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GROUP ANNUITY MORTALITY

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Strange that a harp of thousand strings
Should keep in tune so long!

ISAAC WATTS, *Hymns and Spiritual Songs*

I. INTRODUCTION

FOLLOWING along the trail blazed by Messrs. Jenkins and Lew,[1] the purpose of this paper is to present actuarial tools, based upon group annuity mortality experience, with which the actuary concerned with pension problems may shape or test the mortality basis which, in his judgment, is appropriate to such problems. It is evident that such aids are needed. Jenkins and Lew did not attempt to deal exhaustively with group annuity or pension problems. As indicated by Table 27[2] of their paper, group annuity retired life mortality experience has definitely different characteristics from the individual annuity experience on which the Jenkins and Lew studies for the older ages were based.

The Combined Annuity Mortality Table[3] (originally called the Group Annuity Mortality Table) and the 1937 Standard Annuity Table were both constructed with an eye to group annuity requirements. In both cases, the then most recent group life clerical experience[4] was used for the younger ages (generally below 60) and individual annuity experience for the older ages. Although the use of individual annuity experience for the older ages did provide margins in that area, at the younger ages no basic safety margin was introduced and no provision was made for mortality improvement (even though Mr. Kineke[5] acknowledged the "recent improvement in mortality at the younger ages"). The only margin at the younger ages was that which was implicitly provided by the inclusion of female lives in the group life experience and that which may have been provided by the use of clerical experience. Due to continued mortality improvement at the younger ages, both of these tables, so far as mortality at such ages is concerned, were out of date shortly after coming into gen-

[1] *TSA* I, 369.

[2] *TSA* I, 438–441.

[3] *TASA* XXIX, 118–124.

[4] Cammack's Clerical Mortality Table (1923–26 group life clerical experience) for the Combined Annuity Table, and the 1932–36 group life clerical experience for the 1937 Standard Annuity Table.

[5] *TASA* XXXIX, 8.

eral use. In the light of the present day consciousness of the necessity of providing for mortality improvement at most ages, it was rather startling to the writer, as a "Monday morning quarterback," to reread the account of the construction of these tables.

The mortality table which Mr. Blagden[6] presented recently for group annuity purposes uses the mortality rates at ages under 60 from the Annuity Table for 1949 presented by Messrs. Jenkins and Lew. As the latter stated, this table "is intended to be a conservative representation of current mortality under the kinds of annuities comprising the basic data of the table. The conservatism involved in this table was not designed to cover probable future decreases in mortality rates and is insufficient to do so for most types of annuity." It will be recalled that the basic experience at ages under 55 is the "predominantly clerical" group annuity experience. Therefore, in using the Prudential 1950 Group Annuity Valuation Table the only provision for mortality improvement at ages under 55 is that available in the basic safety margin and in whatever margin is introduced by the use of "predominantly clerical" experience for nonclerical employee categories. At the older ages, this table does have margins over current experience as the mortality rates are roughly equivalent to the 1937 Standard Annuity Table with ages set back one year. Mr. Blagden[7] has acknowledged that the use of this table "is to some extent a stop-gap measure."

The actuarial tools to be presented in this paper consist of (1) an unprojected mortality table representing conservatively the current level of group annuity mortality experience, and (2) a study of two alternative scales of mortality improvement rates associated with the unprojected table.

The unprojected table would, *for mortality rates continuing indefinitely at the present level*, provide an adequate mortality basis for premiums and reserves for deferred annuities beginning at a fixed retirement age (and for immediate annuities provided at the inception of a plan or as supplements at retirement) with respect to a typical group of employed persons where all continue in the group until death including those becoming disabled after entering employment. With static mortality rates, the basic safety margin in the table would suffice to cover a reasonable range of variation in the inherent level of mortality of different groups.

In practice, the actuary must use his judgment as to the appropriateness of the table where different kinds of selective influences or situations may alter the mortality experience—for example:

[6] *TSA* II, 322.

[7] *TSA* II, 330.

i) Selection of annuitants at retirement by the employer.

ii) Options available to the employee to elect freely at retirement a cash settlement of the entire annuity reserve or annuity forms involving the conversion of a substantial part of the annuity on the life of the employee to some form of death benefit.

(In the case of the foregoing situations, the actuary should be prepared for mortality experience similar to that under individual annuities or life income settlement options elected by beneficiaries.)

iii) Annuities purchased at a fixed retirement age excluding those retired for disability prior to such age.

iv) Annuities purchased at retirement under a plan with a variable retirement age, disabled lives being covered at normal retirement age. Here the better lives may be expected to continue working to later retirement ages.

v) A situation such as (iv) but with disabled lives not covered by annuity purchase.

The actuary may also occasionally encounter a group which he believes will have an inherently lower or higher level of mortality than that provided by the range covered by this table.

In general, the writer hopes that this paper will be of value in helping the actuary select a mortality basis for group annuities and pensions which is entirely self-sufficient, thus making it unnecessary to rely upon an abnormally low interest rate assumption or excessive contingency loading in premium rates to offset probable deficiencies in mortality assumptions. If this is achieved, reserves will automatically include desired margins and more accurate reserves will result, particularly for those deferred annuity forms which involve a life contingency only after retirement age. Attention is particularly directed to the comparative reserve studies in Section III. These studies will enable the actuary to consider whether the continued use of the 1937 Standard Annuity Table is appropriate for group annuity purposes, to determine whether an interest differential is a satisfactory means of allowing for mortality improvement, and to examine the adequacy in the future of individual and aggregate reserves computed by static mortality tables.

II. GROUP ANNUITY TABLE FOR 1951

Description of Construction

The construction of the Group Annuity Table for 1951 (G*a*–1951 Table) is described in detail in the Appendix, Section A, and may be briefly described as follows:

a) The intercompany group annuity matured life experience for the years 1946–1950[8] with respect to retirements on or after normal retirement

[8] *TSA* 1951 Reports of Mortality and Morbidity Experience, 109.

date for each sex by lives was used to derive mortality rates at ages 65 and above with an adjustment to allow for mortality after age 65 with respect to retirements prior to normal retirement date.

b) The series of rates for each sex was graduated by a Whittaker-Henderson Type B formula which minimizes a function such that perfect smoothness would be represented by first differences in a geometric series.

c) The resulting mortality rates were adjusted to allow for three years' decrease in mortality according to Jenkins and Lew Projection Scale B and at the same time a margin for annuity purposes was introduced by reducing the mortality rates for males 10% and for females 12½%.

d) These rates, which included age 102, were extended by arbitrary means so as to reach a value of 1.000 at age 110 for each sex.

e) To complete the table for ages under 65, the resulting graduated mortality rates for ages 65 to 110, inclusive, were joined by a 4th order curve to the *a*-1949 Table rates projected one year by Scale B.

f) The entire range of mortality rates so constructed is essentially a 1951 table. The mortality rates below age 56 are identical with the mortality rates of the *a*-1949 Table projected one year.

Comments Regarding Construction

Characteristics of a Table to Be Used for Deferred Annuities

It is important to note that in the intercompany matured life experience for lives retiring on or after normal retirement date, the lives are under observation from normal retirement date whether or not they actually retire. For a table that is to be used for deferred annuities of the usual form under group annuity contracts, it is vital that experience be taken in this manner. The intercompany experience does not include the mortality experience with respect to retirements under certain deposit administration contracts where, with a variable retirement age, the lives do not come under observation until actual retirement. For a table that is to be used for deferred annuities, it is also appropriate that some recognition be given to the mortality experience after normal retirement date with respect to lives retiring before normal date. However, care must be taken in interpreting the experience on early retirements. The early retirement intercompany experience showed high mortality rates persisting several years beyond 65. Although 65 is the common normal retirement age, some plans have a normal retirement age higher than 65, usually 70. Where this is the normal retirement age, the "prior to normal" mortality experience between 65 and 70 will be influenced by the higher mortality rates among these early retirements who will generally be impaired lives and these will

be the only lives that get into the experience. Furthermore, there is some evidence, in the administration of group annuity contracts, that healthy lives are removed from the experience upon termination of employment within the range of early optional retirement ages (the annuity being canceled with a return to the employer) but that the impaired lives remain in the experience by the payment of annuities (the employer being entitled to no return). It was felt that these selective influences would have been pretty well minimized by age 70. Therefore, it was decided to include all

TABLE 1

GRADUATED MORTALITY RATES FOR THE INTERCOMPANY 1946–1950 GROUP ANNUITY MATURED LIFE EXPERIENCE

AGE	1,000 q_x		AGE	1,000 q_x	
	Males	Females		Males	Females
65	28.047	16.064	80	112.432	86.022
66	31.205	17.694	81	122.430	93.964
67	34.524	19.407	82	132.674	102.221
68	37.784	21.441	83	143.094	110.809
69	41.134	23.959	84	153.682	119.822
70	44.939	27.165	85	164.399	129.343
71	49.315	31.159	86	175.312	139.458
72	54.152	35.746	87	186.527	150.258
73	59.336	40.754	88	198.135	161.843
74	64.946	46.128	89	210.225	174.319
75	70.948	51.795	90	222.882	187.807
76	77.558	57.790	91	236.172	202.450
77	85.129	64.225	92	250.179	218.399
78	93.548	71.093	93	265.027	235.818
79	102.737	78.382	94	280.850	254.890
			95	297.806	275.812

the experience after age 70 with respect to lives retiring prior to normal retirement date. Also, after determining the over-all average change in the mortality rates for "on or after normal" retirements at ages 70 and above by including the "early" retirement experience on and after 70, the "on and after normal" rates from 65 to 70 were modified by the same average percentage change.

Comparison of Group Annuity Experience with Other Mortality Experience at Ages 65 and Over

Before deciding on the final form of the Group Annuity Table for 1951, the mortality experience at the older ages from several other available

areas of experience was compared with the graduation of the intercompany 1946–50 group annuity experience (without margin). For convenience of reference, these graduated rates are set down in Table 1.

In studying the comparisons that follow, it should be remembered that the group annuity experience with which comparison is made has been adjusted so that the inclusion of both healthy and impaired lives is contemplated, *i.e.*, no selective influences are assumed to affect the experience either by removal of lives disabled at ages under 60 or 65 or by exclusion after 65 of those continuing actively at work.

Group Life Insurance. A comparison of the group annuity experience with group life insurance experience is made difficult for two reasons, (*a*)

TABLE 2

RATIO OF 1946–50 GROUP LIFE MORTALITY* TO 1946–50 GROUP ANNUITY MORTALITY

Age Group	Clerical (25% Females Assumed)			Nonrated (10% Females Assumed)		
	Premium Waiver	Extended Death Benefit	Instalment Disability	Premium Waiver	Extended Death Benefit	Instalment Disability
66–70	.939	1.124	1.012	.841	.943	.998
71–75	.920	.938	.973	.801	.908	.979
76–80	.833	.887	.942	.758	.914	.989
81–85	.671	.984	.993	.723	.977	.996
86–90	1.417	1.380	1.148	1.154	1.238	1.103
91–95	.....	2.069	.632	.814	1.239	1.278

* *TSA* 1951 Reports of Mortality and Morbidity Experience, 72 (clerical data furnished by Committee). Deaths and exposures for central age pivotal points computed by King's formula.

the lack of knowledge of the precise proportion of females included in the group life experience, and (*b*) no definite knowledge of the varying effect upon the exposures and deaths in the reported experience at the older ages of the three different types of disability clauses usually operative prior to age 60. After examination of a number of clerical groups, it appeared that it was reasonable to assume 25% females for the clerical category. For the larger aggregate of group life insurance, the so-called nonrated, it was assumed there were 10% females. Ten percent was the approximate proportion of females in the group annuity experience, which comprises a cross section of various types of groups and may not be materially different in this respect from the nonrated group life.

A comparison of the 1946–50 group life mortality with the 1946–50 group annuity mortality appears in Table 2. Upon comparing the clerical

with the nonrated and noting the variation by disability clause, it appears that the type of disability clause effective prior to age 60 has a much greater influence on the group life insurance mortality pattern at the older ages than does occupation. After making reasonable allowance for the proportion of females, these figures indicate that the generally accepted lighter mortality experience of clerical groups is spurious and that the lighter experience observed with males and females combined is due principally to the large proportion of females. The disability clauses effective prior to age 60 all have a selective influence on the experience at the older ages by removing impaired lives, although the effect of the extended death benefit clause is quite limited. In the published experience, instalment and waiver disability claims are treated as terminations upon disability and

TABLE 3

RATIO OF 1945–50 CIVIL SERVICE MORTALITY TO 1946–50 GROUP ANNUITY MORTALITY

Age Group	Males	Females
66– 70	1.072	1.229
71– 75	.961	.864
76– 80	.950	.875
81– 85	.961	1.009
86– 90	1.027	.977
91– 95	1.158	.950
96–100	1.050	.947

thus are not continued in the exposures nor are they counted as deaths when death actually occurs. The experience with the premium waiver disability benefit for both the clerical and nonrated has a uniquely light mortality rate after retirement which is difficult to explain. Recognizing that our assumption as to the proportion of females is only an approximation, this group life experience may be used only as a frame of reference. In that light, it appears that our group annuity mortality rates are not excessive and, indeed, they are confirmed as reasonable by the more seasoned group life insurance experience having the total and permanent disability instalment clause.

Federal Civil Service Retirement Plan. Table 3 shows the mortality under the Federal Civil Service Retirement Plan for the period July 1, 1945 to July 1, 1950[9] expressed as a ratio of the 1946–50 group annuity mortality. The civil service experience is only for employees retired for age and thus

[9] Courtesy of John K. Dyer and Robert Armstrong of Special Committee on Non-Insured Pension Plans.

does not include those retired for disability. The mortality up to 75 or 80 is probably lighter than it would be otherwise because of the exclusion of disability retirements. The mortality rates around 65 and 70, however, are probably abnormally high because of the exclusion of the better lives who continue in active employment. With these factors in mind, the group annuity experience appears satisfactorily representative for males but there is some evidence that the group annuity female experience may not

TABLE 4

RATIO OF 1946–49 RAILROAD RETIREMENT MORTALITY TO 1946–50 GROUP ANNUITY MORTALITY

Age	Males
67	1.210
72	1.083
77	1.032
82	.962
87	1.021
92	1.134

TABLE 5

RATIO OF 1946–49 OASI MORTALITY TO 1946–50 GROUP ANNUITY MORTALITY

Age Group	Males	Females
65–69	1.601	1.372
70–74	1.157	.953
75–79	1.060	.919
80–84	.867	.663
85 and over	.994	.816

be entirely reliable. There were about twice as many female deaths in the civil service experience as in the group annuity.

Railroad Retirement Plan. As published in Mr. Niessen's paper,[10] the mortality experience for age retirements (excluding disability) for the years 1946–49 bears the relationship shown in Table 4 to the 1946–50 group annuity experience for male lives. The mortality rates for the railroad retirement experience are higher from 65 to 75 because of the tendency for the better lives to continue actively at work and thus not come under observation in the retirement experience. In the 70's and 80's, there is a reasonably close correspondence to the group annuity experience.

[10] *TSA* III, 399. Experience graduated as mentioned in *TSA* III, 404.

OASI Primary Beneficiaries. From the basic data[11] on which Mr. Shudde's paper[12] was based, the relationship of the 1946–49 mortality experience of primary beneficiaries under OASI to the group annuity experience has been calculated as shown in Table 5. The effect on this retired life mortality experience of the continuance in employment of healthy lives is very pronounced here in the 60's. (Mr. Shudde showed extremely light mortality with respect to those over 65 who were not collecting benefits.) It is unknown how much these figures are affected by inaccuracy in age reporting or by the lag or lack of reporting deaths. As will be evident from comparing the following ratios of the 1941–45 OASI mortality to the 1946–50 group annuity mortality with the ratios shown in Table 5 for the 1946–49 OASI, there are erratic characteristics in the OASI experience

Age Group	Males	Females
75–79	1.089	.800
80–84	1.036	1.048
85–90	.899	.....

which incline one to limit the credibility given to it. In general, our male group annuity experience seems reasonable in the light of this OASI experience, after allowing for uncertainties in the OASI basic data, but there

TABLE 6

RATIO OF 1946–49 ORDINARY INSURANCE MORTALITY (ULTIMATE) TO 1946–50 GROUP ANNUITY MORTALITY—MALES

Age	Ratio
67	1.083
72	1.022
77	.970
82	.947
87	1.001
92	.999

may be some reason for being less certain about the complete reliability of the group annuity female experience.

Ordinary Insurance Experience. The graduation of the intercompany 1946–49 ultimate ordinary insurance experience[13] has the relationship shown in Table 6 to our 1946–50 group annuity experience. This com-

[11] Courtesy of Mr. Shudde.

[12] *TSA* III, 201.

[13] *TSA* II, 507.

parison of the ordinary insurance experience (in which male lives are probably greatly predominant) with the group annuity male experience shows a close parallel and also encourages confidence in the reliability of the group annuity male experience.

