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A RETIREMENT SYSTEM GRANTING UNIT ANNUITIES AND INVESTING IN EQUITIES

ROBERT M. DUNCAN

DURING the past forty years the American economy has been subject to severe fluctuations which have affected the cost of living. Some of these changes have been rapid, others more or less gradual. Some of them have been merely temporary, whereas others have produced far-reaching and long-term effects on the purchasing power of the dollar. The last thirty year period has also been one of unparalleled increase in the establishment of pension plans of all types, so that the problem of best providing for the years of retirement is now assuming greater and greater significance.

Adequate provision for pensions is not easily attained if it implies the objective of a satisfactory income in terms of current purchasing power. Several methods of accomplishing the payment of fixed dollar annuities to retired persons have been developed, whereas little progress has been made in devising techniques to assist in maintaining a more consistent purchasing power of pensions after the actual annuity benefits have commenced. The serious deterioration in the purchasing power of the dollar during the last ten years is almost too obvious to mention, but there have been other periods in our history, such as 1929–1933, when the cost of living has declined rather steadily. However, the persistent though irregular tendency to a lower value of the dollar, which pervades our entire history, is frequently overlooked. What the future holds is, of course, uncertain except to the extent that we can be reasonably assured that the purchasing power of the dollar will continue to change.

The current methods of government financing, taxation, and subsidization, and the implications of the defense program, as well as some developments in private business, such as the cost-plus contracts and escalator wage agreements, all point to continued inflation. To a greater or lesser extent, similar factors have prevailed for a long time in this country, and in other countries too. On the other hand, various combinations of factors have in the past often and sometimes unexpectedly produced business depressions. If past experience is any guide, then, we should not look for a change in the basic pattern of successive inflations and deflations, even though the extents and durations cannot be foreseen, but we may expect a tendency for succeeding inflation levels to be higher as time passes.

Because pension financing covers such long periods, temporary and minor fluctuations in business conditions have little significance. The fact that an employee retiring at 65 who entered a retirement plan at age 30 can expect to have been involved with the objectives of that plan for about fifty years before his death serves to emphasize the importance of attempting to provide him a retirement income which will bear as adequate a relationship as possible to the living costs he must meet after his working years.

The difficulty is that there seems to be no perfect protection against inflation. However, there is good reason to believe that retirement security can be substantially enhanced by broadening the scope and diversification of the investment of funds accumulated during working years.

Teachers Insurance and Annuity Association has formed a new corporation whose objectives are to supplement T.I.A.A. annuities so that the two corporations as a whole may provide pensions which will, in so far as possible, increase when prices are high. Of course, this carries with it the probability of decrease when prices are low. In March this year a charter for the new corporation, the College Retirement Equities Fund, was granted by the State of New York and it is expected that within a few months the Fund will begin operations under the supervision of the State Insurance Department.

The purpose of this paper is to describe this rather novel type of corporation, which presents some unusual and interesting actuarial problems. No attempt will be made here to discuss in detail the tax, management, and many other phases of the new organization. Instead, a general explanation of the Fund will first be presented, followed by a summary of the statistical data compiled in developing it. Finally, there is given a detailed presentation of the actuarial plan to be used, involving the accumulation units and annuity units upon which the Fund's annuities are to be based.

COLLEGE RETIREMENT EQUITIES FUND

The new corporation is a "running mate" to T.I.A.A. and is technically separate except for joint control at the top level. Inasmuch, however, as the Fund's annuities will supplement T.I.A.A. annuities, a good deal of coordination is necessary. For the present at least, both companies will be operated by the same staff, though they are governed by separate operating boards and finance committees.

T.I.A.A.'s principal function is the writing and maintenance of accumulation-type individual deferred annuity contracts to fund college retirement plans. These are life insurance company fixed dollar contracts, supported

almost entirely by investments in debt obligations. The new Fund will issue, similarly, individual deferred annuities, which will supplement the T.I.A.A. contracts. The Fund's assets, however, will be invested, at least initially, entirely in common stocks and other equity-type investments.

The promises in the T.I.A.A. contract are expressed in terms of dollars. For reasons given later, the Fund's contracts will be expressed solely in terms of shares in the Fund, or units, as explained in detail in the last section of this paper. The charter of the Fund does not permit it to make fixed dollar contracts, or to issue life insurance.

For reasons which will be developed later, the Fund will not accept a premium for any participant unless it is accompanied by a premium to T.I.A.A. of at least an equal amount.

As is the case with his T.I.A.A. contract, each individual participant in the Equities Fund will at all times have a contract fully vested as to both employee and employer contributions, so that if he changes jobs he will take with him the entire accumulated contributions. To assure that these accumulations will be used for their intended purpose of providing a retirement or death benefit (equivalent in value to the amount of the total accumulations), there will be, as in T.I.A.A. contracts, no cash or loan values in the Fund's contracts.

In setting up a retirement plan, the educational institution will choose among the following alternatives:

1. Neither the college nor its staff members will participate in the Equities Fund.
2. The college and all of the persons in its retirement plan will participate in the Equities Fund and the college will specify the proportion of total employee and employer contributions, such as one-fourth, one-third or one-half which will go to the Equities Fund.
3. The college will decide that it favors the Equities Fund but will leave the decision as to individual participation entirely up to each staff member. Thus, one person at the college can choose to contribute his full combined premium to T.I.A.A., another three-fourths to T.I.A.A. and one-fourth to the Equities Fund, and so on up to the fifty percent maximum to the new corporation.

Any particular individual will thus have one, two or three parts to his retirement income: (*a*) his Social Security benefit, if he is eligible; (*b*) his T.I.A.A. annuity; (*c*) his Equities Fund unit annuity, if elected.

Inasmuch as college premium payments are generally related to salaries, the regular investment of periodic premiums in a fund involving many institutions and staff members not only takes advantage of the

diversification possible *among issues* of equity investments and *over time*, but will also receive the benefit of *dollar cost averaging*. In a period of fluctuating market prices, the regular investment of equal amounts of dollars results in dollar cost averaging, whereby investments are made during both the ups and downs in the market so as to avoid the possibility of investing only at the high points, and moreover purchases are automatically made at an average price per share bought which is lower than the average of the prices per share prevailing over the period of investment. This is true because of the fact that the average of the "prevailing prices" as shown by market indexes is unweighted by the number of shares bought at such prices, whereas the average of the "prices per share bought" is weighted by the number of shares bought, and more shares will be purchased by an equal number of dollars when prices are low than when prices are high. Although these benefits may tend to be reduced somewhat for participants in times of depression, due to the lowering of the participants' salaries, a substantial dollar cost averaging benefit should still be obtained, if history is any indication for the future. The reason for this lies in large part with the tendency for salary levels to change considerably less violently than prices of equity investments.

The Fund's liability supporting its contracts will at all times have two important and separate parts, one part made up of the portion of the Fund's total net asset amount allocated to participants in the accumulation period during working years, and the other part made up of the portion of the Fund's total net asset amount allocated to participants receiving annuity benefits. Separate internal fund accounts will be maintained for each part, but there will be no segregation of assets. This internal separation is being established because it is planned to use different methods of treating the factors of mortality, expense, dividend income and capital gains and losses in the periods before and after retirement.

