TRANSACTIONS OF SOCIETY OF ACTUARIES
1952 VOL.4 NO. 9

VOL. IV MEETING NO. 9

TRANSACTIONS

JUNE, 1952

SOME PRACTICAL ASPECTS OF THE CALCULATION OF EMPLOYER CONTRIBUTIONS UNDER GROUP ANNUITIES OF THE DEPOSIT ADMINISTRATION TYPE

RAYMOND F. HOUSEMAN

THE benefits provided under deposit administration plans are often complex and require a substantial amount of service from the life insurance company's actuarial staff for the preparation of cost estimates and in the determination of amounts to be paid to the insurance company each year to meet the accruing liabilities. The purpose of this paper is to outline some of the points to be considered in arriving at a method of making such calculations and their practical application.

The deposit administration group annuity plan here considered[1] is one under which payments to the plan are held in a deposit account (referred to as the active life fund) that accumulates at interest and from which are withdrawn amounts to purchase single premium annuities for employees as they retire. Provision may also be made for the withdrawal of funds on other occasions, such as for the purchase of deferred annuity credits upon the completion of vesting requirements, or to provide benefits in the event of death, disability or withdrawal.

The insurance company usually guarantees the purchase rates of the single premium annuities and the interest rate applied to the active life fund. A type of guarantee commonly used runs initially for a period of five years and from year to year thereafter. The guarantee applies to each dollar of contribution when it is paid. Therefore, all contributions made during the initial five years of a plan accumulate at interest and are used to purchase annuities in accordance with the original guarantee in the contract.

Thus, the insurance company assumes the full risk of interest, mortality and expense on retired lives and the interest risk on the active life fund within the limits of its guarantees. The employer assumes the risk of mor-

[1] A more complete description of the various deposit administration plans currently being offered appears in Mr. Blagden's discussion in *TSA* I, 256–264.

tality and withdrawals on active lives. There are various ways used to provide for the expenses of administration for active lives. Under some plans the loading in the annuity single premiums is used to cover all expenses, while under other plans a deduction is made from each employer's contribution as it is paid. Deposit administration plans are usually issued on a participating basis so that any excess interest or savings in mortality and expense are returned in the form of dividends.

When a deposit administration group annuity is issued under a noncontributory pension plan all amounts held in the active life fund are unassigned, that is, no amounts are allocated for the individual employee's benefits. This feature gives the deposit administration group annuity two advantages. First, it permits complete flexibility in the choice of a benefit formula and a funding method. Second, it is usually possible to administer a plan without maintaining a file of records on individual active employees, which is particularly valuable under plans having a high rate of employee terminations.

When a deposit administration group annuity is issued under a contributory pension plan, employee contributions are usually applied in one of two ways, (*a*) used to purchase paid-up deferred annuity credits, or (*b*) held in individual employee accounts to accumulate at interest. Where the latter procedure is adopted the insurance company may either maintain an account for each employee's contributions or it may maintain a single employer account as in a noncontributory case leaving to the employer the responsibility of maintaining individual employee records.

FUNDAMENTAL CONSIDERATIONS

The essentials of the problem of determining a basis for the practical valuation of benefits include the following considerations:

1. Many of the rates that may be used in making calculations, such as withdrawal rates, retirement rates, salary increase index numbers,[2] and under some circumstances interest and mortality rates, are unreliable and cannot be forecasted accurately.
2. Substantial chance fluctuations can be expected due to the small number of lives involved in the average pension case as compared to the number required from a statistical viewpoint for a stable group.

Since arithmetically correct calculations according to a given set of assumptions usually involve a large amount of clerical work, the problem arises of the extent to which approximations can be used to shorten the

[2] These various rates are described in "The Valuation of Self-Insured Retirement Plans" by Joseph C. Noback, *TSA* II, 50.

work that will be consistent with the degree of reliability of the underlying assumptions.

