1876 and the original date benefit at least as early as 1878; also the London Life which was offering the option in both forms as early as 1883.

Conversion as of attained age came first and conversion as of original date was a later development, as may be seen from Table A. In the 40 companies tabulated, the average interval between the dates of introduction of the two forms of the option was six years.

The way for the option was paved by Dr. T. B. Sprague's monumental paper on select mortality tables, presented to the Institute of Actuaries in three parts during 1878–1881. In his quaint language as an advocate of spelling reform, Dr. Sprague put the problem thus:

> "An offis enters into an agreement that it wil upon aplication n years hence, if a person now ov the age x shal be then alive, grant a policy on his life at the ordinary rate ov premium" (*JIA* XXII, 433).

The solution furnished by Dr. Sprague is equivalent (see *TFA* II, 243) to the modern formula (Spurgeon, p. 88)

$$_{n|}O_{[x]} = (P'_{[x]+n} - P'_{[x+n]}) \frac{N_{[x]+n}}{D_{[x]}} \tag{1}$$

Referring to the uses of select tables as explained by Dr. Sprague, the Actuarial Society's "Actuarial Studies, No. 1," page 54, states:

> "Those uses opened a new field of vision to many actuaries, and enabled them to solve problems at which previously some had scarcely even ventured to guess. The result was the immediate acceptance by nearly all actuaries of the Select Table principle."

Although the term conversion option has grown up on this continent under the theory of select life mortality tables, the latter have not found acceptance in America for general life insurance purposes. In Great Britain "Calculations of premiums to be charged by Life Offices are now almost invariably made on the basis of Select Tables" (Spurgeon, p. 45). Premiums, values and dividends in America are almost invariably based on aggregate or ultimate tables, now generally the CSO Table.

The theory of the term conversion option has been developed from the standpoint of attained age conversion, with little attention to the problem of original date conversion. When the latter form of benefit was added, it was evidently reasoned that both benefits could be given for the price of the attained age benefit, which was correct. This was not because the latter benefit was worth nothing, but because the blended premium for the two mutually exclusive benefits was not greater than the premium for the one alone. The deficiency in theory is to be seen in the common formula:

TABLE A

ANALYSIS OF PRESENT U.S. AND CANADIAN COMPANIES
LISTED IN 1891 INSURANCE YEAR BOOK

Number of Companies According to Year in Which They
Commenced to Include Conversion Option
in Term Policies

Year Commenced	Attained Age Conversion	Original Date Conversion
1876–9	1	1
1880–9	1	1
1890–9	15	6
1900–9	16	17
1910–9	5	9
1920–9	1	4
1930–9	1	1
1940–5	....	1
	40	40

$$\text{Cost of original date conversion} = \left.\begin{array}{r}\text{Accumulated difference in}\\ \text{premium (less dividends)}\,.\end{array}\right\}\quad (2)$$

This formula returns to the policyholder the charge made for the option. But a charge to policyholders enjoying an option ought not to be returned to those exercising the option. In the unlikely situation that 100% convert and that the conversions are all as of original date the formula is correct because, as will be seen later, in that case the theoretical charge for the option is zero. But in practice not all conversions are as of original date and less than 100% convert, and a significant charge needs to be made for the conversion option and retained by the company.

Here we must distinguish between "charge for the option," which is the premium for the benefit that the company needs to collect from those who have the option privilege, and "cost of conversion," which is the amount required to establish the permanent policy.

In this paper the theory of the charge for the option is extended to include original date conversion, and a suitable formula is indicated for the cost of original date conversion. The latter is:

Cost of original date conversion = Difference in policy value, loaded. (3)

Policy value is here used in the sense of Spurgeon, Chapter VI. It does not mean the cash value, but is ordinarily the reserve on some stipulated basis.

THEORY

Assume that a group of lives at age x take out annual premium r-year term insurance, and that n years later a fraction k of the survivors convert to ordinary life insurance then dated back m years, the residue receiving the value of the term insurance.

Let the net cost of conversion be the difference in reserve for the policies on the contract basis. By virtue of the conversion privilege, the ordinary life premiums will be the same as for new policyholders on the date of the ordinary life policy.

Let functions indexed c be according to the mortality experienced on the converted lives, which may or may not differ from normal mortality.

Then, using the notation of select life tables, we have $l_{[x]}$ lives insured for n years at normal mortality after which, of the $l_{[x]+n}$ survivors, $kl_{[x]+n}$ are then insured for the remainder of life at mortality denoted by index c, and the remaining $(1-k)l_{[x]+n}$ receive the then value of the term insurance.

Hence the value of the *benefit* is:

$$l_{[x]}A_{\overset{1}{[x]}:\overline{n}|}+v^n l_{[x]+n}[kA^c_{[x]+n}+(1-k)\,{}_nV_{\overset{1}{[x]}:\overline{r}|}]\,.$$

For this the value of the *payment* is:

$$l_{[x]}P_{\overset{1}{[x]}:\overline{r}|}\ddot{a}_{[x]:\overline{n}|}+l_{[x]}\,{}_{n|}O_{[x]}+k\,v^n l_{[x]+n}[{}_mV_{[x+n-m]}-{}_nV_{\overset{1}{[x]}:\overline{r}|}$$
$$+P_{[x+n-m]}\ddot{a}^c_{[x]+n}]\,,$$

where ${}_{n}|O_{[x]}$ denotes the net single premium for the conversion option and

$${}_{m}V_{[x+n-m]} - {}_{n}V_{\overset{1}{[x]}:\overline{r}|} \qquad \text{is the net cost of conversion.} \quad (4)$$

We have, by subtracting $kv^{n}l_{[x]+n}\,{}_{n}V_{\overset{1}{[x]}:\overline{r}|}$ from both sides of a familiar equation:

$$\left.\begin{aligned} l_{[x]}P_{\overset{1}{[x]}:\overline{r}|}\ddot{a}_{[x]:\overline{n}|} - k\,v^{n}l_{[x]+n}\,{}_{n}V_{\overset{1}{[x]}:\overline{r}|} = l_{[x]}A_{\overset{1}{[x]}:\overline{n}|} \\ + v^{n}l_{[x]+n}(1-k)\,{}_{n}V_{\overset{1}{[x]}:\overline{r}|}\,. \end{aligned}\right\} \quad (5)$$

Hence, equating the residue of the *payment* to the residue of the *benefit*, and dividing out by $l_{[x]}$:

$${}_{n}|O_{[x]} + k\,\frac{D_{[x]+n}}{D_{[x]}}\left[{}_{m}V_{[x+n-m]} + P_{[x+n-m]}\ddot{a}^{c}_{[x]+n}\right] = k\,\frac{D_{[x]+n}}{D_{[x]}}\,A^{c}_{[x]+n}$$

$$\begin{aligned} {}_{n}|O_{[x]}\,\frac{D_{[x]}}{k\,D_{[x]+n}} &= A^{c}_{[x]+n} - {}_{m}V_{[x+n-m]} - P_{[x+n-m]}\ddot{a}_{[x+n-m]+m} \\ &\qquad + P_{[x+n-m]}(\ddot{a}_{[x+n-m]+m} - \ddot{a}^{c}_{[x]+n}) \\ &= A^{c}_{[x]+n} - A_{[x+n-m]+m} + P_{[x+n-m]}(\ddot{a}_{[x+n-m]+m} \\ &\qquad - \ddot{a}^{c}_{[x]+n}) \\ &= 1 - d\ddot{a}^{c}_{[x]+n} - 1 + d\ddot{a}_{[x+n-m]+m} + P_{[x+n-m]} \\ &\qquad \times(\ddot{a}_{[x+n-m]+m} - \ddot{a}^{c}_{[x]+n}) \\ &= (d + P_{[x+n-m]})(\ddot{a}_{[x+n-m]+m} - \ddot{a}^{c}_{[x]+n}) \end{aligned}$$

$${}_{n}|O_{[x]} = k\,\frac{D_{[x]+n}}{D_{[x]}}\,\frac{(\ddot{a}_{[x+n-m]+m} - \ddot{a}^{c}_{[x]+n})}{\ddot{a}_{[x+n-m]}}\,. \quad (6)$$

DETERMINATION OF ${}_{n}|O_{[x]}$ ON THEORY OF SELECT TABLE

As above stated, the index c denotes the mortality of the converted lives.

If all $l_{[x]+n}$ survivors convert ($k = 1$), c must denote the mixed mortality of the select table, and therefore

$$A^{c}_{[x]+n} = A_{[x]+n}\,, \qquad \text{if } k = 1\,.$$

Hence, from (37) in the Appendix,

$$A^{c}_{[x]+n} = A_{[x]+n} = FA_{[x+n]} + GA''_{[x]+n}\,, \qquad \text{if } k = 1 \text{ as above}$$

where F and G are weights such that:

$$F:G::l_{[x+n]}:(l_{[x]+n}-l_{[x+n]}) \quad \text{and} \quad F+G=1$$

$$F=\frac{l_{[x+n]}}{l_{[x]+n}} \quad \text{and} \quad G=\frac{l_{[x]+n}-l_{[x+n]}}{l_{[x]+n}}. \tag{7}$$

$k \geqq G$

Assuming that $kl_{[x]+n}$ of the survivors convert where $k<1$, it is proper in the determination of ${}_n|O_{[x]}$ to make the adverse assumption that the effect of the failure of the $(1-k)l_{[x]+n}$ lives to convert is subtracted out of the weight applicable to $A_{[x+n]}$, which is therefore reduced to be proportional to $l_{[x+n]}-(1-k)\,l_{[x]+n}$. This may be interpreted as assuming that lives failing to convert will as a group have select mortality, if $k \geqq G$.