Each participant's share in the "Accumulation Fund" (before retirement) will be expressed in terms of *accumulation units;* and after he has transferred into the "Annuity Fund" at retirement, his share in the latter fund will be expressed in terms of a fixed number of *annuity units* payable at stated intervals. A beneficiary receiving an annuity after the participant's death during the deferred period will also participate in the Annuity Fund through annuity units. The dollar value of both accumulation units and annuity units will change from time to time according to the experience of the two parts of the Fund described above. As will be explained in detail in a later section, the current dollar value of an accumulation unit will not normally be the same as the current dollar value of an annuity unit.

STATISTICAL DATA USED IN DEVELOPMENT OF EQUITIES FUND

While it is impossible to predict the course of common stock prices and dividend income, the historical performance of equities gives some clues as to the type of broad investment program which would have been appropriate for the problem of retirement planning under the various business conditions existing in the United States over the past several decades. By including periods of favorable and unfavorable common stock results, together with periods of increases and decreases in the purchasing power of the dollar, some indications can be gained as to over-all effects under differing business conditions.

Since the problem involves the maintenance of substantial funds for retirement plans over long periods of time, data seem indicated on as wide a base as possible as to the experience of life insurance companies and also as to common stocks.

The investment history of the life insurance companies was selected as the basis for comparisons, for several reasons. American life insurance companies have demonstrated a remarkable record of financial security under adverse as well as favorable conditions. They are regarded as non-speculative, able to provide assured benefits in the form of life insurance and annuities. Although we shall be discussing the results of life insurance investments in providing pensions, which are subject to the long range changes in the purchasing power of the dollar, it should be emphasized that many of the services of life insurance companies are relatively unaffected by such changes. This is true whenever the insurance is paid for on a basis whereby the lump sum benefit bears a close relationship to the purchasing power of the premiums, such as low premium types of individual insurance, group insurance, and the usual accident and health coverages. In addition, life insurance is the best and, in fact, the only method of creating an immediate estate in the event of an early death of the breadwinner. One of the conclusions from the data to follow is the advisability, not only of using equities, but also of establishing a firm base of dollar investments, such as regular annuities and endowments, in planning for individual security.

The method followed in the study was to reconstruct, for various periods from 1880, the approximate experience of a one hundred percent fixed dollar fund, a one hundred percent common stock fund, and a combined fund based on an even division of premiums between the fixed dollar fund and the common stock fund. Individual judgments will differ as to the appropriate percentage of premiums for investment in common stocks, and the 50–50 combined fund is presented with the other two funds mainly for illustrative purposes.

After studying a large mass of background material, the following measures, shown in detail in the Appendix, were selected as most applicable to an accumulating fund built up from periodic investments, covering a wide variety by type and covering a long period by time:

1. *Cost of Living Index.* The index selected was the United States Bureau of Labor Statistics Index. This covers all items for moderate income families, and uses 1935–1939 as the base period of 100. The annual average of the fifteenth day of the month prices was employed in the calculations. It was recognized that this does not coincide exactly with the prices of items purchased by retired lives, but no long series of such a special nature is available and we felt that the more general index should be sufficiently illustrative.
2. *Common Stock Price Index.* The data used for the common stock fund were based on the Cowles Commission All Stock Price Index Series P, covering over 400 stocks, which was extended from its expiry date of 1938 by fitting the Standard and Poor's Weekly Index of composite prices of 416 stocks, so that the entire series uses 1926=100. These indexes include an estimated 75% of the market value of the common stocks listed on all United States exchanges whose issues are sufficiently traded to be available for inclusion in an index.
3. *Common Stock Net Yields.* These were obtained for the years 1880 to 1938 from the Cowles Commission studies of over 400 stocks, and from 1939 from the dividends paid on Standard and Poor's 90 stocks. Yields for the larger group since 1938 are not available, but a high correlation was found between the small and large lists in prior years. These yield rates each year are based on the market values of the stocks. Net yields were assumed to be one-quarter of one percent less than the gross yields.
4. *Life Insurance Company Net Interest Rate.* Gross yields are shown in the Spectator Year Book for the period from 1880 to 1899, and from these an estimated one-fifth of one percent charge for investment expenses was deducted to obtain net yields for this period. Net yields from 1900 to 1905 were estimated from the experience of twenty-eight United States life insurance companies. From 1906 to the present, net yields are available for all United States companies (now over 600) from data prepared by the Institute of Life Insurance.

Although the data shown during the last two decades of the nineteenth century are probably not as satisfactory as later data, because of various estimates that had to be made and because the experience of the railroad and coal industries had a relatively large effect on the indexes, the period is useful for study. Common stock net yields from 1880 to 1899 were below those of life insurance companies in all but two of the years, and during this period the cost of living was fairly level or even declining gently. This

combination of poor common stock performance and declining prices can be expected to happen again, although the causes would undoubtedly be different.

FUND COMPARISONS

Assumptions

In each comparison, the same amount of money is assumed to be invested in each of the funds. All involve annual premiums of $100 to age 65. Because of the form of some of the data, each year shown is assumed to extend from July 1 to June 30 of the next year. An expense charge of 4% (T.I.A.A.'s loading) is deducted before computing all accumulations and annuities. Since the primary purpose of the comparisons was to examine the relative investment experience of the funds, it was decided to use the same annuity mortality assumptions in all cases in order to avoid the complications of interpreting the results which would be introduced by changing mortality levels in different periods. For the same reason, no mortality gain or loss was assumed. The mortality assumption used was that single premium annuity values would be 16/15ths of those produced by the Standard Annuity Table. This had the advantage, when coupled with the 4% expense charge, of simplifying the calculations and also it represents roughly the recent level of the mortality of retired college personnel, with a small amount of conservatism added.

Tables 1 to 4 show the results achieved by two individuals who joined separate hypothetical retirement plans at age 35 in the first year of various periods, paid premiums of $100 per year to age 65 and then retired, reaching age 85 in the last year of the respective periods. The combined 50–50 annuity assumes premiums of $50 per year to the fixed dollar fund and a like amount to the common stock fund.

The last four columns, involving changes in the cost of living, are included in the tables in order to give a basis for comparing the purchasing power of premium dollars paid, with the purchasing power of annuity dollars received each year during retirement, as explained in the footnotes.

1880–1930 Period

The fixed dollar fund obtained by accumulating net premiums of $96 per year at the assumed net interest rates would have grown to $6,305.00 in 1910, and would have provided an annuity of about $587.00 per year on the basis of the average of the interest rates over the period 1910–1930.

The common stock fund obtained by investing $96 net per year would have grown to $9,345.00 in 1910, due mainly to the appreciation in values in the period from 1900 to 1910. The unit annuity received, as later de-

scribed in detail, takes into account not only the usual factors used in determining fixed dollar annuities, but also variations in common stock dividends from an assumed rate of 4% per year, together with changes in common stock market values, both realized and unrealized.