These considerations seem to indicate that rather broad approximations would be quite suitable, although there must be wide differences of opinion on the degree of approximation that would be acceptable. However, the use of approximations has its disadvantages as well as its advantages. Whenever an approximate method is used it is necessary to know at least the limits within which the results will differ from the true values. The determination of such limits requires the use of judgment or possibly a supplementary calculation. Because of the necessity of determining their reliability, approximate methods do not usually lend themselves to routine office procedures. Oftentimes, a more detailed method, if properly organized, will prove to be more efficient than a much shorter method if the latter requires special consideration for each application.

Waiting Period, Nearest Ages and Durations

The use of a waiting period which excludes from the funding employees whose service with the employer as of the date of valuation is less than the waiting period will serve to shorten the calculation work by reducing the number of employees to be considered. Neither withdrawal rates nor salary increase index numbers are generally used with a waiting period. The waiting period eliminates recently hired employees among whom terminations can be expected to be high. While there will be withdrawals among employees included in the funding, the overstatement resulting from using no withdrawal rates will tend to be offset by the understatement from excluding all employees who have not completed the waiting period and the understatement from using no salary increase index numbers.

Another type of approximation involves the use of age nearest birthday or nearest duration. The justification of this device is that where there is a uniform distribution the aggregate approximate values are equivalent to the exact values.

In order to discuss specific applications of these principles a general calculation procedure will be outlined and some of the more common types of benefit formulas and funding methods will be described.

GENERAL CALCULATION PROCEDURE

The data for making the calculation of a premium for a deposit administration group annuity plan are usually in the form of "census sheets" giving for each employee who is actively employed on the date of valuation such information as his name or an identification code number, date of birth, date of employment and earnings. In addition it is desirable to

have similar information on employees who have terminated employment in recent years to assist in arriving at a suitable waiting period or in choosing suitable withdrawal rates.

The census sheets are reviewed to determine whether they meet the insurance company's underwriting requirements and to decide upon the waiting period and suitable actuarial assumptions.

The data on the census sheets are adjusted to eliminate employees to be excluded from the funding and to make changes needed to facilitate the calculations such as replacing the date of birth by the office year of birth. The information as to each employee is then transferred to a card. These cards may then be sorted into desired groupings for the purpose of making additional card entries and to obtain totals necessary for the application of actuarial functions.

An outline of the tables of actuarial functions and their application to several types of pension calculations involving different benefit formulas and funding methods is set forth in an Appendix to this paper. It will be referred to from time to time in connection with the discussion of funding methods.

TYPES OF BENEFIT FORMULAS

The following are brief descriptions of several types of benefit formulas.

Flat Benefit

This formula provides the same monthly pension for all employees regardless of earnings or years of service—for example, a benefit of $40 per month at retirement.

Flat Benefit per Year of Service

This formula provides for a specified amount of monthly pension for each year of service, such as $1.50 per month for each year of service. A variation of this method is the common formula providing $4.00 per month for each year of service up to a maximum of $100 per month for 25 or more years of service.

Flat Percentage of Average Earnings

This formula provides for a monthly pension equal to a specified percentage of average earnings, such as 40% of average monthly earnings.

Flat Percentage of Final Earnings

This formula provides for a monthly pension equal to a specified percentage of final earnings, where final earnings are usually defined as the average earnings of the last few years, such as 5 or 10 years, prior to retirement.

Flat Percentage of Average Earnings for Each Year of Service

This formula provides a monthly pension equal to a specified percentage of average earnings multiplied by the number of years of service. Usually only full years of service are counted. Where service is measured in years and fractions of years the amount of pension may be obtained by applying the percentage to the employee's total earnings rather than multiplying his average earnings by his years of service.

Flat Percentage of Final Earnings for Each Year of Service

This is similar to the flat percentage of average earnings formula except that the specified percentage is applied to the final earnings.

Remarks on Benefit Formulas

Sometimes different percentages are applied to different portions of earnings such as 1% of the first $300 of monthly earnings and 1½% of monthly earnings in excess of $300.

