Revisiting Pension Actuarial Science:
A Five-Part Series

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Abstract for the Series

The current financial model put forth as the market value of public sector pension benefit liabilities is simply the expected cash flows of the accumulated benefit obligation, as defined for current private sector financial reporting, discounted using a risk-free yield curve. This model is in serious need of an overhaul. It fails to faithfully represent the fair value of a currently accrued public sector pension benefit liability in three important ways:

1. Its use of the accumulated benefit obligation cash flows fails to accurately represent the terms of the employment contract which gives rise to the obligation being valued – a violation of labor economics principles.

2. Its use of expected cash flows as if they were fixed fails to recognize the risk premium load, which a fair exit price would include for the potential for adverse cash flow experience – a violation of actuarial finance and pricing principles.

3. Its use of risk-free discount rates fails to adequately reflect the observable and not-so-observable inputs from market participants' behavior – a violation of financial engineering principles.

Parts 1 through 3 in this series propose solutions to these three flaws.

Part 4, "The Residual Benefit Liability," presents an alternate approach to obtaining the fair value of the public sector employer's pension benefit liability. It approaches the task by modeling the real world operation of the pension fund, rather than approaching the task from the perspective of a theoretical construct. This alternate approach dares to model the long-term agency operation of the plan rather than ignoring it in favor of a pass-through approach. The current model ignores the effectiveness (even the existence) of the pension fund itself, while the alternate approach attempts to model the plan's operation in practice over time in order to determine the employer's residual asset or liability.

In spite of these three improvements and the alternate model, we believe the fair value of public sector post-employment benefit liabilities has little to no usefulness in most venues. There are legitimate roles which the market or fair value might play in valuing an individual member's personal wealth, a minor role in the context of certain discussions concerning risk measurement and risk management, and a major role in the context of plan terminations and freezes.

However, for purposes of advance funding, taxpayers, financial reporting, lenders and rating agencies, comparability, and the major part of risk measurement and analysis, the decision-usefulness of market or fair value is negligible, possibly even misleading. Other existing models and methods are far more suitable for these purposes, including conventional actuarial approaches and others that are less conventional or popular, but which should be considered in the actuarial toolbox and have higher decision utility.

Part 5 in this series, "Consider the Measurement Purpose," addresses various purposes for measuring a public sector pension liability and which measures have the most practical usefulness.

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Introduction to the Series

The market value of public sector pension benefit liabilities lacks substantive decision utility. Nevertheless, it is entirely possible that actuaries might be required to calculate a market or fair value of such liabilities. This requirement might be imposed upon actuaries and upon public sector employers and plans by the Governmental Accounting Standards Board or the U. S. Securities and Exchange Commission, the Actuarial Standards Board or, indirectly, by the American Academy of Actuaries. Even less likely, such a requirement could be imposed by public sector plans and employers themselves or by the marketplace.

The model commonly put forth as the market value of pension liabilities is the sum of the expected cash flows of the accumulated benefit obligation (per private sector financial reporting standards) discounted using a risk-free yield curve observed as of the measurement date. We believe this model and its resultant value of the pension liability have limited usefulness, if any. Even viewing this model from the perspectives of financial engineering and actuarial finance, it is a poor representation of the fair value price of the pension liability.

With respect to pension liabilities, David Wilcox (2008), an economist with the Federal Reserve Board, recently testified:

> "The economics of how cash flows with no credit risk should be discounted back to the present are completely unambiguous and utterly noncontroversial. Those cash flows should be discounted back to the present using interest rates that are derived from securities with no credit risk. Every first year MBA student, even as we speak, is having this simple point drilled into their head right now in an introductory finance class. The only factors that matter for the determination of the scale of these obligations are the size of the promised cash flow and their essential characteristic which is that they are free of risk. That's all you need to know. These are riskless cash flows. There's an unambiguous answer as to what their value today is. What I'm trying to suggest, over and over again, is that the analytics of valuing cash flows that have no credit risk in them – those analytics are very straightforward. There's no professional dispute associated with that question. These happen to be really simple cash flows to value. They're free of credit risk. There's only one conceptually right answer to how you discount those cash flows. You use discount rates that are free of credit risk. This is one of those things where it's just really is that simple."

This sounds so easy, even a caveman could do it. This testimony is exactly why we must revisit pension actuarial science. If actuaries will be required to calculate and report the market value of public sector pension benefit liabilities, we must give the current model described by Wilcox an overhaul. What he described is very simple; too simple for financial modeling and too simple for any actuary charged with determining the fair values of public sector pension liabilities.

In his textbook Derivatives, Paul Wilmot (1998) provides sage advice "[E]very financial axiom I've ever seen is demonstrably wrong…The real question is how wrong is the theory, and

how useful is it regardless of its validity. Everything you read in any theoretical finance book, including this one, you must take with a generous pinch of salt."

Parts 1 through 3 in this series will revisit the current model of the market value of public sector pension benefit liabilities and propose three important and substantive improvements to better reflect the principles of labor economics, actuarial finance and pricing, and financial engineering and in a fair value pricing model. Part 4, "The Residual Benefit Liability," will also present an alternative model for the fair value of a public sector employer's pension benefit liability. Part 5 in this series, "Consider the Measurement Purpose," will explore the venues of usefulness for a fair value of public sector pension liabilities and propose more useful measures of the liability appropriate to various purposes.

A. Measurement Attribute Terminology

Recent literature and media coverage have made much of the notion that all of an employer's assets and liabilities should be valued and reported (in financial statements) at market, including its pension liabilities, and regardless of whether the employer is in the private sector or the public sector. While certainly an opinion held by more than a few, it is a fairly narrow and ideological position. There is a rich history and ongoing discussion among accountants and their financial-statement standards setters on measurement attribute models beyond just market value.

Measurement attribute models for the financial reporting of a public sector entity's assets and liabilities are currently under reconsideration. The Governmental Accounting Standards Board (GASB) is deliberating a conceptual framework project on recognition and measurement attributes. These may include initial transaction date-based measurement (initial value) and current financial statement date-based measurement (remeasured value) such as fair value, current acquisition, sale and/or settlement price, replacement costs, and value-in-use. These may vary depending on whether the assets or liabilities are used in the provision of services or not. There may also be an exception considered for assets that will be held to maturity. We are still some time away from having a final concept statement from GASB on recognition and measurement attributes. Even then, we should not be surprised if liabilities for postemployment benefits have a separate type of attribute for recognition and measurement when a new and final accounting standard on the topic is adopted.

Nevertheless, Parts 1 through 4 in this series will be limited primarily to discussions around market or fair values.