The retirement period shown in Table 1 includes the serious rise in the cost of living which began in the first World War and reached a maximum

TABLE 1

AMOUNTS OF ANNUAL ANNUITY 1910–1930
PURCHASED BY ACCUMULATIONS OF $100 PER YEAR 1880–1910

YEAR	AMOUNTS OF ANNUAL ANNUITY			ADJUSTED COST OF LIVING ANNUITY*	PURCHASING POWER COMPARISON†		
	Fixed Annuity	Unit Annuity	Combined Annuity		Fixed Annuity	Unit Annuity	Combined Annuity
1910.....	$587	$ 817	$ 702	$ 698	84%	117%	101%
1911.....	587	813	700	698	84	116	100
1912.....	587	843	715	741	79	114	97
1913.....	587	763	675	727	81	105	93
1914.....	587	732	660	738	80	100	90
1915.....	587	757	672	746	79	102	91
1916.....	587	868	728	801	73	108	91
1917.....	587	802	694	942	62	85	74
1918.....	587	741	664	1,106	53	67	60
1919.....	587	874	731	1,273	46	69	58
1920.....	587	811	699	1,473	40	55	48
1921.....	587	718	652	1,313	45	55	50
1922.....	587	887	737	1,231	48	72	60
1923.....	587	918	752	1,253	47	73	60
1924.....	587	981	784	1,257	47	78	63
1925.....	587	1,212	899	1,290	46	95	71
1926.....	587	1,357	972	1,300	45	104	75
1927.....	587	1,607	1,097	1,275	46	126	86
1928.....	587	2,021	1,304	1,261	47	162	105
1929.....	587	2,535	1,561	1,260	47	203	125
1930.....	587	2,007	1,297	1,228	48	164	106

* The Adjusted Cost of Living column shows the amount of annuity necessary each year so that the Fixed Dollar Annuity would have adjusted to cost of living changes occurring during both the accumulation and the annuity period. The calculation was made by multiplying the Fixed Annuity by the ratio of the Cost of Living Index for the year to the average Cost of Living Index during the premium paying period.

† The Purchasing Power Comparison is obtained by dividing the Amount of Annual Annuity provided by each of the three funds by the Adjusted Cost of Living Annuity; 100% = constant purchasing power.

in 1920 before declining for two years and then leveling off at a plateau considerably above the prewar level. Neither the combined annuity nor the unit annuity supplied a satisfactory adjustment for the increasing lack in purchasing power from 1916 to 1924, although both supplied somewhat more dollars than the fixed annuity. From that point, however, even the combined 50–50 annuity increased sharply, reaching the "adjusted cost of living" annuity late in 1927 and staying above it until 1931.

1890–1940 Period

Over the first thirty years of this period, the fixed dollar fund would have grown to $6,226.00, and the common stock fund to $8,201.00.

The first ten years of retirement overlap with the same period in Table 1, and although the 50–50 combined annuity dropped considerably under the adjusted cost of living annuity in 1932, it recovered fairly rapidly and from 1934 to 1940 was very close to or above the adjusted cost of living

TABLE 2

AMOUNTS OF ANNUAL ANNUITY 1920–1940
PURCHASED BY ACCUMULATIONS OF $100 PER YEAR 1890–1920

YEAR	AMOUNTS OF ANNUAL ANNUITY			ADJUSTED COST OF LIVING ANNUITY*	PURCHASING POWER COMPARISON†		
	Fixed Annuity	Unit Annuity	Combined Annuity		Fixed Annuity	Unit Annuity	Combined Annuity
1920	$567	$ 717	$ 642	$1,208	47%	59%	53%
1921	567	635	601	1,077	53	59	56
1922	567	784	676	1,009	56	77	67
1923	567	811	689	1,028	55	79	67
1924	567	867	717	1,030	55	84	70
1925	567	1,071	819	1,057	54	102	78
1926	567	1,200	883	1,066	53	112	83
1927	567	1,420	994	1,046	54	135	95
1928	567	1,786	1,177	1,034	55	173	114
1929	567	2,240	1,404	1,033	55	217	136
1930	567	1,774	1,170	1,007	56	175	116
1931	567	1,158	863	917	62	126	94
1932	567	637	602	823	69	77	73
1933	567	825	696	779	73	106	90
1934	567	941	754	807	70	116	93
1935	567	1,012	790	827	69	123	96
1936	567	1,417	992	836	68	170	119
1937	567	1,432	1,000	866	65	164	115
1938	567	1,086	826	850	67	128	98
1939	567	1,172	870	838	68	141	105
1940	567	1,106	836	845	67	131	99

*, † See footnotes to Table 1.

annuity. It should be noted that again, as in Table 1, the combined annuity supplied more dollars in every year than the fixed dollar annuity.

1900–1950 Period

Over the first thirty years of this period, the fixed dollar fund would have grown to $6,499.00, and the common stock fund to $14,437.00.

Because the years 1929–1932 showed such a serious drop in common stock prices, it is important to note the effects on the various annuity amounts in Table 3 for a person retiring at about that time—un-

usual though it may have been. The participant in the combined fund would have experienced a rapid decrease in his annuity for a few years after his retirement, but even in 1932 the combined annuity was only moderately less than the fixed dollar annuity, and from that point forward it moved generally ahead of the fixed dollar annuity. Reflecting, however, that the participant received the benefit of the rise in prices during the

TABLE 3

AMOUNTS OF ANNUAL ANNUITY 1930–1950
PURCHASED BY ACCUMULATIONS OF $100 PER YEAR 1900–1930

YEAR	AMOUNTS OF ANNUAL ANNUITY			ADJUSTED COST OF LIVING ANNUITY*	PURCHASING POWER COMPARISON†		
	Fixed Annuity	Unit Annuity	Combined Annuity		Fixed Annuity	Unit Annuity	Combined Annuity
1930	$552	$1,263	$ 907	$ 724	76%	174%	125%
1931	552	824	689	659	84	125	105
1932	552	454	503	592	93	76	85
1933	552	587	570	560	98	104	101
1934	552	670	611	580	95	115	105
1935	552	720	636	595	93	121	107
1936	552	1,009	780	601	92	168	130
1937	552	1,019	786	622	89	164	127
1938	552	773	662	611	90	126	108
1939	552	834	693	602	91	137	114
1940	552	787	670	607	91	129	110
1941	552	736	644	638	86	114	100
1942	552	661	607	706	78	94	86
1943	552	874	714	749	74	117	96
1944	552	951	752	761	72	124	98
1945	552	1,150	851	778	71	148	110
1946	552	1,315	933	844	65	155	110
1947	552	1,169	860	965	57	121	89
1948	552	1,198	875	1,038	53	115	84
1949	552	1,194	874	1,025	54	117	86
1950	552	1,462	1,007	1,038	53	140	97

*, † See footnotes to Table 1.

