

37TH ACTUARIAL RESEARCH CONFERENCE
UNIVERSITY OF WATERLOO
AUGUST 10, 2002

ANALYSIS OF THE DIVERGENCE CHARACTERISTICS OF ACTUARIAL SOLVENCY
RATIOS UNDER THE THREE OFFICIAL DETERMINISTIC PROJECTION
ASSUMPTION SETS FOR THE UNITED STATES SOCIAL SECURITY SYSTEM

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ABSTRACT

Annual reports of the U.S. Social Security Trustees present 75-year financial projections of the system under three different deterministic sets of economic and demographic assumptions or scenarios. The extent of the divergence between these scenarios provides useful information concerning the range of possible outcomes, but, in the absence of stochastic model projections, no probability measure can be assigned to the three scenarios, apart from being described as intermediate, low-cost and high-cost. Demographic research and stochastic model forecasts of U.S. population suggest plausible hypotheses for analysis of the divergence characteristics of the actuarial solvency ratios implied by the three deterministic scenarios. Available data from the 2002 Trustees' report are presented and various statistical measures are developed to aid in the assessment of plausible associated probability measures for the scenario projections.

Overview of U.S Social Security Financial Projections

The financial condition of the U.S. Social Security system, comprising the Federal Old-Age and Survivors Insurance (OASI) and Disability Insurance (DI) Trust Funds, is presented in the annual reports of the Board Trustees. Each year, the trustees report on the financial operations of the trust funds, assumptions about the future and projections of future financial status. The trustees' reports present the results of long-range actuarial estimates, extending up to 75 years, of the annual income rates, cost rates and balances for the OASI trust fund, the DI trust fund, and for the combined OASDI funds.

For the purpose of preparing the long-range actuarial estimates, the Social Security actuaries utilize demographic assumptions and methods relating to mortality, fertility and immigration to develop total population estimates. The Social Security actuaries also utilize economic assumptions and methods relating to productivity, inflation, average earnings, real-wage differentials, labor force, unemployment, gross domestic product and interest rates.

In the introduction to the annual reports, the Trustees state "Although, in general, a greater degree of certainty can be presumed for projections encompassing the next few years than for a period as long as the next 75 years, any estimation of future experience is uncertain. Therefore three alternative sets of demographic, economic, and program-specific assumptions are used to show a range of possible outcomes for all projections. The *Intermediate* set of assumptions reflects the Trustees' best estimates of future experience; the *Low-cost* is more optimistic, and the *High-cost* alternative more pessimistic for the trust funds' future financial outlook."

After projecting the system's income, expenditure and assets at various future points of time within the next 75 years, the Social Security actuaries present the projection results in terms of annual income rates, cost rates and balances. The annual income rate is the ratio of income from revenues, comprising payroll tax contributions and income from the taxation of benefits, to the OASDI taxable payroll for the year. The annual cost rate is the ratio of the cost, comprising outgo and expenditures for benefits, administrative expenses and other disbursements, of the program, to the taxable payroll for the year. In this particular context, the "balance" is simply the difference between the income rate and the cost rate for a specific year.

The next step in preparing the results of the 75-year projections is the development of summarized income rates, cost rates and balances. The summarized rates represent the projected annual figures on a present-value basis for various periods within the overall 75-year projection period. Results are presented for 25-year, 50-year and 75-year projection periods, representing cash-flow from income and costs without regard to the initial trust fund balance, any minimum target level for the trust fund assets or the adequacy of the trust fund to meet scheduled benefit payments.

The next step involves modifying the summarized income rates and cost rates to include the effect of the initial trust fund balance and to maintain a minimum target trust fund balance equal to one year's outgo for benefits and expenses at the end of the projection period. The difference between the summarized income rates and summarized cost rates with these trust fund adjustments is referred to as the "actuarial balance." This actuarial balance is a measure of the surplus or deficit of the system and is widely regarded as the principal quantitative measure of the adequacy of the financial viability of the system. The results of projections of the summarized income rates, cost rates and actuarial balances on this basis for 25, 50 and 75 years on each of the three official deterministic sets of assumptions are shown in the appended Exhibit I. The extent of the increasing divergence of the low-cost and high-cost projections from the intermediate projection as the projection period is extended from 25 years to 50 and 75 years is evident from these results.