Population Experience. The comparison in Table 7 of the 1948 population experience for white lives with the 1946–50 group annuity experience confirms the generally recognized fact that mortality rates in the general population are higher than they are with respect to active and former workers of the country—a point which is also demonstrated by Mr. Shudde's paper.[14] In order to remain in the active labor force, a better

TABLE 7

RATIO OF 1948 WHITE POPULATION MORTALITY TO 1946–50 GROUP ANNUITY MORTALITY

Age	Males	Females
67	1.257	1.448
72	1.165	1.267
77	1.100	1.132
82	1.057	1.190

average standard of health is required than that found in the general population which includes many in an impaired state of health who have been out of the labor force.

Adjustment of 1946–50 Group Annuity Experience Mortality Rates

The 1946–50 group annuity experience represents the average experience for a composite of groups each of which has its own inherent level of mortality. A study of the foregoing comparison of the group annuity experience with that from other areas of experience indicates that there is some difference in the level of mortality experience, higher or lower than the group annuity average. As we follow mortality experience into the older ages and make allowance for the selective influences discussed above, we observe that there are not large differences in the inherent mortality level of different areas of experience. In order to provide a basic safety margin and to allow for groups that have an inherently lighter mortality experience than the average, it was decided to modify the experience mortality rates by discounting those for males by 10% and for females 12½%. This type of margin provides a percentage adjustment of the annuity values (or reserves) which increases with age. This is desirable in view of

[14] *Loc. cit.*

the relative unreliability of our experience at the oldest ages. For example, in the case of the 10% margin, the annuity values (or reserves) are increased by approximately the following percentages:

Age	Percentage
65	4.8%
70	5.7
75	6.7
80	7.8
90	9.7

It is also interesting to note that, since a one-year set forward of the age for a wide range of ages in the male experience table represents approximately a ten percent increase in mortality rates, an approximation of the experience level of rates may be secured from the Group Annuity Table for 1951 by advancing the age one year. This age adjustment may be a convenient device to use where one wishes to make a valuation of an aggregate of many groups using average mortality rates rather than those considered safe for the better groups mortalitywise.

Group Annuity Table for 1951 for Ages under 65

Comparison of Derived Experience Rates of Group Annuity Table for 1951 with 1946–50 Group Life Clerical Experience. It will be recalled that both the Combined Annuity Table and the 1937 Standard Annuity Table were based upon group life clerical experience at the younger ages. Before finally deciding to bridge the intercompany group annuity matured life experience into the *a*–1949 Table rates for the younger ages, study was given to the practicability of adapting the group life clerical experience in view of the much larger volume of data. It was soon evident, however, that the lack of a separation of experience by sex and the complications of the different types of disability clauses made this course not feasible. Before deciding finally on using the *a*–1949 Table at the younger ages, we were interested, at least, in attempting a comparison of the recent group life clerical experience with the experience rates on which the *a*–1949 Table were based. By the process of adjustment outlined in the Appendix, Section B, we arrived at the relationship shown in Table 8 between the 1946–50 group life clerical experience and the derived basic experience rates of the Group Annuity Table for 1951 as adapted from the *a*–1949 Table, *i.e.*, the effect of an assumed safety margin of 10% has been eliminated. Because of the difficulty of making accurate allowance for the proportion of females and for the effect of the disability clauses, this group life experience must again be taken only as a frame of reference. With this in mind, it appears that the adaptation of group annuity active life experience by

Jenkins and Lew for ages under 55 is reasonably confirmed by the group life clerical experience. At ages over 40, the important area for group annuities, the figures in Table 8 indicate that the Group Annuity Table for 1951 may have a welcome inherent margin of conservatism. After study of this comparison we were satisfied to use the *a*-1949 Table as a basis for our group annuity table for ages about 60 and younger.

"Ill-health Terminations" under Group Annuity Contracts. A study of group annuity mortality would be incomplete without some examination of the problem of "ill-health terminations." An explanation of this problem appeared in 1948 in the report of the Committee to Prepare Mortal-

TABLE 8

RATIO OF 1946–50 GROUP LIFE CLERICAL INSURANCE MORTALITY RATES TO DERIVED EXPERIENCE MORTALITY RATES OF THE GROUP ANNUITY TABLE FOR 1951

Age	(1) Premium Waiver	(2) Extended Death Benefit	(3) Instalment Disability	(4) (1), (2) and (3) Combined
28	1.008	1.158	.704	.885
33	1.003	.961	.898	.938
38	.984	.989	.835	.910
43	1.090	1.204	.953	1.056
48	1.167	1.146	.995	1.073
53	1.165	1.233	1.101	1.154
58	1.030	1.166	1.142	1.138
63	1.114	1.207	1.013	1.087

ity Studies on Group Annuities[15] and is reproduced here for the convenience of the reader.

The terminations in ill health are those cases where a surrender value is not paid because evidence of good health is not satisfactory to the company and where for the following reasons the financial effect on the insurance company generally is the same as if the employee had died. In many cases the death of an individual is preceded by a more or less prolonged period of illness or disablement which results in termination of the individual's employment or in payment of a disability benefit to him if one is provided for by his employer. If a surrender value were allowed to the employer when such cases arise under group annuity contracts, mortality experienced by the insurance companies would be lighter than the true mortality of the employees covered, which would be to the companies' financial disadvantage. To avoid this, the contract usually provides a surrender value to the employer on that portion of the annuity which is purchased by the employer and which does not provide a death benefit, only upon

[15] *TASA* XLIX, 203.

presentation of satisfactory evidence of the good health of the employee at the date his employment is terminated.

Assuming that the mortality assumption at ages under 60 in the group annuity premium structure is satisfactory, the financial benefit which the insurance company needs to obtain from the administration of the "ill-health" contract provision is measured by the reserves released from the extra mortality which would have been experienced had all terminated lives remained in the experience, as would be the case where all annuities are vested and none are canceled. The fact is that the 1937 Standard Annuity Table, even with an age setback, has overstated actual mortality rates at ages under 60 or 55 for some time in the past. Consequently, there has been a mortality loss at the younger ages under contracts using this table which has had to be met by excess interest earnings, reserves released on "ill-health" terminations, and any excess of reserves over purchase payment returns allowed. The contract provision requiring evidence of good health of the terminating employee as a condition to the payment of a return to the employer is for the general protection of the insurance company with respect to its mortality guarantees and, by its language, the "ill-health" cases contemplated are not necessarily limited to true total and permanent disability cases. From the legal point of view, the writer believes that the insurance company is on firm grounds in refusing to allow a return to the employer where there is evidence that the ex-employee (i) is in such physical condition that he will be unable to work for an extended period of time or (ii) has a physical impairment, established by medical evidence, that definitely decreases his prospective longevity but does not necessarily meet the usual test of qualification for total and permanent disability benefits under an insurance contract. The financial importance of the ill-health terminations in past experience is indicated by the fact that, in the male experience, there has been approximately one such case to each three deaths and all these ill-health terminations were needed to produce an aggregate mortality experience approximating the expected by the 1937 Standard Annuity Table (103%).[16]

Once a mortality table is put into use which adequately reflects current and prospective experience at ages under 60, it is possible that the ill-health termination contract provision may be administered more liberally, especially at ages under 50. However, the writer would not change the contract provision requiring satisfactory evidence of good health but believes that it is needed to continue to afford protection to the insurance company with respect to its mortality guarantees generally.

[16] 1949 Report of the Committee on Group Mortality and Morbidity, pp. 46–47.

Comparison of Selected Mortality Tables with Group Annuity Table for 1951

Table 9 presents a comparison of the mortality rates of certain selected annuity tables with the mortality rates of the Group Annuity Table for 1951, and thus shows how the mortality rates at various ages from these different annuity tables compare with rates conservatively representing *current mortality* levels under group annuity contracts.

In Table 10, there is a comparison of deferred and immediate annuity values by selected annuity tables with values calculated on the basis of the Group Annuity Table for 1951, with a common interest rate of $2\frac{1}{2}\%$.

Group Annuity Table for 1951—$2\frac{1}{2}\%$

On pages 262–65, l's, d's and q's, together with D's and N's on the $2\frac{1}{2}\%$ interest basis, are shown for the G*a*–1951 Table.

III. GROUP ANNUITY TABLE FOR 1951 WITH PROJECTION

Mortality Decrease Statistics

As a supplement to the statistics regarding decreases in mortality rates presented by Messrs. Jenkins and Lew, the additional data shown in Table 11 will be found to be of interest and significance.

The very substantial decrease in mortality rates for the group life insurance coverage having the waiver of premium disability clause (prior to 60) may be due in part to a larger proportion of females in the experience for the later period taken.

The railroad retirement experience covered a short interval and is influenced materially during the first and second years of retirement by the retirement policy or practice in operation for the two periods compared. It is fair to conclude, however, that the railroad retirement experience does display a general pattern of improvement that is not without significance.

The improvement in female mortality rates is striking and is considerably greater than that of males as was observed in the statistics presented by Jenkins and Lew.

The past presents a picture of a steadily increasing divergence of male and female mortality rates. In seeking to project the future, should we be prepared for this divergence to continue? W. J. Martin, M.D., in a paper appearing in the *Journal of the Royal Statistical Society*,[17] presents mortality statistics of England showing this increasing divergence over a period of a century. Table 12 presents samples of his data showing the ratio of the male death rate to the female death rate.

[17] "A Comparison of the Trends of Male and Female Mortality," *JRSS*, Part III, 1951.

TABLE 9

RATIO OF THE MORTALITY RATES OF SELECTED ANNUITY TABLES TO THE MORTALITY RATES OF THE GROUP ANNUITY TABLE FOR 1951

Age	Combined Annuity Set Back 2 Years	1937 Standard Annuity	1937 Standard Annuity Set Back 1 Year	Prudential 1950 Group Annuity	a-1949 Table Projected 1 Year
	Males				
20	3.101	2.161	2.123	1.013	1.000
25	2.876	2.059	1.974	1.013	1.000
30	2.291	2.084	1.954	1.013	1.000
35	1.958	2.170	2.011	1.012	1.000
40	1.975	2.178	2.019	1.013	1.000
45	1.648	1.777	1.647	1.013	1.000
50	1.362	1.434	1.330	1.013	1.000
55	1.261	1.299	1.204	1.012	1.000
60	1.262	1.270	1.178	1.007	.995
65	1.197	1.177	1.092	.983	.934
70	1.104	1.062	.986	.965	.884
75	1.029	.969	.900	.896	.866
80	.949	.874	.813	.819	.853
85	.941	.850	.792	.803	.911
90	.994	.883	.824	.841	1.039
95	1.056	.926	.866	.885	1.182
100	1.073	.991	.908	.952	1.268
105	.979	1.135	1.009	1.100	1.189
	Females				
20	4.555	3.402	3.394	1.447	1.000
25	3.980	2.689	2.642	1.261	1.000
30	3.288	2.333	2.236	1.148	1.000
35	2.484	2.220	2.082	1.080	1.000
40	2.152	2.228	2.065	1.040	1.000
45	2.146	2.185	2.025	1.016	1.000
50	2.081	2.072	1.921	1.181	1.000
55	2.055	1.998	1.853	1.411	1.000
60	1.820	1.729	1.603	1.348	.946
65	1.564	1.453	1.347	1.152	.902
70	1.370	1.245	1.155	1.040	.899
75	1.060	.942	.875	.856	.803
80	.937	.815	.758	.755	.824
85	.909	.776	.722	.727	.930
90	.906	.760	.707	.718	1.072
95	.888	.734	.685	.699	1.194
100	.831	.681	.637	.651	1.233
105	.731	.635	.582	.610	1.138

TABLE 10

COMPARISON OF ANNUITY VALUES ON MALE LIVES BY SELECTED ANNUITY TABLES AND GROUP ANNUITY TABLE FOR 1951

AGE x		$G\alpha$–1951 TABLE	COMBINED ANNUITY SET BACK 2 YEARS	1937 STANDARD ANNUITY	1937 STANDARD ANNUITY SET BACK 1 YEAR	PRUDENTIAL 1950 GROUP ANNUITY	α–1949 TABLE
	Deferred Annuity ${}_{65-x}\|\ddot{a}_x^{(12)}$ at 2½%						
25	Value	3.334	2.968	3.025	3.210	3.488	3.490
	Ratio*	.....	89.02%	90.73%	96.28%	104.62%	104.68%
35	Value	4.310	3.888	3.954	4.189	4.510	4.512
	Ratio*	.....	90.21%	91.74%	97.19%	104.64%	104.69%
45	Value	5.632	5.175	5.284	5.580	5.895	5.898
	Ratio*	.....	91.89%	93.82%	99.08%	104.67%	104.72%
55	Value	7.681	7.229	7.415	7.779	8.046	8.050
	Ratio*	.....	94.12%	96.54%	101.28%	104.75%	104.80%
	Immediate Annuity $\ddot{a}_x^{(12)}$ at 2½%						
60	Value	13.766	13.316	13.557	13.965	14.212	14.218
	Ratio*	.....	96.73%	98.48%	101.45%	103.24%	103.28%
65	Value	11.492	11.252	11.555	11.950	12.046	12.038
	Ratio*	.....	97.91%	100.55%	103.99%	104.82%	104.75%
70	Value	9.343	9.295	9.649	10.020	10.017	9.893
	Ratio*	.....	99.49%	103.28%	107.25%	107.21%	105.89%
75	Value	7.384	7.496	7.885	8.225	8.185	7.865
	Ratio*	.....	101.52%	106.78%	111.39%	110.85%	106.51%
80	Value	5.713	5.900	6.303	6.604	6.543	6.034
	Ratio*	.....	103.27%	110.33%	115.60%	114.53%	105.62%
85	Value	4.451	4.533	4.928	5.186	5.116	4.465
	Ratio*	.....	101.84%	110.72%	116.51%	114.94%	100.31%

* Ratio to value by $G\alpha$–1951 Table, expressed as a percentage.

GROUP ANNUITY TABLE FOR 1951—MALES—2½%

x	l_x	d_x	D_x	N_x	q_x
5.....	9999.9999	5.5900	8838.5428	290870.8245	.000559
6.....	9994.4099	5.1871	8618.1484	282032.2817	.000519
7.....	9989.2228	4.9347	8403.5859	273414.1333	.000494
8.....	9984.2881	4.8024	8194.5702	265010.5474	.000481
9.....	9979.4857	4.7502	7990.8572	256815.9772	.000476
10.....	9974.7355	4.7579	7792.2474	248825.1200	.000477
11.....	9969.9776	4.8454	7598.5664	241032.8726	.000486
12.....	9965.1322	4.9427	7409.6326	233434.3062	.000496
13.....	9960.1895	5.0399	7225.3245	226024.6736	.000506
14.....	9955.1496	5.1468	7045.5302	218799.3491	.000517
15.....	9950.0028	5.2735	6870.1343	211753.8189	.000530
16.....	9944.7293	5.4099	6699.0176	204883.6846	.000544
17.....	9939.3194	5.5660	6532.0716	198184.6670	.000560
18.....	9933.7534	5.7318	6369.1840	191652.5954	.000577
19.....	9928.0216	5.9072	6210.2527	185283.4114	.000595
20.....	9922.1144	6.1120	6055.1781	179073.1587	.000616
21.....	9916.0024	6.3462	5903.8519	173017.9806	.000640
22.....	9909.6562	6.5998	5756.1692	167114.1287	.000666
23.....	9903.0564	6.8628	5612.0347	161357.9595	.000693
24.....	9896.1936	7.1648	5471.3615	155745.9248	.000724
25.....	9889.0288	7.4959	5334.0491	150274.5633	.000758
26.....	9881.5329	7.8657	5200.0057	144940.5142	.000796
27.....	9873.6672	8.2741	5069.1381	139740.5085	.000838
28.....	9865.3931	8.7309	4941.3562	134671.3704	.000885
29.....	9856.6622	9.2160	4816.5689	129730.0142	.000935
30.....	9847.4462	9.7588	4694.6980	124913.4453	.000991
31.....	9837.6874	10.3689	4575.6541	120218.7473	.001054
32.....	9827.3185	11.0263	4459.3477	115643.0932	.001122
33.....	9816.2922	11.7599	4345.7018	111183.7455	.001198
34.....	9804.5323	12.5596	4234.6299	106838.0437	.001281
35.....	9791.9727	13.4542	4126.0540	102603.4138	.001374
36.....	9778.5185	14.4233	4019.8875	98477.3598	.001475
37.....	9764.0952	15.4956	3916.0569	94457.4723	.001587
38.....	9748.5996	16.6799	3814.4800	90541.4154	.001711
39.....	9731.9197	17.9943	3715.0766	86726.9354	.001849
40.....	9713.9254	19.4279	3617.7633	83011.8588	.002000
41.....	9694.4975	21.2503	3522.4661	79394.0955	.002192
42.....	9673.2472	23.6995	3429.0194	75871.6294	.002450
43.....	9649.5477	26.7196	3337.1886	72442.6100	.002769
44.....	9622.8281	30.2830	3246.7784	69105.4214	.003147
45.....	9592.5451	34.3413	3157.6203	65858.6430	.003580
46.....	9558.2038	38.8541	3069.5766	62701.0227	.004065
47.....	9519.3497	43.7795	2982.5354	59631.4461	.004599
48.....	9475.5702	49.0835	2896.4085	56648.9107	.005180
49.....	9426.4867	54.7396	2811.1269	53752.5022	.005807
50.....	9371.7471	60.6821	2726.6368	50941.3753	.006475
51.....	9311.0650	66.9186	2642.9091	48214.7385	.007187
52.....	9244.1464	73.3800	2559.9166	45571.8294	.007938
53.....	9170.7664	80.0700	2477.6546	43011.9128	.008731
54.....	9090.6964	86.9343	2396.1192	40534.2582	.009563
55.....	9003.7621	93.9633	2315.3221	38138.1390	.010436
56.....	8909.7988	101.0906	2235.2774	35822.8169	.011346
57.....	8808.7082	108.3295	2156.0155	33587.5395	.012298