1920's when he was in the premium period, this chart emphasizes the importance of continuing investment, year after year, if the long-range advantages of dollar cost averaging and diversification of common stock purchases over time are to be obtained through the medium of common stocks. And, moreover, the purchasing power of the combined annuity fell in 1932 only to 85% of the purchasing power of the premiums when paid.

1910–1950 Period

Over the first thirty years of this period, the fixed dollar fund would have grown to $6,150.00 and the common stock fund to $7,434.00.

It is interesting to note that the cost of living in 1940 was slightly lower than the average cost of living of the entire accumulation period from 1910 to 1940, but that it rose rapidly during the ensuing ten years. This table shows the material loss in purchasing power which has had to be faced by those individuals who retired in 1940 and who depended mainly on fixed dollar annuities. Although the combined annuity would not have adjusted

TABLE 4

AMOUNTS OF ANNUAL ANNUITY 1940–1950
PURCHASED BY ACCUMULATIONS OF $100 PER YEAR 1910–1940

YEAR	AMOUNTS OF ANNUAL ANNUITY			ADJUSTED COST OF LIVING ANNUITY*	PURCHASING POWER COMPARISON†		
	Fixed Annuity	Unit Annuity	Combined Annuity		Fixed Annuity	Unit Annuity	Combined Annuity
1940.....	$505	$ 650	$ 577	$490	103%	133%	118%
1941.....	505	608	556	514	99	119	109
1942.....	505	546	525	569	89	96	93
1943.....	505	722	613	604	84	120	102
1944.....	505	786	645	613	83	129	106
1945.....	505	950	727	628	81	152	117
1946.....	505	1,086	795	681	74	159	117
1947.....	505	966	735	778	65	124	95
1948.....	505	989	747	837	61	120	91
1949.....	505	987	746	826	61	119	90
1950.....	505	1,207	856	837	61	146	104

*, † See footnotes to Table 1.

perfectly, it has followed the changes in the cost of living fairly well, with the variations in either direction tending to about counterbalance each other.

In this example the impact of inflation on the individual's annuity was confined entirely to the period after retirement; a system such as relating the retirement benefit to final average salary would not have protected the participant against these changes.

Annuities from Accumulations over Shorter Periods

The foregoing comparisons are all based on thirty year accumulation periods. Obviously, there will be some persons who, for one reason or another, may start saving for their retirement at age 45 or even 55. Comparable figures were prepared, therefore, for twenty year and ten year ac-

cumulations, but in the interest of economy of space, all the details will not be presented here. The data again indicated that the combined annuity adjusted much more closely to the cost of living than the fixed dollar annuity.

Table 5 shows a summary of the results for the ten year accumulation periods. The accumulation period 1920–1930 and the following twenty year annuity period should be noted, since here the accumulation was made during rising prices and the annuity commenced when common stock prices were falling very rapidly. The year by year comparisons show

TABLE 5

COMPARISON OF ANNUAL FIXED ANNUITY AND AVERAGE COMBINED ANNUITY RESULTING FROM INVESTMENT OF $100 A YEAR FOR 10 YEARS

Accumulation Period	Annuity Period	Percent Average Combined Annuity to Fixed Annuity	Percent Needed to Compensate for Cost of Living Changes*
1880–1890.....	1890–1910	105%	103%
1890–1900.....	1900–1920	129	135
1900–1910.....	1910–1930	121	167
1910–1920.....	1920–1940	125	132
1920–1930.....	1930–1950	115	95
1930–1940.....	1940–1950	122	133

* Percent average cost of living during retirement to average cost of living during premium-paying stage.

that during the twenty years of annuity payments the combined annuity exceeded the fixed income annuity in all but four years.

The figures for annuities resulting from twenty year accumulation periods are not included, since they lead to no different conclusions from those for the longer and shorter periods.

Purchasing Power Comparisons

Tables 6 and 7 summarize the previous data in the form of percentile groups related to the purchasing power that would have resulted if the price level had been stable throughout the accumulation and annuity periods. Although there is overlapping in the annuity periods, this is on a uniform basis and extends from 1920 to 1950 in Table 6 and from 1900 to 1950 in Table 7.

During these periods, the fixed dollar annuity never exceeded 110% of the purchasing power base. It was between 90% and 110% on only ten of

the seventy possible occasions and less than 90% on sixty of the seventy occasions. The combined annuity was above 110% on thirteen of the seventy occasions, within the 90% to 110% range thirty-three times, and below on twenty-four occasions. Further analysis of the individual percentages revealed the disturbing fact that in half of the cases the annuitant would have received from his fixed dollar annuity less than two-thirds

TABLE 6

YEAR BY YEAR PURCHASING POWER OF ANNUITIES RESULTING FROM PREMIUMS OF $100 A YEAR FOR 30 YEARS
NUMBER OF YEARS IN WHICH THE AMOUNT OF ANNUAL ANNUITY WAS A GIVEN PERCENTAGE OF CONSTANT PURCHASING POWER VALUE

Accumulation Period	Annuity Period	Fixed Annuity			Combined Annuity		
		Under 90%	90% to 110%	Over 110%	Under 90%	90% to 110%	Over 110%
1880–1910....	1910–1930	20	0	0	11	8	1
1890–1920....	1920–1940	20	0	0	8	7	5
1900–1930....	1930–1950	12	8	0	5	11	4
1910–1940....	1940–1950	8	2	0	0	7	3
Totals...	.....	60	10	0	24	33	13

TABLE 7

PURCHASING POWER OF ANNUITIES RESULTING FROM PREMIUMS OF $100 A YEAR FOR 10 YEARS
NUMBER OF YEARS IN WHICH THE AMOUNT OF ANNUAL ANNUITY WAS A GIVEN PERCENTAGE OF CONSTANT PURCHASING POWER VALUE

Accumulation Period	Annuity Period	Fixed Annuity			Combined Annuity		
		Under 90%	90% to 110%	Over 110%	Under 90%	90% to 110%	Over 110%
1880–1890....	1890–1910	4	16	0	0	15	5
1890–1900....	1900–1920	17	3	0	3	12	5
1900–1910....	1910–1930	18	2	0	14	5	1
1910–1920....	1920–1940	18	2	0	8	7	5
1920–1930....	1930–1950	4	5	11	0	5	15
1930–1940....	1940–1950	8	2	0	4	6	0
Totals...	.....	69	30	11	29	50	31

of the purchasing power criterion. This would have happened in only 13% of the cases for the combined annuity.

Table 7 gives comparable figures for the ten year accumulations.

Further study of the individual percentages for this table showed that in 44 of the 110 instances the purchasing power of the fixed annuity was below 80%, as against 18 occasions for the combined annuity.

Periods Used in Comparisons

The selection of the dates for the beginning and end of the various accumulation and annuity periods, of course, affects the results, but some periods had to be selected and the ten, twenty and thirty year periods seemed representative. By using periods terminating in calendar years which end with zero, we were able to include five reasonably typical periods and two that were not. The periods ending in 1920 reflected the peak of the post-World War I inflation where all the funds showed poor results. The periods ending in 1930 were at a point near the speculative high of the stock market in 1929. Other periods and durations were studied, and where accumulation plus annuity periods extended over reasonable lengths of time we found no material differences from the patterns already shown.