Usually and historically, proponents of a financial economics approach to public pension benefit liability measurement use the term, "market value" of liabilities. In this paper we will use the term "market value" of liabilities to refer to the current model proposed by those with a financial economics approach to public pension benefit liability measurement, primarily because that is the term they have used over the last few years. Unfortunately, the term market value is not the current term of preference in financial reporting circles. We should be using the term "fair value." This is significant because we are endeavoring to assign prices to liabilities that are not traded in any market.

We get most of what we know about the specifics of the terms "market value" and "fair value" from the worlds of financial reporting and pricing. The valuation of an employer's assets and liabilities is usually undertaken for the purposes of financial reporting or pricing (for financial transaction purposes). Hence, in developing a faithful model and definition for "fair value," actuaries should be looking to the world of financial reporting for its terminology; thus, our preference for "fair value."

The GASB sets generally accepted accounting principles (GAAP) and standards for governmental entities. One current project in the research phase addresses fair value measurement. GASB also has a major project being deliberated on postemployment benefit accounting and financial reporting. In short, it may be a long time before the GASB adopts amendments to existing standards for financial reporting of liabilities for public sector postemployment benefit obligations, including their measurement attribute, recognition and disclosure requirements.

The Financial Accounting Standards Board (FASB), which sets GAAP standards for private sector and not-for-profit entities, recently adopted a major standard on fair value measurements, Statement of Financial Accounting Standard (SFAS) No. 157.

Paragraph C50 of SFAS No. 157 states that FASB deliberately chose not to use the term "fair market value." Instead, FASB chose "fair value" for the purpose of financial reporting of certain assets and liabilities. SFAS No. 157 represents the most current authority on fair value for U. S. reporting entities. While it neither applies to postemployment benefit liabilities nor to certain other assets and liabilities, yet SFAS No. 157 is useful in guiding our opinions of a fair value model for pensions.

Paragraph 7 of SFAS No. 157 states, "A fair value measurement assumes that the asset or liability is exchanged in an orderly transaction between market participants to sell the asset or transfer the liability at the measurement date…..Therefore, the objective of a fair value measurement is to determine the price that would be received to sell the asset or paid to transfer the liability at the measurement date (an exit price)."

Since there is no real market for public sector pension liabilities, there is no true mark-to-market concept. It is more of a mark-to-model concept. The accounting field, by way of FASB pronouncements gives some guidance on fair value, which provides some secular help as we develop a model for the fair value of the public pension benefit liability.

In the end, fair value is about pricing. In the absence of a market to observe, any acceptable model for fair value of public pension benefit liabilities must envision the market players and pricing principles they might likely employ. The field of financial engineering has developed models, not the least of which is the original Black-Sholes model and its variants, for pricing and valuation of financial instruments in the marketplace. This too will be useful as we seek to propose a true fair value model. Our job is to imagine the operation of a market (a gedanken experiment) where public pension benefit liabilities are bought and held or sold for gain, and apply financial engineering principles to develop a fair value model that describes the operation and prices in such an hypothetical market.

Particularly for uncharted or illiquid markets, Emanuel Derman (2004) observes, "So much of financial modeling is an exercise of the imagination…To estimate the value of an illiquid security, you find a set of similar liquid securities, with known market prices, whose payouts match those of illiquid security under all circumstances. The best estimate for the value of the illiquid security is then the value of the set of liquid securities with the same payout…Models are only models, not the thing itself. We cannot, therefore, expect them to be truly right. Models are better regarded as a collection of parallel thought universes you can explore. Each universe should be consistent, but the actual financial and human world, unlike the world of matter, is going to be infinitely more complex than any model we make of it….You must always ask: Does the model give you a set of plausible variables to describe the world…A little hubris is good. Catastrophes strike when people allow theories to take on a life of their own and hubris evolves into idolatry. Somewhere between these two extreme's a little north of common sense but still south of idolatry, lies the wise use of conceptual models. It takes judgment to draw the line.

It is instructive to examine different valuation techniques. Paragraph 18 of SFAS No. 157 requires "Valuation techniques consistent with the market approach, income approach, and/or cost approach shall be used to measure fair value." Even under the income approach (Life Practice Council, 2008), a risk-neutral approach has a number of apparently unrealistic properties, is merely one tool for valuing financial instruments, and may be relevant when the exit market consists of financial institutions other than insurance companies, many of which typically use risk-neutral methods to price their products.

However, in the case of exchange transactions to transfer a pension liability, the principal market is, arguably, the current single premium group annuity market where the players are limited to a handful of insurance companies, while the most advantageous market may be the other public sector pension funds. Imagine a market whose market participants are hundreds of public sector pension funds (including large statewide plans) which buy and hold or sell pension liabilities from each other for gain.

Finally, there is also the valuation premise to consider: whether the fair value should be based upon a value-in-exchange or a value-in-use premise. Part 5 of this series addresses the valuation premise.

Much more can and should be explored before all these relevant valuation parameters are chosen for fair valuation of public sector pension benefit liabilities. This is the job of standards setters.

Under a fair value model, the public sector pension benefit liability is viewed as if it were a financial instrument, with no market and whose fair value must be derived on a theoretical basis. In the private sector, the values placed on illiquid financial securities have a significant effect on the company's earnings, its stock price, and the bonuses of the traders that management them (Derman 2004). Over the last several months we have seen first-hand how these values have a significant effect even on the continued existence of the company itself. The implications in the public sector are no less serious. With so much riding on public sector postemployment benefit calculations, a dose of humility is in order. Such models of fair value are mere theoretical

contrivances. In terms of financial reporting criteria, the market or fair value fails in the categories of relevance, reliability and interperiod equity, especially considering their magnitude. Other measurement attributes (besides fair value) are more appropriate. Again, refer to Part 5 for more details.

A fair value attribute for public pension benefit liabilities was seriously considered by the GASB over 25 years ago, during the decade leading up to the adoption of Statement Nos. 25 and 27 (1994). GASB's board members chose to go in another direction. In current deliberations[1], the GASB has indicated it is open to the idea of a mixed attribute model for assets and liabilities reported in financial statements. In the event GASB chooses, again, not to apply a fair value attribute to pension liabilities and not to include such a value in disclosures, this issue should be dead on arrival.

However, it could go the other way with the GASB. Separately, the Actuarial Standards Board (ASB), through its actuarial standards of practice (ASOPs), could impose a requirement upon actuaries to calculate a marked-to-model fair value and to include such a calculation in relevant actuarial communications. Either of those two actions might keep the fair value attribute for public pension liabilities alive.