Hence:

$$A^c_{[x]+n}=F_kA_{[x+n]}+G_kA''_{[x]+n}, \quad \text{for} \quad k<1 \tag{8}$$

where:

$$F_k:G_k::[l_{[x+n]}-(1-k)\,l_{[x]+n}]:(l_{[x]+n}-l_{[x+n]}) \quad \text{and} \quad F_k+G_k=1.$$

Hence:

$$F_k=\frac{l_{[x+n]}-(1-k)\,l_{[x]+n}}{kl_{[x]+n}} \tag{9}$$

$$G_k=\frac{l_{[x]+n}-l_{[x+n]}}{kl_{[x]+n}}. \tag{10}$$

From (8), (9), (10):

$$A^c_{[x]+n}kl_{[x]+n}=[l_{[x+n]}-(1-k)\,l_{[x]+n}]\,A_{[x+n]}+(l_{[x]+n}-l_{[x+n]})\,A''_{[x]+n}$$

[from (31)] $\quad = [l_{[x+n]}-(1-k)\,l_{[x]+n}]\,A_{[x+n]}+l_{[x]+n}A_{[x]+n}-l_{[x+n]}A_{[x+n]}$

$$kA^c_{[x]+n}=A_{[x]+n}-(1-k)\,A_{[x+n]} \tag{11}$$

$$k(1-d\ddot{a}^c_{[x]+n})=1-d\ddot{a}_{[x]+n}-(1-k)(1-d\ddot{a}_{[x+n]})$$

$$k\ddot{a}^c_{[x]+n}=\ddot{a}_{[x]+n}-(1-k)\,\ddot{a}_{[x+n]}. \tag{12}$$

From (9): $\quad F_k=0 \quad \text{if} \quad l_{[x+n]}=(1-k)\,l_{[x]+n}$

$\quad$ if $\quad k=G \quad$ [See (7)].

Hence (11) and (12) hold for values of $k \nless G$.

From (6), (12):

$${}_n|O_{[x]}=\frac{D_{[x]+n}}{D_{[x]}}\,\frac{[k\ddot{a}_{[x+n-m]+m}-\ddot{a}_{[x]+n}+(1-k)\,\ddot{a}_{[x+n]}]}{\ddot{a}_{[x+n-m]}}. \tag{13}$$

$k \gtreqqless G$

Assume that the effect of those failing to convert is to eliminate the weight applicable to $A_{[x+n]}$ above, so that

$$A^c_{[x]+n} = A''_{[x]+n}, \qquad \text{if} \qquad k \gtreqqless G .$$

This may be interpreted as assuming that no lives convert which as a group will have select mortality, and that those converting are as a group considered to have the substandard mortality represented by $A''_{[x]+n}$.

From above,

$$\frac{1 - A^c_{[x]+n}}{d} = \frac{1 - A''_{[x]+n}}{d}$$

$$\ddot{a}^c_{[x]+n} = \ddot{a}''_{[x]+n} . \tag{14}$$

Hence, from (6), (14), (35):

$$_{n|}O_{[x]} = k\,\frac{D_{[x]+n}}{D_{[x]}}\,\frac{\left(\ddot{a}_{[x+n-m]+m} - \dfrac{l_{[x]+n}\ddot{a}_{[x]+n} - l_{[x+n]}\ddot{a}_{[x+n]}}{l_{[x]+n} - l_{[x+n]}}\right)}{\ddot{a}_{[x+n-m]}} . \tag{15}$$

$m = 0$ (ATTAINED AGE CONVERSION, OPTION PREMIUM DESIGNATED $_{n|}{}^aO_{[x]}$)

$k \geqq G$ From (13),

$$\left.\begin{aligned} _{n|}{}^aO_{[x]} &= \frac{D_{[x]+n}}{D_{[x]}}\,\frac{\ddot{a}_{[x+n]} - \ddot{a}_{[x]+n}}{\ddot{a}_{[x+n]}} \\ &= \frac{D_{[x]+n}}{D_{[x]}}\left[\frac{1}{\ddot{a}_{[x]+n}} - \frac{1}{\ddot{a}_{[x+n]}}\right]\ddot{a}_{[x]+n} \\ &= (P_{[x]+n} - P_{[x+n]})\,\frac{N_{[x]+n}}{D_{[x]}} . \end{aligned}\right\} \tag{16}$$

$k \gtreqqless G$ From (15),

$$\left.\begin{aligned} _{n|}{}^aO_{[x]} &= k\,\frac{D_{[x]+n}}{D_{[x]}}\,\frac{l_{[x]+n}(\ddot{a}_{[x+n]} - \ddot{a}_{[x]+n})}{(l_{[x]+n} - l_{[x+n]})\,\ddot{a}_{[x+n]}} \\ \text{(from above)} \qquad &= (P_{[x]+n} - P_{[x+n]})\,\frac{N_{[x]+n}}{D_{[x]}}\,\frac{k\,l_{[x]+n}}{l_{[x]+n} - l_{[x+n]}} . \end{aligned}\right\} \tag{17}$$

$m = n$ (ORIGINAL DATE CONVERSION, OPTION PREMIUM DESIGNATED ${}_{n|}{}^{o}O_{[x]}$)

$k \geqq G$ From (13),

$$\left.\begin{aligned} {}_{n|}{}^{o}O_{[x]} &= \frac{D_{[x]+n}}{D_{[x]}} \frac{(1-k)(\ddot{a}_{[x+n]} - \ddot{a}_{[x]+n})}{\ddot{a}_{[x]}} \\ &= \frac{D_{[x]+n}}{D_{[x]}}(1-k)\left[\frac{1}{\ddot{a}_{[x]+n}} - \frac{1}{\ddot{a}_{[x+n]}}\right]\frac{\ddot{a}_{[x]+n}\ddot{a}_{[x+n]}}{\ddot{a}_{[x]}} \\ &= (P_{[x]+n} - P_{[x+n]})\frac{N_{[x]+n}}{D_{[x]}}(1-k)\frac{\ddot{a}_{[x+n]}}{\ddot{a}_{[x]}}. \end{aligned}\right\} \quad (18)$$

$k \lesseqgtr G$ From (15),

$$\left.\begin{aligned} {}_{n|}{}^{o}O_{[x]} &= k\,\frac{D_{[x]+n}}{D_{[x]}} \frac{l_{[x+n]}(\ddot{a}_{[x+n]} - \ddot{a}_{[x]+n})}{(l_{[x]+n} - l_{[x+n]})\,\ddot{a}_{[x]}} \\ \text{(from above)} \quad &= (P_{[x]+n} - P_{[x+n]})\frac{N_{[x]+n}}{D_{[x]}} \frac{kl_{[x+n]}}{l_{[x]+n} - l_{[x+n]}} \frac{\ddot{a}_{[x+n]}}{\ddot{a}_{[x]}}. \end{aligned}\right\} \quad (19)$$

RELATION BETWEEN OPTION VALUES

Let

$$Q = (P_{[x]+n} - P_{[x+n]})\frac{N_{[x]+n}}{D_{[x]}}.$$

This, on a net basis, agrees with (1), the textbook formula.

Attained Age Conversion

$k \geqq G$ From (16),

$$ {}_{n|}{}^{a}O_{[x]} = Q \quad (20)$$

which is constant for all values of k down to G.

$k \lesseqgtr G$ From (17),

$$ {}_{n|}{}^{a}O_{[x]} = Q\,\frac{kl_{[x]+n}}{l_{[x]+n} - l_{[x+n]}} \quad (21)$$

which decreases with k.

For $k = G$, the two expressions have the same value.

Original Date Conversion

$k \geqq G$ From (18),

$$ {}_{n|}{}^{o}O_{[x]} = Q(1-k)\frac{\ddot{a}_{[x+n]}}{\ddot{a}_{[x]}} \quad (22)$$

which is zero for $k = 1$ and with decrease in k the value increases to a maximum for $k = G$.

$k \lesseqgtr G$ From (19),

$$ {}_{n|}{}^{o}O_{[x]} = Q\,\frac{kl_{[x+n]}}{l_{[x]+n} - l_{[x+n]}} \frac{\ddot{a}_{[x+n]}}{\ddot{a}_{[x]}} \quad (23)$$

which decreases with k.

For $k = G$, the two expressions have the same value.

EXAMPLE

As an example, values for $x = 40$, $n = 5$, have been calculated by the $O^{[M]}$ Table with interest at $2\frac{1}{2}\%$, the minimum rate tabulated in the official volumes. These appear in Table B.

TABLE B

ILLUSTRATION OF VARIATION IN NET SINGLE PREMIUM FOR CONVERSION OPTION WITH VARIATION IN PERCENTAGE CONVERTING

According to Select Mortality Table Theory

PER \$1,000 INSURANCE $\quad O^{[M]}\ 2\frac{1}{2}\%$ $\quad x=40;\ n=5$

% Converting	Attained Age Conversion	Original Date Conversion	$\frac{3}{4}$ Attained Age and $\frac{1}{4}$ Original Date Conversions
k	$_{5\rvert}{}^{a}O_{[40]}$	$_{5\rvert}{}^{o}O_{[40]}$	$.75\ _{5\rvert}{}^{a}O_{[40]} + .25\ _{5\rvert}{}^{o}O_{[40]}$
	(1)	(2)	(3)
100%	\$14.76	....	\$11.07
90	14.76	\$ 1.35	11.41
80	14.76	2.70	11.75
70	14.76	4.05	12.08
60	14.76	5.41	12.42
50	14.76	6.76	12.76
40	14.76	8.11	13.10
30	14.76	9.46	13.44
20	14.76	10.81	13.77
10	14.76	12.16	14.11
2.17	14.76	13.22	14.38
2	13.62	12.20	13.27
1	6.81	6.10	6.63

$$G = 1 - \frac{l_{[45]}}{l_{[40]+5}} = .0217\,.$$

The values in columns (1) and (2), except in the last two lines, have been calculated by Formulae (16) and (18). The last two lines have been included to illustrate values calculated by Formulae (17) and (19).

Inasmuch as the conversion option commonly permits a choice of attained age conversion or original date conversion, blended values are illustrated in column (3) on the assumption that the choice will be made in the ratio of 3:1.

NON-SELECT TABLE

In terms of aggregate or ultimate mortality tables (6) becomes:

$$ {}_{n|}O_x = k\,\frac{D_{x+n}}{D_x}\,\frac{\ddot{a}_{x+n}-\ddot{a}^c_{x+n}}{\ddot{a}_{x+n-m}} \tag{24} $$

whence:

$$ \left.\begin{aligned} {}_{n|}{}^{a}O_x &= k\,\frac{D_{x+n}}{D_x}\,\frac{\ddot{a}_{x+n}-\ddot{a}^c_{x+n}}{\ddot{a}_{x+n}} \\ &= k\,\frac{D_{x+n}}{D_x}\left[\frac{1}{\ddot{a}^c_{x+n}}-\frac{1}{\ddot{a}_{x+n}}\right]\ddot{a}^c_{x+n} \\ &= k\,\frac{D_{x+n}}{D_x}\left(P^c_{x+n}-P_{x+n}\right)\ddot{a}^c_{x+n} \end{aligned}\right\} \tag{25} $$

$$ \left.\begin{aligned} {}_{n|}{}^{o}O_x &= k\,\frac{D_{x+n}}{D_x}\,\frac{\ddot{a}_{x+n}-\ddot{a}^c_{x+n}}{\ddot{a}_x} \\ &= k\,\frac{D_{x+n}}{D_x}\left(P^c_{x+n}-P_{x+n}\right)\frac{\ddot{a}^c_{x+n}\ddot{a}_{x+n}}{\ddot{a}_x}. \end{aligned}\right\} \tag{26} $$

Here the mortality table, although used to assess mortality charges through premiums, policy values and dividends, does not distinguish degrees of insurability and offers no theory for determination of the values of ${}_{n|}{}^{a}O_{[x]}$ and ${}_{n|}{}^{o}O_{[x]}$. These are therefore to be found through suitable assumptions as to k and c according to experience.