Effects of the Funds during Deflation and Inflation

By selecting two sharply contrasting periods of deflation and inflation, Table 8 illustrates the effects that would have been achieved in accumulating funds for retirement over periods that (1) coincide with the worst periods from the standpoint of common stocks and (2) coincide with periods during the latter part of which there was a substantial inflation.

This table illustrates the widely different results that would have been achieved in different periods by individuals who had chosen to invest either entirely in a fixed dollar fund or entirely in a common stock fund.

Conclusions from Fund Comparisons

Since a person cannot be sure at the time he starts to invest for retirement as to the relative success of either the fixed dollar or common stock funds, it would seem prudent to consider a middle course. Although we have already shown that such a combined fund would not have shown perfect correlation with the cost of living, it nevertheless indicates that, on the average, it had a historical advantage over either of the other two methods in providing some hedge against both inflation and deflation.

Single Premium Annuities

Although single premium annuities can be administered along similar lines, there is a greatly increased risk to the individual introduced by rea-

son of the particular level of common stock prices at the moment the single premium payment is made. As an extreme example, the investment in a single premium annuity in 1932 would have produced a unit annuity under which the annual payments would have been about four times the size of the annual payments received after 1932 under a unit annuity purchased for the same single premium in 1929. Again, in the interest of space, tables are not shown on the results of investing in single premiums at various dates, but the reader can readily make the proportional adjustments to the accumulations and annuities shown in the previous section

TABLE 8

BALANCING EFFECTS OF FIXED DOLLAR AND COMMON STOCK FUNDS DURING THE 1932 DEFLATION AND THE 1950 INFLATION

ACCUMULATION PERIOD	PERCENT COST OF LIVING END OF PERIOD TO AVERAGE FOR PERIOD	PURCHASING POWER COMPARISON 100% = CONSTANT PURCHASING POWER		
		Fixed Dollar Fund	Common Stock Fund	Combined Fund
Deflation:				
1902–1932	103%	97%	73%	85%
1912–1932	89	112	78	95
1922–1932	81	123	65	94
1927–1932	84	119	46	83
Inflation:				
1920–1950	142	70	144	107
1930–1950	144	69	133	101
1940–1950	125	80	132	106

of "Fund Comparisons," since the annuity resulting from a single premium will vary in the same proportion as those shown in Tables 1–4.

Hence, although the year-by-year changes in the unit annuity would probably provide some correlation with changes in the cost of living, the level of payments would depend on the particular year in which the single premium was paid. This risk is minimized, of course, in the annual premium accumulating fund, for reasons already given.

Moving Average of Capital Gains and Losses

Because the year-to-year fluctuations in the cost of living index are generally less marked than the fluctuations in common stock prices, investigations were made as to the possibility of obtaining a better correlation

between the unit annuity and the cost of living by means of a graduation of the capital gains and losses in the annuity period. The allied problem of the effects of short-term or even daily fluctuations in market values was also studied. Several methods of treating these questions were investigated, but none produced material advantages. Hence, it was decided to base annuity unit values on market prices as of the end of March, which will be the end of the Fund's fiscal year.

ACCUMULATION UNITS

Premium payments made will buy accumulation units. The dollar value of each such unit will vary from month to month by reason of both realized and unrealized capital gains and losses and expenses of operation. Each premium payment, after deducting an expense charge, will therefore buy a different number of units as the value of a unit changes from time to time. Dividend earnings on accumulation units already held will be used to purchase additional accumulation units for the participant. Although there is no compelling objection to allowing dividend earnings to increase the value of each existing accumulation unit, it was thought that the purchase of additional accumulation units through dividends had two practical advantages:

1. Each participant will see his fund of units increased from time to time by reason of dividend earnings.
2. In the absence of capital gains and losses, the number of additional accumulation units bought from time to time by a series of regular premiums will be fairly constant, instead of a decreasing number of additional units which would result if dividends were to regularly increase the dollar value of each accumulation unit.

Accumulation units are valued by prorating C.R.E.F. funds allocated to such units (the Accumulation Fund) among the total number of outstanding units.

In order to avoid the multiplicity of individual calculations for each participant each month and to relieve the Fund of the requirement of a complete balance sheet on the first day of each month, all calculations involving accumulation units will be made only once a year, but the final effect will be the same as if such calculations were made each month. This is to be accomplished by a *series of monthly factors* which will be calculated each month to produce the rates of dividend earnings and capital gains and losses. By the appropriate combination of monthly factors it will be possible to calculate the accumulated effect at the end of each year (and at any other time) of the premiums, expenses, dividend earnings and capital gains and losses allocable to any participant and to determine the current

value as well as the total number of accumulation units then owned by him.

The calculations will be made by means of two factors, d_n and g_n, which will be determined from the following formulae:

Let d_n = net monthly dividend rate credited for month n to that part of the total funds eligible to participate in dividend earnings.

$$d_n = \frac{D_n^C - \frac{1}{24}\, e_y^D\,(A_{n-1}^C + A_n^C)}{A_{n-1}^C - L_{n-1}^C + E_n^F}, \tag{1}$$

where D_n^C = total dividends received in month n.

e_y^D = investment expense rate for the current fiscal year y (per dollar of mean assets). In the initial period, C.R.E.F. expenses will be covered by a management contract with T.I.A.A., expressed in terms of expense rates for the various types of expense.

A_{n-1}^C = value of total assets at end of month $n-1$.

L_{n-1}^C = value of total liabilities at end of month $n-1$, excluding Accumulation and Annuity Funds.

E_n^F = net of all funds added to, less withdrawals from, the Accumulation and Annuity Funds during the month n, as of the first day of month n.

Let $$g_n = \frac{\text{unrealized and realized capital gain in month } n}{A_{n-1}^C - L_{n-1}^C + E_n^F}. \tag{2}$$

Certain other factors will be found to be convenient. Some of these involve the accumulation of premiums for fractions of years. All premiums received within the usual grace period will start to participate in the Fund as of the end of the month in which they are received. They will be said to be "applied" when they begin to participate. All payments from the Fund will be made on the first day of a month.

Let a_y^P = current value of accumulation units at end of year y which have been purchased by a person from equal monthly premiums throughout such year, together with net capital gains and dividend income on such premiums only.

$$a_y^P = \mathrm{P}\,(1 - e_y^A)\left[1 + \sum_{m=1}^{11} \prod_{n=m}^{11} (1 + g_{n+1} + d_{n+1})\right], \tag{3}$$

where P = gross premium applied as of last day of each month.

e_y^A = operating expense rate in accumulation period for year y (per dollar of premiums). See remarks on e_y^D under formula (1).

$\prod_{n=m}^{11}$ = product of factors involving n, where n varies from m to 11.