Different percentages may be applied to earnings for service prior to the effective date of a pension plan than for service after that date.

Since the average earnings for service prior to the effective date would generally be less than the current earnings on the effective date due to the upward trend in earnings, a percentage of average earnings for service prior to the effective date may be approximated by using a lower percentage of current earnings in order to eliminate the necessity of going through past earnings records to determine average earnings.

FUNDING METHODS

Four funding methods will be considered. They are the entry age normal method, the unit credit method, the individual funding to normal retirement age method, and the aggregate method.

Entry Age Normal Method

Under this funding method an employee's pension is purchased by a level annual premium based upon his age at entry (usually his age when employment commenced or upon completing minimum service requirements). This premium will be called the normal premium both in reference to the premium for an individual employee and in reference to the aggregate premium for an entire group.

When a pension plan is first established the normal premium is based upon the same entry age as though the pension plan had been in force at least from the date of entry of the employee having the longest period of service. Therefore, since the normal premium is based upon employees' entry ages instead of their attained ages, there is a liability outstanding

at the effective date equal to the forborne annuity value of all prior due normal premiums. This liability will be referred to as the initial accrued liability. The corresponding liability representing the forborne annuity value of past due normal premiums as of any subsequent premium due date will be referred to as the accrued liability as of that due date. The accrued liability minus the amount of the active life fund will be referred to as the unfunded accrued liability.

Where the entry age normal method of funding is adopted the same premium calculation procedure may be used regardless of the benefit formula. The estimated amount of each employee's pension at retirement is first determined, and the subsequent calculations are based upon that amount. An illustration of an application of this method is given in the Appendix.

The entry age normal method is somewhat laborious to apply. The principal difficulty arises from the necessity of obtaining both the entry age and the attained age for each employee. Several approximations are available to shorten the work.

An approximation based upon the assumption that each employee enters at one of a limited number of average entry ages is apparently used to some extent. While this approximation should give reasonably accurate results it does not seem that it would result in much labor saving.

If there is a minimum age requirement for the inclusion of employees in the funding, such as age 30 for example, entry ages can be dispensed with and the normal premium obtained by a single multiplication of the level annual premium factor for the minimum age by the total pension benefits. The accrued liability involves only the attained age since the normal premium for the minimum age is used in all accrued liability factors.

Where this approximation is used in the case of an employee who actually entered at an age higher than the minimum age, a normal premium based upon the minimum age would be too low. However, if the accrued liability is calculated as the present value of benefits minus the present value of future normal premiums for the minimum age, the accrued liability is automatically increased by an amount exactly sufficient to offset the deficiency in future normal premiums. Errors resulting from the use of this approximation, therefore, change the relative amounts of the normal premium and the accrued liability. Calculating the accrued liability as the forborne annuity value of past due normal premiums from the minimum entry age would give the same result.

The data used in pension calculations are not very well suited to the use of this type of an approximation. The attained age distribution of employees varies widely from group to group, and the entry ages do not

usually follow any system nor do they show any evident relationship to attained ages. There are no inherent reasons why these age distributions should follow any general law since they depend in each case upon the pattern of growth of the individual company and the requirements of the work in which the employer is engaged.

Since the unfunded accrued liability consists of the accrued liability minus the amount in the active life fund, differences between actual and assumed experience are absorbed by the unfunded accrued liability. A variation of the entry age normal method known as the frozen initial liability method[3] provides for the differences between actual and assumed experience to be absorbed by the normal premium and spread over the future service of the employees in the plan. According to this method the first year normal premium and the initial accrued liability are determined in the same manner as in the entry age normal method. In subsequent years the unfunded accrued liability consists of the initial accrued liability less any payments made toward its amortization accumulated at the valuation rate of interest. The normal premium for subsequent years is determined by applying to the current annual earnings of the covered employees a ratio consisting of the present value of benefits minus the unfunded accrued liability and minus the amount in the active life fund divided by the present value of future earnings to retirement. As shown in the Appendix this variation involves no particular calculation difficulties after the first year since entry ages are not involved.