This problem is considered in detail by Mr. Griffin in his paper, "A New Approach to the Problem of Term-Insurance Conversion Costs" (*RAIA* XXXI, 374).

COST OF CONVERSION

In (4) it was assumed that the cost of conversion to be paid for the difference in plan is

$$ {}_{m}V_{[x+n-m]} - {}_{n}V_{[x]:\overline{r}|}^{1}. $$

Let m = O (attained age conversion)

The first term vanishes and there of course is no cost of conversion, but the second term shows that the reserve on the term plan should be a credit in the transaction.

Let $m = n$ (original date conversion)

The cost is

$$ {}_{n}V_{[x]} - {}_{n}V_{[x]:\overline{r}|}^{1} \tag{27} $$

which is *not* equal to

$$ \left(P_{[x]} - P_{[x]:\overline{r}|}^{1} - {}^{o}P\right)\frac{N_{[x]}-N_{[x]+n}}{D_{[x]+n}} \tag{28} $$

except when ${}^{o}P$ is zero. ${}^{o}P$ represents the annual extra charge paid with the term premium for the conversion option. Inasmuch as ${}^{o}P$ is zero (22) only on the unrealistic assumption of $k = 1$, and since in practice $k \neq 1$, formula (28) is not suitable and the theoretical cost requires to be determined by (27).

On the basis of a non-select table, (27) and (28) are in terms of x instead of $[x]$.

Needless to say, accumulation of the difference in premium at a rate of compound interest is in practice intended as an approximation to accumulation with benefit of interest and survivorship according to (28), on the assumed reserve basis. Accumulation in terms of gross premiums and dividends is the same in nature, the assumption of interest and survivorship then requiring to be consistent with the dividend basis, and an assumption being made as to provision for expense and contingencies. For the reasons stated, consistency requires the cost to be determined according to (27) and not (28).

OTHER PLANS

The foregoing analysis assumes conversion to the ordinary life plan. The form of analysis, however, applies to the other usual permanent plans and the conclusions apply equally to them. However, in determining the charge for the conversion option, in which there is room for difference of opinion as to the appropriate amount in practice, it may be regarded as sufficient to consider ${}_{n|}O_{[x]}$ in terms only of ordinary life. An example of the difficulty is exhibited in Mr. Moir's paper, "Office Premiums," *TFA* II, where on page 244 he shows that the charge by one table may be double that by another. In his example, the explanation is that the select period in the $O^{[M]}$ table is ten years and in the $O^{[NM]}$ table is five years.

LOADED DIFFERENCE IN POLICY VALUE

The difference in policy value, which will ordinarily be taken as the difference in reserve, is the *net* cost of conversion as of original date. Such amount constitutes the net consideration for the policy change to permanent plan, in the nature of a net premium taking account of assumed interest and mortality.

As in the case of all other premiums, this amount requires to be loaded for expenses and contingencies. The loading may be related to that used for single premium plans, or it may be related to the expense charge made in the dividend formula. An appropriate loading is 10%.

An important advantage of the loaded difference in policy value is that the costs are figures that do not vary from time to time. They therefore lend themselves to compilation in permanent form.

MORTALITY EXPERIENCE AFTER CONVERSION

Mortality experience after conversion leaves no doubt that term conversion is a valuable option for which a significant charge needs to be made. The experience of one company is exhibited in Table C. Data were available for only attained age conversions. Experience on regular new issues is shown for comparison.

TABLE C

THE NORTHWESTERN MUTUAL LIFE INSURANCE COMPANY
MORTALITY EXPERIENCE AFTER TERM CONVERSION TO PERMANENT PLAN WITHOUT EVIDENCE OF INSURABILITY

EXPERIENCE OF POLICY YEAR	ON REGULAR NEW ISSUES				ON TERM CONVERSIONS							
	Policy Years 1–5		Policy Years 6–10		Policy Years 1–5 after Conversion				Policy Years 6–10 after Conversion			
	A/E Pols. (1)	A/E Amts. (2)	A/E Pols. (3)	A/E Amts. (4)	A/E Pols. (5)	A/E Amts. (6)	(5)/(1) Pols. (7)	(6)/(2) Amts. (8)	A/E Pols. (9)	A/E Amts. (10)	(9)/(3) Pols. (11)	(10)/4 Amts. (12)
	Ratio: Actual/Expected Mortality by A.M. Ultimate Table											
1943–4.....	48%	41%	55%	56%	71%	85%	*148%*	*207%*	65%	60%	*118%*	*107%*
1944–5.....	62	44	74	65	61	66	*98*	*150*	54	52	*73*	*80*
	Ratio: Actual/Expected Mortality by CSO Table											
1945–6.....	40%	35%	54%	52%	50%	55%	*125%*	*157%*	58%	62%	*107%*	*119%*
1946–7.....	25	25	36	43	43	44	*172*	*176*	61	61	*169*	*142*
1947–8.....	30	31	39	41	39	33	*130*	*106*	56	56	*144*	*137*
					Average of above		*135%*	*159%*			*122%*	*117%*

PREMIUMS FOR TERM CONVERSION OPTION

Where a company offers a variety of term plans, in which the difference in premium will often be small, the charge for the conversion option must be consistent, if anomalous premium rates are to be avoided.

In one office, it is found that satisfactory net charges are obtained by the formula: 20% of CSO 2% net premium plus $1.00 per $1,000 insured. This applies to term plans up to ten years which are convertible for the full term. For 15 and 20 year term with conversion limited to 10 years, the foregoing 20% becomes 15% and 10% respectively.

The presence of these latter two plans illustrates the practical difficulty of following the method suggested in *JIA* LIV, 136, of finding the charge as the difference in regular premiums for term policies of five year difference in term. The 10 and 15 year term rates would be alike, which would be difficult to understand.

The office net premium becomes the CSO 2% net premium loaded as above. This is appropriate recognition that for a convertible term policy the net premium requires to be a premium for insurance plus a premium for conversion option.

Gross term premiums are found by the regular premium loading formula applied to the office net premium. The dividend formula also applies to term plans, the same as permanent plans, the loading being the difference between gross and office premium. A special mortality gain factor may be used to reflect mortality on term insurance differing somewhat from permanent plans. The effective charge for the option will accordingly be the net amount plus the expense charge in the dividend scale, as illustrated in Table D.

TABLE D

EFFECTIVE ANNUAL CHARGE FOR TERM CONVERSION OPTION PER $1,000 INSURED

Age at Issue	5 Year Term Convertible for 5 Years	10 Year Term Convertible for 10 Years
20	$1.64	$1.69
30	1.93	2.05
40	2.59	2.87
50	4.15	4.79
60	7.73	......

COMPARISON OF METHODS FOR COST OF CONVERSION AS OF ORIGINAL DATE

For illustration, figures are shown for one company for $1,000 five year term insurance converted at the end of three years to ordinary life insurance as of original date, issue age 40.

Year	5 Year Term			Ordinary Life		
	Premium	Dividend	Net Payment	Premium	Dividend	Net Payment
1	$13.80	$4.51	$ 9.29	$32.55	$5.35	$27.20
2	13.80	4.63	9.17	32.55	5.65	26.90
3	13.80	4.76	9.04	32.55	5.95	26.60
	......	......	$27.50	......	......	$80.70

Difference in net payment	OL	$80.70	
	T	27.50	$53.20
Interest at 5%			5.68
Cost: By common accumulation formula (2)			$58.88

The term policy includes an effective annual charge of \$2.59 for the conversion option, also an excess annual mortality charge in the dividend. These ought not to be credited back to the policyholder in the cost calculation; they should be retained by the company.

The term policy net payments are as follows:

Year	Net Payment (As Above)	Deduct Charges for:		Net Payment on Normal Insurance Basis
		Conversion Option	Extra Mortality	
1.	\$ 9.29	\$2.59	\$.31	\$ 6.39
2.	9.17	2.59	.33	6.25
3.	9.04	2.59	.35	6.10
	\$27.50	. . . .	. . . .	\$18.74

With this adjustment the common form of calculation gives:

Difference in net payment	OL.	\$80.70	
	T.	18.74	\$61.96
Interest at 5%.			6.53
Cost: By adjusted formula (2).			\$68.49

Reserve on policy basis, CSO 2%:

Ordinary Life.	\$62.35	
5 Year Term.	1.40	
Difference.		\$60.95
Loading 10%.		6.10
Cost: By loaded difference in policy value, formula (3)		\$67.05

The cost by formula (3) is not only proper, as shown above and by the theoretical development, but it is also easy to determine and gives consistent results.

ATTAINED AGE *VS.* ORIGINAL DATE

The theory shows that attained age conversion is more valuable than conversion as of original age, ${}_{n|}{}^{a}O_{[x]}$ being greater than ${}_{n|}{}^{o}O_{[x]}$ as shown by (20), (21) compared with (22), (23). Under attained age conversion the net amount at risk is greater. Contrary to popular notion, the insured does not gain by original date conversion unless through financial selection. The fact that he obtains the premium rate of a younger age is offset by the payment required for the cost of conversion. Original date conversion therefore is usually not advisable and should be limited to a brief period. A further reason for limiting original date conversion to a brief period is its nature as constituting an option on policy form.

Attained age conversion, with suitable charge for the option, will preferably run the full term period of the policy, but a limit such as ten years may well be made on long term policies.