The Concept of Actuarial Solvency

While the level of the actuarial balance reported by the Trustees is a well-established measure of the financial viability of the Social security system, it does not convey any indication of the extent of the solvency of the system, i.e. the degree to which asset-income is projected to be available to meet liability-outgo. The Trustees routinely report the calendar year in which the trust fund is projected to become "exhausted." In more conventional actuarial terms, this is the end year of the period for which the system is at least 100% solvent.

To arrive at a measure of conventional actuarial solvency, it is necessary to remove the present value of the minimum target fund balance of one year's outgo for benefits and expenses from the reported summarized cost rates. The Trustees do in fact report this exact adjustment to the summarized cost rates excluding the minimum target fund ending balance to arrive at a "summarized disbursement rate." It is then possible to derive a conventional measure of actuarial solvency by comparing the summarized income rates to the summarized disbursement rates over 25, 50 and 75 years. These actuarial solvency percentage ratios are shown in the appended Exhibit II. This exhibit shows the summarized present values of income and disbursement rates and the corresponding solvency ratio percentages at five-year intervals for projection periods from 25 to 75 years on each of the three deterministic projection bases. From this exhibit, it may be seen that the solvency ratio exceeds 100% for the full 75-year projection period on the low-cost basis. The solvency ratio exceeds 100% for more than 40 years on the intermediate basis and exceeds 100% for more than 25 years on the high-cost basis.

It should be noted that the solvency ratios, as computed, are based on income rates and disbursement rates that are related to the payroll tax rates and scheduled benefits under the present Social Security law. Obviously, scheduled benefits under present law may only be paid if there are sufficient assets in the trust fund from which to make the

full scheduled benefit payments. Also, the income rate includes a small element of income from the taxation of scheduled benefits that would be realized only if the scheduled benefits are in fact paid. If the trust fund assets were to ever become exhausted, the scheduled benefits under the present law would either be reduced or delayed or, the present law might be amended in some respect to modify payroll taxes and/or scheduled benefits. However, the concept of solvency, based on the present values of scheduled income and disbursement rates, is a valid measure of the capacity of the projected asset-income stream to meet the projected liability-outgo stream. This solvency measure is adopted as the basis for further analysis in this paper, particularly with regard to the nature of the divergence characteristics of the actuarial solvency ratios under each of the three official deterministic assumption sets or scenarios.

Interestingly, the word “solvency” does not seem to appear anywhere in the 208-page 2002 annual report of the Board of Trustees. The term “actuarial balance” used by the trustees is, perhaps, somewhat of a misnomer, since it does not measure the balance of asset-income and liability-outgo. Strictly speaking, “actuarial balance” as used by the trustees, is a modified measure of actuarial surplus or deficit for the system, subject to the constraint of maintaining an additional liability for the minimum trust fund balance equal to one year’s estimated outgo at the end of a projection period.

If the actuarial balance concept, as used in the trustees’ reports, were to be modified by removing the requirement for maintaining the minimum trust fund balances, and expressed as a percentage of the summarized income rates, the modified actuarial balance percentage would then become the complement of the actuarial solvency ratios presented in this paper. From an actuarial and statistical standpoint, the actuarial solvency percentage ratio is a preferable, more meaningful, readily comprehensible measure, and is less likely to be misinterpreted or misrepresented than the actuarial balance concept utilized in the trustees’ reports.

Divergence of Actuarial Solvency Ratios

The extent of the divergence between the low-cost and high-cost solvency ratios provides a useful measure of the range of plausible outcomes around the intermediate best estimate. This divergence in the solvency ratio percentages is 20.25% for 25 years, 25.12% for 50 years and 29.96% for 75 years, as shown in the appended Exhibit III.

The semi-range or one half of the divergence between the low-cost and high-cost solvency ratios is a practical measure of the plausible expected variation around the intermediate solvency ratio. At 25 years, the solvency ratio could be regarded as 114.41% plus or minus 10.13%; at 50 years as 95.11% plus or minus 12.56% and at 75 years as 88.80% plus or minus 14.98%, as shown in the appended Exhibit III.

To put the semi-range divergence percentage (SRD) into a standardized perspective relative to the intermediate solvency ratio percentage (ISR), it is useful to compute the $(SRD)/(ISR)$ percentage ratios; these are 8.85% at 25 years, 13.21% at 50 years and

16.87% at 75 years. By converting this sequence of ratios to a base of 100 at 25 years, it is possible to develop an index of the increasing divergence characteristics at successive points along the projection period; this index is 149 at 50 years and 191 at 75 years. This index is a quantitative measure of the phenomenon sometimes referred to as “an increasing funnel of doubt.” The reciprocal of this index might appropriately be described as an index of relative reliability or credibility. The values of this index of relative credibility (IRC) are 100 at 25 years, 67 at 50 years and 52 at 75 years. These various statistics are also presented in Exhibit III. A similar analysis of the divergence characteristics of the actuarial balance is presented in Exhibit I.