GROUP ANNUITY TABLE FOR 1951—MALES—2½%—*Continued*

x	l_x	d_x	D_x	N_x	q_x
58.....	8700.3787	115.7324	2077.5619	31431.5240	.013302
59.....	8584.6463	123.4386	1999.9280	29353.9621	.014379
60.....	8461.2077	131.6141	1923.0937	27354.0341	.015555
61.....	8329.5936	140.4869	1847.0048	25430.9404	.016866
62.....	8189.1067	150.2947	1771.5641	23583.9356	.018353
63.....	8038.8120	161.3229	1696.6348	21812.3715	.020068
64.....	7877.4891	173.8326	1622.0357	20115.7367	.022067
65.....	7703.6565	188.1079	1547.5534	18493.7010	.024418
66.....	7515.5486	204.3703	1472.9418	16946.1476	.027193
67.....	7311.1783	220.1542	1397.9395	15473.2058	.030112
68.....	7091.0241	233.9045	1322.7754	14075.2663	.032986
69.....	6857.1196	246.4654	1247.9438	12752.4909	.035943
70.....	6610.6542	259.8185	1173.7453	11504.5471	.039303
71.....	6350.8357	274.2481	1100.1108	10330.8018	.043183
72.....	6076.5876	288.4921	1026.9315	9230.6910	.047476
73.....	5788.0955	301.4672	954.3189	8203.7595	.052084
74.....	5486.6283	313.1603	882.5504	7249.4406	.057077
75.....	5173.4680	322.9641	811.8800	6366.8902	.062427
76.....	4850.5039	331.5174	742.6310	5555.0102	.068347
77.....	4518.9865	339.5205	674.9994	4812.3792	.075132
78.....	4179.4660	345.5875	609.0589	4137.3798	.082687
79.....	3833.8785	348.6759	545.0709	3528.3209	.090946
80.....	3485.2026	347.4015	483.4135	2983.2500	.099679
81.....	3137.8011	341.0978	424.6120	2499.8365	.108706
82.....	2796.7033	329.9523	369.2236	2075.2245	.117979
83.....	2466.7510	314.3553	317.7199	1706.0009	.127437
84.....	2152.3957	295.0353	270.4689	1388.2810	.137073
85.....	1857.3604	272.7571	227.7024	1117.8121	.146852
86.....	1584.6033	248.5228	189.5257	890.1097	.156836
87.....	1336.0805	223.2858	155.9037	700.5840	.167120
88.....	1112.7947	197.8404	126.6820	544.6803	.177787
89.....	914.9543	172.8523	101.6191	417.9983	.188919
90.....	742.1020	148.8612	80.4110	316.3792	.200594
91.....	593.2408	126.0963	62.7132	235.9682	.212555
92.....	467.1445	105.1827	48.1788	173.2550	.225161
93.....	361.9618	86.3366	36.4203	125.0762	.238524
94.....	275.6252	69.6684	27.0567	88.6559	.252765
95.....	205.9568	55.2016	19.7246	61.5992	.268025
96.....	150.7552	42.8831	14.0858	41.8746	.284455
97.....	107.8721	32.6014	9.8332	27.7888	.302223
98.....	75.2707	24.2007	6.6940	17.9556	.321515
99.....	51.0700	17.4928	4.4310	11.2616	.342526
100.....	33.5772	12.2712	2.8422	6.8306	.365462
101.....	21.3060	8.3208	1.7595	3.9884	.390538
102.....	12.9852	5.4275	1.0462	2.2289	.417979
103.....	7.5577	3.4017	.5941	1.1827	.450096
104.....	4.1560	2.0331	.3187	.5886	.489201
105.....	2.1229	1.1413	.1588	.2699	.537605
106.....	.9816	.5866	.0716	.1111	.597619
107.....	.3950	.2653	.0281	.0395	.671554
108.....	.1297	.0988	.0090	.0114	.761722
109.....	.0309	.0269	.0021	.0024	.870434
110.....	.0040	.0040	.0003	.0003	.999999

GROUP ANNUITY TABLE FOR 1951—FEMALES—2½%

x	l_x	d_x	D_x	N_x	q_x
5.....	9999.9999	3.3500	8838.5428	300135.0894	.000335
6.....	9996.6499	2.7491	8620.0799	291296.5466	.000275
7.....	9993.9008	2.3086	8407.5214	282676.4667	.000231
8.....	9991.5922	2.0383	8200.5650	274268.9453	.000204
9.....	9989.5539	1.9080	7998.9191	266068.3803	.000191
10.....	9987.6459	1.8877	7802.3330	258069.4612	.000189
11.....	9985.7582	2.0471	7610.5935	250267.1282	.000205
12.....	9983.7111	2.2164	7423.4471	242656.5347	.000222
13.....	9981.4947	2.3856	7240.7797	235233.0876	.000239
14.....	9979.1091	2.5646	7062.4869	227992.3079	.000257
15.....	9976.5445	2.7435	6888.4604	220929.8210	.000275
16.....	9973.8010	2.9123	6718.6010	214041.3606	.000292
17.....	9970.8887	3.1009	6552.8188	207322.7596	.000311
18.....	9967.7878	3.2894	6391.0057	200769.9408	.000330
19.....	9964.4984	3.4876	6233.0700	194378.9351	.000350
20.....	9961.0108	3.6955	6078.9154	188145.8651	.000371
21.....	9957.3153	3.9132	5928.4490	182066.9497	.000393
22.....	9953.4021	4.1406	5781.5796	176138.5007	.000416
23.....	9949.2615	4.3777	5638.2190	170356.9211	.000440
24.....	9944.8838	4.6443	5498.2811	164718.7021	.000467
25.....	9940.2395	4.9204	5361.6716	159220.4210	.000495
26.....	9935.3191	5.2061	5228.3099	153858.7494	.000524
27.....	9930.1130	5.5211	5098.1173	148630.4395	.000556
28.....	9924.5919	5.8654	4971.0076	143532.3222	.000591
29.....	9918.7265	6.2290	4846.8973	138561.3146	.000628
30.....	9912.4975	6.6315	4725.7107	133714.4173	.000669
31.....	9905.8660	7.0530	4607.3650	128988.7066	.000712
32.....	9898.8130	7.5231	4491.7898	124381.3416	.000760
33.....	9891.2899	8.0317	4378.9035	119889.5518	.000812
34.....	9883.2582	8.5787	4268.6320	115510.6483	.000868
35.....	9874.6795	9.1835	4160.9043	111242.0163	.000930
36.....	9865.4960	9.8359	4055.6435	107081.1120	.000997
37.....	9855.6601	10.5554	3952.7805	103025.4685	.001071
38.....	9845.1047	11.3416	3852.2410	99072.6880	.001152
39.....	9833.7631	12.1939	3753.9544	95220.4470	.001240
40.....	9821.5692	13.1413	3657.8531	91466.4926	.001338
41.....	9808.4279	14.1830	3563.8624	87808.6395	.001446
42.....	9794.2449	15.3084	3471.9112	84244.7771	.001563
43.....	9778.9365	16.5655	3381.9363	80772.8659	.001694
44.....	9762.3710	17.9237	3293.8607	77390.9296	.001836
45.....	9744.4473	19.4304	3207.6226	74097.0689	.001994
46.....	9725.0169	21.0936	3123.1479	70889.4463	.002169
47.....	9703.9233	22.9110	3040.3647	67766.2984	.002361
48.....	9681.0123	24.9092	2959.2063	64725.9337	.002573
49.....	9656.1031	27.1240	2879.6021	61766.7274	.002809
50.....	9628.9791	29.5610	2801.4765	58887.1253	.003070
51.....	9599.4181	31.8605	2724.7570	56085.6488	.003319
52.....	9567.5576	34.4145	2649.4766	53360.8918	.003597
53.....	9533.1431	37.2555	2575.5575	50711.4152	.003908
54.....	9495.8876	40.4240	2502.9192	48135.8577	.004257
55.....	9455.4636	43.9490	2431.4774	45632.9385	.004648
56.....	9411.5146	48.0175	2361.1471	43201.4611	.005102
57.....	9363.4971	52.7820	2291.8054	40840.3140	.005637

GROUP ANNUITY TABLE FOR 1951—FEMALES—2½%—*Continued*

x	l_x	d_x	D_x	N_x	q_x
58.....	9310.7151	58.3316	2223.3040	38548.5086	.006265
59.....	9252.3835	64.7389	2155.4878	36325.2046	.006997
60.....	9187.6446	72.0036	2088.2008	34169.7168	.007837
61.....	9115.6410	80.1083	2021.3031	32081.5160	.008788
62.....	9035.5327	88.9819	1954.6730	30060.2129	.009848
63.....	8946.5508	98.5015	1888.2179	28105.5399	.011010
64.....	8848.0493	108.5125	1821.8816	26217.3220	.012264
65.....	8739.5368	118.8315	1755.6468	24395.4404	.013597
66.....	8620.7053	129.2330	1689.5369	22639.7936	.014991
67.....	8491.4723	139.7442	1623.6186	20950.2567	.016457
68.....	8351.7281	151.9847	1557.9500	19326.6381	.018198
69.....	8199.7434	166.8976	1492.2911	17768.6881	.020354
70.....	8032.8458	185.5427	1426.2605	16276.3970	.023098
71.....	7847.3031	208.1654	1359.3334	14850.1365	.026527
72.....	7639.1377	232.7492	1290.9994	13490.8031	.030468
73.....	7406.3885	257.5868	1221.1368	12199.8037	.034779
74.....	7148.8017	281.7557	1149.9189	10978.6669	.039413
75.....	6867.0460	304.2719	1077.6558	9828.7480	.044309
76.....	6562.7741	324.9361	1004.7862	8751.0922	.049512
77.....	6237.8380	343.7548	931.7437	7746.3060	.055108
78.....	5894.0832	360.0872	858.9241	6814.5623	.061093
79.....	5533.9960	373.3178	786.7803	5955.6382	.067459
80.....	5160.6782	382.6436	715.8097	5168.8579	.074146
81.....	4778.0346	387.5655	646.5710	4453.0482	.081114
82.....	4390.4691	388.0033	579.6342	3806.4772	.088374
83.....	4002.4658	384.0086	515.5215	3226.8430	.095943
84.....	3618.4572	375.9722	454.6935	2711.3215	.103904
85.....	3242.4850	364.2219	397.5112	2256.6280	.112328
86.....	2878.2631	349.1189	344.2533	1859.1168	.121295
87.....	2529.1442	331.0270	295.1191	1514.8635	.130885
88.....	2198.1172	310.3478	250.2365	1219.7444	.141188
89.....	1887.7694	287.5073	209.6645	969.5079	.152300
90.....	1600.2621	262.9727	173.3977	759.8434	.164331
91.....	1337.2894	236.8928	141.3688	586.4457	.177144
92.....	1100.3966	210.2847	113.4890	445.0769	.191099
93.....	890.1119	183.6666	89.5623	331.5879	.206341
94.....	706.4453	157.5578	69.3482	242.0256	.223029
95.....	548.8875	132.4663	52.5674	172.6774	.241336
96.....	416.4212	108.8737	38.9083	120.1100	.261451
97.....	307.5475	87.2146	28.0348	81.2017	.283581
98.....	220.3329	67.8522	19.5948	53.1669	.307953
99.....	152.4807	51.0524	13.2298	33.5721	.334812
100.....	101.4283	36.9634	8.5856	20.3423	.364429
101.....	64.4649	25.5990	5.3237	11.7567	.397100
102.....	38.8659	16.8348	3.1314	6.4330	.433150
103.....	22.0311	10.4192	1.7317	3.3016	.472930
104.....	11.6119	6.0168	.8905	1.5699	.518156
105.....	5.5951	3.1923	.4186	.6794	.570545
106.....	2.4028	1.5181	.1754	.2608	.631813
107.....	.8847	.6225	.0630	.0854	.703676
108.....	.2622	.2066	.0182	.0224	.787851
109.....	.0556	.0493	.0038	.0042	.886054
110.....	.0063	.0063	.0004	.0004	.999999

In discussing this paper, R. D. Clarke, M.D., stated:

It seemed that the mortality sex differential was made up of a fundamental biological difference overlaid by an occupational difference. . . . But whereas the special risks inherent in many occupations were being reduced by improved conditions of work, there was at present very little being done to alleviate the occupational risks of the professional and administrative class—risks which might be summed up as the price of responsibility. He [the speaker] had just referred to the possible outcome of using anti-coagulants in the treatment of coronary thrombosis. But while this kind of medical discovery might play its part in re-

TABLE 11

AVERAGE YEARLY RATE OF DECREASE IN MORTALITY RATES
(Geometric Basis)

Age Group	Intercompany Group Annuity on and after Normal Retirement	Federal Civil Service Retirement Plan—Age Retirements	Age Group (Assumed All Males)	Railroad Retirement Plan		
				First Year of Retirement	Second Year of Retirement	Ultimate—3d Year and and after
	From 1938–40 to 1946–50	From 1935–40 to 1945–50		From 1943–46 to 1946–49		
Males:						
61–65	1.44%	−1.83%	65–69	−1.26%	1.83%	1.18%
66–70	1.00	−0.15	70–74	2.86	9.09	−1.06
71–75	2.41	1.25	75–79	2.46	11.97	0.02
76–80	1.69	1.29				
81–85	−0.15	1.57				
86–90	1.13	0.14				
91–95	.....	0.63				
Females:						
61–65	3.36	1.72				
66–70	6.08	2.36				
71–75	−1.46	1.89				
76–80	2.74	2.77				
81–85	.....	2.44				
86–90	.....	4.01				

Total Number of Deaths Included in Experience

Males:						
First Period	1,422	8,997	First Period	2,033	1,755	10,481
Second Period	11,729	17,900	Second Period	2,709	2,567	21,030
Females:						
First Period	90	653				
Second Period	764	1,730				

TABLE 11—*Continued*

Age Group (Males and Females Combined)	Intercompany Group Life Insurance			
	Instalment T. & P. Disability		Waiver of Premium T. & P. Disability	
	Clerical	Nonrated	Clerical	Nonrated
	From 1922–29 to 1946–50		From 1932–39 to 1946–50	
66–70	1.27%	0.70%	2.18%	2.47%
71–75	0.87	0.56	2.35	3.06
76–80	0.76	0.13	2.16	3.00
81–85	−0.07	0.58	4.23	3.58
86–90	−0.22	−0.38	4.27	−1.52
91–95	.....	−2.37	.....	−1.50
Total Number of Deaths Included in Experience				
First Period	1,368	14,211	811	5,688
Second Period	3,365	36,217	697	14,712

Sources: Group Annuity: 1938–40: Second and Fifth Reports of Committee on Mortality Investigation
1946–50: *TSA* 1951 Report of Mortality and Morbidity Experience, 111.
Civil Service: Courtesy of J. K. Dyer and R. H. Armstrong of Special Committee on Non-Insured Pension Plans
Railroad Retirement: 1943–46: *TASA* XLIX, 301
1946–49: *TSA* III, 409
Group Life Insurance: 1922–29: Combined Group Mortality Experience; 1932–39: *TASA* XLI, 425–26
1946–50: *TSA* 1951 Report of Mortality and Morbidity Experience, 74–75, for nonrated; clerical experience from Committee.