For example,

$$\prod_{n=9}^{11}(1+g_{n+1}+d_{n+1}) = (1+g_{10}+d_{10})\ (1+g_{11}+d_{11})\ (1+g_{12}+d_{12}).$$

Let B_y^P = same as a_y^P, except that dividend income is excluded.

$$B_y^P = \mathrm{P}(1-e_y^A)\left[1+\sum_{m=1}^{11}\prod_{n=m}^{11}(1+g_{n+1})\right]. \tag{4}$$

Although (3) and (4) provide for equal monthly premiums, any increments or decrements in the regular premiums during the year can be easily handled by a similar formula for such increments or decrements.

Certain other factors involve the accumulation of dividend income or termination equities. These are:

Let γ_y^C = total net dividend income in the year y with net capital gains thereon to end of year.

$$\gamma_y^C = \left\{\sum_{m=1}^{11}\left[D_m^C - \frac{e_y^D}{24}(A_{m-1}^C + A_m^C)\right]\prod_{n=m}^{11}(1+g_{n+1})\right\} + D_{12}^C - \frac{e_y^D}{24}(A_{11}^C + A_{12}^C). \tag{5}$$

This assumes dividends received are invested as of the end of the month they are received.

Let γ_y^A = portion of (5) credited to Accumulation Fund.

$$\gamma_y^A = \left\{\sum_{m=1}^{11} h_m^A\left[D_m^C - \frac{e_y^D}{24}(A_{m-1}^C + A_m^C)\right]\prod_{n=m}^{11}(1+g_{n+1})\right\} + h_{12}^A\left[D_{12}^C - \frac{e_y^D}{24}(A_{11}^C + A_{12}^C)\right] \tag{6}$$

where $$h_m^A = \frac{F_{m-1}^A + E_m^A}{A_{m-1}^C - L_{m-1}^C + E_m^F}.$$

F^A_{m-1} = The Accumulation Fund at the end of month $m-1$. Its calculation is derived later in the section titled "Fund Accounts."

E^A_m = The net of all funds added to, less withdrawals from, the Accumulation Fund during the month m, as of the first day of month m.

Let V^{TPA}_m = current value of accumulation units held as of the end of month m by a person whose accumulation units are terminated for any reason as of such date.

$$V^{TPA}_m = V^{PA}_{y-1}\prod_{n=1}^{m}(1+g_n+d_n) + \mathrm{P}(1-e^A_y) \times\left[1+\sum_{x=1}^{m-1}\prod_{n=x}^{m-1}(1+g_{n+1}+d_{n+1})\right], \tag{7}$$

where V^{PA}_{y-1} = current value of accumulation units at end of year $y-1$ held by a person at that time.

Let

$$T^{PA}_y = V^{TPA}_m\prod_{n=m}^{11}(1+g_{n+1}), \tag{8}$$

where T^{PA}_y = current value of accumulation units held as of the end of month m by a person whose accumulation units are terminated for any reason as of such date, together with net capital gains thereon to the end of year y.

Value of One Accumulation Unit

Let V^A_y = current value of one accumulation unit at end of year y.

At the end of the first year of operation, it is planned to set the value of V^A at \$10.00. The first unit value established can be any empirical figure and this amount was decided on as a practical compromise between a small figure to produce a sizable number of units for a typical participant and a larger figure to minimize the arithmetic in calculations involving the number of units as the total fund grows and to provide a reasonable number of significant digits in fund valuations.

Subsequent to the first year of operation, the value of an accumulation unit at the end of any year will be obtained by dividing (1) the dollar amount of that part of the accumulation fund at the end of the year which would correspond to the accumulation units that were in force at the beginning of year by (2) the number of such units in force at the beginning of the year. This formula, which eliminates the effects of the current year's

dividend income, premiums and terminations, was used because the new accumulation unit value at the end of the year depends on a number of units and the part of the fund then supporting them. Also, in order to find the net number of all units added to the fund during the year it is necessary to know the unit value at the end of the year. However, the unit value is a function of a number of units and hence it is necessary to exclude the effects of those factors affecting the change in the *number* of units during the year. Bearing in mind that all dividend income and premiums during the year are used to purchase additional units, and making the necessary adjustment for the terminations during the year, the formula becomes

$$V^A_{y+1} = \frac{f^A_{y+1}\,(A^C_{y+1} - L^C_{y+1}) - (\gamma^A_{y+1} + \Sigma B^P_{y+1} - \Sigma T^{PA}_{y+1})}{\Sigma N^{PA}_y}, \qquad (9)$$

where f^A_{y+1} = proportion which the Accumulation Fund is of the total of the Accumulation and Annuity Funds at the end of year $y+1$. Its calculation is derived later in the section titled "Fund Accounts."

N^{PA}_y = number of accumulation units held by a person at the end of year y.

Total Number of Accumulation Units at End of Fiscal Year

At the end of the first year of operation, assuming this to be at the end of year y, the current value of all accumulation units will be equal to

$$\Sigma V^{PA}_y = \Sigma B^P_y + \gamma^A_y - \Sigma T^{PA}_y .$$

Hence, the total number of accumulation units at the end of the first year will be given by

$$\Sigma N^{PA}_y = \frac{\Sigma B^P_y + \gamma^A_y - \Sigma T^{PA}_y}{10}. \qquad (10)$$

After the first year, the formula becomes

$$\Sigma N^{PA}_{y+1} = \Sigma N^{PA}_y + \frac{\Sigma B^P_{y+1} + \gamma^A_{y+1} - \Sigma T^{PA}_{y+1}}{V^A_{y+1}}. \qquad (11)$$

Number of Accumulation Units Added for a Person at the End of Each Fiscal Year

At the end of the first year of operation, this will be equal to $a^P_y \div 10$ (see formula 3). This includes units purchased by both premiums and dividends.

For any person remaining in the accumulation period throughout any

subsequent fiscal year, the number of accumulation units added to his account at the end of such year, say ending at $y + 1$, will be made up of two parts.

The first part, a^P_{y+1} / V^A_{y+1}, will come from premiums paid during the year, together with dividend income and capital gains or losses on such premiums and income. The balance of the accumulation units added to the person's account arises from dividend income, with capital gains or losses on such income, earned on the person's accumulation units held at the end of the previous year. The complete formula for both parts, then, is

$$N^{PA}_{y+1} - N^{PA}_{y} = \frac{a^P_{y+1} + N^{PA}_{y} \cdot V^A_y \left[\prod_{n=1}^{12} (1 + g_n + d_n) - \prod_{n=1}^{12} (1 + g_n) \right]}{V^A_{y+1}}. \quad (12)$$

ANNUITY UNITS

At the time of an individual's retirement, the number of accumulation units held by him are exchanged for a contract promising to pay him each month the current value of a fixed number of annuity units. The number of annuity units established at an individual's retirement is, in general, obtained by dividing the dollar value of his accumulation units at retirement by the present value of a life annuity, payable in the amount of the current value of one annuity unit per month. An adjustment is made to take into account anticipated future expenses. The number of annuity units so established will be payable to the annuitant each month during his lifetime, and the check received by the annuitant in any month will be the current value of the annuity units then payable. This involves assumptions as to interest (or dividends), mortality and expense. Inasmuch as any variations from these assumptions in the actual experience of the Fund will be automatically compensated for each time a new current annuity unit value is calculated, it is planned to inject little, if any, conservatism in these assumptions. Consistent with this approach, it is planned, for instance, to use approximate current mortality levels with appropriate assumptions as to future improvement.