These simple descriptive statistics and indices are a helpful first step in the analysis of the divergence characteristics. The next step is to focus on whether the low-cost and high-cost projection results are symmetrically distributed around the intermediate projection results. For this purpose, the high divergence and low divergence are computed relative to the ISR at each point along the 75-year projection and their difference computed as the asymmetric delta measure (ADM). The ADM is 0.13 at 25 years, 0.26 at 50 years and 0.43 at 75 years. These ADM values indicate that the extent of the asymmetric characteristics of the projections of solvency ratios is quite small and, when expressed as a percentage of the ISR, the $(ADM)/(ISR)$ amounts to 0.11% at 25 years, 0.27% at 50 years and 0.48% at 75 years. At a level of less than one-half of one percent, the $(ADM)/(ISR)$ ratio indicates that the low-cost and high-cost solvency ratios are reasonably symmetrically distributed around the ISR. These statistical measures and indices are also presented in Exhibit III.

The extent of the divergence of the solvency ratios, between the three different deterministic sets of economic and demographic assumptions or scenarios underlying the projections, provides useful information concerning the range of plausible outcomes, but, in the absence of stochastic model projections, no probability measure can be assigned to the three scenarios. While it is feasible to generate parameterized stochastic models to represent the complex set of actuarial, demographic and economic elements necessary to develop the financial projections, it is of interest to note that other academic research has previously addressed the problem of relating stochastic model methodology to the results of the high, medium and low population forecasts for the United States. The paper by Ronald D. Lee and Shripad Tuljapurkar, entitled “Stochastic Population Forecasts for the United States: Beyond High, Medium and Low” presents a method for making stochastic population forecasts that provide consistent probability intervals. The authors blend mathematical demography and statistical time series methods to estimate stochastic models of fertility and mortality based on U.S. data back to 1900 and then use the theory of random matrix products to forecast various demographic measures and their associated probability intervals to the year 2065. Their projected total population sizes agree quite closely with the Census medium projections and their 95 percent probability intervals are close to the Census high and low scenarios.

Virtual Stochastic Model

The work of Lee and Tuljapurkar suggests that it may be feasible to develop a virtual stochastic model for the actuarial solvency ratios under the three official deterministic projection assumption sets, so that the intermediate projection results are approximately at the 50th percentile of a probability distribution and the high-cost and low-cost results are approximately represented by some percentile at the tails of the probability distribution, such as the 95th and 5th percentiles or the 99th and 1st percentiles.

This virtual stochastic model may be developed based on the assumption that such a model provides a reasonable fit to the three projection results, that the cross-sectional distributions for each projection period are symmetrical about the central intermediate values and that a Gaussian normal distribution represents the shape of the cross-sectional distributions. With this construction of a virtual stochastic model, it is possible to examine the implications of various plausible hypotheses as to where, on the cross-sectional probability distributions, the high-cost and low-cost solvency ratios might lie. The cross-sectional probability distributions will be uniquely defined by mean and variance parameters. The intermediate values along the time-period projection are assumed to be the means of the respective cross-sectional distributions and the variance or standard deviation may be derived from the semi-range divergence (SRD) based on the points on the cross-sectional distribution where the high-cost and low-cost values are assumed to lie.

The appended Exhibit IV shows the implied standard deviation of the solvency ratios for projection periods from 25 years to 75 years for a series of illustrative assumptions as to the semi-range divergence corresponding to various Gaussian probability points. For example, the cross-sectional standard deviations of the solvency ratios for 95/50/5 probability points are 6.16 at 25 years, 7.64 at 50 years and 9.11 at 75 years. Exhibit IV shows the full array of implied standard deviations for all projection periods from 25 years to 75 years with Gaussian probability points set at 75/25, 80/20, 85/15, 90/10, 95/5, 97.5/2.5, 99/1, 99.5/0.5, 99.75/0.25, 99.9/0.1 and 99.95/0.05. While there is no precise way of uniquely defining the exact probability points on the cross-sectional distributions for the high-cost and low-cost solvency ratios, it is possible to explore the implications of plausible hypotheses using the virtual stochastic model. The model is essentially based on three types of assumptions, namely, demographic, economic and statistical. In many respects, the process and judgment required to make the statistical assumptions for the virtual stochastic model are similar to the process and judgment required to make the demographic and economic assumptions. In this context, the actuarial assumptions for the virtual stochastic model embrace the integration of demographic, economic and statistical disciplines.