TABLE 12

Male Death Rate Expressed as a Percentage of the Female Death Rate in England

Period	Ages						
	25–34	35–44	45–54	55–64	65–74	75–84	85 and Over
1841–45	94%	101%	114%	111%	111%	109%	106%
1861–65	99	109	122	118	112	109	110
1881–85	104	117	127	122	116	113	112
1901–05	117	121	130	128	119	115	110
1921–25	113	129	132	133	128	120	113
1941–45	169	147	153	165	144	130	109

ducing excess mortality, what was even more necessary was the prevention of diseases like coronary thrombosis by relieving the nervous strain which would appear to cause them. Ultimately, he believed, that any substantial or permanent reduction in this high ratio of male to female mortality in the professional and administrative class must come from the psychiatrist and the neurologist.

Advances in the field of psychosomatic medicine, suggested by Dr. Clarke, may not only narrow this difference between male and female mortality rates but also become an important influence working for improving male mortality rates in addition to the already recognized advances and expected progress in medical treatment, care and prevention of organic conditions.

Biologists and anthropologists tell us that women have a natural physical advantage over men. Ashley Montagu, chairman of the Department of Anthropology, Rutgers University, wrote as follows recently:

> In the sex cells there are twenty-four chromosomes, but only one of these is a sex chromosome. There are two kinds of sex chromosomes, X and Y. Half the sperm cells carry X and half carry Y chromosomes. All the female ova are made up of X-chromosomes. When an X-bearing sperm fertilizes an ovum the offspring is always female. When a Y-bearing chromosome fertilizes an ovum the offspring is always male. And this is what makes the difference between the sexes. So what? Well, the sad fact is that the Y-chromosome is but an iota, the merest bit, of a remnant of an X-chromosome; it is a crippled X-chromosome. The X-chromosomes are fully developed structures; the Y-chromosome is the merest comma. It is as if in the evolution of sex a particle one day broke away from an X-chromosome, and thereafter in relation to X-chromosomes could produce only an incomplete female—the creature we now call the male! It is to this original chromosomal deficiency that all the various troubles to which the male falls heir can be traced. . . . Women . . . are fundamentally more resistant than men. With the exception of the organ systems subserving the functions of reproduction women suffer much less frequently than men from the serious disorders which affect mankind. . . . Women are both biologically stronger and emotionally better shock absorbers than men.[18]

A discussion of this biological advantage and an explanation of the more rapid rate of mortality improvement among women are found in these quotations from Amram Scheinfeld's book entitled "The New-You and Heredity":[19]

> As childhood proceeds, and as the chief hazards are reduced for a while, the differences between the sexes in mortality diminish considerably, but with male casualties still always in the lead. Then with maturity the curve goes sharply

[18] Ashley Montagu, "The Natural Superiority of Women," *Saturday Review of Literature*, March 1, 1952.

[19] Amram Scheinfeld, *The New-You and Heredity*, pp. 177, 191–92.

up again, becoming more marked in the middle and older ages, where in almost every major affliction, except diabetes, cancers peculiar to women and goiter, the male death rate is much higher. Further, the more that environmental factors for the two sexes have been improved and equalized, the proportionately greater has been the advantage to women, and the more apparent it has become that females genetically are better constructed, have a more efficient internal chemical system, and in various other ways are biologically better adapted to resist most of the modern human afflictions. . . . Further, peculiar as it may seem, we have the situation mentioned earlier that as environments have improved, the genetic disadvantage of males has become even more marked in comparison with females. Why? Because where conditions are very bad, the female's extra margin of resistance isn't sufficient to make much difference. But the more conditions improve, the more that slight advantage of the female may count in preventing a disease from developing, or in permitting her to pull through in a serious illness or accident where a male might succumb.

The writer believes that the matter of future mortality rates among females in connection with annuity contracts and pensions needs to be studied with particular care. Although females have constituted in the past a small minority of the coverage under group annuity contracts and pension plans generally, we should be prepared for their representing a more significant part. They now constitute 20% to 25% of the number of retired lives in clerical groups.

Some actuaries have questioned the assumption made by Messrs. Jenkins and Lew in their Projection Scales A and B that there will be no mortality improvement in the future at ages 90 and older. Since the part of mortality tables for ages over 90 is commonly an empirical projection to the limiting age selected, we are not sure how well such tables reflect current true mortality at these ages to say nothing of future mortality. A very valuable study[20] of mortality of the aged by Paul Vincent appeared recently, in which he observes:

> The benefit of the progress realized, up to now, in the fight against mortality seems to extend through the whole duration of life; but it decreases with age, until at the more advanced ages it becomes unnoticeable.
>
> No human being seems, in the present state of things, to be able to exceed the age of 110 years, and it is extremely doubtful whether a death at that age has ever been observed with any certainty.

Vincent developed mortality rates at ages over 80 (or 85), male and female, for four countries, comparing two periods of observation for each except for Sweden where there were three periods of observation. Thus there were ten comparisons of historical periods. As to the mortality rates

[20] "La mortalité des vieillards," published in *Population*, Sixth Year, April–June 1951, Number 2, p. 181.

at ages 90 and over, six of the comparisons (for Sweden and the Netherlands) showed no significant decrease in mortality rates from an earlier period to a later period. Of the remaining four comparisons, two, applicable to males and females of France, showed mortality decreases of 6% or 7% for ages 90 to 96 for a period comparison of 1920–29 and 1929–38. The remaining two comparisons, applicable to males and females of Switzerland, showed mortality decreases for the early 90's of 5% to 7% for a period comparison of 1876–1914 and 1914–1948. There were no significant decreases in mortality rates at ages over 96 or so.

Vincent presents a set of reliable mortality rates at the most advanced ages, males and females combined. The comparison in Table 13 of

TABLE 13

COMPARISON OF GA–1951 TABLE MORTALITY RATES WITH VINCENT'S RATES FOR OLDEST AGES

Age x	$1{,}000q_x$			Ratio of Vincent's to GA–1951	
	GA–1951		Vincent's		
	Males	Females		Males	Females
85.....	146.852	112.328	195.7	1.33	1.74
90.....	200.594	164.331	280.3	1.40	1.71
95.....	268.025	241.336	371.5	1.39	1.54
100.....	365.462	364.429	500.0	1.37	1.37
105.....	537.605	570.545	715.0	1.33	1.25
110.....	1000.000	1000.000	1000.0	1.00	1.00

Vincent's mortality rates with those of the Group Annuity Table for 1951 indicates that there is a margin in the table to cover considerable future improvement over the mortality levels found by Vincent.

In considering the prospects for mortality improvement in this country, it is of interest to compare the mortality rates of the Group Annuity Table for 1951 with those prevailing today in those countries of the world having the lowest rates. According to the *United Nations Demographic Yearbook, 1949–50*, Norway and the Netherlands have the most favorable mortality rates in the world, at least for males. A study of Table 14 will indicate that we have considerable room and possibilities for improvement in this country.

Projection Scales

After study of the additional data presented in this paper on mortality improvement at the older ages, it seemed important to make studies based

upon a higher rate of mortality decrease for ages over 60 than Projection Scale B. Furthermore, if Projection Scale B is appropriate for individual annuities, some actuaries may feel that we should be prepared for the gap between individual annuity mortality and group annuity mortality at ages over 65 to become narrowed in the future. The expansion of medical care and better hygiene and nutrition could have greater influence in the future upon the classes covered by group annuity contracts than upon those purchasing individual annuities. Accordingly, projection studies presented in this paper are based upon two scales, Projection Scale B and an alternative scale which we shall call Projection Scale C, shown in Table 15.

TABLE 14

COMPARISON OF SELECTED POPULATION MORTALITY RATES WITH RATES OF GROUP ANNUITY TABLE FOR 1951

AGE	$1{,}000q_x$					
	MALES			FEMALES		
	GA–1951	Norway 1948	Netherlands 1949	GA–1951	Norway 1948	Netherlands 1949
40–44	2.5	3.6	2.9	1.6	2.8	2.5
45–49	4.6	5.1	4.5	2.4	3.5	3.8
50–54	7.9	7.2	7.6	3.6	5.1	5.9
55–59	12.3	10.8	10.9	5.6	7.2	9.3
60–64	18.4	15.8	18.2	9.8	11.7	15.4
65–69	30.1	23.0	29.8	16.5	19.5	27.7
70–74	47.5	43.0	51.8	30.5	35.6	49.9
75–79	75.1	71.3	87.3	55.1	62.5	83.0

Projection Scale C is $1\frac{1}{3}$ times Scale B, subject to a maximum annual rate of 1.25%. A good case could probably be made for using even greater rates for females. However, different scales for males and females would involve increased complexities in application and the actuary can still use the device of rating down the age if his judgment so dictates. Furthermore, it could be argued that the class with the higher mortality rates (males) has greater margin for improvement than the class with the lower rates (females) and that, the doctors having more to work on, males may experience greater rates of improvement than females in the future.

Comparison of Reserves on Certain Static Annuity Bases with Reserves on the Group Annuity Table for 1951, with Projection, $2\frac{1}{2}$% Interest

In order to illustrate the extent to which deferred and immediate life (no return upon death) annuity values are increased by introducing an

allowance for mortality improvement beyond current mortality levels, Table 16 shows the percentage increase of annuity values in 1952 on the Group Annuity Table for 1951 with Projection on Scales B and C, over annuity values on the Group Annuity Table for 1951, both at $2\frac{1}{2}\%$ interest.

The use of a lower interest rate in computing reserves has been considered by actuaries as a convenient method of allowing for future de-

TABLE 15

AVERAGE RATES OF DECREASE PER YEAR (GEOMETRICAL BASIS) ASSUMED IN PROJECTING THE GROUP ANNUITY TABLE FOR 1951

Age	Projection Scale B	Projection Scale C
20–50	1.25%	1.25%
60	1.20	1.25
65	1.10	1.25
70	.95	1.25
75	.75	1.00
80	.50	$.66\frac{2}{3}$
85	.25	$.33\frac{1}{3}$
90	0	0

TABLE 16

PERCENTAGE INCREASE IN ANNUITY VALUES BY REASON OF ASSUMED FUTURE DECREASES IN MORTALITY RATES

Age in 1952	Ret. Age	Males		Females	
		Proj. B	Proj. C	Proj. B	Proj. C
25	65	23.5%	28.0%	14.7%	17.7%
35	65	17.9	21.5	11.4	14.0
45	65	12.2	14.9	8.1	10.1
55	65	6.8	8.6	4.9	6.2
65	65	2.5	3.3	2.1	2.7
75	75	1.3	1.7	1.1	1.5

creases in mortality. The eventual ineffectiveness of this device is illustrated in Table 17, where a $\frac{1}{2}\%$ difference in interest rate is assumed to allow for mortality improvement.

It is evident that the difference in interest rate is reasonably satisfactory at issue in 1952 but the 2% "unprojected" reserves become progressively deficient (or less excessive in the case of females in 1962) as we follow them into the future.

TABLE 17

RATIO OF RESERVES ON THE GROUP ANNUITY TABLE FOR 1951, 2% INTEREST, TO RESERVES ON THE SAME TABLE WITH PROJECTION, 2½% INTEREST

AGE IN 1952	RET. AGE	CALENDAR YEAR OF COMPARISON				
		1952	1962	1972	1982	1992
		Projection Scale B				
Males:						
25	65	102.55%	97.74%	93.43%	90.57%	91.09%
35	65	102.32	97.59	94.02	93.35	91.84
45	65	102.36	97.94	95.84	93.99	96.28
55	65	102.39	98.58	96.31	97.12	.....
65	65	101.58	98.80	97.97	.....	.....
75	75	101.47	98.84	.....	.....	.....
		Projection Scale C				
25	65	98.97%	94.32%	90.16%	87.41%	88.14%
35	65	99.27	94.67	91.23	90.78	88.85
45	65	99.94	95.63	93.73	91.49	95.16
55	65	100.73	97.06	94.38	96.21	.....
65	65	100.79	97.54	97.33	.....	.....
75	75	101.05	98.49	.....	.....	.....
		Projection Scale B				
Females:						
25	65	111.00%	105.74%	100.94%	96.94%	94.89%
35	65	108.78	103.70	99.29	96.57	94.36
45	65	106.78	101.91	98.41	96.11	98.42
55	65	104.84	100.39	98.00	99.10	.....
65	65	102.57	100.01	99.77	.....	.....
75	75	102.17	100.49	.....	.....	.....
		Projection Scale C				
25	65	108.12%	103.01%	98.33%	94.44%	92.58%
35	65	106.38	101.40	97.10	94.55	91.90
45	65	104.89	100.11	96.75	94.05	97.51
55	65	103.52	99.20	96.40	98.38	.....
65	65	101.91	98.98	99.27	.....	.....
75	75	101.80	100.19	.....	.....	.....

In examining the comparison in Table 18 of reserves on certain static tables with reserves on a table with projection, it should be kept in mind that the Group Annuity Table for 1951 was designed to cover a reasonably wide range of differing inherent levels of current mortality experience. The mortality rates in the portion of the table under 55 or 60 may be assumed to be 10% under the predominantly clerical intercompany group annuity experience (which experience is not greatly different from the "all other"[21] or nonclerical experience) and the rates in the portion over 65 are 10% (12½% for females) under the average intercompany group annuity experience for all occupational classes. The comparison is based on deferred and immediate life annuities with no return upon death. For ages 25 to 55 in 1952 the annuity is a deferred annuity issued in 1952 beginning at 65, and for ages 65 and 75 in 1952 the annuity is an immediate annuity issued in 1952. To conserve space, the comparison for females is shown only for Projection Scale C and for three static annuity bases. The general trend for Scale B and the other annuity bases may be sensed by studying the other figures presented.

The reader will observe that, in general, the reserve deficiencies of the static annuity bases for each issue age in 1952 (following across along a straight line) display a fair degree of decrease with advancing calendar years in the neighborhood of attained age 65 and a marked decrease around attained ages 75 and 85. Probably the most important factor accounting for this phenomenon is the margin in the mortality rates at the oldest ages possessed by the 1937 Standard Annuity Table and the Prudential 1950 Group Annuity Valuation Table. Furthermore, as reserves on two different annuity bases are followed to the end of the tables, they are bound to approach each other more and more closely, becoming equal at the end if the tables have the same terminating age. However, even though these reserves ultimately reach a fairly satisfactory level, there is an important consideration that should not be overlooked. During the period that reserves on a static basis may be seriously deficient, surplus that may be only apparent must be retained to cover mortality losses that probably will be experienced during this period.

The reader will also note that for a given *attained age* followed into the future (along an upward diagonal), the reserves on the static bases become progressively more deficient. For example, for attained age 65, males, Projection Scale C, static basis Standard Annuity set back one year and 2¼% interest, the ratios progress decennially: 102.82% in 1952, 99.01% in 1962, 95.62% in 1972, 92.61% in 1982, and 89.91% in 1992.

As noted, the foregoing comparisons, in the case of deferred annuities,

[21] 1949 Report of the Committee on Group Mortality and Morbidity, pp. 46–47.

TABLE 18

RATIO OF RESERVES ON CERTAIN STATIC ANNUITY BASES TO RESERVES ON THE GROUP ANNUITY TABLE FOR 1951 WITH PROJECTION, INTEREST 2½%

AGE IN 1952	CALENDAR YEAR OF COMPARISON				
	1952	1962	1972	1982	1992
	PROJECTION SCALE B				
	Standard Annuity 2%				
Males:					
25	93.19%	89.78%	87.76%	87.55%	91.71%
35	93.98	91.67	90.89	93.99	98.30
45	96.15	94.67	96.50	100.60	108.75
55	98.98	99.25	103.07	109.70	.....
65	102.27	105.75	110.67	.....	.....
75	108.60	111.65	.....	.....	.....
	Standard Annuity Set Back 1 Year 2½%				
25	87.82%	86.60%	86.48%	87.81%	92.92%
35	90.65	90.33	91.16	95.23	101.07
45	94.74	94.96	97.77	103.43	113.39
55	99.27	100.56	105.98	114.38	.....
65	103.62	108.73	115.39	.....	.....
75	111.66	116.41	.....	.....	.....
	Standard Annuity 2½%				
25	82.73%	81.68%	81.82%	83.65%	89.79%
35	85.50	85.46	86.84	92.03	96.84
45	89.64	90.45	94.48	99.11	107.70
55	94.56	97.17	101.55	108.64	.....
65	100.14	104.18	109.60	.....	.....
75	107.00	110.58	.....	.....	.....
	Standard Annuity 2¼%				
25	73.49%	74.35%	76.32%	79.95%	87.95%
35	77.83	79.71	83.00	90.13	95.43
45	83.61	86.45	92.54	97.66	106.68
55	90.38	95.17	100.07	107.61	.....
65	98.07	102.66	108.56	.....	.....
75	105.43	109.52	.....	.....	.....
	Prudential 1950 Group Annuity 2½%				
25	84.73%	84.81%	85.15%	86.75%	91.68%
35	88.79	88.94	90.06	93.96	99.06
45	93.29	93.81	96.47	101.38	110.74
55	98.08	99.22	103.88	111.70	.....
65	102.24	106.57	112.69	.....	.....
75	109.45	113.69	.....	.....	.....