Recently, the term "economic value" was used in high profile pronouncements[2] by the American Academy of Actuaries (AAA) in regard to a request to the ASB. This is an unfortunate shift in terminology because "economic value" has little if any historical authoritative or definitive basis for use. We are unaware of any standard setting body or other governing authority, which has defined economic value in any meaningful way. We think we know what the AAA meant (namely, the current market value model) since their statements were the culmination of a number of iterations and revisions among AAA staff and committees, which did use the term market value. Nevertheless, it appears to be a deliberate shift in terminology.

Fortunately, the ASB is a quasi-independent standard-setting body and has always been careful to define its own terms in sufficient detail for practicing actuaries. Whether and how the ASB will address this issue remains to be seen. Settling this will take time, and if a model with the label "economic value" is to have any contextual meaning, it must address the same three improvements and alternate model we propose in this series.

The use of the term economic value might be a convenient shift in order to deflect the reasons being posited, which the current model's failures in satisfying an honest measure of market or fair value. These failings have been raised in various literature and venues, and are set

[1] The Governmental Accounting Standards Board met on Nov. 4-6, 2008 to discuss, among other topics, the progress of its project on Conceptual Framework: Recognition and Measurement Attributes. According to Boaz (2009), the Board tentatively supported staff's recommendation as a path forward and as an approach that would not exclude a mixed-attribute model at this time. Each element would be evaluated as to the appropriate measurement attribute to use. Of course, this position of the Board could change.

[2] The Public Interest Committee (PIC) of the American Academy of Actuaries (AAA) issued a formal statement shortly after its meeting on Sept. 11, 2008. It stated that "it is in the public interest for retirement plans to disclose consistent measures of the economic value of plan assets and liabilities." Similarly, with input from the PIC, the AAA Board of Directors (BOD) asked the Actuarial Standards Board (ASB) "to develop standards for consistently measuring the economic value of pension plan assets and liabilities."

forth in substantive detail in Parts 1 through 4 of this series. In any event, for the purpose of this series, the term market value will continue to be used for the current model, and can serve as a surrogate tem for economic value because they both mean exactly the same thing in their actual usage by proponents.

One of the reasons market value and fair value of pension benefit liabilities have only limited utility in real world applications is that they are more of a theoretical construct, which interests only ideological purists. Nevertheless, it is possible that actuaries might be required to calculate the fair value of the public pension benefit liabilities.

Recognizing that possibility, we must be true to the term "fair value" in our model.

B. Improve the Current Model

The current model generally accepted as the market value of the public sector pension benefit liability as of a given measurement date is simply the present value of the expected cash flows of the accumulated benefit obligation (ABO per SFAS No. 87) as of the measurement date, discounted using a risk-free yield curve observed at the measurement date.

In financial engineering, we have learned that no model is perfect. In fact, some are found not to be even close. We must continually explore ways to improve, recalibrate and revisit our financial pricing models to ensure they fairly represent the fair values of the same or similar assets and liabilities under examination. Immature, erroneous or inappropriately applied models spell doom for financial and other institutions, which rely upon them and disclose them to various publics. There are serious unintended consequences for a company or government, even for a whole sector or the entire economy for wholesale reliance on flawed financial pricing models or, worse yet, on the wrong metric for the purpose at hand.

The current model for the market value of public sector pension benefit liabilities needs serious improvements, even an overhaul. The current model may be unambiguous, simple for first-year finance students to understand or, in its simplistic form, may be consistent with simple models used to price simple financial instruments. These are not reasons to cling to it. In fact, these qualities should be red flags signaling us to revisit the model.

Pension plans and other postemployment benefit (OPEB) plans have many moving parts, at least as many as collateralized mortgage obligations traded in foreign currencies, swaptions or weather derivatives. Both are highly complex structures with many economic, demographic and behavioral variables having separate and sometimes correlated distributions, with many complex contract terms in different contracts, and with many principals and agents. Adequately and honestly pricing the fair market value of public sector pension and OPEB benefit liabilities is far more complex than portrayed by the current market value of liabilities model.

In a practical sense, the fair value of a public sector pension benefit liability has limited utility. This will be explored in Part 5 of this series, "Consider the Measurement Purpose." Nevertheless, if actuaries are going to be required to calculate a fair value of public sector

pension benefit liabilities, then we must improve the model in ways that align it with best practices in financial engineering. We propose three areas of improvement:

1. Revise the benefits being valued to better reflect the employer's benefit obligation in its voluntary exchange transaction with its employees. Identifying the benefits that have been earned to date under the terms of the employment contract (whether implicit or explicit) is the first step toward assigning a fair value pursuant to proper financial engineering principles. The ABO does not reflect the contract being valued. Refer to Part 1 in this series, "The Contractual Benefit Obligation."

2. Build risk premiums into the fair value to better reflect the price required to protect market players from various non-investment-related risks. Pricing an obligation requires a fair and full recognition of the risks and risk premiums built into an exit price. Given the amount of dialogue and monologue that has transpired in recent years about recognition of risk in pension valuations, a great void has existed concerning demographic and other non-investment risks. These include longevity risks and retirement rate risks, as well as cost of living and other risks. Fair value pricing must not be built upon mere expected benefit cash flows, but must include premium margins for absorbing the material risks that the cash flows may very well exceed expected values. While there are various other risks that should be considered, we will address only longevity and retirement rate risk. Refer to Part 2 in this series, "Risk-Adjusted CBO Cash Flows."

3. Recognize market observables in setting the discount rates. This is a more controversial assumption, which must be addressed and improved for pricing a revised and risk-adjusted fair exit value of the liability. We present theoretical arguments as well as propose observables from the single premium group annuity markets, from high quality corporate bonds, and from the behavior and risk tolerance of public sector pension trustees. Refer to Part 3 in this series, "A Market-Related Discount Rate."

Again, while its utility is highly questionable, if actuaries will be required to calculate and publish the "fair value" of public sector pension benefit liabilities, then the current "market value" model must be overhauled in favor of one based on more careful rigor and integrity, and more faithful to current economics, actuarial and financial pricing principles.

C. Measure the Employer's Liability

The current model totally ignores the operation of the public sector pension fund, essentially treating it as if it did not exist. The current model values the pension liability the same, whether the obligation is funded or unfunded. But the public sector pension fund is the five-ton elephant in the room, which the current model ignores in the name of "pass through."