POLICY LOANS

Agents sometimes mistakenly urge original date conversion with borrowing of funds to cover the cost. The only advantage in original date, as against attained age, conversion is as a means of using available funds. If the policyholder does not have the funds, the transaction is against his interest. The company therefore should refuse, or at least discourage, such conversion with the aid of a policy loan.

AGENTS' COMMISSIONS

An additional defect of the common method of determining the cost of original date conversion as an accumulated difference in premiums less dividends, has been its tendency to imply that commissions should be adjusted to correspond with the premiums. This has caused pressure by agents for original date conversions often without proper consideration of the insured's interests.

The cost determined as the loaded difference in policy value avoids any suggestion of reopening past commissions.

It is desirable for agents' commissions in cases of term conversion to be on a simple and consistent basis. A suggestion is as follows:

Attained Age Conversion

(1) Term commissions on premiums due before request is made for conversion to be unaffected by conversion.

(2) Permanent plan commissions to be the same as for new issues.

Original Date Conversion

(1) Term commissions on premiums due before request is made for conversion to be unaffected by conversion.

(2) No commissions or collection fees to be allowed on the amount collected to effect conversion.

(3) Permanent plan commissions to be allowed only on regular premiums carrying the policy forward after conversion, commencing with the one due on the date to which the

policy was paid through term premiums due before request was made for conversion; the first year commission rate to apply to such regular permanent plan premium payment for one year, and renewal commission rates to apply thereafter, the same as if there had not been a prior term policy.

For consistency with so-called preliminary or short term, it may in practice be advisable, on attained age conversion in the first policy year, to stop the term commission at the month of conversion.

OPTION ON POLICY FORM

The original date conversion privilege will normally guarantee the permanent plan as originally available. This constitutes an option not only as to insurability but also as to policy form. The form as of original date may be more advantageous than currently offered for new issues at time of conversion. The theoretical charge takes no account of that feature, but some protection is given the company in the deterrent offered by the necessity of paying a cost of conversion.

As for attained age conversion, this should be to the policy form as then currently offered for new issues. No justification is seen for an option on the permanent form current at issue of the term policy which may no longer be current at conversion. It would give an advantage over new policyholders with no basis for a charge. It would also be objectionable because of irregularities produced in the company records, a cause of errors and expense.

AUTOMATIC OPTION

In addition to the usual options to convert to any permanent plan at any time within stated periods, the term policy may provide for automatic conversion as of attained age at the end of the term period. On payment of the permanent premium, the insurance will thus commence on permanent plan simultaneously with expiry of the term plan. In an office where policy changes and conversions are handled only on a prepaid basis, such automatic conversion would mean primarily the automatic billing of the first permanent plan premium which, when paid, would result in issue and delivery of the new policy. For the smooth operation of such a provision, it is advisable that the usual grace period be made applicable to such premium. Thus the term policy would afford insurance for the term period plus the grace period if the permanent plan premium is not paid. It may be assumed that the probability of claims during that period will be small because impaired lives will be concerned to see that the premium is paid promptly.

APPENDIX

In order to avoid interruption of the argument in the paper, certain demonstrations as therein referred to are given in this Appendix.

From the method of constructing select mortality tables we have:

$$A_{[x+n]} < A_{[x]+n} \tag{29}$$

and

$$\ddot{a}_{[x+n]} > \ddot{a}_{[x]+n} .$$

Also we have:

$$l_{[x]+n} A_{[x]+n} = l_{[x+n]} A_{[x+n]} + (l_{[x]+n} - l_{[x+n]})\, A''_{[x]+n} \tag{30}$$

if

$$A''_{[x]+n} = \frac{l_{[x]+n} A_{[x]+n} - l_{[x+n]} A_{[x+n]}}{l_{[x]+n} - l_{[x+n]}} . \tag{31}$$

From (29), (31):

$$\begin{aligned} A''_{[x]+n} &> \frac{l_{[x]+n} A_{[x]+n} - l_{[x+n]} A_{[x]+n}}{l_{[x]+n} - l_{[x+n]}} \\ &> A_{[x]+n} . \end{aligned} \tag{32}$$

From (31):

$$A''_{[x]+n} = \frac{v\,(d_{[x]+n} - d_{[x+n]}) + v^2\,(d_{[x]+n+1} - d_{[x+n]+1}) + \dots}{(d_{[x]+n} - d_{[x+n]}) + (d_{[x]+n+1} - d_{[x+n]+1}) + \dots}$$

$$\frac{+\, v^b\,(d_{[x]+n+b-1} - d_{[x+n]+b-1}) + 0}{+\,(d_{[x]+n+b-1} - d_{[x+n]+b-1}) + 0} .$$

Hence

$$v^b < A''_{[x]+n} < v \quad \text{(\textit{Hall \& Knight Higher Algebra}, Art. 14)} \tag{33}$$

where b is the select period of the mortality table.

Therefore from (29), (32), (33):

$$A_{[x+n]} < A_{[x]+n} < A''_{[x]+n} < v . \tag{34}$$

This shows that $A''_{[x]+n}$ is a quantity of the order of a single premium for assurance, and as such corresponds to mortality higher than the select mortality represented by $A_{[x+n]}$ and the mixed mortality represented by $A_{[x]+n}$.

From (31):

$$1 - d\,\ddot{a}''_{[x]+n} = \frac{l_{[x]+n}\,(1 - d\,\ddot{a}_{[x]+n}) - l_{[x+n]}\,(1 - d\,\ddot{a}_{[x+n]})}{l_{[x]+n} - l_{[x+n]}}$$

$$\ddot{a}''_{[x]+n} = \frac{l_{[x]+n}\ddot{a}_{[x]+n} - l_{[x+n]}\ddot{a}_{[x+n]}}{l_{[x]+n} - l_{[x+n]}} . \tag{35}$$

From (34):

$$\frac{1-A_{[x+n]}}{d} > \frac{1-A_{[x]+n}}{d} > \frac{1-A''_{[x]+n}}{d} > \frac{1-v}{d}.$$

Hence

$$\ddot{a}_{[x+n]} > \ddot{a}_{[x]+n} > \ddot{a}''_{[x]+n} > 1\,. \tag{36}$$

This shows that $\ddot{a}''_{[x]+n}$ is a quantity of the order of a single premium for an annuity-due and as such corresponds to mortality higher than the select mortality represented by $\ddot{a}_{[x+n]}$ and the mixed mortality represented by $\ddot{a}_{[x]+n}$.

From (34) and (36) we may regard $A''_{[x]+n}$ and $\ddot{a}''_{[x]+n}$ as an assurance and an annuity-due, consistent with each other, on a substandard basis of mortality.

(30) shows that:

The value of the assurance to the $l_{[x]+n}$ mixed lives at the mixed rate $A_{[x]+n}$ is equal to the value at the select rate $A_{[x+n]}$ to a portion of the group plus the value to the residue at the substandard rate $A''_{[x]+n}$ according to (31), if it be assumed that the respective portions of the group are in the ratio $l_{[x+n]}:(l_{[x]+n}-l_{[x+n]})$. A corresponding statement applies to the annuities-due.

It follows that

$$A_{[x]+n} = F A_{[x+n]} + G A''_{[x]+n} \tag{37}$$

and

$$\ddot{a}_{[x]+n} = F \ddot{a}_{[x+n]} + G \ddot{a}''_{[x]+n}$$

where F and G are weights such that:

$$F:G::l_{[x+n]}:(l_{[x]+n}-l_{[x+n]}) \qquad \text{and} \qquad F+G=1$$

and therefore:

$$F=\frac{l_{[x+n]}}{l_{[x]+n}} \qquad \text{and} \qquad G=\frac{l_{[x]+n}-l_{[x+n]}}{l_{[x]+n}}.$$

DISCUSSION OF PRECEDING PAPER

HAROLD A. GARABEDIAN:

Mr. Fassel's mathematical demonstration of the character of the conversion option and of methods to determine premium charges for the option now leaves us with a quite complete literature on the theoretical aspects of that phase of the term conversion problem.

The paper is also satisfying in its completeness, since the author is not silent on any phase of his subject, giving us for full measure a summarized one-company mortality experience following conversion at attained age and some illustrative premium charges for the option. We are, however, still short on the matter of these experiences, and it would be desirable to have a large and detailed body of data for use in more accurate appraisal of the cost of these options which we do know is very appreciable. It is to be hoped that the informal discussions at this meeting will be fruitful in this respect.

Relative to the cost of conversion, as distinguished from the premium charges for the option, certain statements in the paper stimulate reflection upon the propriety of the methods in current use when applied to original date term conversions. Upon examination, all of these methods seem more or less defective when we apply the principle that the net cost of conversion (*i.e.*, before any surcharges for expenses or other purposes) should approximate as closely as practicable a difference in values based upon asset shares.

Under the "accumulated difference in premium (less dividend)" method, the author reminds us of two major defects. One of these is that this method does not properly allow for the force of survivorship as well as for inherently different properties in the gross premiums and dividends under the two plans. The other is that the method automatically and improperly refunds the charges for the cost of the option included in the term premiums. Even when the accumulation is made with an interest rate of 5% compounded, the resulting value is sometimes less than the difference in reserves or cash values, thus requiring the imposition of a special minimum control on the cost based on one of the latter differences.

The author notes the strong implication to agents in the above method for the payment of the full difference in commissions, which is avoided when charges are based upon a difference in policy values. Undoubtedly for this reason original date changes are sometimes forced in instances where an attained age method would as well meet the desire of the policyholder to preserve the guarantee of continued insurability. We fully considered the commission aspects of this problem in the company with which I am connected when contracts were revised in connection with adoption of the standard legislation and found that it is not easy to resist the field view that a commission adjustment should be made upon the exercise of an original date term conversion option. This view springs mainly from the considerations (1) that many such conversions represent a natural fulfillment of plans contemplated at the time of the sale of term insurance when the insured's finances were temporarily impaired and (2) that the offer of the option by the company carries with it the obligation to pay for the field services rendered upon the exercise of it. The continued popular use of the "accumulated difference in premium (less dividend)" method, despite its defects, attests to the general agreement with this view.

The "difference in reserve" method is of course free of the two major defects alluded to, as the author points out. Moreover, in the absence of cash values in wide areas under term policies, which in turn may reflect actual negations in asset values, the "difference in reserve" method may give generally better results than the "difference in cash value" method in approximating the difference in asset values in term conversions, though otherwise such an advantage could hardly be claimed.