Because the month to month fluctuations in the cost of living generally tend to be fairly small, it is planned to change the current dollar value of the annuity unit only once a year, thus giving annuitants a better opportunity to budget their expenses over the ensuing twelve month period. This also has an advantage from the standpoint of the Fund's operating expenses, since it will avoid the multiplicity of individual calculations for each participant each month.

The value of each annuity unit will vary from year to year in accordance with dividend earnings, capital gains and losses, mortality experience, and expenses. Normally the value of an annuity unit will not be the same as the value of an accumulation unit, since, in addition to the factors affecting the value of accumulation units, the value of annuity units will also be affected by the actual mortality experience of C.R.E.F. annuitants.

The value of an annuity unit at the end of each fiscal year is, in general, obtained by dividing the part of the allocated funds then supporting annuity units (the total Annuity Fund less the provisions made for meeting all future expenses of the Annuity Fund) by the present value of the total number of annuity units expected to be paid over the future lifetimes of all participants then receiving annuity payments, in accordance with the assumptions made as to mortality, dividend earnings, and expense rates for the future.

Aside from the introduction of the mortality element, different treatment will be accorded to accumulation units and annuity units as to expenses and dividend earnings, as they affect the values of such units. During the accumulation period, expenses will be assessed as a percentage of premiums paid. During the annuity period, expenses will be assessed as a different percentage of annuity payments made. Any variation in the expenses assessed against the Annuity Fund in any fiscal year from the anticipated provisions for expenses will automatically increase or decrease the value of existing annuity units. Dividend earnings which are greater or less than those assumed in fixing the number of annuity units will be used to increase or decrease the value of existing annuity units, whereas all dividend earnings allocable to accumulation units will be used to buy additional accumulation units.

Capital gains and losses allocable to the Annuity Fund throughout any fiscal year will automatically be taken into account in determining the annuity unit value for the succeeding fiscal year, since they will either increase or decrease the Annuity Fund and will be distributed over the future lifetimes of all annuitants in the Annuity Fund at the time the new annuity unit value is determined. This point is developed further in later sections.

Because the determination of both accumulation and annuity unit values depends on subsequent analysis of any fiscal period, there may be a lag of up to a month or two in announcing changes in unit values. In the case of accumulation units this will not affect the number of units credited as of any date while the participant remains in the accumulating period. However, it may mean the temporary continuation of the previous year's

value in the case of annuity units. This should not produce any serious consequences, however, because changes in annuity unit values will usually be gradual. Even in periods of rapidly changing prices, however, such "overpayments" or "underpayments" for a month or two will have relatively little effect on subsequent annuity unit values, since any such temporary "overpayments" or "underpayments" would be spread over the entire future lifetimes of all annuitants at the next valuation date.

Because the valuation of accumulation unit values and annuity unit values depends on the actual assets, no unassigned surplus or general contingency reserve is necessary for the Equities Fund nor are they desirable.

The calculations for annuity units can be made from the following formulae:

Value of One Annuity Unit

Let V_y^B = current value of one annuity unit at end of year y. During the first year of operation, it is planned to set the value of V^B at \$10.00. Again, the first annuity unit value established can be any empirical figure and was fixed at \$10.00 for practical reasons similar to those used in fixing the first accumulation unit value.

For all years of operation, the value of one annuity unit at the end of year y, for use in the following twelve months, will be given by

$$V_y^B = \frac{(1 - e_{y+1}^B) F_y^B}{12 \Sigma N_x^{PB} \ddot{a}_x^{(12)}}, \tag{13}$$

where e_{y+1}^B = operating expense rate for fiscal year ending in calendar year $y + 1$ (per dollar of anticipated annuity payments before such expense deduction). See remarks on e_y^D under formula (1).

F_y^B = net assets allocated at y for annuity units. This is the "Annuity Fund" and will be derived later.

N_x^{PB} = number of annuity units payable each month to a person now age x.

The denominator of formula (13) will be suitably modified for types of annuities involving more than single life contingencies.

Determination of the Number of Annuity Units When a Person Commences His Annuity Payments at End of Any Fiscal Year y

With the first annuity payment to be made as of y, the formula for converting his accumulation into annuity units is

$$N_x^{PB} = \frac{(1 - e_{y+1}^B)\ (V_y^{PA})}{12\ (V_y^B)\ \ddot{a}_x^{(12)}}. \tag{14}$$

The denominator of formula (14) will be suitably modified for types of annuities involving more than single life contingencies. The same formula can be used for any settlement option involving life contingencies where the beneficiary is to commence receiving benefit payments at the end of any fiscal year y, inasmuch as the benefit at the death of the participant is equivalent in value to the accumulation. In this case, of course, the age x in $\ddot{a}_x^{(12)}$ of the denominator is the beneficiary's age.

The formula for the number of annuity units per month, purchased by a single premium at the end of year y, with the first annuity payment to be made as of one month after y, is

$$N_x^{PB} = \frac{(1 - e_{y+1}^B)\,(\mathrm{NP})}{12\,(V_y^B)\,a_x^{(12)}}, \tag{15}$$

where NP = gross premium paid less loading. It is anticipated that the loading charged will cover only the first year acquisition expenses.

Determination of the Number of Annuity Units When a Person Commences His Annuity Payments at an Interim Month in the Fiscal Year

An adjustment is necessary to formulas (14) and (15) in order to preserve equity among all annuitants, regardless of the date they entered the Annuity Fund. Inasmuch as persons in the Annuity Fund at a given date who entered the Annuity Fund in prior years will receive annuity payments in the current fiscal year which are not affected by the current year's capital gains or losses or by the difference between the current year's net dividend income rate and the assumed rate used in the valuation of annuity units, the use of a "Conversion Value" will be found useful for transfers from the Accumulation Fund to the Annuity Fund at interim dates in the fiscal year. This "Conversion Value" will be equivalent to what the current value of a person's accumulation units would be at the date of transfer, if it is assumed that after the first day of the current fiscal year there were no capital gains or losses and the net dividend income was the rate (i) assumed in the valuation of annuity units.

If we define $(\mathrm{CV})_m^{PA}$ as the "Conversion Value" of a person's accumulation units at the end of an interim month m in the fiscal year ending at $y + 1$, the formula for converting his accumulation units into annuity units is

$$N_x^{PB} = \frac{(1 - e_{y+1}^B)\,(\mathrm{CV})_m^{PA}}{12\,(V_y^B)\,\ddot{a}_x^{(12)}}, \tag{16}$$

where x is the age at the end of the interim month m, and

$$(\mathrm{CV})_m^{PA} = \frac{V_y^{TPA}(1+i)^{m/12}}{\prod_{n=1}^{m}(1+d_n+g_n)}. \tag{17}$$

Similar calculations will be made for beneficiaries commencing annuity payments under any settlement option involving life contingencies and the term $\ddot{a}_x^{(12)}$ will be suitably modified for other forms on annuity.