Part 4 in this series, "The Residual Benefit Liability," demonstrates that the public sector pension fund is too important to ignore. There is a very serious contract in place between the

public sector employer and the independent pension fund. The current model ignores this contract as well. A public sector employer owes only a residual pension benefit to employees, after the pension fund has paid all it can. Therefore, rather than price a first-dollar obligation as the current model does, an alternate model should be employed to price a fair value of the public sector employer's residual liability, as the payer of last resort. To do so, we must model the operation of the pension fund until its depletion, and then assign a fair value to the residual payment obligation.

The agency cost or benefit inherent in the pension fund's operation over time should be modeled before identifying the tail of the cash flow that must be settled. That tail is the employer's benefit liability.

D. Consider the Measurement Purpose

It is unrealistic to think that one measure of the liability should be used for all purposes; just as one calculation can never communicate useful risk information. Different purposes require different treatments.

Examples of this abound in the world all around us. The methods and degree of care employed in building a fence depends on the purpose of the fence. The rigor applied to composing music depends on the purpose of the end result. Why should we think that one measure of pension liabilities should ever apply to all purposes? As examples from the actuarial world, insurance company reserve calculations differ depending on the purpose; mortality and future lifetimes might be calculated differently for pension valuations of impaired lives as compared to personal injury litigation (which itself may differ depending on defendant or plaintiff); or methods and margins employed for calculating the per-member, per-month price charged by an HMO wanting to introduce a new product into market or capture market share might be different that HMO's methods and margins for en existing product in the same market, both of which might be different from those employed for reserving purposes.

Actuarial valuations of pension benefits are needed for numerous purposes, some of which should have the same measurement methods and some need to be different. The careful actuary will match the method with the purpose. Following are various purposes actuaries may encounter and which should force the actuary to consider carefully which methods are appropriate for the purpose at hand:

A. Advance funding
B. Taxpayers
C. Financial reporting
D. Lenders and rating agencies
E. Comparability
F. Risk measurement and analysis
G. Personal wealth
H. Plan terminations and freezes

The current model of the market value of the liability, the fair value with improvements and/or the alternate fair value model for residual employer benefit liability have some usefulness in the last two venues listed and a little usefulness in discussions of risk. However, for the other venues listed, market or fair value have little to no usefulness. These purposes will be examined in Part 5 of this series, “Consider the Measurement Purpose.”

Special Thanks and References for the Series

Special Thanks

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Part 1: Fair Value of the Liability – The Contractual Benefit Obligation

An understanding of the terms of a financial instrument, in all its complexities, is fundamental to its fair valuation. Financial engineering and pricing require a careful analysis of the specific terms of the financial contract and the amounts, conditions, likelihood and timing of payments due in the future. Financial instruments often have complex contract provisions. These must be identified and considered carefully in developing and applying the pricing model. Certainly, risk margins must be built in, but it all starts with the pricing imperative to model the contract terms themselves as closely as possible.

An understanding of the voluntary exchange transaction that occurs between employer and employee (i.e., the terms of the instrument) is fundamental to the fair valuation of public sector pension benefit liabilities. That is the specific contract which we are to price, in developing and applying a financial and actuarial model for valuing public pension benefit liabilities. Again, it all starts with the pricing imperative to closely model the contract terms themselves. No more; no less. As we will see below, the current model's use of the accumulated benefit obligation (ABO) misses the mark on this important point.

Generally speaking, in exchange for an employee's creditable service for a given period of time (e.g., a year), the employer agrees to compensate the employee. All under the terms of the exchange transaction between the employer and employee:

- Some of this compensation is paid immediately by the employer to the employee.
- Some is paid by another party under a separate agreement between the employer and the other party.
- Some is paid immediately by the employer to another party, which in turn provides benefits or payments to the employee later or provides insurance coverage during that period.
- Some compensation is deferred and paid by the employer to the employee at a later date.

The contract may be implicit or explicit. For fair value, what must be valued is the part of that exchange, which represents the employer's future obligation for benefit earned to date.

Some payments made by the employer to the employee may not even be as compensation for prior services rendered pursuant to the contract. Such payments might, more appropriately, be considered unilateral payments to maintain goodwill, encourage future employment longevity, or for political reasons. These may be paid immediately or may be deferred or transferred to another party for later payment. Deferred payment promises of these types, once embedded in the contract (even if not specifically negotiated but granted unilaterally), become part of an employee's or retiree's contract rights and must be valued along with the more usual type of deferred payment promises as part of the contractual benefit obligation (CBO).

A. Benefit Contract Terms

The actuary's job is to identify the amounts, conditions, likelihood and timing of those deferred benefit payments arising out of the voluntary exchange transaction between employee and employer. True to the principles of pricing financial instruments under fair value models, we must model the contract terms carefully.

A complicating feature of valuing pension liabilities (as compared to typical financial instruments' static contract terms) is the dynamic nature of the pension payoff promises as they accrue over time. If a financial option can be exercised at a later date, the total amount of payment (expected or risk-adjusted) must be factored into the pricing process. Pension benefit amounts and rights accrue over time in accordance with the contract. The terms of the pension contract automatically and dynamically change the amount, conditions, likelihood and timing of payment as each year of creditable service is rendered. So the amount, conditions, likelihood and timing of payments must be factored into the pension pricing model at a measurement date. These factors must be determined based on what has been earned by the employee under the terms of the contract as of the measurement date, not based on what might be earned by the employee in the future conditioned on future employment.

This is an important fair value pricing principle often ignored in the name of other worthy goals not associated with fair value pricing.

B. Current Private Sector Treatments

Current Private Sector Accounting

As previously mentioned, the currently accepted model for the market value of the public sector pension liability appears to be the expected ABO discounted using at risk-free yield curve.

The Financial Accounting Standards Board (FASB) issued Statement of Financial Accounting Standards (SFAS) No. 87 (Employer's Accounting for Pensions) in December 1985. That accounting standard defined three types of benefit measures for disclosure, the projected benefit obligation (PBO), the accumulated benefit obligation (ABO) and the vested benefit obligation (VBO).

FASB does not describe the PBO, ABO or VBO as fair value. There are reasons unique to the history and perspective of the FASB and its purposes that gave rise to terminology and measurement attributes for pension valuations that are not equivalent to market value or fair value of the pension benefit liability. While the accounting standard does discuss fair value of plan assets, it does not refer to fair value of plan liabilities. It deliberately does not use a fair value attribute model. This may change in the future, but that appears unlikely. FASB makes an exception for corporate pension liabilities and describes its own model for such calculation without attempting to fold its model into an attribute framework.

Standards setters often make exceptions to their conceptual framework, and pensions are an oft-excepted liability. Strict uncompromising ideologues find no place in standards setting because real world situations seldom conform to simple, unified theories.