TABLE 18—*Continued*

Age in 1952	Calendar Year of Comparison				
	1952	1962	1972	1982	1992
	Projection Scale C				
	Standard Annuity 2%				
Males:					
25	89.93%	86.63%	84.69%	84.50%	88.75%
35	91.18	88.93	88.19	91.40	95.10
45	93.88	92.44	94.38	97.92	107.49
55	97.37	97.72	101.01	108.68	.....
65	101.48	104.40	109.94	.....	.....
75	108.15	111.26	.....	.....	.....
	Standard Annuity Set Back 1 Year 2½%				
25	84.75%	83.56%	83.45%	84.75%	89.91%
35	87.95	87.63	88.45	92.61	97.78
45	92.50	92.72	95.62	100.68	112.08
55	97.67	99.01	103.86	113.31	.....
65	102.82	107.34	114.63	.....	.....
75	111.20	116.00	.....	.....	.....
	Standard Annuity 2½%				
25	79.84%	78.82%	78.96%	80.73%	86.89%
35	82.96	82.91	84.25	89.49	93.69
45	87.52	88.32	92.40	96.47	106.46
55	93.03	95.68	99.52	107.63	.....
65	99.36	102.86	108.88	.....	.....
75	106.55	110.18	.....	.....	.....
	Standard Annuity 2¾%				
25	70.92%	71.74%	73.65%	77.16%	85.10%
35	75.51	77.33	80.53	87.65	92.32
45	81.64	84.42	90.50	95.06	105.44
55	88.92	93.71	98.06	106.61	.....
65	97.31	101.35	107.84	.....	.....
75	105.00	109.13	.....	.....	.....
	Prudential 1950 Group Annuity 2¾%				
25	81.76%	81.84%	82.17%	83.73%	88.72%
35	86.13	86.29	87.38	91.38	95.83
45	91.08	91.60	94.35	98.67	109.46
55	96.49	97.69	101.79	110.66	.....
65	101.45	105.21	111.95	.....	.....
75	108.99	113.29	.....	.....	.....

TABLE 18—*Continued*

Age in 1952	Calendar Year of Comparison				
	1952	1962	1972	1982	1992
	Projection Scale C				
	Standard Annuity Set Back 1 Year 2½%				
Females:					
25	90.58%	89.19%	88.48%	89.57%	94.21%
35	92.10	91.24	92.10	96.22	104.82
45	94.38	94.95	98.45	107.27	123.26
55	98.19	100.94	109.95	124.36	.....
65	103.70	112.90	125.48	.....	.....
75	116.11	126.65	.....	.....	.....
	Standard Annuity 2½%				
25	76.47%	77.21%	78.66%	81.97%	89.22%
35	79.73	81.11	84.29	91.12	99.11
45	83.91	86.90	93.24	101.43	116.19
55	89.86	95.60	103.96	117.22	.....
65	98.21	106.75	118.28	.....	.....
75	109.78	119.38	.....	.....	.....
	Prudential 1950 Group Annuity 2½%				
25	86.96%	87.09%	87.35%	88.64%	93.53%
35	89.93	90.07	91.14	95.63	102.88
45	93.18	93.96	97.75	105.30	120.61
55	97.17	100.21	107.93	121.69	.....
65	102.96	110.82	122.78	.....	.....
75	113.97	123.92	.....	.....	.....

are based upon annuities without return at death prior to retirement age. The group annuity business is also concerned with plans where there is no mortality risk (or small risk) prior to retirement, *i.e.*, where annuities are purchased only at retirement as under deposit administration contracts and where deferred annuities are purchased providing for the return of contributions with or without interest upon death prior to retirement. For a plan where immediate annuities are purchased at a fixed retirement age 65 (as may be the case under some deposit administration plans), the reserve comparison may be made by starting with the ratios for 1952 at age 65 and following the diagonal upwards for successive decennial calen-

dar years of the future. For example, the comparative figure for an immediate annuity to be purchased in 1972 at age 65 is that for calendar 1972 for a deferred annuity purchased in 1952 at age 45. In the case of deferred annuities with return to be purchased in 1952 at age 45, the appropriate comparison is made by taking the same ratio as above for 1972 and increasing it by recognizing any difference between the interest rate of the basis being compared (assuming it to be lower) and $2\frac{1}{2}\%$.

TABLE 19

"THREE-TO-ONE" CONTRIBUTORY PLAN RATIO OF RESERVES ON CERTAIN ANNUITY BASES TO RESERVES ON GROUP ANNUITY TABLE FOR 1951 WITH PROJECTION, $2\frac{1}{2}\%$ INTEREST, IN 1952

Age in 1952	Standard Annuity 2%	Standard Annuity Set Back 1 Year $2\frac{1}{2}\%$	Standard Annuity $2\frac{1}{2}\%$	Standard Annuity $2\frac{1}{2}\%$
	Projection Scale B			
Males:				
25	101.16%	94.38%	90.42%	80.90%
35	100.29	95.78	91.76	83.99
45	100.37	98.01	93.94	87.97
55	101.11	100.72	96.79	92.71
	Projection Scale C			
Males:				
25	98.00%	91.44%	87.60%	78.38%
35	97.63	93.24	89.33	81.77
45	98.26	95.95	91.97	86.13
55	99.64	99.26	95.39	91.36

Group annuity contracts are also written involving a combination of annuities with return and without return at death, as in the case of contributory plans where employee contributions are returned upon death but employer's are not. To illustrate reserves under such a plan, a single situation has been selected where employees are contributing $3.00 a month for each dollar of monthly annuity credit for a year of service. For ease of calculation it has been assumed that employee contributions are returned without interest and that the normal form of annuity is a five year certain and for life. The figures in Table 19 are shown for males only and are based upon reserves in 1952.

Under group annuity contracts, reserve aggregates are probably more important than individual life reserves as the contracts are treated as a whole for surplus distribution purposes. Aggregate reserve comparisons have been worked out (Table 20) using a model office distribution for one year's new issue. Based upon the experience of the writer's company, the following was taken as the distribution of annuities placed in force during one year.

Central Age	Assumed Annual Annuity Income Put in Force in 1952
25	$ 27,580
35	182,750
45	288,300
55	351,770
62	123,600
67	26,000

It was assumed that for ages 25 to 65, deferred life annuities without return beginning at 65 were issued, and for ages 65 and over immediate life annuities. Male lives only were assumed. Based upon our experience, it was further assumed that the annual persistency rate would be .935 up to 65 and that, after 65, the persistency rate would follow the 1937 Standard Annuity Table survival rates.

If a persistency rate of .975 had been used, tests indicate that the percentage ratios shown in the table would have been decreased by about 1.25 for 1972 and 1.5 to 1.7 for 1992. These ratios would all be increased by including in the new issue deferred annuities with return of contributions upon death.

Comparison of Reserves on Certain Static Annuity Bases with Reserves on 1951 "Experience" Table with Projection, 2¾% Interest

Recalling that the Group Annuity Table for 1951 with Projection is designed to be adequate for the "best" group mortalitywise (or most of the "best" groups), it seemed desirable to examine reserves which the actuary may consider as an aggregate valuation basis for the average of a wide distribution of groups having inherent mortality ranging from high mortality to low mortality. Since the Group Annuity Table for 1951 at ages over 65 was based upon the average intercompany experience, an "average" reserve basis may be determined by removing the basic margin that was introduced. For this purpose, it was assumed that for the full range of ages, mortality rates equal to 10/9ths of those in the table may be taken as 1951 average experience rates. Mortality improvement must also be taken into account and this was done by using our two alternative projec-

tion scales B and C. An "experience" interest rate of 2¾% was assumed. Table 21 shows, for males only, the ratio of reserves for deferred annuities without return beginning at 65 and immediate annuities at 65 and 75 (ages in 1952) on certain annuity bases, to reserves computed on the "experience" basis just described. In order to examine aggregate reserves (Table 22) on these bases, an existing total amount in force was taken as it

TABLE 20

MODEL OFFICE TRACING OF RESERVES ON ONE YEAR'S NEW ISSUE RATIO OF RESERVES ON CERTAIN ANNUITY BASES TO RESERVES ON GROUP ANNUITY TABLE FOR 1951 WITH PROJECTION—2¼% INTEREST

Annuity Basis	1952	1962	1972	1982	1992
	Projection Scale B				
Standard Annuity 2%	98.36%	97.94%	98.72%	100.05%	101.81%
Standard Annuity Set Back 1 Year 2¼%	97.96	98.83	100.60	102.73	105.32
Standard Annuity 2¼%	93.38	94.78	96.53	98.43	100.51
Standard Annuity 2½%	88.74	91.79	94.42	96.86	99.25
Prudential 1950 Group Annuity 2½%	96.54	97.40	98.85	100.81	103.06
	Projection Scale C				
Standard Annuity 2%	96.59%	96.05%	96.57%	97.75%	99.45%
Standard Annuity Set Back 1 Year 2¼%	96.19	96.92	98.40	100.36	102.88
Standard Annuity 2¼%	91.70	92.96	94.42	96.16	98.18
Standard Annuity 2½%	87.14	90.02	92.36	94.63	96.95
Prudential 1950 Group Annuity 2½%	94.80	95.53	96.79	98.49	100.67

might have been derived from a steadily increasing volume of new issue in the past. The assumed distribution of annuities without death benefit, retirement age 65, males, is as follows:

Central Age	Total Annuity Income in Force in 1952
25	$ 27,580
35	189,791
45	336,757
55	437,292
65	262,365
75	75,917
85	9,551

TABLE 21

RATIO OF RESERVES ON CERTAIN ANNUITY BASES TO RESERVES ON 1951 "EXPERIENCE" TABLE WITH PROJECTION—2¾% INTEREST

AGE IN 1952	CALENDAR YEAR OF COMPARISON				
	1952	1962	1972	1982	1992
	PROJECTION SCALE B				
	Standard Annuity Set Back 1 Year 2¼%				
Male:					
25	104.77%	100.73%	97.99%	96.60%	98.63%
35	105.82	102.69	100.57	101.22	108.46
45	108.13	105.08	104.07	111.16	123.93
55	110.21	107.20	114.07	125.08	.....
65	110.63	117.21	126.29	.....	.....
75	120.58	127.52	.....	.....	.....
	Standard Annuity 2¼%				
25	87.67%	86.48%	86.47%	87.95%	93.35%
35	90.85	90.63	91.57	95.81	102.41
45	95.42	95.67	98.50	104.96	116.59
55	100.34	101.46	107.71	117.68	.....
65	104.71	110.67	118.81	.....	.....
75	113.85	119.97	.....	.....	.....
	Prudential 1950 Group Annuity 2¼%				
25	101.08%	98.65%	96.48%	95.44%	97.32%
35	103.64	101.12	99.36	99.88	106.30
45	106.47	103.81	102.69	108.95	121.03
55	108.08	105.77	111.81	122.16	.....
65	109.16	114.88	123.34	.....	.....
75	118.18	124.54	.....	.....	.....
	PROJECTION SCALE C				
	Standard Annuity Set Back 1 Year 2¼%				
25	101.11%	97.20%	94.56%	93.23%	95.44%
35	102.66	99.62	97.57	98.44	104.94
45	105.58	102.60	101.78	108.20	122.49
55	108.42	105.55	111.79	123.92	.....
65	109.78	115.72	125.46	.....	.....
75	120.09	127.07	.....	.....	.....

TABLE 21—*Continued*

Age in 1952	Calendar Year of Comparison				
	1952	1962	1972	1982	1992
	Projection Scale C—*Continued*				
	Standard Annuity 2½%				
25	84.61%	83.45%	83.44%	84.88%	90.33%
35	88.14	87.92	88.84	93.18	99.08
45	93.17	93.41	96.33	102.17	115.24
55	98.71	99.90	105.56	116.59	.....
65	103.90	109.26	118.03	.....	.....
75	113.38	119.55	.....	.....	.....
	Prudential 1950 Group Annuity 2½%				
25	97.55%	95.20%	93.10%	92.11%	94.18%
35	100.55	98.10	96.40	97.13	102.85
45	103.96	101.36	100.43	106.05	119.63
55	107.12	104.14	109.57	121.02	.....
65	108.32	113.42	122.53	.....	.....
75	117.70	124.10	.....	.....	.....

TABLE 22

MODEL OFFICE TRACING OF RESERVE FOR TOTAL IN FORCE IN 1952—MALES
RATIO OF RESERVES ON CERTAIN ANNUITY BASES TO RESERVES ON 1951
"EXPERIENCE" TABLE WITH PROJECTION—2¾% INTEREST

Annuity Basis	Calendar Year of Comparison				
	1952	1962	1972	1982	1992
	Projection Scale B				
Standard Annuity Set Back 1 Year 2½%	110.14%	108.64%	110.68%	111.30%	114.47%
Standard Annuity 2½%	100.59	101.18	103.97	104.93	107.85
Prudential 1950 Group Annuity 2½%	108.54	107.02	108.81	109.18	111.09
	Projection Scale C				
Standard Annuity Set Back 1 Year 2½%	108.40%	106.67%	108.36%	108.80%	111.89%
Standard Annuity 2½%	99.00	99.35	101.79	102.57	105.41
Prudential 1950 Group Annuity 2½%	106.83	105.08	106.53	106.72	109.46

As before, an annual persistency rate of .935 was assumed up to age 65 and a survival rate after 65 according to the 1937 Standard Annuity Table.

Equivalent Age Setback of Male Table, with Projection, to Reproduce Reserves on Female Table, with Projection

Group annuity rate structures, with many variations in annuity and death benefits, are so complicated that there is great advantage if one can use the male table with an age setback for females. Table 23 shows the number of years the age should be set back on the male Group Annuity Table for 1951 with Projection C, in order to reproduce reserves computed on the corresponding female table. (Note that, in this setback process, both the unprojected q's and the projection scales are set back.) The figures for Projection Scale C only are shown, as tests indicated they would be practically the same for either projection scale. It would appear that a five-year setback in age of the male table would be reasonably satisfactory unless the actuary wished to make allowance for a greater rate of mortality improvement for females than that assumed in the projection scales. Even a five-year setback results in stronger reserves in the future for females as indicated by the decreased equivalent for 1962 and 1972.

Comparison of the Complete Expectation of Life by the Group Annuity Table for 1951 with Projection, with That of Certain Other Annuity Tables

For the convenience of reference, the complete expectation of life according to the Group Annuity Table for 1951, with and without projection, is set forth in Table 24 along with the complete expectation of life from a number of annuity tables.

Concluding Comments

As indicated in our introduction, it has not been our purpose to draw specific conclusions or to develop specific recommendations from this study. There clearly was a need for an examination of the application to group annuity problems of the principles and approach developed by Jenkins and Lew. However, it remains for the individual actuary to determine whether the use of a static table, with or without an extra-conservative interest rate, or a table with projection using a self-sufficient interest rate, will best serve his purposes. In that consideration, the following points will be among those he may want to have in mind.

1. Will a static table provide the desired order of equity as between groups with differing age distributions?

2. Will a static table provide satisfactory immediate annuity rates to be used for deposit administration contracts where rates at a fixed level may be guaranteed for purchases extending over a period of fifteen to twenty-five years?

TABLE 23

NUMBER OF YEARS SETBACK OF MALE GROUP ANNUITY TABLE FOR 1951 WITH PROJECTION C, WHICH WILL REPRODUCE RESERVES OF FEMALE TABLE WITH PROJECTION

AGE IN 1952	RESERVE IN THE CALENDAR YEAR		
	1952	1962	1972
	Immediate Life Annuities		
25	4.5 Years	.....	.....
35	4.6	.....	.....
45	4.8	.....	.....
55	4.7	4.5 Years	4.2 Years
60	4.4	.....	.....
65	4.0	3.8	3.5
70	3.6	.....	.....
75	3.2	3.0	2.8
	Deferred Annuities to 55 (no return)		
25	4.4 Years	.....	.....
35	4.5	.....	.....
45	4.7	.....	.....
	Deferred Annuities to 65 (no return)		
25	4.1 Years	.....	.....
35	4.3	.....	.....
45	4.5	4.2 Years	4.0 Years
	Deferred Annuities to 75 (no return)		
25	3.5 Years	.....	.....
35	3.6	.....	.....
45	3.8	.....	.....
55	3.9	.....	.....
65	3.7	.....	.....