Exhibit V presents a concise summary of the stochastic model parameters and key statistical measures for the illustrative case where the high-cost/intermediate/low-cost projections are represented by 95/50/5 probability points on the cross-sectional distributions. This exhibit shows for each projection period from 25 to 75 years, the

mean, standard deviation, coefficient of variation (equal to the standard deviation divided by the mean) denoted as SDM, a standardized series of SDM indices based on an SDM of 100 for the 25-year projection period, the cross-sectional bandwidths corresponding to the interval between the 5th and 95th percentiles of the cross-sectional distributions and a series of standardized relative bandwidth (RBW) indices based on an RBW of 100 for the 25-year projection.

Similar sets of parameters and statistics may be readily developed for any other illustrative choice of plausible probability points for the high-cost and low-cost solvency ratios. With this parameterized virtual stochastic model, constructed from demographic, economic and statistical assumptions, it is possible to address many practical problems regarding the interpretation of the diverging results presented in the trustees' reports that have very significant policy and planning implications.

Practical Applications of the Virtual Stochastic Model

The stochastic model for projected solvency ratios has many practical applications. Some specific questions that may be evaluated and quantified with reference to the model are:

- What is the probable range of the actuarial solvency ratios for various projection periods?
- What is the probability that the solvency ratio will fall within a specified range at various future times?
- What is the probability that the solvency ratio will not fall below a specified limit?
- How relatively credible are the intermediate estimates for longer projection periods compared to shorter projection periods?
- What is the standard deviation of the best estimates for various projection periods?
- What is the probable error associated with the best estimates?
- What are the outer limits of credible estimate ranges at various probability levels?
- What is the inter-quartile range of the solvency ratio estimates?
- What are the amounts of the actuarial balance corresponding to any specific point, range or limits of the solvency ratios?
- How reasonable is it to focus exclusively on the 75-year intermediate best estimate of the actuarial balance for major policy decisions regarding the future financing and structure of the Social Security system?

The virtual stochastic model presented in this paper may be a practical means of addressing the recommendation of the Social Security Advisory Council that stochastic modeling should be used as a tool for recognizing explicitly the uncertainty surrounding the trustees' demographic and economic assumptions and so permit policy analyses to be conducted in a way that more realistically incorporates uncertainty into measures of long-term financial viability.

Acknowledgements and Dedication

I would like to acknowledge the philosophical wisdom of the John Ruskin quotation “The work of science is to substitute facts for appearances and demonstrations for impressions” that has served as an inspiration and guiding principle for this paper.

I also acknowledge the work of Ronald D. Lee and Shripad Tuljapurkar as providing the insight and conceptual framework for linking deterministic population projections and stochastic models.

I also acknowledge the experience, knowledge and collegiality of my fellow members of the Social Insurance Committee of the American Academy of Actuaries with whom I have worked on various related Social Security issues and whose professional objectivity has been a valued influence in my development of ideas for this paper.

Finally, this paper is dedicated to the mission of improving the understanding of Social Security financial projections among policymakers, government officials, researchers in policy think-tanks, and writers and commentators in the media. The quality of the work of the Social Security actuaries presented in the Board of Trustees’ annual reports is excellent and merits closer study than often is apparently given by those to whom this paper is dedicated. It is the sincere hope of the author that readers of the Social Security Board of Trustees’ annual reports will in future years reflect upon the concepts of solvency and the nature of the divergence in the Trustees’ official financial projections.

References

The 2002 Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds.

Ronald D. Lee and Shripad Tuljapurkar. Stochastic Population Forecasts for the United States: Beyond High, Medium, and Low. *Journal of the American Statistical Association*, December 1994, Volume 89, No. 428.