Presumably this method of spreading the premiums is intended for a benefit formula that depends upon earnings, and it seems that it would be more appropriate for other types of benefit formulas involving service but not earnings to use, as the denominator of the ratio, the present value of 1 per year per employee during his future working lifetime and apply the ratio to the number of employees. In other words, use an average rate per employee instead of an average rate per $1 of earnings.

The entry age normal method has the advantage that it is not necessary to maintain individual employee records for the purposes of premium calculation. Each year the premium is calculated independently from a set of census sheets of the then active employees.

Another advantage of the entry age normal method is that it allows wide latitude in the amount of each annual contribution towards a pension plan. The annual contribution will consist of the normal premium plus an amount to be applied to amortize the unfunded accrued liability. Under a

[3] In *TSA* I, 273, Mr. William S. Rae questions the appropriateness of the name "frozen initial liability" and points out that this method may be applied to methods of funding other than the entry age normal method.

deposit administration group annuity a further limitation is imposed by the necessity of maintaining a sufficient amount in the active life fund to purchase single premium annuities for currently retiring employees.

Unit Credit Method

Under this method a deferred single premium annuity credit is purchased or otherwise provided annually for each employee.

The methods of premium calculation fall in two general classes depending upon the benefit formula. The first method which is applicable to the flat benefit per year of service and the flat percentage of average earnings per year of service formulas consists of purchasing or otherwise providing for each employee the single premium value of the amount of pension credit accrued for that year. The second method applies to all benefit formulas and consists of estimating the total amount of each employee's pension at retirement and then determining the unit amount to be purchased each year by dividing the pension by the employee's total years of service at retirement. Of course, it would also be possible to purchase the pension by unequal amounts that vary according to some plan—for example, such as to result in a level annual premium.

The premium for the amount of benefit accruing during the year subsequent to the premium due date will be referred to as the current service premium.

The initial past service liability is defined as the single premium value as of the effective date of all pension benefits accrued on account of service prior to the effective date. Similarly, the past service liability as of any subsequent premium due date is the single premium value of the benefits resulting from service prior to that date.

Where the first method of premium calculation is used in connection with the flat benefit per year of service formula the amount of an employee's past service benefit would be the flat benefit multiplied by the number of years of past service. The information needed to determine this could be obtained from the census sheets and there would be no need to maintain individual employee records. Even if there were a subsequent revision of the benefit formula this would not cause a great deal of difficulty since the increment in the flat benefit rate from the date of revision could be added to the past service calculated at the old rate. However, if the flat percentage of average earnings per year of service formula were used, the earnings of each employee could change each year, so it seems that it would be necessary to maintain individual employee records of past earnings or accrued benefits. Where benefits are not actually purchased from the active life fund until an employee retires, it may be

practical to have the employer furnish census sheets containing a record of past earnings of each employee in addition to the other data. It is also possible to apply the second method to these benefit formulas using estimates of past and future earnings based upon current earnings.

Calculations under the second method are rather straightforward and can best be described by following through the steps shown in the example in the Appendix.

Deviations of actual experience from that assumed would be absorbed by the unfunded past service liability, which is the past service liability minus the amount in the active life fund. However, in the event of a blanket increase in benefits due to a general salary increase or otherwise, the increase in an employee's benefits would increase the current service premium under the first method where the additional benefit would be purchased each year as it accrues. Under the second method where the amount to be purchased each year consists of the estimated pension at retirement divided by the number of years of service at retirement, the increase would be spread uniformly over his entire period of service, thus increasing both his past service liability and his current service premium in the same proportion.

Individual Funding to Normal Retirement Age Method

Under this method pensions are provided by use of a level annual premium calculated individually for each employee. The premium for each employee's pension benefits is determined on the basis of his attained age as of the effective date of the plan or later date of entry into the plan. Where the amount of pension depends upon earnings it is assumed that the employee's earnings at entry will remain at the same annual rate to retirement. In the event that his pension is increased at a later date an additional annual premium for the amount of the increase based upon his then attained age is determined.