On the other hand the "difference in reserve" method cannot appeal to a company whose current contracts have reflected its desire to accept fully the principles of the standard legislation. Except for a few states it is of course not now necessary to make any reference in the contract whatever to reserves, which omission is not only a natural counterpart to the use of the adjusted premium method of determining policy cash values, but is also desirable as an aid in the cause of removing the fallacy, so often reiterated, of the concept of the valuation reserve as individual property of the policy. Perpetuation of the use of the "difference in reserve" method in policy conversions operates as a deterrent to this cause.

We seem, therefore, to be confronted with somewhat of a dilemma in choosing a wholly satisfactory practical method for determining the cost of conversion under original date term conversions. All things considered, the "difference in cash value" method would appear to be the least objectionable, even though use of it should necessitate the introduction of special adjustments to cover deficiencies arising from negative asset values. An ideal solution would of course be the discontinuance of the option of original date conversion in term policies, leaving as the only one available the attained age option, which seems wholly sufficient for the purpose of guaranteeing continued insurability and which as well would eliminate possible financial selection on policy form, to which the author of this excellent paper refers.

DONALD B. CHENEY:

Mr. Fassel's paper on the "Term Conversion Option" is a very timely one. The question of the proper cost of conversion is a very troublesome one in rate making and surplus distribution and there is but scant material available on this subject.

One of the principal points of the paper is that the mortality on the lives which convert can be approximated if we know the rate of conversion and assume that all those that do not convert are select lives. We used this principle in the current revision of our Term premiums because we do not as yet have any very complete actual mortality experience on the lives that convert. The cost of conversion which we adopted assumed that those who do not convert at the end of the term will thereafter experience select mortality which grades into our ultimate experience table at duration 15. It further assumed that those who do convert will experience bad enough mortality so that the total death rate on those who convert and those who do not convert will equal the rates shown in our ultimate experience table.

Nevertheless we are still of the opinion that a more realistic cost of conversion should be based on the rate of conversion and the actual mortality on the lives that convert, according to the latest experience. There does not seem to be any very reliable substitute for this procedure.

For original age conversions Mr. Fassel recommends the loaded difference in reserve method for determining the charge for conversion. He feels that an appropriate loading is 10%. He also states that the cost by the loaded difference in policy value formula not only is proper but also is easy to determine and gives consistent results.

He probably intended that these statements be considered more in the nature of his personal impressions than of facts. They would not necessarily agree with the results of other companies' studies. For example, our charge consists approximately of the difference in asset shares (excluding the conversion fund), plus commissions and taxes on the increase in premiums on the new policy plus the expense for making the change. The conversion privilege in our new Term policies provides that we may charge the difference in premiums with 5% compound interest or the cash surrender value (not reserve) loaded not more than 12%. This allows some latitude by kind and duration which our tests indicate advisable.

Mr. Fassel thinks an original date conversion is usually not to the insured's advantage and, if allowed, should be limited to a brief period. While he is rightly concerned about the insured's best interests, possibly he is overlooking the lower premium rate scales, more liberal benefits and more generous settlement options contained in the older policies of some companies.

He also feels that a company should refuse or at least discourage original date conversions which require a policy loan. We all know that encumbrances are detrimental as they have a tendency to lead to early lapse. However, as a practical matter I don't see how we could refuse such requests. The insured can always accomplish the same result by borrowing elsewhere to pay the charge in cash and then by repaying this loan from the proceeds of a loan on the new policy.

Mr. Fassel suggests that no commissions or collection fees be allowed on the amount collected to effect an original date conversion but that first year and renewal commissions be allowed on future premiums. As a practical matter most companies may find that, if they are to encourage conversions and have a contented agency staff, it is necessary to allow the excess of the commissions that would have been allowed on the new policy over the commissions that have already been paid on the Term policy.

So far as the premiums for the cost of conversion in Table D are concerned, it would be interesting to know what rate of conversion and rate of mortality on the lives that convert were used in arriving at these results. The costs at the lower ages are considerably higher than might reasonably be expected. Some actuaries are of the opinion that the cost of conversion should be lower at the younger ages, as many young men start their insurance programs with term insurance with the expectation of converting to permanent insurance when their incomes improve. This in itself should reduce the possible adverse mortality that might otherwise result.

WARD VAN B. HART:

Mr. Fassel's paper emphasizes the immense importance of the "Term Conversion Option." Companies which have minimized the importance of this option, or apparently have ignored it entirely, will not derive much encouragement from either the figures in his Table C showing the actual mortality experienced or his Table D showing the effective annual charges for the option. As will be mentioned later, the figures shown in Table D are more or less of the same order of magnitude as those developed in our own Company by an entirely different method of attack in connection with a recent rate investigation.

I regret, however, that he has seen fit to construct such an imposing mathematical edifice. I am not critical necessarily of the fact that, after several pages of mathematical formulas, he ends up with an annual net charge of 20% of the CSO 2% net premium plus $1 per thousand. The use of simple working formulas following an elaborate investigation is well known to us in nonparticipating rate-making. Rather, I dislike having the student reach the conclusion that the wealth of formulas shown in the paper constitute a definitive method of assessing once and for all the cost of this important option. He indirectly in several places uses the technique of assuming that $l_{[x]+n}$ ultimate lives consist of $l_{[x+n]}$ select lives plus $l_{[x]+n} - l_{[x+n]}$ damaged lives. The pitfalls inherent in this type of reasoning have been mentioned in various papers in the past.* The trouble is that sometimes this technique gives a correct result and sometimes it gives a fallacious result, and the student must be extremely careful in handling it. For instance, the well-known textbook formula quoted by Mr. Fassel as #1 of the paper can be correctly derived (on the assumption that all lives convert) without using the technique in question, which however was Dr. Sprague's original method of deriving the formula. On the other hand, Mr. Fassel almost implies in one or two places that $A''_{[x]+n}$ is actually the single premium for whole life insurance on the impaired lives who are converting. Really it is nothing but a mechanical device which under certain conditions will give the correct result. The late Mr. J. F. Little rather concisely expressed the matter over 35 years ago in a somewhat similar connection by likening this reasoning to: "If one farm of 50 acres produced 1,000 bushels of wheat and another of 70 acres produced 1,100 bushels, the latter must consist of 50 acres of land of quality equal to that of the first farm, producing 20 bushels per acre, plus a further 20 acres producing only 5 bushels per acre."

In using select tables, we have to remember that the only part of the mortality table that has contact with reality is the rate of mortality as derived from experience after having been adjusted in the process of graduation. The "Number Living" is only a mechanical tool which must not be misused.

In preliminary investigations leading up to our present rates charged for nonparticipating 5 Year Term and 10 Year Term insurance, two years ago we arrived at the following effective annual charges for the conversion option loaded for the percentage expenses of commissions and taxes.

Age at Issue	5 Yr. Term	10 Yr. Term
25.	$ .89	$1.00
35.	1.65	1.94
45.	3.37	4.07
55.	6.68	8.26

* *TFA* III, 384; *JIA* LIV, 123; *TASA* XIII, 359.

The 10 Year Term policy is convertible during the first 7 years only. The foregoing figures are based on the following percentages of the mortality assumed in our corresponding preliminary investigation of non-participating life and endowment rates:

During Term period	115%
First 10 years after conversion	115
Thereafter	105

A word of caution needs to be used in interpreting the above charges for the conversion option. If lower mortality than 115% has been assumed during the term period, the total gross premiums eventually charged for the term insurance might have been less but the charge for the conversion option would have been more. As a matter of fact our mortality experienced on term insurance during the term period has barely exceeded the corresponding mortality on life and endowment policies and the 115% assumption during the term period is at least partially in the nature of a safety margin. Again, if 115% mortality had been assumed in the third period—that is, after 10 years after conversion—the charge for the option would have been higher since our rather plausible assumption that mortality on converted policies will eventually drop toward the general level of our Company mortality operates in our formula to lower the cost of the conversion option. In our method, therefore, the Company can include its safety margin in any one of three places.

To date, three methods of attack on the problem of assessing the value of this option appear in actuarial literature:

the classical approach, as presented by Mr. Fassel;

the ingenious method of assuming a frequency distribution of impaired lives, presented by Mr. F. L. Griffin, *RAIA* XXXI, 374; and

the method of attempting to guess as intelligently as possible the probable future level of mortality on term conversions and of applying it to an assumed proportion of lives converted, as mentioned by Mr. Jenkins and myself in the discussion of Mr. Griffin's paper, *RAIA* XXXII, 131.

Briefly, the third method consists with us in first constructing a double decrement table for the term period allowing for both death and other discontinuances (really a triple decrement table since the other discontinuances are further subdivided into lapses and conversions) and then calculating as of the end of each policy year of the term policy the single premium for the expected excess mortality per policy converted. This is in the nature of an additional one year renewable term benefit varying by age and duration and can eventually be combined with the normal deaths included in our usual gross premium term formula. I presume, although Mr. Fassel does not mention it specifically, that in arriving at his eventual empirical formula he must also in some way have to bring in the cost of conversion at each year of the term policy. The figures shown in his Table B, for instance, relate only to those converting at the end of the fifth year.

His inclusion of the necessity of giving some weight to the matter of original date conversion as well as attained age is novel and at least of theoretical interest. Our point of view has been that the theoretical considerations are outweighed by certain other very important factors which are exactly the same as those involved in a noncontractual change from a less expensive plan to a more expensive plan. In our Company, we found it necessary about 15 years ago to greatly curtail this latter type of change and now it is hardly permitted at all. To all intents and purposes, the original date provision still given in our 5 and 10 Year Term policies is a relic still promised in these contracts which has practically entirely disappeared elsewhere.

EARL M. MACRAE:

Mr. Fassel's paper is particularly timely because of the increased sale of term insurance in recent years and its probable continued increase because of pending developments in extension of old age benefits under Social Security and the growth of employee retirement plans.

I do not wish to discuss the mathematical aspect of Mr. Fassel's paper in which he has covered, in his usual comprehensive manner, the development of the actuarial formulas involved in determining the proper charge for extra mortality on term conversions. My comments will be along more general lines.

First, though, I want to comment on a reference in the introductory remarks of the paper to the use of mortality tables in the calculation of premiums and dividends. "Premiums, values and dividends in America are almost invariably based on aggregate or ultimate tables, now generally the CSO Table." I believe that this statement may lead to some confusion in the mind of a student since he knows that there is an appreciable gain because of selection and may wonder why the actuary in computing premiums or dividends does not take into account mortality savings arising from selection. The answer that they are applied to amortize all or part of the extra expenses incident to issuance of the policy is hardly satisfactory. If the saving resulting from selection, and the first year and renewal costs, are commensurable, why not take them into account in the calculation of premiums and dividends?