None of FASB's three benefit obligation measures is consistent with fair value and the employment contract principles in terms of the benefits valued. In addition, FASB's benefit obligation measures are not consistent with current fair value principles in terms of non-investment risk premiums. Finally, they are not entirely consistent with current fair value models in terms of the discount rates used, including other margins.

The rhetorical linkage of fair value of all assets and liabilities sounds like a worthy goal in conversation, but many believe that certain assets and liabilities should not have a fair value measurement attribute. The accounting profession through its standards-setting bodies has legitimate reasons for mixed attribute models. PBO, ABO and VBO may have some similarities to fair value, but they are not fair value.

FASB's PBO applies expected future pay increases for pay-related pension plans. This is clearly not consistent with the fundamental principle of measuring the amounts, conditions, likelihood and timing of payment earned under the terms of the employment contract at the date of measurement. It may be consistent with FASB's objectives, but it is not consistent with a fair value of the contract. Under the contract, a pay-related pension benefit formula provides an earned right only to the formula factors through the measurement date. Future pay would be used only for future measurement dates to determine the proper contractual amounts earned at those later dates. Fair value of the pension benefit liabilities requires the determination of the contractual benefits earned at the measurement date, and no more.

FASB's ABO recognizes only the earnings to the measurement date and, therefore, is more in line with valuing the contractual benefits than the PBO. However, the ABO (and PBO) includes three features that are inconsistent with the employment contract being valued and, thus, inconsistent with a fair value of the pension benefit obligation.

1. ABO and PBO include the value of future increases in the employee's vested percentage that might occur after the measurement date. This, too, is inconsistent with a fair value of the contractual benefit obligation -- another overstatement (slight though it may be) of the terms of the labor contract in place. At the measurement date, an employee who has not rendered enough qualifying service under the terms of the contract has earned no rights to deferred compensation for retirement on account of those prior years of service. That is the way the contract works. At future measurement dates, upon reaching the years of service required for vesting, there is a sudden jump in the exit liability owed by the employer for future retirement benefits, enough to make up for all prior years of service. Then each measurement date thereafter, the progression is naturally much smoother. That is how the contract works, the voluntary exchange transaction between the employee and the employer has punctuated discontinuities, which should not disturb a faithful application of financial engineering principles to price the contractual benefits. A nonvested employee has no contractual retirement benefits

until he or she works enough years to earn the nonforfeitable right to the retirement benefit.

2. ABO and PBO load the benefit obligation for active employees with expected benefits, "accrued" ratably as of the measurement date, for duty and nonduty disabilities that might occur in the future, even though the disability has not occurred as of the measurement date[3]. This is a natural feature of the traditional unit credit cost method of funding (projected and unprojected), which was borrowed for private sector accounting purposes such as PBO, ABO, VBO and for private sector funding purposes such as current liability and funding target. This might be appropriate for funding benefits under a traditional unit credit cost method (Pension Research Committee, 1991). It might also be appropriate in the minds of those accountants preferring a more linear progression or other smoothing to avoid discontinuities in the benefit incidence, but a fair value of the benefit contract terms knows of no such technique. This treatment of future ancillary benefits is not appropriate to a fair value of the benefits actually earned to date under the terms of the contract. An employee does not earn a nonforfeitable disability right until he or she is disabled under the terms of the contract.

   Employment contracts usually call for coverage under a group life insurance policy. There is no claim on benefits unless and until the employee dies. The current year's compensation (including the group term premium for the year) makes sufficient provision for one-year term costs to account for that possibility. But, if the event does not occur, there is no exit obligation to value. Each day he works, he has the right to "coverage" for benefits payable in the case of death.

   Indeed, all those who had become disabled prior to the measurement date, and therefore are receiving or entitled to future disability pensions, have a contractual benefit promise that must be valued. The same feature of FASB's ABO also applies to duty and nonduty death benefits. Again, if as of the measurement date an employee has not become disabled or died from duty and nonduty causes, then there is no disability or death benefit earned as of the measurement date (except what may attach itself to a vested deferred retirement benefit earned). Even the Internal Revenue Code Section 411(d)(6) does not attach future ancillary benefit rights to the current accrued retirement benefit for private sector plans at any given measurement date.

   The fair value of the pension benefit earned for the coming year should include a one-year term cost for duty and nonduty disability and death benefits. Including a level funding type of cost in the valuation to account for disability and death benefits that might arise in the future (after the measurement date) is not consistent with the contract and compensation terms of the current exchange.

[3] Statement of Financial Accounting Standards No. 87, Paragraphs 17 and 42b, and Footnote 10.

3. ABO and PBO load the benefit obligation with the value of future early retirement subsidies which employees might earn in the future[4], even though they may not have yet achieved sufficient service to earn a right to such subsidies at the measurement date. Again, this is not consistent with the terms of the contract and what the employee has actually earned as of the measurement date. Certainly, if an employee has earned (as of the measurement date) enough creditable service to satisfy the service requirement for a subsidized early retirement, then he has indeed earned the subsidy right and it has attached itself to his then-current accrued retirement benefit. However, if the employee has not earned (as of the measurement date) sufficient service to satisfy the service requirement for a subsidized early retirement, then the fair value of his contractual benefit obligation should not yet include any early retirement subsidy.

FASB's VBO has none of these failings. It uses only the earnings to the measurement date and recognizes the extent vested as of the measurement date. It does not include any liabilities for disabilities and deaths not yet occurred at the measurement date, and it does not include the value of early retirement subsidies unless the employees eligible for early retirement subsidies at the measurement date. This makes VBO more consistent with labor economics than ABO or PBO. However, it still has a remaining flaw that disqualifies it.

All three of FASB's measures of the obligation (VBO, ABO and PBO) ignore or override the employment contract in the issue of certain complex accrual patterns, such as formulas which are backloaded, or are the greater of two formulas, or which limit the service credits[5]. As one such example of a backloaded pattern, consider an employer-employee contract that states that the employee's retirement benefit formula is 2 percent of final average pay for each of the first 20 years of service plus 3 percent of final average pay for years in excess of 20, and consider an employee who has 21 years at the current measurement date and will retire at 35 years. FASB's measurement rules require the benefit to accrue linearly from zero to 35 years. For the current measurement date, all three of FASB's benefit obligation measurements require the employee's benefit for service to date to be valued at 51 percent of average pay (21/35 times 85 percent). However, under the actual terms of the employer-employee contract, the employee has earned a retirement right to 43 percent of average pay (20 times 2 percent plus one times 3 percent). The 43 percent answer represents the contractual benefit obligation of the employer, and should be used to measure the fair value of the pension obligation.