UNITED STATES SOCIAL SECURITY SYSTEM

SUMMARIZED INCOME RATES, COST RATES AND ACTUARIAL BALANCES
FOR 25-YEAR, 50-YEAR AND 75-YEAR PROJECTION PERIODS
AS A PERCENTAGE OF TAXABLE PAYROLL

DETERMINISTIC PROJECTION BASIS	PROJECTION PERIOD YEARS	INCOME RATE %	COST RATE %	ACTUARIAL BALANCE %	DIVERGENCE FROM INTERMEDIATE	DIVERGENCE FROM LOW TO HIGH	SEMI-RANGE DIVERGENCE LOW TO HIGH SRD	SRD AS % OF ACTUARIAL BALANCE
<u>LOW COST BASIS</u>	25	14.17	11.87	2.30	1.07			
	50	13.74	12.92	0.82	1.77			
	75	13.60	13.16	0.44	2.31			
<u>INTERMEDIATE BASIS</u>	25	14.21	12.98	1.23		2.38	1.19	97
	50	13.82	14.77	-0.95		4.03	2.01	212
	75	13.72	15.59	-1.87		5.44	2.72	145
<u>HIGH COST BASIS</u>	25	14.28	14.36	-0.08	-1.31			
	50	13.92	17.13	-3.21	-2.26			
	75	13.87	18.87	-5.00	-3.13			

SOURCE: TABLE IV.B8 OF THE 2002 ANNUAL REPORT OF THE BOARD OF TRUSTEES OF
THE FEDERAL OLD-AGE AND SURVIVORS INSURANCE AND DISABILITY TRUST FUNDS

UNITED STATES SOCIAL SECURITY SYSTEM

EXHIBIT II

SUMMARIZED INCOME RATES AND DISBURSEMENT RATES
AS A PERCENTAGE OF TAXABLE PAYROLL
AND SOLVENCY RATIO PERCENTAGES

PROJECTION PERIOD YEARS	LOW COST BASIS			INTERMEDIATE BASIS			HIGH COST BASIS		
	INCOME RATE	DISBURSEMENT RATE	SOLVENCY RATIO %	INCOME RATE	DISBURSEMENT RATE	SOLVENCY RATIO %	INCOME RATE	DISBURSEMENT RATE	SOLVENCY RATIO %
25	14.17	11.39	124.41	14.21	12.42	114.41	14.28	13.71	104.16
30	14.06	11.74	119.76	14.07	12.94	108.73	14.18	14.43	98.27
35	13.96	12.05	115.85	13.98	13.41	104.25	14.10	15.10	93.38
40	13.88	12.32	112.66	13.91	13.83	100.56	14.03	15.72	89.25
45	13.80	12.54	110.05	13.86	14.20	97.61	13.97	16.29	85.76
50	13.74	12.73	107.93	13.82	14.53	95.11	13.92	16.81	82.81
55	13.69	12.88	106.29	13.79	14.81	93.11	13.89	17.28	80.38
60	13.65	12.98	105.16	13.76	15.04	91.49	13.86	17.71	78.26
65	13.62	13.04	104.45	13.74	15.22	90.28	13.85	18.08	76.60
70	13.60	13.07	104.06	13.73	15.36	89.39	13.86	18.40	75.33
75	13.60	13.05	104.21	13.72	15.45	88.80	13.87	18.68	74.25

SOURCE: 25, 50 AND 75 YEAR INCOME AND DISBURSEMENT RATES FROM TABLE IV.B8 OF THE 2002 ANNUAL REPORT OF THE BOARD OF TRUSTEES OF THE FEDERAL OLD-AGE AND SURVIVORS INSURANCE AND DISABILITY INSURANCE TRUST FUNDS. INCOME AND DISBURSEMENT RATES FOR OTHER PROJECTION PERIODS ARE ESTIMATED, EXCEPT INCOME RATES ON INTERMEDIATE BASIS FOR OTHER PROJECTION PERIODS ARE FROM TABLE IV.B6 OF THE TRUSTEES' REPORT.