My company has for several years used a "profit analysis" approach in computing premium and dividend scales. We use a select mortality table based on company experience with margin for contingencies. Each plan of insurance is treated separately and profits arising out of each policy year are discounted at a reasonably conservative rate of interest to date of issue using mortality, persistency and expense factors which vary by plan of insurance. Nonparticipating premiums at quinquennial ages are adjusted so as to provide a satisfactory prospective profit margin. The calculations are usually made to cover a period of not more than twenty years after issue. This approach is realistic, it gives us a clear picture of the incidence of earnings and it enables us to make the adjustments in premiums, cash values and dividends which are required in order to produce the desired profit margins or contingency funds. It is also very easy to determine the effect upon profit margins of a change in the average size of policy, due, for example, to an increase or decrease in the minimum amount which the company will issue.

The experience of Mr. Fassel's company for the past five years indicates a substantial extra mortality in the five years following conversion with considerable improvement during the sixth to tenth years. The experience of my company on policies converted from term insurance during the years 1937–1946 inclusive, exposed to the 1947 anniversary, shows mortality of 156% of the mortality under all plans of insurance for the same period. The corresponding ratio in Mr. Fassel's company for the first five policy years after conversion is 159% and since the greater part of our exposure was in the five policy years after conversion, the results in the two companies seem to agree very closely. Our experience on converted term insurance is not extensive, although we have issued a relatively large volume of term insurance for several years. This is because most of our term business has been written either on long term or on renewable term plans. The experience quoted was based on expected deaths amounting to $567,000. Our present convertible term rates do not provide for higher than normal mortality either before or after conversion. Our renewable term rates provide for mortality which increases at the average rate of five percent per year, an assumption which is conservative since the degree of anti-selection at date of renewal of term (usually five years) should be materially less than upon conversion to a permanent plan. Our recent mortality investigation showed that our mortality on term business over the period mentioned had been less than on other plans. Although based on a relatively small exposure, this result is of interest since it is contrary to the results of other published experiences.

From the standpoint of a stock company, the important question as regards excess mortality is not entirely whether converted term business shows a higher mortality than business originally written on permanent plans, but whether or not term business with a conversion option is profitable to the company after due allowance is made for excess mortality in the years following conversion and excess mortality, if any, during the term period. It may be expedient for the company to adopt convertible term rates which will produce a lower margin of profit than business on permanent plans since it may be assumed that only a small proportion of the business written on term plans would be written in the company on other plans if the term plans were not available.

Mr. Fassel recommends charging difference in reserves loaded 10% on original date conversions rather than difference in premiums accumulated at interest. This is undoubtedly sound when net level premium reserves are carried. On a less stringent reserve basis, it may be to the company's advantage to charge difference in premiums with interest.

I feel sure that Mr. Fassel's paper will prove to be of intense interest to those companies which issue a considerable volume of term insurance. He has given us the tools with which to work and each company can use these tools in solving related problems which are peculiar to the individual company.

DONALD D. CODY:

I should like to present an outline of the practices of the Equitable Society under its convertible term insurance policies to show a particular application of some of the ideas set forth by Mr. Fassel in his thought-provoking paper which is so timely because of the growing importance of term policies in many companies. Incidentally, many of our procedures are similar to those so well described by Mr. Griffin in *RAIA* XXXI, 374, although our own statistics have been sufficiently dependable so that we have not had to have recourse to any of Mr. Griffin's more empirical procedures.

Our convertible, nonrenewable term insurance policies are in three forms:

(1) A 2 Year Initial Term policy, written as a unit with the permanent policy to which conversion is automatic.

(2) A 5 Year Term policy, with privilege of retroactive or attained age conversion to any form of permanent insurance within 5 years. The type of converted policy must be specified only if the disability waiver of premium feature is attached; such feature may be continued on the converted policy and provides for waiver of premiums on both the term and permanent policies in case of disability during the term period.

(3) 10, 15, and 20 Year Term policies, with privilege of conversion to any form of permanent insurance on a retroactive basis during the first seven years and on an attained age basis within a period running from is-

sue to three years prior to the expiry date but not beyond age 65. The term portion of our Double Protection to Age 65 policy has a somewhat similar provision. The disability waiver of premium feature provides for waiver only of the term premium and may not be continued on the permanent policy without evidence of insurability.

In designing our premium and dividend scales we take account of the following items:

(1) Term insurance mortality rates, which generally run above rates on permanent forms.
(2) Post-conversion extra mortality relative to select mortality on permanent policies as implicitly assumed in dividends paid under the converted policy. This extra mortality as described by Mr. Fassel arises from anti-selection by insured electing to convert and, in the case of attained age conversion, from the normal deterioration of the average vitality of the group with duration from the medical examination.
(3) Conversion rates. It is to be expected that as conversion rates increase, post-conversion extra mortality will decrease due to lessened anti-selection.
(4) Rates of nonrenewal on both term and permanent policies. The former rates are very important in determining the level and rapidity at which conversion costs are to be charged in the term period and the latter rates affect the level of the post-conversion costs.
(5) Expenses saved on the permanent policy relative to newly issued business due to absence of new underwriting and to the higher average size of converted policies relative to normal permanent policies.

Our procedure consists of determining for each issue age and policy year of the term policy the cost of post-conversion extra mortality reduced by expense savings. The cost of retroactive conversions is relatively unimportant since extra mortality arises only from moderate anti-selection and not at all as a result of duration from medical examination and since the number of such conversions is small. Term insurance dividends are then determined and tested in asset share calculations so as to bring out reasonable contributions to surplus at all issue ages on the various term plans. The 3-factor dividend formula now in use involves excess interest and claims-gain factors consistent with those on permanent forms, but has special charges in the form of percentages of weighted premiums and constants per $1,000 of face amount in the loading return factor to account for expenses, select mortality, extra term mortality, post-conversion mortality costs, and contingency provisions. We use a single 3-factor formula for all term plans in which, of course, premium and reserve functions apply to the particular policy.

The change basis of retroactive conversions is difference in premiums with 5% interest less arithmetical difference in dividends, except that the annual difference in dividends is reduced by an annual amount to provide for the term policy's share of the extra term mortality and the conversion costs charged in the policy years since issue. This annual amount is roughly the annual equivalent on a multiple decrement table basis of the excess of the asset share at the end of the conversion period computed taking account of term and post-conversion extra mortality costs over the corresponding asset share taking no account of such costs. The commissions paid on conversion are the difference in back commissions on the permanent and term policies. It is seen therefore that our basis of change essentially is difference in asset shares with due account given to proper allocation of all mortality and conversion costs.

TABLE A

CONVERSIONS FROM 2 YEAR INITIAL TERM

Policy Years after Conversion	By Duration			Age at Conversion	By Age		
	No. of Deaths	Mortality Ratio			No. of Deaths	Mortality Ratio	
		Policies	Amounts			Policies	Amounts
1–2	37	164%	171%	To 29	37	84%	91%
3–5	76	109	98	30–39	155	124	119
6–10	259	109	109	40–49	197	102	101
11 and over	162	98	105	50 and over	145	108	116
All	534	108%	109%	All	534	108%	109%

Although each company's term conversion experience depends heavily on the way in which term insurance is marketed and serviced, it may be of interest to record our post-conversion mortality experience under attained age conversions from 2, 5 and 10 Year Term policies, from anniversaries of the converted policies in 1940 to those in 1945, where such conversions occurred in years 1930 to 1944. Tables A, B, and C show the ratio of actual deaths to expected deaths under our 1940–1945 select table by amounts for standard, premium-paying, medically-examined issues, with duration based on date of conversion. Deaths due to the war are excluded from the actual statistics and from the select table. The number of actual deaths by policies are given as a measure of the statistical dependability. In *TASA* XL, 479–482, may be found a corresponding experience for years 1932 to 1938.

TABLE B

CONVERSIONS FROM 5 YEAR TERM

	Conversions in Years 1–4			Conversions in Year 5		
	No. of Deaths	Mortality Ratio		No. of Deaths	Mortality Ratio	
		Policies	Amounts		Policies	Amounts
Policy Years after Conversion:						
1–2	34	167%	215%	46	193%	174%
3–5	71	147	151	110	199	196
6–10	111	95	83	228	136	124
11 and over	130	103	108	139	131	137
All	346	111%	109%	523	148%	140%
Age at Conversion:						
to 29	18	127%	201%	7	....	....
30–39	67	100	100	59	128%	125%
40–49	132	103	103	210	150	133
50 and over	129	127	114	247	153	150
All	346	111%	109%	523	148%	140%

TABLE C

CONVERSIONS FROM 10 YEAR TERM

	Conversions in Years 1–4			Conversions in Years 5–7		
	No. of Deaths	Mortality Ratio		No. of Deaths	Mortality Ratio	
		Policies	Amounts		Policies	Amounts
Policy Years after Conversion:						
1–2	12	178%	359%	24	241%	164%
3–5	17	114	106	34	157	119
6–10	34	117	137	110	143	162
11 and over	35	116	142	117	101	128
All	98	121%	150%	285	127%	143%
Age at Conversion:						
to 29	2	80%	93%	2	198%	237%
30–39	8			38		
40–49	36	113	113	100	114	125
50 and over	52	143	194	145	125	145
All	98	121%	150%	285	127%	143%

ROBERT M. DUNCAN:

In Mr. Fassel's welcome analysis of the single premium for the term conversion option for all effective dates of conversion by an ingenious application of select mortality theory, he obtains values for attained age conversions which are independent of the period of term coverage. In reviewing published experiences on mortality after conversion, there is evidence to indicate that, in practice, the level of mortality on attained age conversions tends to be significantly higher for election at the end of the term period than for all conversions before expiry and also that most conversions take place at expiry (*e.g.*, Appendices to Mr. Griffin's paper in *RAIA* XXXI, 394–396).

This raises the question of applying the theory in calculating the single premium for the option arising from various percentages of attained age conversions at or near durations of, say, ten years for equal ages at issue on 10 and 20 Year Term policies, when conversion is limited to ten years in each case. There would presumably be more anti-selection at that time from the ten year plan group than from the twenty year plan group, since some of the lives in bad health in the twenty year group might not convert at the end of the ten year period.