TABLE 24

COMPLETE EXPECTATION OF LIFE IN YEARS—$\mathring{e}_x$

AGE	1951 GROUP ANNUITY	GA–1951 PROJ. B	GA–1951 PROJ. C	STANDARD ANNUITY	PRUDENTIAL 1950	a–1949	a–1949 PROJ. B
		In 1952					In 1950
Males:							
25	48.81	52.24	52.88	46.53	49.48	49.41	52.91
35	39.24	41.81	42.30	37.38	39.92	39.85	42.39
45	29.94	31.66	32.04	28.78	30.64	30.57	32.22
55	21.51	22.50	22.76	21.02	22.28	22.20	23.11
60	17.72	18.41	18.61	17.55	18.56	18.48	19.09
65	14.21	14.65	14.79	14.40	15.12	15.01	15.40
70	11.12	11.38	11.46	11.60	12.12	11.86	12.07
75	8.49	8.62	8.66	9.17	9.57	9.09	9.19
Females:							
25	53.77	56.58	57.14	51.18	54.30	54.55	57.39
35	44.09	46.24	46.71	41.91	44.68	44.88	46.99
45	34.60	36.12	36.48	33.00	35.22	35.41	36.84
55	25.49	26.41	26.67	24.78	26.30	26.33	27.16
60	21.15	21.82	22.01	21.02	22.28	22.02	22.60
65	17.10	17.54	17.68	17.55	18.56	17.94	18.31
70	13.37	13.62	13.71	14.40	15.12	14.18	14.38
75	10.18	10.30	10.34	11.60	12.12	10.82	10.91
		In 1972					
Males:							
45		33.30	33.92				
55		23.92	24.45				
65		15.66	16.08				
75		9.12	9.33				
Females:							
45		37.34	37.91				
55		27.52	28.01				
65		18.40	18.79				
75		10.78	10.98				
		In 1992					
Males:							
65		16.61	17.27				
75		9.61	9.96				
Females:							
65		19.21	19.79				
75		11.24	11.57				

3. Where an abnormally low interest rate is used to offset some deficiency in a static mortality table, are equities seriously distorted as between annuity forms with no mortality risk before retirement and those with full mortality discount before retirement?

4. As a practical matter, can a company perform the voluminous calculations involved in the determination of premium rates and reserves based upon a projected mortality table, including a year-to-year increase in reserve factors? What is the relative position of the large company and the small company in this respect?

5. How important is it, from a sales viewpoint, to have premium rates based upon an interest rate, loading and mortality table which appear reasonable, each by itself, to the lay businessman?

6. Are reserves, computed on a static table, a satisfactory basis to be used in surplus distribution, having in mind differing age distributions and character and degree of mortality risk?

With no intention of indicating a final conclusion, the writer believes that for a company with adequate machine facilities, there is much to be gained by using a mortality table with projection together with a loading and interest rate which appear reasonable to the layman. There is a feeling of security in that no matter what the annuity form or the age distribution, whether it be an immediate annuity or deferred annuity, the rates and reserves will have margins where they are needed and belong and the best order of equity is automatically achieved.

In our preliminary study of projected tables, we examined the application of Sternhell's functions for the calculation of rates and reserves. It was our conclusion that the many complicated forms of annuities commonly found under group annuity contracts made the calculation work altogether too complicated. However, upon examining the work involved in using multiple generation tables, we believe that modern machines make it possible and practicable to calculate rates and reserves directly from such generation tables using the usual actuarial formulas. The work done in preparing the figures presented in this paper has strengthened us in this belief.

For the actuary who may wish to get his pencil out and examine some of the figures in this paper, sample generation tables for males, projection scale C, are included in the Appendix. Without undue time or labor, mortality tables on this basis for all ages in 1952 have been calculated on punch cards.

I wish to acknowledge my great indebtedness to a number of my associates for their assistance in the preparation of this paper. I want to ac-

knowledge, in particular, my obligation to Howard Hennington and Robert Link for helpful suggestions and ideas, to Kingsland Camp for his skillful graduation and description thereof in the Appendix, and to Felicitas Reich for her able direction of extensive machine operations and calculations.

APPENDIX

A. GRADUATION AND CONSTRUCTION OF THE G*a*–1951 TABLE

Basic Data for Ages 65 and over

The basic data for ages 65 and over were the exposed and deaths by lives at attained ages, in the calendar years 1946–1950, on matured group annuities. The experience by lives was used since there was no significant difference between the experience by lives and that by amount of income. The data were compiled by the Committee on Group Mortality and Morbidity.[1] For attained ages 70 and over, both exposed and deaths were taken as the sum of the two classes (*a*) with income commencing on or after normal retirement date, and (*b*) with income commencing prior to normal retirement date; thus the crude mortality rates were simply the ratios at each age:

$$\frac{\text{Deaths in both classes } (a) \text{ and } (b)}{\text{Exposed } \text{" } \text{" } \text{" } \text{" } \text{" } \text{"}}.$$

Inclusion of class (*b*) below age 70 would have increased the combined mortality rate more than seemed conservative so the average effect above that age was extended below it by multiplying the deaths in class (*a*) by the ratio:

$$\frac{\text{Sum of Deaths (70 \& over) for both classes} \div \text{sum of corresponding expected on both classes, by } a\text{–1949 Table}}{\text{Deaths (70 \& over) for class } (a) \div \text{corresponding expected by } a\text{-1949 Table}}.$$

This ratio came out as 1.017 and 1.023 on the male and female tables respectively. Thus the crude mortality rates for ages 65-69 were taken as:

$$\frac{\text{Deaths of class } (a) \text{ modified as described}}{\text{Exposed in class } (a)}.$$

[1] Data for quinquennial age groups are published in *TSA* 1951 Reports of Mortality and Morbidity Experience, 111–12.

These rates are exhibited in the tables headed 1946–1950 Group Annuity Experience on Male and Female Retired Lives.

The Graduation for ages 65 and over

It is desirable, in producing a useful mortality table, to involve as little preconceived shaping and graduator's personal bias as possible. Inspection of the crude rates, sprinkled with zero death entries and sometimes zero exposed above 96 male and 92 female, indicated that a Makeham fit would be arbitrary and, at least at the higher ages, a summation process or a Whittaker-Henderson "A" graduation would be unacceptable. This left the Whittaker-Henderson "B" process as the only one with real promise of scientific impartiality, that could utilize all available evidence, show fidelity to the supporting data where they were heavy, but permit only negligible deviation from the main trend wherever the data were light. The present job may be the first published use of a "B" formula that constrains first differences towards a geometric progression, thus making the main trend resemble Makeham's first law at points where the data are light. It involves little more labor than would a "B" formula with second differences in the expression minimized. (The accepted "B" process, introduced by Mr. Henderson, has third differences in the function minimized, and is at least twice as laborious.)

In symbols, the process used minimizes

$$\sum_{65}^{\omega} W_x (q_x - q_x'')^2 + g_2 \sum_{66}^{\omega-1} (r^{-1} \cdot \delta q_{x+1/2} - r \cdot \delta q_{x+1/2})^2,$$

wherein q_x represents the graduated, q_x'' the crude rates, the symbol ω the highest age in the range graduated, and the weights are

$$W_x = (\text{Exposed lives at age } x) \div p_x^a q_x^a$$

in which p_x^a and q_x^a are from preliminary experimental smoothed rates. g_2 is the graduating coefficient; 1.5×10^6 appeared from trials to be the most nearly satisfactory value to use for g_2 with the above weighting formula. r^2 was approximated by graphing on semilogarithmic paper; more accurate r^2 values would have appreciably affected only the highest ages in the range graduated, where nevertheless the results will probably be more nearly true than the approximate second difference series that the laborious third-order formula would produce.

The crude series, the results by the graduations, and the second differences are given in columns below; also the expected and the actual deaths, and their ratios by 5-year age groups. The degree of success of the com-

promise achieved between fidelity and smoothness may be inferred from the following summary:

	Male Table		Female Table	
g_2	1.5×10^6		1.5×10^6	
r^2 (analogous to Makeham c)	1.095		1.104	
	By Individual Ages	By Age Groups (Chiefly in 5's)	By Individual Ages	By Age Groups (Chiefly in 5's)
Number of Terms in Range Graduated	38	7	39	6
Number of Sign Changes in Deviations Column	18	5	20	3

Multiples of Standard Deviation	Ideal Percentage within Those Multiples	Actual Percentages			
½	38%	37%	43%	37%	33%
1	68	60	43	66	67
1½	87	71	71	90	83
2	95	87	86	97	83
2½	99	97	100	97	83
3	100	97	.....	100	100
3½	100	100	.....	100	.....

The Difference Series

The graduation process, as noted above, constrains first differences towards a geometric series with a ratio quite certainly near to the c value that would best fit a Makeham curve to the data. Since the process also allows for the varying weight of data at different points of the series, the practical effect at the ages near 65 with heavy data is to minimize second differences and thus smooth only first differences, while at the high ages with light data the nearly unhampered geometric trend imposed on the first differences tends to make all difference-orders geometric. The over-all errors, and the deviations at most of the important annuity ages, were in the conservative direction.

Remembering this, and noting that the first differences of these graduated q_x results are positive throughout and form a slowly decreasing percentage of q_x (from around 11% to around 6% on male rates, from around 14% to around 8% on female rates), it is pretty clear that second differ-

1946–1950 GROUP ANNUITY EXPERIENCE ON MALE RETIRED LIVES

Age	Crude Data $1{,}000q_x''$	Graduated Rates $1{,}000q_x$	Second Differences	Expected Deaths	Actual Deaths	Actual ÷ Expected
65	27.883	28.047	.....			
66	31.061	31.205	.161			
67	35.151	34.524	− .059	4,906.45	4,911.29	1.001
68	38.245	37.784	.090			
69	40.439	41.134	.455			
70	44.233	44.939	.571			
71	49.282	49.315	.461			
72	55.159	54.152	.347	3,920.79	3,933	1.003
73	58.531	59.336	.426			
74	67.284	64.946	.392			
75	72.644	70.948	.608			
76	69.505	77.558	.961			
77	85.796	85.129	.848	2,234.95	2,183	.977
78	87.529	93.548	.770			
79	105.120	102.737	.506			
80	120.495	112.432	.303			
81	121.207	122.430	.246			
82	139.461	132.674	.176	1,055.69	1,100	1.042
83	137.736	143.094	.168			
84	177.591	153.682	.129			
85	176.872	164.399	.196			
86	166.697	175.312	.302			
87	182.138	186.527	.393	321.99	317	.985
88	192.131	198.135	.482			
89	171.789	210.225	.567			
90	252.174	222.882	.633			
91	361.111	236.172	.717			
92	272.727	250.179	.841	64.33	82	1.275
93	458.333	265.027	.975			
94	259.403	280.850	1.133			
95	533.333	297.806	1.299			
96	555.556	316.061	1.487			
97	.....	335.803	1.694	26.64	16	.601
98	.....	357.239	1.910			
99	.....	380.585	2.138			
100	97.752	406.069	2.378			
101	.....	433.931	2.628	.....	.....	.....
102	181.818	464.421	.....			
Over-all comparison				12,530.84	12,542.29	1.001

1946-1950 GROUP ANNUITY EXPERIENCE ON FEMALE RETIRED LIVES

Age	Crude Data $1{,}000\,q_x''$	Graduated Rates $1{,}000q_x$	Second Differences	Expected Deaths	Actual Deaths	Actual ÷ Expected
65	15.505	16.064	.....			
66	19.977	17.694	.083			
67	18.630	19.407	.321	287.61	288.61	1.003
68	21.937	21.441	.484			
69	21.932	23.959	.688			
70	21.021	27.165	.788			
71	31.636	31.159	.593			
72	39.825	35.746	.421	258.88	259	1.000
73	39.766	40.754	.366			
74	53.098	46.128	.293			
75	58.867	51.795	.328			
76	42.244	57.790	.440			
77	62.579	64.225	.433	169.86	160	.942
78	56.672	71.093	.421			
79	83.665	78.382	.351			
80	113.025	86.022	.302			
81	93.956	93.964	.315			
82	173.913	102.221	.331	73.62	98	1.331
83	88.819	110.809	.425			
84	116.788	119.822	.508			
85	125.000	129.343	.594			
86	148.939	139.458	.685			
87	142.857	150.258	.785	26.70	30	1.124
88	206.897	161.843	.891			
89	294.118	174.319	1.012			
90	153.846	187.807	1.155			
91	200.000	202.450	1.306			
92	500.000	218.399	1.470	9.92	6	.605
93	.....	235.818	1.653			
94	500.000	254.890	1.850			
95	.....	275.812	2.067			
96	.....	298.801	2.303			
97	.....	324.093	2.561	.....	.....	.....
98	.....	351.946	2.843			
99	.....	382.642	3.152			
100	.....	416.490	3.491			
101	.....	453.829	3.860	.....	.....	.....
102	.....	495.028	4.265			
103	.....	540.492	.....			
Over-all comparison				826.59	841.61	1.018

ences behaving as follows cannot be of major significance:

(1) One of them (male, 67) is negative but small
(2) Only two or three, in the female series, are as much as 20% of the corresponding first differences
(3) Outside of these, second differences are seldom as much as 10% of corresponding first differences

Nevertheless the essential impartiality of the graduation process is beyond doubt, and recurrence of some of these details when exactly the same process is applied to other material is at least interesting. The negative second difference at 67 male appeared also in such a graduation of this same material before we could add in the experience for the year 1950. The same process, applied to the Railroad Retirement Board's experience for the years 1946-1949 among nondisability annuitants, showed three small negative second differences at ages 66, 67 and 68. As an interesting speculation, it is possible that men find enough difficulty of adjustment to new living habits to cause just a little higher mortality at the normal retirement age of 65 than would otherwise occur, with the result that for a short interval thereafter the always positive first difference, or increase in the rate of mortality, does not itself increase. This persistent feature of three graduation jobs on male data does not appear among results for females above age 65.

All the graduations, male and female, whether of group annuity or railroad retirement data, show curiously low (but positive) second differences about some age in the late 70's or early 80's. It is hard to believe that the recurrence of this feature has no significance. One is led to speculate as to whether it dates the extinction of a "less durable" class of lives from the exposed, or indicates a general inclination of those surviving to the fourscore milestone to improve their vitality by relinquishing all responsibilities and completely trusting thereafter in Providence, their annuity incomes and their bank accounts.

Whatever the explanations, these peculiarities seem to be genuine features of the data and should be retained. Not impossibly, future experiences, graduated by methods equally scientific or more so, will yield valuable and instructive corroborative or contrasting features. Such a possibility is much more important than the mathematical smoothness of a fitted Makeham curve, for example—especially since premiums, reserves and dividends will be computed from them for entire group annuity contracts and not for individual lives. Furthermore, the very operation of deriving financial functions tends in itself to produce smoother series than the q_x from which they are derived.

Bridging Technique and Termination of Table

The graduated rates for ages 65 and over were adjusted to allow for three years' decrease in mortality by Projection Scale B and the resulting

male rates were then discounted 10% and the female rates 12½%. These rates were next bridged to the *a*-1949 Table projected one year. To determine the mortality rates between the ages 55 and 65, we simply joined the G*a*-1951 q_x rates with margin for ages 65 and 66 by a simple four-order binomial curve, to the rates for ages 53, 54 and 55 by the *a*-1949 Table projected one year. Thus first differences were reasonable near 65, and both first and second, near 55.

As the graduated rates with margin did not reach a q_x equal to unity, they were extended by a third-order curve to reach the value of 1.000 at age 110.

B. CALCULATIONS FOR GROUP LIFE INSURANCE CLERICAL EXPERIENCE COMPARISON FOR AGES UNDER 65

For the purpose of comparing the mortality rates at ages under 65 of the Group Annuity Table for 1951 with recent clerical group life experience, the group life experience rates for the period 1946-50 (centering on 1948) were brought forward to 1951 by discounting for three years' mortality decrease according to Projection Scale B and then further discounted by 10% to correspond to the assumed basic safety margin contained in the table for ages under 65.

It was also necessary to take appropriate account of disability claims in the group life experience and to allow for the proportion of females in that experience.

As to disability claims, the adjustment required is to estimate the number of excess deaths that would have been realized had the disabled lives continued in the experience. The percentages of ill-health cases counted as deaths by Jenkins and Lew in the group annuity experience[2] were based on the assumption that the extra mortality would be measured by the group conversion experience. However, the mortality experience among disabled lives is considerably higher than that under group conversions. With this in mind, the following percentages of disability claims were set down as crudely representing the excess deaths that would have arisen had the disabled lives remained in the experience until death.

Central Age	Percentage of Disability Claims Counted As Deaths
28	7½%
33	10
38	12½
43	15
48	20
53	30
58	40

[2] *TSA* 1, 374.

The group life experience, so adjusted, was then compared with a composite q_x derived from the Group Annuity Table for 1951 assuming the following proportions of females. These proportions were arrived at by examining the sex distribution by age of a number of large banks and insurance companies.

Central Age	Assumed Proportion of Females
28	50%
33	33⅓
38	30
43	30
48	30
53	30
58	25
63	25

C. JOINT LIFE ANNUITY VALUES BASED ON THE GROUP ANNUITY TABLE FOR 1951 WITH PROJECTION

Joint annuity values are needed under group annuity contracts almost exclusively for the purpose of converting a single life annuity to an annuity involving two lives. There is rarely an occasion to determine the consideration for a two-life annuity newly purchased. Any lack of mathematical precision in establishing the proper equivalence between a single life annuity and a two-life annuity will be adjusted in an equitable fashion through the process of surplus distribution which reflects the actual mortality experience of a contract. Since there is always a substantial employer contribution to a group annuity plan, the employer ultimately pays for the actual benefits paid whether they are more or less than the mathematically precise amount determined by actuarial considerations. However, reasonable precision should be sought as individual employee equities are involved.