These characteristics of VBO, ABO and PBO, which fail the test of contractual benefits, are a part of FASB's measurement model because of a desire to attribute costs to all years of service in a smoothed ratable fashion. FASB did not want discontinuities in its reporting model.

Financial engineering, however, regularly deals with punctuated liabilities over time and discontinuities in the liability progressions. Such discontinuities should not disturb those who calculate or use a fair valuation of the contractual benefit obligation. It is what it is. An alternate approach (such as VBO, ABO or PBO) may be entirely appropriate for other purposes. A

[4] Statement of Financial Accounting Standards No. 87, Paragraph 42 and Footnote 9.

[5] Statement of Financial Accounting Standards No. 87, Paragraphs 40, 42, and Footnote 8. See also A Guide to Implementation of Statement 87 on Employer's Accounting for Pensions, Q&A 45.

standards-setting body may choose to adopt a different measurement attribute identical to current private sector financial reporting. If so, it should not be called fair value (or market value). It is something else.

The current model of the market value of pension liabilities is simply the expected ABO discounted using a risk-free yield curve. A corrected or improved model for fair value of the liability is based on the contractual benefit obligation, not an artificially smoothed pattern.

Current Private Sector Funding

At least the current liability and funding target of the Internal Revenue Code use only the earnings to the measurement date and do not require this linear override, valuing the benefits just as they accrue per the contract formula. However, they do apply the traditional unit credit funding features of recognizing the future possibility of duty and nonduty-connected disability and death in the calculation, and they ignore vesting. It is a traditional, unprojected unit credit cost method using the accrued benefits.

Notice that the proponents of the current model for market value of liability disclosures have latched onto private sector concepts of accounting and funding for identifying the benefits to value. However, these concepts do not value the contractual benefits earned to date under the voluntary exchange transaction, which occurs between employer and employee, and thus, should not be part of a fair value model. We should follow the labor economics principles more closely.

C. Contractual Benefit Obligation (CBO) for Pensions

Staying true to financial engineering and pricing principles requires using the contractual obligation to determine what benefits to value. We must not rely on other worthy goals, being reminded that we are pricing the fair value of the contractual benefit obligation here. That is the proper exit liability to value. That is the starting point for the process. This means that the CBO calculations must involve the following features and processes.

1. Those members who are currently in pay status, as of the measurement date, regardless of reason (including in-service duty and nonduty disability and death), should be valued according to the benefit amount and form applicable. This is nothing new.

2. The CBO as of a given measurement date should have zero values for active employees who have not yet (as of such measurement date) earned a nonforfeitable (vested) right to a retirement benefit. For contributory plans with refund features, however, the fair value of the liability for such employees is no less than the accumulated employee contributions as of the measurement date together with any interest credited.

3. For employees with a nonforfeitable vested interest who do not yet have sufficient service to have earned, as of the measurement date, a right to a subsidized early retirement benefit, the value of the CBO is determined by modeling their

decrements until final retirement age, as the contractually accrued and vested retirement benefit calculated as of the measurement date payable at the later of the date for commencement of vested deferred benefits (again, based solely on service at the measurement date in applying the eligibility conditions) or the date of decrement.

If the decrements are a function of service, future service should be assumed for the purpose of decrement probabilities, but not for the purpose of benefit eligibilities, amounts or subsidies. The value of any death benefit associated with vested deferred retirement benefits should also be included (not to be confused with duty or nonduty active employee death benefits). Again, for contributory plans with refund features, the fair value of the CBO for such employees is no less than the accumulated employee contributions as of the measurement date together with any interest credited.

4. For employees who do have sufficient service to have earned, as of the measurement date, a right to a subsidized early retirement benefit (but not necessarily the age), the value of the CBO is typically determined by:

    a. modeling their decrements until their earliest early retirement age (associated with the service earned as of the measurement date), as the retirement benefit contractually accrued as of the measurement date payable at an assumed early retirement commencement date together with the early retirement reduction associated with such age, the number of years for early retirement reduction being based upon the terms of the plan and the service earned as of the measurement date; and
    b. modeling their retirement decrements after the earliest early retirement age (associated with the service earned as of the measurement date) until normal retirement age, as the retirement benefit contractually accrued as of the measurement date but reduced for early retirement at and payable at the time of decrement,
    c. modeling their retirement decrements after normal retirement age until final retirement age, as the retirement benefit contractually accrued as of the measurement date, payable at the time of decrement.

5. One-year term normal costs for duty and nonduty during-employment disability and death benefits provided by the plan should be added to the value of retirement benefits accruing during the coming year to obtain the total normal cost for the year.

Revising the benefits valued (to be the contractual benefits) is the first of three improvements to the current model of market value of liabilities, which are presented herein. The other two are risk adjustments to the contractual cash flows discussed in Part 2, “Risk-Adjusted CBO Cash Flows,” and a discount rate that reflects market prices discussed in Part 3, “A Market-Related Discount Rate,” in the series.

D. Case Study Plan

The contractual benefit obligation is more faithful to the economics principles of labor contracts. To illustrate the difference between the CBO and ABO, consider a case study plan.

Figures 1 and 2 below present the plan provisions and actuarial assumptions relevant to this comparison.

Figure 1

Summary of Case Study Plan Provisions	
Normal (unreduced) Retirement Date (NRD) Eligibility	Age 60 with five years of service, or 30 years of service regardless of age. No DROP provisions.
Normal (unreduced) Retirement Date (NRD) Benefit	2 percent of final average pay for each of the first 20 years plus 3 percent of final average pay for each year in excess of 20.
Early (reduced) Retirement Eligibility	Age 50 with 15 years of service
Early (reduced) Retirement Reduction	3 percent for each year by which actual retirement precedes NRD
Vesting Eligibility	Five-year cliff vesting
Vesting Benefit	Accrued benefit payable at NRD, or a refund of contributions with interest
Nonduty Disability Eligibility	10 years of service
Nonduty Disability Benefit	The greater of accrued benefit or 25 percent of pay, payable immediately
Duty Disability Eligibility	From date of hire.
Duty Disability Benefit	The greater of accrued benefit or 42 percent of pay, payable immediately.
Nonduty Death Eligibility	10 years of service
Nonduty Death Benefit	Accrued benefit payable immediately to beneficiary.
Duty Death Eligibility	From date of hire.
Duty Death Benefit	The greater of accrued benefit or 50 percent of pay, payable immediately to beneficiary
Cost of Living Increase	Increase in consumer price index, not to exceed 3 percent per year