This type of funding is most applicable for pension benefits issued in connection with group level premium life insurance or a pension trust. Since it is necessary to maintain an individual record for each employee for the life insurance, this same record could also be expanded to include the pension benefit data. An original card would probably be maintained for the original amount of pension and a supplementary card would be added for each increase in benefit in order to permit sorting the cards by the issue age of the different amounts of pension.

The first year premium is merely the sum of the premiums for the individual employees. For subsequent years a method must be used that corrects for the differences between actual and assumed experience. This

could be accomplished by determining such subsequent premiums as the gross level premium initial reserve (gross single premium value of benefits minus the present value of future premiums not including the premium currently due) for the benefits minus the amount in the active life fund, *i.e.*, the amount that should be in the active life fund minus the amount actually in it. This assumes no deductions for expenses are made at the time the premium is paid. Where such deductions are made,the formula would require a modification such as subtracting the past deductions accumulated at the valuation rate of interest.

The Aggregate Method

Under this method of funding, the premium for a year is determined by applying a ratio known as the aggregate ratio to the annual earnings of the covered employees. The aggregate ratio for the first year consists of the present value of the pension benefits divided by the present value of future annual earnings. For subsequent years the numerator of this ratio consists of the present value of pension benefits minus the amount in the active life fund, and the denominator consists of the present value of future annual earnings.

Like the frozen initial liability method it seems that this definition of the aggregate method was intended to apply to a benefit formula that depends upon earnings. Where the benefit formula involves service but not earnings it would seem more suitable to use the present value of 1 per year per employee during his working lifetime as the denominator of the aggregate ratio and apply the ratio to the number of employees rather than to covered earnings.

Of all the funding methods the aggregate method is probably the most convenient to use to calculate a premium for a deposit administration group annuity. The method automatically corrects for deviations between actual and assumed experience. It does not require the maintenance of records for individual employees. Each year's premium can be determined by a relatively simple calculation using the then current year's census sheets.

One characteristic of the aggregate method is that the introduction of withdrawal rates into the assumptions may produce a larger premium than where such rates are not included. This will occur if the effect of the withdrawal rates reduces the denominator of the aggregate ratio more than it reduces the numerator.

The past service liability is not defined under the aggregate method, and when it is necessary to furnish a figure for this item it is often calculated by the entry age normal method.

APPENDIX

This Appendix outlines an office method of calculation for the various methods of funding and benefit formulas considered in the paper.

The following symbols refer to the prepared tables of factors used in the calculations.

$12\ddot{a}_y'^{(12)}$ is the gross annuity single premium for retirement age y to provide \$1 per month pension based upon interest, mortality, and expense assumptions used for retired lives.

$\theta_{x:\overline{y-x}|} = 12\ddot{a}_y'^{(12)} \cdot D_y/D_x$ is the present value at age x of \$1 monthly income commencing at age y where D_y/D_x is based upon interest, mortality and withdrawal (if used) rates assumed for active lives and $12\ddot{a}_y'^{(12)}$ is based upon the interest, mortality, and expense assumptions used for retired lives.

$\pi_{x:\overline{y-x}|} = 12\ddot{a}_y'^{(12)} \cdot D_y/(N_x - N_y)$ is the level annual premium at age x to provide \$1 monthly income commencing at age y where $D_y/(N_x - N_y)$ is based upon interest, mortality, and withdrawal (if used) rates assumed for active lives, and $12\ddot{a}_y'^{(12)}$ is based upon the interest, mortality and expense assumptions used for retired lives.

$\ddot{a}_{x:\overline{y-x}|}$ is the net single premium for an annuity due based upon interest, mortality and withdrawal (if used) rates assumed for active lives.