In constructing the Group Annuity Table for 1951, we considered it more important to maintain fidelity to the observed experience (hence the Whittaker-Henderson graduation) than to force the experience into a Makeham or Gompertz curve for the questionable advantage of great precision in computing equivalent two-life annuities.

It may also be noted that if a table such as the Standard Annuity Table is used with 2% or 2¼% interest to introduce more *mortality margin* in the rates or reserves instead of using some other mortality table having adequate mortality margins with say, 2½% interest, the equivalent annuity values derived from the Standard Annuity Table are only approxi-

mately correct at best. As a matter of fact, the ratios of single life annuity values to two-life annuity values based on the Standard Annuity Table with $2\frac{1}{2}\%$ interest and those based on the Standard Annuity Table with an age setback and $2\frac{1}{4}\%$ interest, are practically identical. The setback in age operates to increase the ratio and the lower interest rate operates in the other direction.

Although time and facilities did not permit us to make final calculations based upon the Group Annuity Table for 1951 with Projection, we were able to form certain preliminary opinions based upon a study of an earlier version of such table with projection.

1. To avoid formidable complexities in practice, it appears practically mandatory to use the male table for females with an appropriate setback in age. This may necessitate rating the female age back more than five years in computing equivalent joint annuities where there is a female contingent annuitant.

2. If the male table is so used, it is probable that for the range of ages normally encountered, *i.e.*, between 55 and 75, reasonable accuracy could be attained by assuming either the Gompertz or Makeham Law to apply.

3. The ratios of single life annuity values to two-life annuity values on a mortality table with projection increase in time. For example, to convert a single life annuity to a joint and survivor annuity for two males age 65 in 1955, the income is reduced to 79.4% of the single life income; if conversion takes place in 1975, the ratio is 80.5% and in 1995, it is 81.6%. This is a variation that must be considered in writing a contract, and one approach is to guarantee the lower figure as a minimum.

4. In view of the high speed multiplying facilities available in the new electronic punch card machines (which were extensively employed in the calculations for this paper), the best solution may be the accurate calculation of two-life annuities for the normal range of age combinations without relying upon the equivalent equal age two-life principles or the Gompertz single life principle. (Modern machines appear to be minimizing the importance of actuarial devices designed to save voluminous calculation work. It may become more important for a young actuary to understand the "604" multiplier than Gompertz's or Makeham's Law!)

D. GENERATION TABLES

On the following pages appear sample generation mortality tables for ages in 1952: 35, 45, 55, 60, 65, 70, and 75.

GENERATION TABLE—MALE—AGE (a) IN 1952: 35

Ga-1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_{a}l_x$	${}_{a}d_x$	${}_{a}D_x$	${}_{a}N_x$	${}_{a}q_x$
34.....	9999.9999	12.8100	4319.0534	113627.6535	.001281
35.....	9987.1899	13.5526	4208.3129	109308.6001	.001357
36.....	9973.6373	14.3421	4100.0997	105100.2872	.001438
37.....	9959.2952	15.2178	3994.3451	101000.1875	.001528
38.....	9944.0774	16.1790	3890.9675	97005.8424	.001627
39.....	9927.8984	17.2348	3789.8897	93114.8749	.001736
40.....	9910.6636	18.3843	3691.0346	89324.9852	.001855
41.....	9892.2793	19.8538	3594.3295	85633.9506	.002007
42.....	9872.4255	21.8674	3499.6251	82039.6211	.002215
43.....	9850.5581	24.3604	3406.7058	78539.9960	.002473
44.....	9826.1977	27.2677	3315.3961	75133.2902	.002775
45.....	9798.9300	30.5433	3225.5569	71817.8941	.003117
46.....	9768.3867	34.1405	3137.0760	68592.3372	.003495
47.....	9734.2462	38.0122	3049.8653	65455.2612	.003905
48.....	9696.2340	42.1204	2963.8591	62405.3959	.004344
49.....	9654.1136	46.4170	2879.0088	59441.5368	.004808
50.....	9607.6966	50.8728	2795.2845	56562.5280	.005295
51.....	9556.8238	55.4582	2712.6668	53767.2435	.005803
52.....	9501.3656	60.1436	2631.1465	51054.5767	.006330
53.....	9441.2220	64.9084	2550.7232	48423.4302	.006875
54.....	9376.3136	69.7223	2471.4020	45872.7070	.007436
55.....	9306.5913	74.5737	2393.1948	43401.3050	.008013
56.....	9232.0176	79.4230	2316.1152	41008.1102	.008603
57.....	9152.5946	84.2771	2240.1850	38691.9950	.009208
58.....	9068.3175	89.1960	2165.4219	36451.8100	.009836
59.....	8979.1215	94.2718	2091.8271	34286.3881	.010499
60.....	8884.8497	99.6525	2019.3805	32194.5610	.011216
61.....	8785.1972	105.5014	1948.0304	30175.1805	.012009
62.....	8679.6958	112.0115	1877.6941	28227.1501	.012905
63.....	8567.6843	119.3821	1808.2561	26349.4560	.013934
64.....	8448.3022	127.8313	1739.5706	24541.1999	.015131
65.....	8320.4709	137.5623	1671.4625	22801.6293	.016533
66.....	8182.9086	148.7816	1603.7350	21130.1668	.018182
67.....	8034.1270	159.7345	1536.1715	19526.4318	.019882
68.....	7874.3925	169.3624	1468.9067	17990.2603	.021508
69.....	7705.0301	178.3175	1402.2571	16521.3536	.023143
70.....	7526.7126	188.0925	1336.3948	15119.0965	.024990
71.....	7338.6201	201.7533	1271.2178	13782.7017	.027492
72.....	7136.8668	217.4817	1206.1166	12511.4839	.030473
73.....	6919.3851	233.4670	1140.8416	11305.3673	.033741
74.....	6685.9181	249.8795	1075.4619	10164.5257	.037374

GENERATION TABLE—MALE—AGE (a) IN 1952: 35—*Continued*

Attained Age x	${}_{a}l_x$	${}_{a}d_x$	${}_{a}D_x$	${}_{a}N_x$	${}_{a}q_x$
75.....	6436.0386	266.0916	1010.0171	9089.0638	.041344
76.....	6169.9470	284.4592	944.6429	8079.0467	.046104
77.....	5885.4878	304.0855	879.1133	7134.4038	.051667
78.....	5581.4023	324.1120	813.3582	6255.2905	.058070
79.....	5257.2903	343.3747	747.4404	5441.9323	.065314
80.....	4913.9156	360.0279	681.5826	4694.4919	.073267
81.....	4553.8877	373.0727	616.2391	4012.9093	.081924
82.....	4180.8150	381.6457	551.9554	3396.6702	.091285
83.....	3799.1693	384.9204	489.3367	2844.7148	.101317
84.....	3414.2489	383.0139	429.0328	2355.3781	.112181
85.....	3031.2350	375.5094	371.6131	1926.3453	.123880
86.....	2655.7256	362.4534	317.6368	1554.7322	.136480
87.....	2293.2722	344.6696	267.5958	1237.0954	.150296
88.....	1948.6026	322.4119	221.8315	969.4996	.165458
89.....	1626.1907	296.1001	180.6123	747.6681	.182082
90.....	1330.0906	266.8082	144.1230	567.0558	.200594
91.....	1063.2824	226.0060	112.4027	422.9328	.212555
92.....	837.2764	188.5220	86.3522	310.5301	.225161
93.....	648.7544	154.7435	65.2771	224.1779	.238524
94.....	494.0109	124.8687	48.4946	158.9008	.252765
95.....	369.1422	98.9393	35.3530	110.4062	.268025
96.....	270.2029	76.8606	25.2464	75.0532	.284455
97.....	193.3423	58.4325	17.6243	49.8068	.302223
98.....	134.9098	43.3755	11.9979	32.1825	.321515
99.....	91.5343	31.3529	7.9418	20.1846	.342526
100.....	60.1814	21.9940	5.0942	12.2428	.365462
101.....	38.1874	14.9136	3.1536	7.1486	.390538
102.....	23.2738	9.7280	1.8751	3.9950	.417979
103.....	13.5458	6.0969	1.0647	2.1199	.450096
104.....	7.4489	3.6440	.5712	1.0552	.489201
105.....	3.8049	2.0455	.2847	.4840	.537605
106.....	1.7594	1.0515	.1284	.1993	.597619
107.....	.7079	.4754	.0504	.0709	.671554
108.....	.2325	.1771	.0162	.0205	.761722
109.....	.0554	.0482	.0038	.0043	.870434
110.....	.0072	.0072	.0005	.0005	.999999

GENERATION TABLE—MALE—AGE (a) IN 1952: 45

Ga-1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_aI_x$	${}_ad_x$	${}_aD_x$	${}_aN_x$	${}_aq_x$
44.....	9999.9999	31.4700	3374.0376	74909.3643	.003147
45.....	9968.5299	35.2388	3281.3848	71535.3267	.003535
46.....	9933.2911	39.3756	3190.0343	68253.9419	.003964
47.....	9893.9155	43.8202	3099.8918	65063.9076	.004429
48.....	9850.0953	48.5216	3010.8901	61964.0158	.004926
49.....	9801.5737	53.4480	2922.9838	58953.1257	.005453
50.....	9748.1257	58.5277	2836.1412	56030.1419	.006004
51.....	9689.5980	63.7672	2750.3542	53194.0007	.006581
52.....	9625.8308	69.0942	2665.6138	50443.6465	.007178
53.....	9556.7366	74.5043	2581.9317	47778.0327	.007796
54.....	9482.2323	79.9637	2499.3200	45196.1010	.008433
55.....	9402.2686	85.4384	2417.7983	42696.7810	.009087
56.....	9316.8302	90.8950	2337.3929	40278.9827	.009756
57.....	9225.9352	96.3464	2258.1358	37941.5898	.010443
58.....	9129.5888	101.8314	2180.0529	35683.4540	.011154
59.....	9027.7574	107.4935	2103.1576	33503.4011	.011907
60.....	8920.2639	113.4568	2027.4296	31400.2435	.012719
61.....	8806.8071	119.9399	1952.8222	29372.8139	.013619
62.....	8686.8672	127.1236	1879.2455	27419.9917	.014634
63.....	8559.7436	135.2611	1806.5802	25540.7462	.015802
64.....	8424.4825	144.5557	1734.6659	23734.1660	.017159
65.....	8279.9268	155.2486	1663.3178	21999.5001	.018750
66.....	8124.6782	167.5227	1592.3226	20336.1823	.020619
67.....	7957.1555	179.4100	1521.4541	18743.8597	.022547
68.....	7777.7455	189.6992	1450.8780	17222.4056	.024390
69.....	7588.0463	199.1483	1380.9669	15771.5276	.026245
70.....	7388.8980	209.3940	1311.9253	14390.5607	.028339
71.....	7179.5040	222.9954	1243.6552	13078.6354	.031060
72.....	6956.5086	238.2187	1175.6364	11834.9802	.034244
73.....	6718.2899	253.3736	1107.6858	10659.3438	.037714
74.....	6464.9163	268.5979	1039.9127	9551.6580	.041547
75.....	6196.3184	283.2709	972.3975	8511.7453	.045716
76.....	5913.0475	299.4072	905.3106	7539.3478	.050635
77.....	5613.6403	316.4241	838.5075	6634.0372	.056367
78.....	5297.2162	333.3379	771.9447	5795.5297	.062927
79.....	4963.8783	348.9606	705.7254	5023.5850	.070300
80.....	4614.9177	361.5234	640.1102	4317.8596	.078338
81.....	4253.3943	370.0708	575.5758	3677.7494	.087006
82.....	3883.3235	373.9485	512.6803	3102.1736	.096296
83.....	3509.3750	372.6009	452.0109	2589.4933	.106173
84.....	3136.7741	366.2748	394.1654	2137.4824	.116768
85.....	2770.4993	354.8511	339.6483	1743.3170	.128082
86.....	2415.6482	338.6207	288.9224	1403.6687	.140178
87.....	2077.0275	318.4810	242.3626	1114.7463	.153335
88.....	1758.5465	294.8643	200.1952	872.3835	.167675
89.....	1463.6822	268.3017	162.5634	672.1883	.183306

GENERATION TABLE—MALE—AGE (a) IN 1952: 45—*Continued*

Attained Age x	${}_a l_x$	${}_a d_x$	${}_a D_x$	${}_a N_x$	${}_a q_x$
90.....	1195.3805	239.7862	129.5264	509.6249	.200594
91.....	955.5943	203.1163	101.0187	380.0985	.212555
92.....	752.4780	169.4287	77.6065	279.0798	.225161
93.....	583.0493	139.0713	58.6659	201.4733	.238524
94.....	443.9780	112.2221	43.5831	142.8074	.252765
95.....	331.7559	88.9189	31.7725	99.2243	.268025
96.....	242.8370	69.0762	22.6894	67.4518	.284455
97.....	173.7608	52.5145	15.8393	44.7624	.302223
98.....	121.2463	38.9825	10.7828	28.9231	.321515
99.....	82.2638	28.1775	7.1375	18.1403	.342526
100.....	54.0863	19.7665	4.5783	11.0028	.365462
101.....	34.3198	13.4032	2.8342	6.4245	.390538
102.....	20.9166	8.7427	1.6852	3.5903	.417979
103.....	12.1739	5.4794	.9569	1.9051	.450096
104.....	6.6945	3.2750	.5134	.9482	.489201
105.....	3.4195	1.8383	.2558	.4348	.537605
106.....	1.5812	.9450	.1154	.1790	.597619
107.....	.6362	.4272	.0453	.0636	.671554
108.....	.2090	.1592	.0145	.0183	.761722
109.....	.0498	.0433	.0034	.0038	.870434
110.....	.0065	.0065	.0004	.0004	.999999

GENERATION TABLE—MALE—AGE (*a*) IN 1952: 55

G*a*-1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_{a}l_x$	${}_{a}d_x$	${}_{a}D_x$	${}_{a}N_x$	${}_{a}q_x$
54.....	9999.9999	95.6300	2635.7928	46330.7130	.009563
55.....	9904.3699	102.0744	2546.9139	43694.9202	.010306
56.....	9802.2955	108.4526	2459.1857	41148.0063	.011064
57.....	9693.8429	114.8042	2372.6607	38688.8206	.011843
58.....	9579.0387	121.1653	2287.3769	36316.1599	.012649
59.....	9457.8734	127.7097	2203.3599	34028.7830	.013503
60.....	9330.1637	134.5783	2120.5931	31825.4231	.014424
61.....	9195.5854	142.0166	2039.0299	29704.8300	.015444
62.....	9053.5688	150.2530	1958.5747	27665.8001	.016596
63.....	8903.3158	159.5474	1879.0930	25707.2254	.017920
64.....	8743.7684	170.1450	1800.4093	23828.1324	.019459
65.....	8573.6234	182.3010	1722.3172	22027.7231	.021263
66.....	8391.3224	196.2143	1644.5811	20305.4059	.023383
67.....	8195.1081	209.5407	1566.9520	18660.8248	.025569
68.....	7985.5674	220.8808	1489.6456	17093.8728	.027660
69.....	7764.6866	231.1004	1413.1141	15604.2272	.029763
70.....	7533.5862	242.1144	1337.6152	14191.1131	.032138
71.....	7291.4718	255.8723	1263.0506	12853.4979	.035092
72.....	7035.5995	270.7510	1189.0026	11590.4473	.038483
73.....	6764.8485	285.1722	1115.3622	10401.4447	.042155
74.....	6479.6763	299.2703	1042.2869	9286.0825	.046186
75.....	6180.4060	312.4133	969.9003	8243.7956	.050549
76.....	5867.9927	326.3249	898.4125	7273.8953	.055611
77.....	5541.6678	340.7904	827.7570	6375.4828	.061496
78.....	5200.8774	354.6426	757.9056	5547.7258	.068189
79.....	4846.2348	366.7000	688.9998	4789.8202	.075667
80.....	4479.5348	375.2058	621.3319	4100.8204	.083760
81.....	4104.3290	379.2523	555.4041	3479.4885	.092403
82.....	3725.0767	378.4007	491.7884	2924.0844	.101582
83.....	3346.6760	372.3545	431.0551	2432.2960	.111261
84.....	2974.3215	361.5109	373.7517	2001.2409	.121544
85.....	2612.8106	346.0041	320.3165	1627.4892	.132426
86.....	2266.8065	326.3657	271.1203	1307.1727	.143976
87.....	1940.4408	303.5548	226.4249	1036.0524	.156436
88.....	1636.8860	278.1413	186.3452	809.6275	.169921
89.....	1358.7447	250.7414	150.9085	623.2823	.184539
90.....	1108.0033	222.2588	120.0586	472.3738	.200594
91.....	885.7445	188.2694	93.6347	352.3152	.212555
92.....	697.4751	157.0442	71.9338	258.6805	.225161
93.....	540.4309	128.9057	54.3777	186.7467	.238524
94.....	411.5252	104.0192	40.3974	132.3690	.252765
95.....	307.5060	82.4193	29.4501	91.9716	.268025
96.....	225.0867	64.0270	21.0309	62.5215	.284455
97.....	161.0597	48.6759	14.6816	41.4906	.302223
98.....	112.3838	36.1331	9.9946	26.8090	.321515
99.....	76.2507	26.1178	6.6158	16.8144	.342526