Figure 2

Summary of Relevant Valuation Information	
Discount Rate	2.82 percent, the single discount rate equivalent to the Treasury STRIPS yield curve observed on Dec. 31, 2008.
Mortality Table	1994 Group Annuity Mortality Table for pre- and post-retirement for valuations.
Retirement Rates	24 percent at age 50, then, 7 percent, 7 percent, 7 percent, 11 percent, 11 percent, 11 percent, 11 percent, 8 percent, 8 percent, then 60 percent at age 60, then 30 percent for each year through age 69, then 100 percent at age 70; also 100 percent at 35 years of service regardless of age
Turnover and Disability Rates	Based on a recent experience study
Market Value of Plan Assets at 12/31/2008	$380,717,255
Price Inflation	3.0 percent per year compounded annually
Salary Increases	Service-based, from 14 percent to 4 percent annual increases

Figures 3 and 4 present comparisons of the ABO and CBO values for our case study plan. For this comparison, the mortality table used was 1994 GAM (males and female) and 2.82 percent for the discount rate. This discount rate represents the single discount rate producing a present value of the CBO expected cash flows which is equal to the their present value when using the full U.S. Treasury STRIPS yield curve (yields above 30 years equal to the yield for 30 years) as of Dec. 31, 2008.

Figure 3, below, pulls the layers away a bit to reveal how the ABO and CBO progress through an employee's career, and how the ABO and CBO differ for employees with different service. Notice the two discontinuities that exist at five and 15 years. This treatment may not be appropriate for certain other purposes, but it entirely appropriate for calculation a fair value of the contractual pension benefit liability.

Figure 3

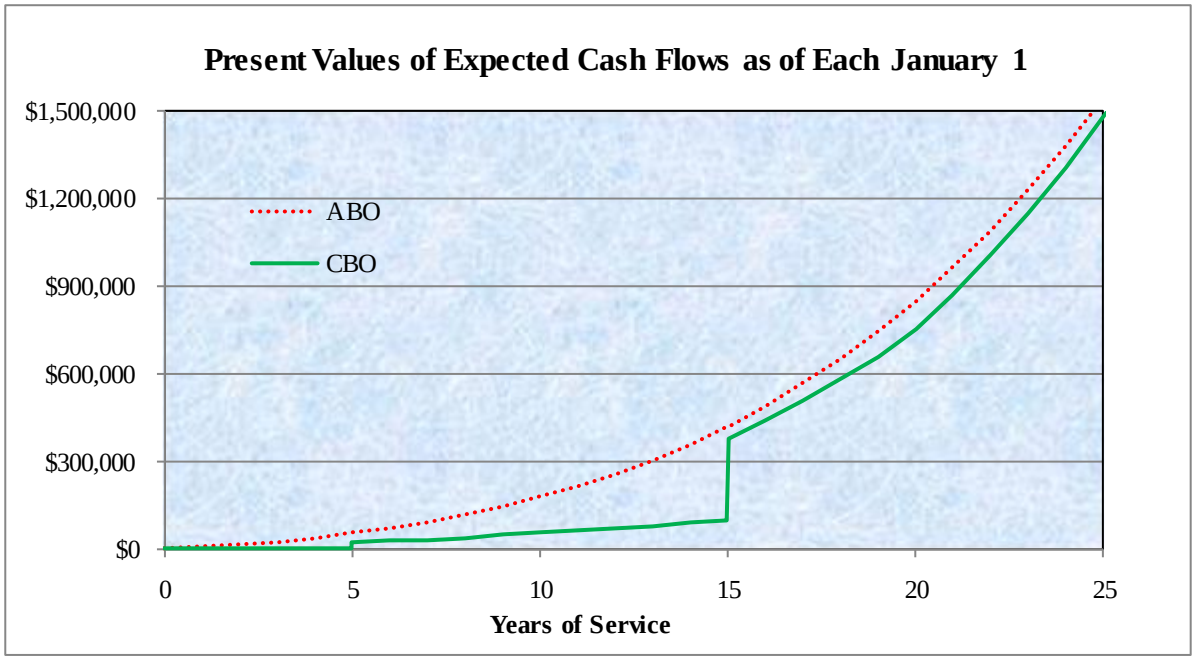
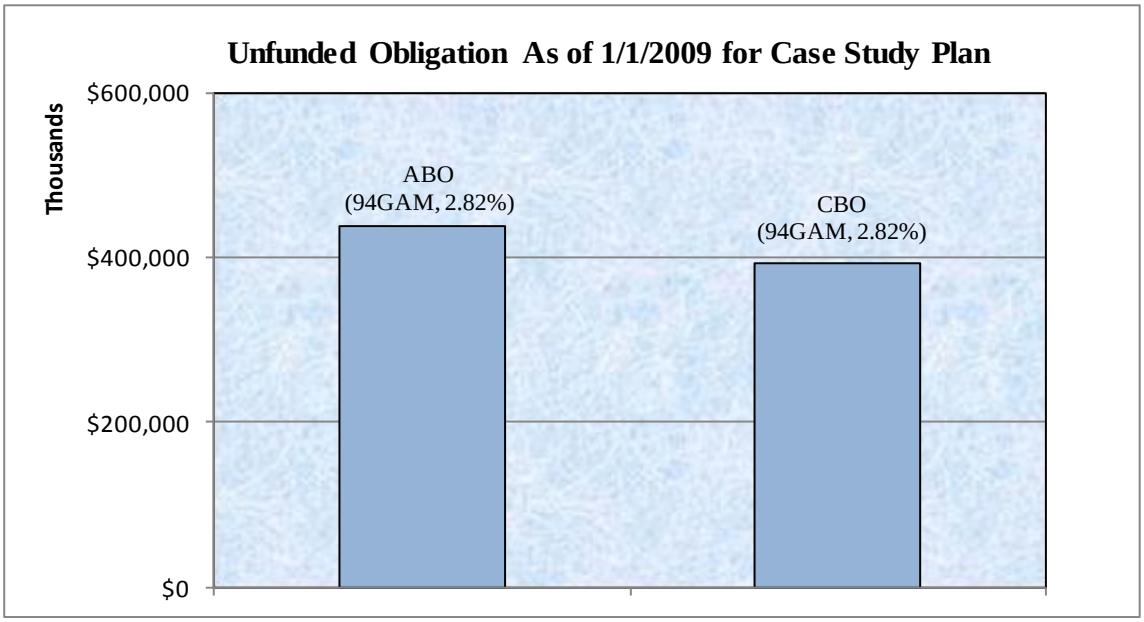


Figure 4



The total present value of ABO (and CBO, respectively) minus the market value of assets was calculated to derive the unfunded obligation as of Jan. 1, 2009.