In addition it is necessary to have factors to provide for the death benefit under modified cash refund annuities used in contributory plans that provide for the return of an employee's contributions, usually with interest, in the event of death prior to retirement and provide for the return of any excess of the accumulated amount of his contributions at retirement, less the amount of annuity payments received, upon death after retirement. Determining these factors in the form of a premium per \$1 of employee contribution is convenient since it does not involve the ratio of employee contributions to benefits.

For convenience the term "contract anniversary" is used to refer to the day and month of issue of the contract in calendar years prior to the year of issue of the contract as well as the calendar years subsequent to the year of issue of the contract.

The determination of ages, durations, and dates of retirement often proves troublesome. For example, where the office year of birth and office year of employment are determined so as to produce the age nearest birthday and years of past service in completed years respectively when subtracted from the calendar year of valuation, it is necessary to determine whether or not a satisfactory value for the years of service at retire-

ment will be obtained by using the office year of birth plus the age at retirement and minus the office year of employment. The resulting figure would represent the number of years from the contract anniversary in the office year of employment to the contract anniversary nearest the employee's retirement age. This would be correct only where complete years of service are counted and where retirements take place on such contract anniversary, otherwise the results would be approximate.

The principle generally followed in these illustrations is that the exact age or duration is used where it would affect the amount of pension or determine the inclusion or exclusion of employees. An example of the first instance is where the amount of pension is a function of the years of service. An example of the second is where the exact age rather than the nearest age determines whether or not an employee is included in the funding, such as the inclusion of all employees who have passed their 30th birthdays.

Where an approximation would affect only the duration of a discount factor, approximations are usually used that will average out to the exact value on the basis of a uniform distribution.

Entry Age Normal Method

The calculation procedure follows the same general pattern regardless of the benefit formula. It consists of determining each employee's estimated pension at retirement and his normal premium. The normal premium for the entire group is the sum of the normal premiums for the individual employees. The accrued liability is determined as the present value of benefits minus the present value of future normal premiums. The present value of benefits is obtained by applying $\theta_{x:\overline{y-x}|}$ factors to the monthly amounts of pension tabulated by attained age groups, and the present value of future normal premiums is obtained by applying $\ddot{a}_{x:\overline{y-x}|}$ factors to the normal premiums also tabulated by attained age groups.

The items to enter on the individual cards will vary under different benefit formulas by the entries required to determine the estimated pension benefit at retirement. The card entries for a benefit formula based upon a percentage of earnings for each year of service would be as follows:

CARD ENTRIES

Item No.

1. Identification Code
2. Sex
3. Calendar Year of Birth

4. Office Year of Birth (year during which the contract anniversary nearest the date of birth occurred)
5. Calendar Year of Entry
   a) Also enter "b" if the date of entry precedes the employee's birthday in the calendar year of entry
   b) Also enter "a" if the date of entry precedes the contract anniversary in the calendar year of entry
6. Earnings
7. Entry Age—Age nearest birthday on contract anniversary next following the date of entry (where the card contains "a" this is Item 5 minus Item 4; where the card does not contain "a" this is 1 plus Item 5 minus Item 4)
8. Years of Service at Retirement in completed years from the date of entry to the employee's birthday in the calendar year of retirement (where the card contains "b" use Item 3 plus retirement age, minus Item 5; where the card does not contain "b" use Item 3 plus retirement age, minus Item 5 minus 1)
9. Estimated Monthly Pension (benefit percentage $\times$ Item 6 $\times$ Item 8—use $\frac{1}{12}$ of the benefit percentage where Item 6 represents annual earnings)
10. Normal Premium (Item 9 $\times \pi_{x:\overline{y-x}|}$ where the entry age is determined from Item 7)

Sort the cards by sex, attained age (office year of birth) and retirement age (which can usually be done without entering the retirement age on each card) and tabulate the amount of normal premium and the amount of estimated monthly pension for each attained age group. The normal premium is the sum of the normal premiums for all attained age groups. The present value of benefits is obtained by applying the appropriate $\theta_{x:\overline{y-x}|}$ factor to the estimated monthly pension for each attained age group and adding the products. The present value of future normal premiums is obtained by applying the appropriate $\ddot{a}_{x:\overline{y-x}|}$ factor to the normal premium for each attained age group and adding the products. The accrued liability is the difference between the two totals, *i.e.*, the present value of benefits minus the present value of future normal premiums.