This point can perhaps be recognized by the use of a different weighted average duration at conversion for each of these plans, provided the pattern of mortality likely to be experienced under each group conforms fairly closely with the select table used to assess the mortality on converted policies in accordance with the theory outlined in the paper. In practice, the charge for the option would probably be handled by an approximate method.

This characteristic of conversion mortality for elections at expiry does, however, add emphasis to the problem of assessing the various mortality costs by plan while maintaining premiums consistent among the plans offered.

B. FRANKLIN BLAIR:

We are indebted to Mr. Fassel for bringing to our attention this important subject, on which so little has previously been published in actuarial journals. His paper is an interesting combination of the theoretical and the practical. It is fortunate that both these aspects have been covered, as it is difficult to understand the practical aspects unless one is conversant with the theoretical and in actual use it is essential to temper theory with practical considerations.

The formulas developed in the paper for ${}_{n|}O_{[x]}$ are of limited practical value because lapses have not been taken into consideration in their development. As many companies have found higher than average lapse rates on term policies, it is important that they be taken into consideration in calculating either the net single premium for the conversion option or the annual charge for this option. Moreover, the "select mortality table theory," on which these formulas rest, seems artificial and open to serious question.

The choice of the now obsolete O^{M} Table as the basis for the figures in Mr. Fassel's Table B seems unfortunate. In Table 1 are shown figures similar to those in Table B except for the use, as indicated, of more modern mortality tables.

The large differences in the figures given in Table 1 for these two modern tables when $k \geqq G$ show the "leverage" which different methods of handling the select period (in the development of the mortality table)can exert on the cost of conversion, using the theoretical formulas developed by Mr. Fassel.

Incidentally, the ratio of attained age conversions to original date conversions has been much higher in the Provident Mutual than the ratio of 3:1 used by Mr. Fassel. A summary of our recent experience on nonrenewable term policies and on term riders is given in Table 2.

Mr. Fassel's Table C shows that Northwestern Mutual mortality on term conversions has been higher than the mortality on regular new issues. However, in interpreting Table C, it is well to bear in mind the disadvantages of both the A. M. Ultimate and the CSO Tables as measuring rods for recent select experiences. As the average age at issue is presumably higher on term conversions than on regular new issues, the expected mortality should be based on a select table which is reasonably parallel at all ages with the mortality actually experienced on regular new issues.

A summary of a recent investigation of Provident Mutual experience on conversions of 5 and 10 year term policies is shown in Table 3. The important features of this table are the high mortality on conversions made in the last 6 months of the conversion period (even when the expected mortality is taken as select from the original term date) and the lower mortality on conversions of 10 year term as compared with 5 year term. This may possibly be due to stricter underwriting of 10 year term.

In determining the basis of our term premiums and dividends, we have found it desirable to take into consideration the expense savings at the time of conversion arising from the fact that evidence of insurability is not required on the new insurance. As the average policy is smaller at the younger than at the older ages, the resultant savings are greater at the younger ages. In some cases our calculations have indicated that the expense savings more than offset the extra mortality on conversions at the younger ages.

As a result, the "effective annual charges for term conversion option" used in the Provident Mutual are much lower at the younger and middle ages than the figures shown in Mr. Fassel's Table D. However, at the upper ages our results are not too far different from Mr. Fassel's. Our "charges" have not been based on the "select mortality table theory." Instead, using multiple decrement tables we have calculated the net cost for

TABLE 1

ILLUSTRATION OF VARIATION IN NET SINGLE PREMIUM FOR CONVERSION OPTION WITH VARIATION IN % CONVERTING

According to Select Mortality Table Theory

Per $1,000 Insurance	$x=40,\ n=5$		
% Converting $=k$	Attained Age Conversion ${}_5\vert{}^aO_{[40]}$ (1)	Original Date Conversion ${}_5\vert{}^oO_{[40]}$ (2)	3/4 Attained Age and 1/4 Original Date Conversions .75(1)+.25(2) (3)
	Miller's Ordinary Select 1930–39 Mortality Table—2½% Interest		
90%	$ 3.18	$ .29	$ 2.46
50	3.18	1.43	2.74
10	3.18	2.57	3.03
1	3.18	2.83	3.09
0.3935	3.18	2.85	3.10
0.2500	2.02	1.81	1.97
	$G=.003935$		
	Elston's* Joint 15 Year Select and Joint 1939–44(15) Tables—2½% Interest		
90%	$11.38	$1.03	$ 8.79
50	11.38	5.17	9.83
10	11.38	9.31	10.86
1.8443	11.38	10.15	11.07
1.0000	6.17	5.50	6.00
0.3935	2.43	2.17	2.37
0.2500	1.54	1.38	1.50
	$G=.018443$		

* Mr. Elston has stated (*TASA* XLVIII, 264) that these tables do not constitute a select and ultimate mortality table "in the ordinary sense of the term." Nevertheless, it seemed reasonable to use them for the purpose of calculating these illustrative figures. The select mortality rates for quinquennial ages were obtained by interpolation.

extra mortality assumed on the basis of our past experience, such as that shown in Table 3.

On page 187, Mr. Fassel states that in one office, "satisfactory net charges are obtained by the formula: 20% of CSO 2% net premium plus $1.00 per $1,000 insured." It would be helpful to have more explanation as to how this formula was determined.

TABLE 2

CONVERSIONS OF PROVIDENT MUTUAL TERM INSURANCE AS A PERCENTAGE OF AMOUNT IN FORCE AT BEGINNING OF POLICY YEAR

Observed from 1936 to 1945 Anniversaries

Policy Year	Attained Age Conversions (1)	Original Date Conversions (2)	(1) As a Multiple of (2) (3)	Attained Age Conversions (4)	Original Date Conversions (5)	(4) As a Multiple of (5) (6)
	5 Year Term Policies Convertible for 5 Years			10 Year Term Policies Convertible for 7 Years		
1	10.9%	1.8%	6.1	6.0%	0.9%	6.7
2	17.3	1.3	13.3	11.0	0.8	13.8
3	15.2	1.5	10.1	12.7	0.6	21.2
4	14.5	1.4	10.4	9.1	0.8	11.4
5	46.3	8.6	5.4	9.6	1.1	8.7
6	....	....	....	11.2	2.1	5.3
7	....	....	....	45.7	4.4	10.4
	10 Year Double Protection Riders Convertible for 7 Years			Family Maintenance Riders Convertible for 7 Years*		
1	8.4%	1.4%	6.0	4.0%	0.6%	6.7
2	17.4	1.5	11.6	8.4	0.3	28.0
3	15.2	1.4	10.9	8.7	0.4	21.8
4	12.8	1.0	12.8	8.2	0.6	13.7
5	11.2	2.0	5.6	9.5	0.7	13.6
6	18.4	1.3	14.2	9.9	0.5	19.8
7	45.3	4.7	9.6	31.2	2.1	14.9

* 10, 15 and 20 year term periods combined.

On the next page Mr. Fassel also states that "Gross term premiums are found by the regular premium loading formula applied to the office net premium. The dividend formula also applies to term plans, the same as permanent plans. . . ." These statements presumably apply to the "one office" previously mentioned. However, they are probably not indicative of general practice; many, if not most, companies use different loading formulas for term premiums than for other premiums and different return of loading formulas for term dividends than for other dividends.

TABLE 3

COMPARISON OF MORTALITY ON ATTAINED AGE CONVERSIONS IN THE PROVIDENT MUTUAL WITH CORRESPONDING MORTALITY ON OTHER LIFE AND ENDOWMENT

Attained Age Conversions of Term Policies Issued in 1919–1944 Exposed between 1936 and 1945 Anniversaries. Expected Mortality Based on Select and Ultimate Mortality on Other Life and Endowment (Excluding Conversions) during the Same Period

Time of Attained Age Conversion	Actual Deaths		Ratio of Actual Deaths to Expected Deaths—by Amounts	
	Number	Amount in Thousands	Select from Original Term Date	Select from Conversion Date
	5 Year Term Policies			
More than 6 Months before End of Conversion Period.	536	$3,078	109%	117%
In Last 6 Months of Conversion Period.	304	1,606	130	152
Total	840	$4,684	115%	127%
	10 Year Term Policies			
More than 6 Months before End of Conversion Period.	264	$1,547	99%	112%
In Last 6 Months of Conversion Period.	89	390	100	134
Total	353	$1,937	99%	115%
	5 and 10 Year Term Policies Combined			
More than 6 Months before End of Conversion Period.	800	$4,625	105%	115%
In Last 6 Months of Conversion Period.	393	1,996	123	148
Total	1,193	$6,621	110%	123%

There is another sentence which might cause confusion in the minds of some students, because the statement seems too broad. This sentence, on page 178, reads "Premiums, values and dividends in America are almost invariably based on aggregate or ultimate tables, now generally the CSO Table." As regards premiums, this is true for participating ordinary insurance, but it is certainly not true for nonparticipating insurance. And as regards dividends, the implication is misleading. Even though the final calculation of the dividends is usually done by applying a two-factor or three-factor formula to functions based on an ultimate mortality table, probably most, if not all, companies actually take into consideration mortality on a select basis in determining the level of their dividends. I feel that select mortality tables are used much more in America than Mr. Fassel implies.

I am in complete agreement with Mr. Fassel's comment that original date conversions "should be limited to a brief period." Original date conversions are comparatively infrequent and it is very difficult to state a contractual basis for the cost of conversion which will be fair to both the insured and the company under all conditions. As a result of these disadvantages, the Provident Mutual has limited original date conversions in its current 15 and 20 year term policies to the first 7 years, although of course attained age conversions are available for considerably longer periods.

JAMES E. HOSKINS:

Mr. Fassel's assumption that original date converts have worse mortality than a cross section of those eligible to convert may perhaps not be accepted as axiomatic by all readers. Indeed it might be thought that only select lives would use the more expensive original date conversion privilege if attained age conversion were also available, as it customarily is. Even if some policyholders chose between the two conversion privileges without regard to their insurability, it might seem that at the worst the original date converts would have no worse mortality than a cross section of the eligible policyholders, so that the original date privilege should not affect the premium calculation as far as mortality is concerned.

The original date conversion privilege, however, gives a right to the rate scale and policy provisions prevailing at the original date, and this right may sometimes come to have very great value, although the author alludes to it only briefly. The potential great value of this right is the reason why the original date privilege is sometimes omitted from nonparticipating Term policies, or limited to a short period.