The similar calculation for a person paying a gross single premium during an interim month m in the fiscal year ending at $y+1$, is

$$N_x^{PB} = \frac{(1-e_{y+1}^{B})(\mathrm{NP})(1+i)^{m/12}}{12(V_y^{B})\,a_x^{(12)}\cdot\prod_{n=1}^{m}(1+d_n+g_n)}. \tag{18}$$

The excess of the current value of accumulation units converted over the value of $(\mathrm{CV})_m^{PA}$ given in formula (17) is to be included in the Annuity Fund, which will be used in the new valuation of annuity units at the end of the current fiscal year. The similar excess in the case of gross single premiums will be likewise applied. These excesses may, of course, be negative.

The formulas for N_x^{PB} at interim dates are approximate, in that mortality fluctuations are ignored in the part of the current year prior to the interim date. Any error so introduced is minimal, however, and it was decided for practical reasons to omit this factor from the calculations.

In the early years when there will be only a comparatively few participants in the Annuity Fund, it may be advisable to apply appropriate adjustments if actual annuity mortality exceeds considerably the mortality expected from the death rates assumed in the calculation of annuity unit values. This can probably be best handled on the basis of judgment at the ends of the early fiscal years, with the objective of making an equitable actuarial distribution over a short period of years. After the number of participants in the Annuity Fund becomes sufficient, mortality adjustments would be automatically made through the foregoing formulas for calculating annuity unit values. This has the effect of spreading the mortality fluctuations in any one year over the future lifetimes of all participants in the annuity period.

FUND ACCOUNTS

The general relationship at all times is given by

$$F_y^A + F_y^B = A_y^C - L_y^C, \tag{19}$$

where F_y^A is the Accumulation Fund and F_y^B is the Annuity Fund.

Accumulation Fund at End of Any Month m

F_m^A = (F_{m-1}^A + net of funds added to, less withdrawals from, F_{m-1}^A during the month m as of the first day of month m) $(1 + g_m + d_m)$ + net of all other funds added to, less withdrawals from, the Accumulation Fund as of the end of month m. (20)

The value of f_m^A can thus be obtained each month by the formula

$$f_m^A = \frac{F_m^A}{A_m^C - L_m^C}. \tag{21}$$

Once a year, a check on the numerical accuracy of the operations of the Accumulation Fund should be made by testing the following relationships at the end of the fiscal year:

$$F_y^A = \sum V_y^{PA} = \sum N_y^{PA} \cdot V_y^A. \tag{22}$$

Annuity Fund at End of Any Month m

F_m^B = (F_{m-1}^B + net of funds added to, less withdrawals from, F_{m-1}^B during the month m as of the first day of month m) $(1 + g_m + d_m)$ + net of all other funds added to, less withdrawals from, the Annuity Fund as of the end of month m. (23)

The value of f_m^B can thus be obtained each month by the formula

$$f_m^B = \frac{F_m^B}{A_m^C - L_m^C}. \tag{24}$$

I wish to express my gratitude to all who assisted in the preparation of the paper, particularly to Joseph B. Maclean and Wilmer A. Jenkins for their critical reading of various sections, and to William C. Greenough, Vice President of T.I.A.A., with whom it was my pleasure to work in compiling the historical data.

APPENDIX

Yields and Index Prices*

Year	Life Ins. Co. Net Yields	Common Stock Net Yields	Common Stock Price Index	Cost of Living Index
1880	5.28%	4.53 %	41.9	57
1881	5.31	4.59	50.3	59
1882	5.33	4.93	47.5	61
1883	5.32	5.44	45.4	57
1884	5.25	6.06	38.1	54
1885	5.21	4.84	37.0	53
1886	5.17	3.60	43.2	54
1887	5.24	3.99	44.5	54
1888	5.21	3.93	41.9	55
1889	5.03	3.63	42.8	55
1890	4.95	3.76	42.4	55
1891	5.08	4.03	40.5	54
1892	5.06	3.91	44.7	54
1893	4.86	4.78	38.4	53
1894	4.79	4.37	35.3	52
1895	4.79	3.72	36.4	52
1896	4.69	3.90	34.1	52
1897	4.64	3.65	35.8	53
1898	4.65	3.47	40.7	53
1899	4.59	2.96	50.6	54
1900	4.33	4.25	49.5	56.6
1901	4.35	3.60	63.1	58.0
1902	4.38	3.46	67.7	56.4
1903	4.38	4.40	58.1	62.2
1904	4.40	3.93	56.7	61.5
1905	4.40	3.28	72.3	61.5
1906	4.26	3.71	77.6	63.6
1907	4.50	5.13	63.1	67.2
1908	4.46	4.68	62.6	64.3
1909	4.54	4.06	78.2	64.3
1910	4.55	4.59	75.2	67.9
1911	4.59	4.67	74.3	67.9
1912	4.59	4.60	76.7	72.1
1913	4.67	5.12	68.5	70.7
1914	4.69	4.76	65.0	71.8
1915	4.77	4.73	66.8	72.5
1916	4.80	5.37	76.3	77.9
1917	4.81	7.57	68.3	91.6
1918	4.72	6.99	60.7	107.5
1919	4.66	5.50	70.7	123.8
1920	4.83	5.88	64.2	143.3
1921	5.02	6.24	55.2	127.7
1922	5.12	5.55	67.7	119.7
1923	5.18	5.69	69.0	121.9
1924	5.17	5.62	72.8	122.2
1925	5.11	4.94	89.7	125.4
1926	5.09	5.07	100.0	126.4
1927	5.05	4.52	118.3	124.0
1928	5.05	3.73	149.9	122.6
1929	5.05	3.23	190.3	122.5

* Sources shown on page 322.

APPENDIX—*Continued*

Year	Life Ins. Co. Net Yields	Common Stock Net Yields	Common Stock Price Index	Cost of Living Index
1930	5.05%	4.01%	149.8	119.4
1931	4.93	5.33	94.7	108.7
1932	4.65	6.44	48.6	97.6
1933	4.25	3.80	63.0	92.4
1934	3.92	3.67	72.4	95.7
1935	3.70	3.63	78.3	98.1
1936	3.71	4.10	111.0	99.1
1937	3.69	4.62	111.8	102.7
1938	3.59	4.08	83.3	100.8
1939	3.54	4.20	90.1	99.4
1940	3.45	5.95	83.8	100.2
1941	3.41	7.74	75.8	105.2
1942	3.40	5.60	65.7	116.5
1943	3.29	4.73	87.0	123.6
1944	3.19	4.42	94.5	125.5
1945	3.07	3.42	115.1	128.4
1946	2.92	4.07	132.5	139.3
1947	2.88	5.07	116.5	159.2
1948	2.96	5.77	117.8	171.2
1949	3.04	6.48	115.0	169.1
1950	3.09	6.96	138.6	171.2