GENERATION TABLE—MALE—AGE (a) IN 1952: 55—*Continued*

Attained Age x	${}_{a}l_{x}$	${}_{a}d_{x}$	${}_{a}D_{x}$	${}_{a}N_{x}$	${}_{a}q_{x}$
100.....	50.1329	18.3217	4.2436	10.1986	.365462
101.....	31.8112	12.4235	2.6271	5.9550	.390538
102.....	19.3877	8.1037	1.5620	3.3279	.417979
103.....	11.2840	5.0789	.8870	1.7659	.450096
104.....	6.2051	3.0355	.4758	.8789	.489201
105.....	3.1696	1.7040	.2371	.4031	.537605
106.....	1.4656	.8759	.1070	.1660	.597619
107.....	.5897	.3960	.0420	.0590	.671554
108.....	.1937	.1475	.0135	.0170	.761722
109.....	.0462	.0402	.0031	.0035	.870434
110.....	.0060	.0060	.0004	.0004	.999999

GENERATION TABLE—MALE—AGE (a) IN 1952: 60

Ga–1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_al_x$	${}_ad_x$	${}_aD_x$	${}_aN_x$	${}_aq_x$
59.....	9999.9999	143.7900	2329.6568	35391.4048	.014379
60.....	9856.2099	151.4012	2240.1548	33061.7480	.015361
61.....	9704.8087	159.6150	2151.9452	30821.5932	.016447
62.....	9545.1937	168.6922	2064.9288	28669.6480	.017673
63.....	9376.5015	178.9318	1978.9614	26604.7192	.019083
64.....	9197.5697	190.5920	1893.8505	24625.7578	.020722
65.....	9006.9777	203.9450	1809.3718	22731.9073	.022643
66.....	8803.0327	219.2043	1725.2706	20922.5355	.024901
67.....	8583.8284	233.7291	1641.2776	19197.2649	.027229
68.....	8350.0993	245.9522	1557.6461	17555.9873	.029455
69.....	8104.1471	256.8609	1474.8933	15998.3412	.031695
70.....	7847.2862	268.5655	1393.3138	14523.4479	.034224
71.....	7578.7207	282.6863	1312.8087	13130.1341	.037300
72.....	7296.0344	297.6344	1233.0156	11817.3254	.040794
73.....	6998.4000	311.9047	1153.8692	10584.3098	.044568
74.....	6686.4953	325.6123	1075.5547	9430.4406	.048697
75.....	6360.8830	338.1064	998.2228	8354.8859	.053154
76.....	6022.7766	351.0014	922.1105	7356.6631	.058279
77.....	5671.7752	364.3095	847.1911	6434.5526	.064232
78.....	5307.4657	376.7451	773.4383	5587.3615	.070984
79.....	4930.7206	387.0714	701.0113	4813.9232	.078502
80.....	4543.6492	393.5255	630.2249	4112.9119	.086610
81.....	4150.1237	395.1997	561.6011	3482.6870	.095226
82.....	3754.9240	391.7625	495.7289	2921.0859	.104333
83.....	3363.1615	383.0473	433.1785	2425.3570	.113895
84.....	2980.1142	369.5461	374.4796	1992.1785	.124004
85.....	2610.5681	351.5208	320.0416	1617.6989	.134653
86.....	2259.0473	329.6266	270.1923	1297.6573	.145914
87.....	1929.4207	304.8678	225.1389	1027.4650	.158010
88.....	1624.5529	277.8895	184.9412	802.3261	.171056
89.....	1346.6634	249.3455	149.5667	617.3849	.185158
90.....	1097.3179	220.1154	118.9007	467.8182	.200594
91.....	877.2025	186.4538	92.7317	348.9175	.212555
92.....	690.7487	155.5297	71.2401	256.1858	.225161
93.....	535.2190	127.6626	53.8533	184.9457	.238524
94.....	407.5564	103.0160	40.0078	131.0924	.252765
95.....	304.5404	81.6244	29.1661	91.0846	.268025
96.....	222.9160	63.4096	20.8281	61.9185	.284455
97.....	159.5064	48.2065	14.5400	41.0904	.302223
98.....	111.2999	35.7846	9.8982	26.5504	.321515
99.....	75.5153	25.8660	6.5520	16.6522	.342526

GENERATION TABLE—MALE—AGE (a) IN 1952: 60—*Continued*

Attained Age x	${}_al_x$	${}_ad_x$	${}_aD_x$	${}_aN_x$	${}_aq_x$
100.....	49.6493	18.1449	4.2027	10.1002	.365462
101.....	31.5044	12.3037	2.6017	5.8975	.390538
102.....	19.2007	8.0255	1.5470	3.2958	.417979
103.....	11.1752	5.0299	.8784	1.7488	.450096
104.....	6.1453	3.0063	.4713	.8704	.489201
105.....	3.1390	1.6875	.2348	.3991	.537605
106.....	1.4515	.8674	.1059	.1643	.597619
107.....	.5841	.3923	.0416	.0584	.671554
108.....	.1918	.1461	.0133	.0168	.761722
109.....	.0457	.0398	.0031	.0035	.870434
110.....	.0059	.0059	.0004	.0004	.999999

GENERATION TABLE—MALE—AGE (a) IN 1952: 65

Ga-1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_{a}l_x$	${}_{a}d_x$	${}_{a}D_x$	${}_{a}N_x$	${}_{a}q_x$
64.....	9999.9999	220.6700	2059.0771	26285.8830	.022067
65.....	9779.3299	235.8090	1964.5263	24226.8059	.024113
66.....	9543.5209	253.0655	1870.3958	22262.2796	.026517
67.....	9290.4554	269.3953	1776.3888	20391.8838	.028997
68.....	9021.0601	282.9636	1682.8087	18615.4950	.031367
69.....	8738.0965	294.9282	1590.2673	16932.6863	.033752
70.....	8443.1683	307.7197	1499.1148	15342.4190	.036446
71.....	8135.4486	322.5461	1409.2468	13843.3042	.039647
72.....	7812.9025	337.8690	1320.3653	12434.0574	.043245
73.....	7475.0335	352.2161	1232.4547	11113.6921	.047119
74.....	7122.8174	365.7139	1145.7392	9881.2374	.051344
75.....	6757.1035	377.6748	1060.4023	8735.4982	.055893
76.....	6379.4287	389.6300	976.7153	7675.0959	.061076
77.....	5989.7987	401.8616	894.6941	6698.3806	.067091
78.....	5587.9371	412.9094	814.3105	5803.6865	.073893
79.....	5175.0277	421.4750	735.7450	4989.3760	.081444
80.....	4753.5527	425.7139	659.3395	4253.6310	.089557
81.....	4327.8388	424.7125	585.6498	3594.2915	.098135
82.....	3903.1263	418.2551	515.2947	3008.6417	.107159
83.....	3484.8712	406.3081	448.8548	2493.3470	.116592
84.....	3078.5631	389.4813	386.8507	2044.4922	.126514
85.....	2689.0818	368.1810	329.6670	1657.6415	.136917
86.....	2320.9008	343.2078	277.5902	1327.9745	.147877
87.....	1977.6930	315.6378	230.7717	1050.3843	.159599
88.....	1662.0552	286.2026	189.2105	819.6126	.172198
89.....	1375.8526	255.6059	152.8086	630.4021	.185780
90.....	1120.2467	224.7148	121.3852	477.5935	.200594
91.....	895.5319	190.3498	94.6693	356.2083	.212555
92.....	705.1821	158.7795	72.7287	261.5390	.225161
93.....	546.4026	130.3301	54.9785	188.8103	.238524
94.....	416.0725	105.1686	40.8438	133.8318	.252765
95.....	310.9039	83.3300	29.7755	92.9880	.268025
96.....	227.5739	64.7345	21.2633	63.2125	.284455
97.....	162.8394	49.2138	14.8438	41.9492	.302223
98.....	113.6256	36.5323	10.1050	27.1054	.321515
99.....	77.0933	26.4065	6.6889	17.0004	.342526
100.....	50.6868	18.5241	4.2905	10.3115	.365462
101.....	32.1627	12.5608	2.6561	6.0210	.390538
102.....	19.6019	8.1932	1.5793	3.3649	.417979
103.....	11.4087	5.1350	.8968	1.7856	.450096
104.....	6.2737	3.0691	.4811	.8888	.489201
105.....	3.2046	1.7228	.2398	.4077	.537605
106.....	1.4818	.8856	.1082	.1679	.597619
107.....	.5962	.4004	.0425	.0597	.671554
108.....	.1958	.1491	.0136	.0172	.761722
109.....	.0467	.0406	.0032	.0036	.870434
110.....	.0061	.0061	.0004	.0004	.999999

GENERATION TABLE—MALE—AGE (*a*) IN 1952: 70

G*a*-1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_{a}l_x$	${}_{a}d_x$	${}_{a}D_x$	${}_{a}N_x$	${}_{a}q_x$
69.....	9999.9999	359.4300	1819.9242	19007.7898	.035943
70.....	9640.5699	374.1698	1711.7177	17187.8656	.038812
71.....	9266.4001	390.5046	1605.1536	15476.1479	.042142
72.....	8875.8955	406.8977	1500.0090	13870.9943	.045843
73.....	8468.9978	421.8916	1396.3358	12370.9853	.049816
74.....	8047.1062	435.6301	1294.4155	10974.6495	.054135
75.....	7611.4761	447.3569	1194.4803	9680.2340	.058774
76.....	7164.1192	458.5466	1096.8545	8485.7537	.064006
77.....	6705.5726	469.8997	1001.6090	7388.8992	.070076
78.....	6235.6729	479.6542	908.7027	6387.2902	.076921
79.....	5756.0187	486.3548	818.3457	5478.5875	.084495
80.....	5269.6639	487.9972	730.9265	4660.2418	.092605
81.....	4781.6667	483.5795	647.0625	3929.3153	.101132
82.....	4298.0872	473.0518	567.4378	3282.2528	.110061
83.....	3825.0354	456.5295	492.6683	2714.8150	.119353
84.....	3368.5059	434.7899	423.2847	2222.1467	.129075
85.....	2933 7160	408 4319	359.6578	1798.8620	.139220
86.....	2525.2841	378.4568	302.0354	1439.2042	.149867
87.....	2146.8273	346.0793	250.5075	1137.1688	.161205
88.....	1800.7480	312.1561	204.9995	886.6613	.173348
89.....	1488.5919	277.4795	165.3300	681.6618	.186404
90.....	1211.1124	242.9419	131.2310	516.3318	.200594
91.....	968.1705	205.7895	102.3482	385.1008	.212555
92.....	762.3810	171.6585	78.6279	282.7526	.225161
93.....	590.7225	140.9015	59.4380	204.1247	.238524
94.....	449.8210	113.6990	44.1567	144.6867	.252765
95.....	336.1220	90.0891	32.1906	100.5300	.268025
96.....	246.0329	69.9853	22.9880	68.3394	.284455
97.....	176.0476	53.2056	16.0478	45.3514	.302223
98.....	122.8420	39.4955	10.9247	29.3036	.321515
99.....	83.3465	28.5483	7.2314	18.3789	.342526
100.....	54.7982	20.0267	4.6385	11.1475	.365462
101.....	34.7715	13.5796	2.8715	6.5090	.390538
102.....	21.1919	8.8578	1.7074	3.6375	.417979
103.....	12.3341	5.5515	.9695	1.9301	.450096
104.....	6.7826	3.3181	.5201	.9606	.489201
105.....	3.4645	1.8625	.2592	.4405	.537605
106.....	1.6020	.9574	.1169	.1813	.597619
107.....	.6446	.4329	.0459	.0644	.671554
108.....	.2117	.1613	.0147	.0185	.761722
109.....	.0504	.0439	.0034	.0038	.870434
110.....	.0065	.0065	.0004	.0004	.999999

GENERATION TABLE—MALE—AGE (a) IN 1952: 75

Ga-1951 TABLE WITH PROJECTION C—2½%

Attained Age x	${}_{a}l_x$	${}_{a}d_x$	${}_{a}D_x$	${}_{a}N_x$	${}_{a}q_x$
74	9999.9999	570.7700	1608.5478	13399.8278	.057077
75	9429.2299	582.7547	1479.7431	11791.2800	.061803
76	8846.4752	593.4039	1354.4298	10311.5369	.067078
77	8253.0713	604.0836	1232.7584	8957.1071	.073195
78	7648.9877	612.4774	1114.6601	7724.3487	.080073
79	7036.5103	616.8275	1000.3960	6609.6886	.087661
80	6419.6828	614.7231	890.4393	5609.2926	.095756
81	5804.9597	605.0045	785.5360	4718.8533	.104222
82	5199.9552	587.8081	686.5033	3933.3173	.113041
83	4612.1471	563.5121	594.0491	3246.8140	.122180
84	4048.6350	533.1566	508.7494	2652.7649	.131688
85	3515.4784	497.6546	430.9788	2144.0155	.141561
86	3017.8238	458.3592	360.9454	1713.0367	.151884
87	2559.4646	416.7499	298.6571	1352.0913	.162827
88	2142.7147	373.9144	243.9294	1053.4342	.174505
89	1768.8003	330.8170	196.4512	809.5048	.187029
90	1437.9833	288.4508	155.8138	613.0536	.200594
91	1149.5325	244.3389	121.5205	457.2398	.212555
92	905.1936	203.8143	93.3568	335.7193	.225161
93	701.3793	167.2958	70.5722	242.3625	.238524
94	534.0835	134.9976	52.4283	171.7903	.252765
95	399.0859	106.9650	38.2207	119.3620	.268025
96	292.1209	83.0953	27.2943	81.1413	.284455
97	209.0256	63.1723	19.0539	53.8470	.302223
98	145.8533	46.8940	12.9711	34.7931	.321515
99	98.9593	33.8961	8.5861	21.8220	.342526
100	65.0632	23.7781	5.5074	13.2359	.365462
101	41.2851	16.1234	3.4094	7.7285	.390538
102	25.1617	10.5171	2.0272	4.3191	.417979
103	14.6446	6.5915	1.1511	2.2919	.450096
104	8.0531	3.9396	.6176	1.1408	.489201
105	4.1135	2.2114	.3078	.5232	.537605
106	1.9021	1.1367	.1388	.2154	.597619
107	.7654	.5140	.0545	.0766	.671554
108	.2514	.1915	.0175	.0221	.761722
109	.0599	.0521	.0041	.0046	.870434
110	.0078	.0078	.0005	.0005	.999999

Machine Procedure for Constructing Generation Tables

For those interested in the machine procedure followed in constructing the generation tables, an outline is presented here. The "604 multiplier" mentioned is the IBM Electronic Calculating Punch—Type 604.

These symbols were added to the usual notation: a was used to identify the generation (either the year of birth or the age in a particular calendar year is convenient); r_x was defined as the complement of the projection factor for age x.

The steps of the machine procedure were then as follows for each mortality basis:

1. One master card was hand-punched for each attained age x, showing r_x, v^x, and the q_x of the Ga-1951 Table.
2. One master card was hand-punched for each a, showing the index l_x to be used for that generation at the youngest age x.
3. An appropriate number of blank cards was filed behind each x master card. This file was run through the 604 multiplier to punch ${}_aq_x$'s onto the blank cards such that each ${}_aq_x$ was the ${}_aq_x$ of the preceding blank card multiplied by r_x, with the q_x of the master card as the starting value. At the same time the a for each blank card was computed and punched, and the x, v^x, and mortality basis code from the master card were gang-punched onto the blank cards.
4. The cards were then sorted to combine all cards with the same a and to arrange x in ascending order behind the a master card.
5. This file was run through the multiplier to compute and punch l_x's and d_x's starting with the l_x from the master card, and computing each succeeding l_x from the l_x and ${}_aq_x$ of the preceding card.
6. Within each a group the cards were then sorted in descending order of x behind the a master card.
7. This file was run through the multiplier to compute and punch D_x's and N_x's, the master cards serving only to start N_x as a new accumulation of the D_x for each new a.