It may have surprised some readers to learn that under the conditions in Table B the conversion clause has the greatest aggregate cost if only

2.17% convert, and that if only attained age conversion is allowed or only attained age conversion exercised it is no more costly to have 20% or 50% of the eligibles convert than to have 2.17% convert. It might have been expected also that since conversion cost is caused by two factors, the proportion who convert and the degree of selection exercised by those who convert, it might be represented by a second degree curve rather than by two straight lines. These results arise from the assumptions about the selection exercised by those who convert and it is possible that those assumptions should be re-examined.

The formula given by Mr. Fassel is for the cost of conversions occurring at the end of n years. It is not clear to me how the theory would be extended to the practical situation where conversion is permitted at any time within a period of years rather than at a single point, or whether Mr. Fassel assumes that only conversions at the end of the conversion period enter into the cost.

No mention is made in the paper of offsets to the conversion cost from the fact that the conversion does not have to be underwritten and from the possibility that converted policies may be of higher average size than new issues on permanent plans.

Mr. Fassel makes no distinction in his theory between conversion privileges which run for the full period of the Term policy and those which expire much earlier. Some actuaries believe that the latter type, under which seriously impaired lives may be inclined to continue the Term insurance because of its low premium, may largely avoid anti-selection.

CHARLES F. B. RICHARDSON:

Mr. Fassel discusses two main problems in this paper; firstly, the cost of the conversion option and the manner of charging for it and, secondly, the amount to be charged for original date conversion.

The theoretical development appears to proceed upon the theory that, each year, all the lives which do not convert are select lives. That assumption seems to me to be invalid and is not supported by any actual mortality experience that I have seen on nonconverted term insurance. On the contrary, the mortality on term insurance is generally higher than that on regular policies.

We have attacked the problem by considering the following factors which affect the cost of the conversion option:

(1) The rate of conversion.

(2) Mortality after conversion as compared with that on regular business, on a select basis measuring the select period for the conversion date.

(3) Lapse rates both before and after conversion.

(4) Expense of issuing the converted policy, less the savings in underwriting and issue expenses as compared with a new policy.

Mr. Fassel ignores the last two items.

Mr. Fassel gives his company's experience in terms of the American Men Ultimate Table and the CSO Table. Since both these tables are ultimate tables and also grossly overstate mortality at the younger ages, they are not a satisfactory measure of the extra mortality.

We have measured our experience in terms of a *Select* Table, with the select period starting at the conversion date, based on a table derived from our experience on standard policies issued during the period in which the term policies were converted. This seems to us a realistic basis on which to measure the extra mortality, since term policies converted at the attained age receive the same dividends as new policies issued at the age at conversion.

The bulk of the experience on *automatic* conversions arose from 1–10 year term policies convertible automatically at the end of the term period, issued 1931–38, and on 1–3 year term policies similarly convertible issued 1938–47, most of the experience being on the latter group. The results were as follows, on policies converted from 1943–47, exposed from date of conversion to the anniversary in 1948:

AUTOMATIC TERM CONVERSIONS

(By Amount of Insurance)

Policy Year since Conversion	No. of Deaths	Ratio to Mutual Life 1942–5 Select Table	Probable Error
1–2	43	213%	±22%
3–5	42	114	±12
1–5	85	157	±11

On regular term policies, we investigated the experience on attained age conversions (original date conversions account for only 5% of the total converted). Nearly all this experience arises from 5, 10, 15 and 20 year term policies (mostly 5 and 10 year term) issued 1938–47, the conversion period expiring 3 years before the end of the term, except on 5 year term where the conversion period is 3 years. The mortality experience after conversion, on policies converted 1943–47, exposed to 1948 anniversaries, was as follows:

ATTAINED AGE CONVERSIONS

(By Amount of Insurance)

	No. of Deaths	Ratio to Mutual Life 1942–5 Select Table	Probable Error
Policy Year since Conversion:			
1	28	162%	±20%
2	33	194	±22
3	29	121	±15
4	10	57	±12
5	13	99	±18
1–5	113	137%	± 9%
Age at Date of Conversion:			
20–29	2	119%	±56%
30–39	7	102	±26
40–49	37	214	±23
50–59	41	113	±12
60 and Over	26	116	±15
Total	113	137%	± 9%

We next computed rates of conversion, which appeared to depend upon the number of years before the conversion period expires. The following table shows the graduated rates of conversion on regular (non-automatic) term policies:

Years before Conversion Privilege Expires	Rate of Conversion as % of Business in Force
1	28%
2	15
3	13
4	11
5	10
6	9
7	9
8	8.5
9	8
10	7.5

We computed the cost of the conversion option on regular term policies, converted at the attained age, on the following bases:

1. Mortality after conversion was assumed at the following rates, in terms of our own Select Mortality Table on recent issues: Year 1, 200%; year 2, 180%; year 3, 160%; year 4, 140%; years 5–10, 120%; years 11 on, standard.

2. Conversion to the ordinary life plan was assumed, and we used the same average policy ($4,200) as on new issues, although the average converted policy is for $8,000.

3. Conversion rates were assumed in accordance with the above table.

4. Lapse rates on the term insurance, in addition to the above conversion rates, were assumed at Linton's A rates, and on the converted term policy at Linton's A rates, both of these being close to actual experience.

5. Expenses on the converted policy were the difference between (a) our actual functional unit costs for new policies, and (b) the costs of carrying through the conversion operation, resulting in a net saving of approximately $2.50 per M.

The calculation was performed as follows:

(a) We computed the asset share at the end of 10 years on a regular ordinary life policy, using actual rates of mortality, lapse and expense.

(b) The asset share on a term policy converted to ordinary life was also computed from date of conversion to the end of 10 years after conversion, on the bases stated above.

(c) The difference between (a) and (b), discounted back ten years with allowance for mortality and lapse rates on the bases assumed equals the *single* premium cost of making each conversion, as of the conversion date. Then single premium costs worked out as follows:

Age at Conversion	Single Premium Conversion Cost
25	$ .50
30	1.40
35	2.90
40	5.00
45	8.20
50	13.40
55	21.90
60	35.10

We then solved for the level annual premium required on each persisting policy, but not longer than the period of conversion, to accumulate the single premiums indicated above, allowing for actual mortality, lapse and conversion rates, using a triple decrement table. These worked out as follows:

Plan	Convertible Within	Level Annual Charge at Issue Age		
		25	40	55
5 Year Term.	3 years	$0.13	$1.03	$4.50
10 Year Term.	7 years	0.13	0.84	3.60
20 Year Term.	15 years	0.16	0.80	. . . .

While the conversion periods in our policies are shorter than those used by Mr. Fassel, the cost of conversion which we found is very substantially less than his figures. For example, he shows a cost of $2.59 per M on 5 year term at age 40 as compared with $1.03 in the above table.

Coming now to the question of original date conversions, our current policy forms do not guarantee such changes, although we allow them by current practice. We do not guarantee the terms of original date changes in any of our policies. So far as the charge is concerned, we formulated our change rules on the basis of the difference in asset shares and expressed the result as closely as possible in terms of a simple formula for easy administration. In the first five years we charge the difference in gross premiums with $4\frac{3}{4}\%$ interest, ignoring dividends, and after the fifth year the charge is the difference in reserves with a loading grading down from 8% to 5% according to duration. In the first 5 years we pay the full difference in commissions and thereafter a percentage of the cost to make the change grading down from 5% to 2%. Original date conversions are only about 5% of the total conversions made.

The conversion option in term insurance represents a substantial item which is liable to be overlooked in computing premiums and dividends and Mr. Fassel has done a real service in drawing attention to it.

(AUTHOR'S REVIEW OF DISCUSSION)

ELGIN G. FASSEL:

The trouble taken by so many members in contributing to the very full discussion that the paper has received is appreciated. Also, its value has been much increased by the additional information which has been furnished for a number of offices.

At the time of preparing the paper I felt apologetic for the elaboration of the theoretical treatment which makes that portion of the paper hard reading, notwithstanding that as much as possible has been placed in an appendix. This was for the very reason to which one speaker refers, *viz.*, to avoid pitfalls in drawing conclusions from the theory of select tables, as exemplified in Mr. Little's illustration of the "two farms" as has been quoted, and in the solutions on three pages of the text book "Actuarial Theory" which were discarded as erroneous (see Recommendations of Educational Committee, Fourth Edition 1923, page 11).

Several speakers allude to a discontinuation of the original date privilege as perhaps the best answer to the difficulties it presents. It is quite possible that with an appropriate charge made for the privilege enjoyed, this option will fall away in favor of attained age conversion, which after all represents the natural form taken by an option as to insurability.

My reference to the basing of premiums, values and dividends in America on aggregate or ultimate tables was in the sense that the formulae, however arrived at, are generally applied to such tables in determining the numerical results. I believe there are some exceptions in the case of non-participating premiums.

It is certainly true, as pointed out by several speakers, that underwriting expense saved on the permanent policy is a credit in the conversion. This does not enter the theory in—as styled by one speaker—the "classical approach," but is to be taken into account in fixing the charge, in much the same way as net premiums are hardly ever the proper actual or gross premiums.

The formula in the paper is for conversion occurring at duration n, and n is tacitly assumed to be the period of the term policy, inasmuch as the lives failing to convert are considered as a group to have select mortality. The cost of the option will be smaller if n is less than the term period, because it is to be assumed that those failing to convert will include impending deaths that will become claims before the term policy expires. As for the charge to be made for exercising the option at any time up to duration n, I would consider that this should be the same as for the option at duration n; the annual premium might be subject to increase because those converting early withdraw from the group paying premiums to the end of the period.

One speaker suggests that, with two options available, the a lives selecting one option may perhaps show no worse than average mortality, and that the adverse mortality may perhaps all be among the b lives electing the other option, c being the remaining lives not choosing either option. His thought is that a be excused from a charge because not electing a valuable benefit. On the same principle, c should similarly be excused.

This would mean a sufficient charge to be collected retrospectively from b only, which I would not think to be practicable. It seems to me that the preferable view is to charge $a + b + c$ prospectively for the privilege of exercising an option and that the charge to all should be the same regardless of the choice eventually made.

In column (1) of Table B, all but the last two values are constant because the lives failing to convert are assumed as a group to have select mortality, the extra mortality of the total number of lives being concentrated in the group that do convert whether large or small.