The amount to be entered in Item 6 will depend upon the benefit formula. If the pension is based on average earnings, the employee's current earnings might be considered to be a suitable approximation. This may tend to overstate the average earnings for the older employees and understate the average earnings for the younger employees due to the upward trend in earnings. Where a final earnings formula is used, this item would probably be the current earnings multiplied by a salary increase index factor.

Similar principles are followed for the preparation of cards where other types of benefit formulas are involved.

Under the frozen initial liability method the card entries referring to

entry ages and individual normal premiums are not needed after the first year which substantially shortens the process. Earnings and estimated monthly pension benefits would be tabulated by attained age groups. The present value of future earnings is obtained by applying the appropriate $\ddot{a}_{x:\overline{y-x}|}$ factor to the earnings for each attained age group and adding the products. The normal premium is equal to the present value of benefits minus the unfunded accrued liability and minus the amount in the active life fund divided by the present value of future earnings.

Unit Credit Method

Flat percentage of average earnings per year of service calculated according to the first method referred to in the description of this funding method.

The general procedure is to calculate the premium as though the benefit were 1% of earnings for each year of service and then multiply the resulting current service premium and past service liability by the actual benefit percentages.

CARD ENTRIES

Item No.

1. Identification Code
2. Sex
3. Office Year of Birth (year during which the contract anniversary nearest the employee's date of birth occurred)
4. Office Year of Entry (year during which the contract anniversary on or next subsequent to the employee's date of entry occurred)
5. Earnings
6. Years of Past Service in completed years (calendar year of valuation minus Item 4)
7. Past Service Benefit on a 1% basis (.01×Item 5×Item 6)

Sort cards by Sex, Attained Age (Office Year of Birth) and Retirement Age, and tabulate the totals of Item 5 and Item 7 for each group.

The current service premium is obtained by applying $\theta_{x:\overline{y-x}|}$ factors to 1% of the totals of Item 5 for each group, adding the resulting products and applying the benefit percentage to the total.

The past service liability is obtained by applying $\theta_{x:\overline{y-x}|}$ factors to the totals of Item 7 for each group, adding the resulting products and applying the benefit percentage to the total.

Unit Credit Method

Flat benefit formula calculated according to the second method mentioned in the description of this funding method.

CARD ENTRIES

Item No.

1. Identification Code
2. Sex
3. Office Year of Birth (year during which the contract anniversary nearest the employee's date of birth occurred)
4. Office Year of Entry (year during which the contract anniversary on or next subsequent to the employee's date of entry occurred)
5. Years of Past Service in Completed Years (calendar year of valuation minus Item 4)
6. Years of Total Service at Retirement in completed years from the employee's date of entry to the contract anniversary in the year nearest the employee's retirement age (Item 3 plus retirement age, minus Item 4)
7. Unit Benefit (flat benefit divided by Item 6)—the flat benefit replaces the "estimated pension at retirement."
8. Past Service Benefit (Item 5 multiplied by Item 7)

Sort cards by Sex, Attained Age (Office Year of Birth) and Retirement Age.

The premium for the current year's unit benefit is obtained by multiplying the totals of Item 7 for each group by the appropriate value of $\theta_{x:\overline{y-x}|}$ and adding the resulting products.

The lump sum past service liability is similarly obtained by multiplying the total of Item 8 for each group by the appropriate value of $\theta_{x:\overline{y-x}|}$ and adding the resulting products.

This list of illustrations could be extended indefinitely. However, it seems that a sufficient number of examples have been given to illustrate the principles that could be applied to other benefit formulas and to the Individual Funding to Normal Retirement Age Method and the Aggregate Method.