UNITED STATES SOCIAL SECURITY SYSTEM

EXHIBIT III

ANALYSIS OF SOLVENCY RATIO PERCENTAGES

PROJECTION PERIOD YEARS	INTERMEDIATE BASIS SOLVENCY RATIO % ISR	SOLVENCY RATIO % DIVERGENCE HIGH TO LOW	SEMI RANGE DIVERGENCE HIGH TO LOW SRD	HIGH TO INTERMEDIATE DIVERGENCE RANGE	LOW TO INTERMEDIATE DIVERGENCE RANGE	ASYMMETRIC DELTA MEASURE ADM	SRD/ISR RATIO %	SRD/ISR RATIO INDEX	INDEX OF RELATIVE CREDIBILITY IRC	ADM/ISR RATIO %
25	114.41	20.25	10.13	10.25	10.00	0.13	8.85	100	100	0.11
30	108.73	21.49	10.75	10.46	11.03	0.29	9.89	112	89	0.27
35	104.25	22.47	11.23	10.87	11.60	0.37	10.77	122	82	0.35
40	100.56	23.41	11.71	11.31	12.10	0.39	11.64	132	76	0.39
45	97.61	24.29	12.15	11.85	12.44	0.29	12.45	141	71	0.30
50	95.11	25.12	12.56	12.30	12.82	0.26	13.21	149	67	0.27
55	93.11	25.91	12.95	12.73	13.18	0.23	13.91	157	64	0.25
60	91.49	26.90	13.45	13.23	13.67	0.22	14.70	166	60	0.24
65	90.28	27.85	13.93	13.68	14.14	0.21	15.43	174	57	0.23
70	89.39	28.73	14.37	14.06	14.67	0.30	16.08	182	55	0.34
75	88.80	29.96	14.98	14.55	15.41	0.43	16.87	191	52	0.48

UNITED STATES SOCIAL SECURITY SYSTEM

EXHIBIT IV

PROJECTION PERIOD YEARS	SEMI RANGE DIVERGENCE HIGH TO LOW SRD	IMPLIED STANDARD DEVIATION OF SOLVENCY RATIOS										
		IF SEMI-RANGE DIVERGENCE (SRD) CORRESPONDS TO GAUSSIAN PROBABILITY POINTS										
		75/50 and 50/25	80/50 and 50/20	85/50 and 50/15	90/50 and 50/10	95/50 and 50/5	97.5/50 and 50/2.5	99/50 and 50/1	99.5/50 and 50/0.5	99.75/50 and 50/0.25	99.9/50 and 50/0.1	99.95/50 and 50/0.05
25	10.13	15.03	12.03	9.77	7.90	6.16	5.17	4.36	3.93	3.61	3.28	3.08
30	10.75	15.95	12.77	10.37	8.39	6.53	5.48	4.62	4.17	3.83	3.48	3.27
35	11.23	16.66	13.34	10.83	8.76	6.83	5.73	4.83	4.36	4.00	3.63	3.41
40	11.71	17.37	13.91	11.29	9.13	7.12	5.97	5.03	4.55	4.17	3.79	3.56
45	12.15	18.03	14.43	11.72	9.48	7.39	6.20	5.22	4.72	4.33	3.93	3.69
50	12.56	18.64	14.92	12.11	9.80	7.64	6.41	5.40	4.88	4.47	4.06	3.82
55	12.95	19.21	15.38	12.49	10.10	7.87	6.61	5.57	5.03	4.61	4.19	3.93
60	13.45	19.96	15.97	12.97	10.49	8.18	6.86	5.78	5.22	4.79	4.35	4.09
65	13.93	20.67	16.54	13.43	10.87	8.47	7.11	5.99	5.41	4.96	4.51	4.23
70	14.37	21.32	17.07	13.86	11.21	8.74	7.33	6.18	5.58	5.12	4.65	4.37
75	14.98	22.23	17.79	14.45	11.68	9.11	7.64	6.44	5.82	5.34	4.85	4.55

UNITED STATES SOCIAL SECURITY SYSTEM

VIRTUAL STOCHASTIC MODEL SOLVENCY RATIO PARAMETERS AND STATISTICS
BASED ON ASSUMED 95/50/5 GAUSSIAN PROBABILITY DISTRIBUTION POINTS

PROJECTION PERIOD YEARS	MEAN ISR	STANDARD DEVIATION	COEFFICIENT OF VARIATION SDM	RELATIVE SDM INDEX	CROSS SECTIONAL BANDWIDTH DIVERGENCE	RELATIVE BANDWIDTH INDEX RBW
25	114.41	6.16	0.0538	100	20.25	100
30	108.73	6.53	0.0601	112	21.49	106
35	104.25	6.83	0.0655	122	22.47	111
40	100.56	7.12	0.0708	132	23.41	116
45	97.61	7.39	0.0757	141	24.29	120
50	95.11	7.64	0.0803	149	25.12	124
55	93.11	7.87	0.0845	157	25.91	128
60	91.49	8.18	0.0894	166	26.90	133
65	90.28	8.47	0.0938	174	27.85	138
70	89.39	8.74	0.0978	182	28.73	142
75	88.80	9.11	0.1026	191	29.96	148