E. Contractual Benefit Obligation (CBO) for OPEBs

While this paper's primary focus is measuring the liabilities of public sector pension plans, the financial reporting treatment for other postemployment benefits (OPEBs) is and will likely be the same as for pensions. Although, standards-setting boards often make exceptions to the application of concepts and principles, it would be a serious error to build momentum for the proper model for fair value of public sector pension liabilities without regard to how that model might apply to public sector OPEBs.

Governmental entities' unfunded OPEB liabilities, as measured pursuant to GASB Statement No. 45, are often of the same order of magnitude as unfunded pensions. While having similarities to pensions, the nature of OPEBs and the nature of the employment contract for OPEBs (whether implicit or explicit) are different from pensions, while remaining at least as complex and illusory.

Some have specifically excluded OPEBs from the public and private discussions and limited them to pensions in the interest of narrowing the focus of the already wide-ranging topic of public sector pension finance. That may have been a reasonable approach for the past. However, the valuation of OPEB liabilities cannot be treated as an afterthought, added on to whatever becomes the accepted or standard model for pensions. We must begin to include OPEBs in the primary discussions. Having said all that, this paper focuses on pensions, but will include brief discussions of OPEBs at relevant points.

Contractually, in considering the voluntary exchange transaction between employee and employer, identifying that component of compensation earned to date which relates to OPEBs is challenging. Some employers have detailed plan provisions, documented and duly adopted. As actuaries are now valuing the OPEB obligations of many governmental employers for the first time, it is becoming clear to labor and management that precious little has been reduced to writing. Both FASB and GASB have embraced the concept of the "substantive plan," the plan as understood between employee and employer. GASB included language concerning moral and political obligations that may exist between the employer and the employee/retiree, which go beyond the terms of any collective bargaining agreement, statutory requirements, or written plan documents or booklets, which might exist.

Progress is being made in this area. However, when it comes to identifying what an employee has "earned" to date, the implicit and explicit contract terms are little help because they primarily cover the eligibility conditions, the benefit levels and contribution requirement once the employee actually retires. Furthermore, few employers provide vested deferred OPEB subsidies to those employees who terminate prior to reaching eligibility for early retirement pension benefits. Determining the CBO earned through a given measurement date may not be fully answerable.

A reasonable way to help determine what nonforfeitable right to future OPEBs an employee has actually earned, as of a given measurement date, is to ask these questions:

1. If the employee were to terminate employment (other than by death or disability) as of the measurement date, what right to future OPEBs would he have? That is not an irrelevant question for determining the contractual benefit obligation and its fair value as of a measurement date if we are to be true to the concept of "fair value" of the current contract obligation. It may indeed be an irrelevant question if we are determining another form of benefit measurement for other purposes, such for funding, accounting (under a mixed-attribute model), comparability or lenders and rating agencies. But the question may provide some real insight for determining the fair value of the current OPEB CBO.

2. Does the employer have the right to alter or amend the eligibility conditions, the benefit plan (vendors, copays, deductibles), or the future level of contributions required from retirees? Can the employer terminate the program unilaterally?

3. Since "fair value" imagines a market in which an employer discharges or settles the voluntary exchange obligation, which the employee had earned for service to the measurement date, how much of the total future obligation had really been earned under the terms of the voluntary exchange as of the date of measurement?

These questions have legal, accounting and funding answers, and they illustrate only one of the challenges (benefit accrual rights) that exist in attempting to contrive a logical measure of the current contractual right to an OPEB while maintaining faithfulness to fair value principles in financial engineering.

Furthermore, if an employer holds the unilateral right to cut back or terminate OPEBs or raise the contributions required from retirees, then we should question whether they really are part of the voluntary exchange transaction for prior service.

Part 2: Fair Value of the Liability – Risk-Adjusted CBO Cash Flows

With respect to pension liabilities, David Wilcox (2008), an economist with the Federal Reserve Board, recently testified, "You'll need to study the behavior of participants and see whether the choices that they make are systematically related to actual market conditions. If they are not, then you can treat expected values as if they were known with certainty. And the reason for that is because those are so-called unpriced factors, if they're not systematically related to market conditions."

Whether the cash flows are or are not systematically related to actual (financial) market conditions is a narrow view of risk, often taken by non-actuaries to simplify their world. In pricing pension liabilities, there are a number of noninvestment risks that must be considered.

Developing the fair value of a pension benefit liability is about pricing. Understanding the noninvestment risks inherent in projected cash flows is fundamental to actuarial finance and pricing principles. It is patently wrong and dangerous to price an asset or liability by assuming that expected cash flows are certain. Pension valuation systems are built on models often relying on a number of assumptions derived from experience studies and each of these assumptions can be a source of a risk. Any fair value of an exit liability should include risk premiums for the possibility of adverse or worse-than-assumed experience.

We often divide pension and OPEB assumptions into three broad categories:

- Economic assumptions (rates of investment return, discount rates, price and wage inflation, salary scales, medical trend, Medicare payments, etc.),
- Demographic assumption (mortality rates, retirement and DROP rates, termination patterns, disability, marriage rates, etc.) and
- Behavioral assumptions (option election rates, retiree medical acceptance and lapse rates, etc.)

In this section we focus on two demographic risks – longevity and retirement rates. Risk premiums for these uncertainties will be analyzed and useful tools provided for inclusion in the pricing of fair values of pension liabilities.

A. Longevity Risks

We begin with an overview of mortality (or one should say, longevity) risk. In this section we will address stochastic risks resulting from random deviations of experience from the expected best estimate (or mean) mortality rates, as well as systemic risks resulting from nonrepresentative sample subsets and from mortality improvements beyond our best estimates.

The arithmetic of annuity calculations when mortality rates are certain has been known for the better part of two centuries. One might even say that most actuarial students can calculate annuity factors when given the interest rate and the mortality rates. However, there is a lot more

to computing pension plan liabilities than calculating the present value of expected cash flow associated with annuities. As a matter of fact, first year MBA finance students can calculate the present value of a well-defined and certain cash flow if given the discount rate. It is the modeling of that cash flow that presents actuarial challenges. Any such present value calculations thereafter seem trivial in comparison.

When comparing the effect of interest discount rates and mortality on the prices of annuities, it has been common to start with the observation that the price of a unit pure endowment, issued at age *x* for *n* years, is a function of the force of interest (i.e., continuously compounded forward interest rate) δ_x and the force of mortality $\mu_x(s)$ (representing the force of mortality at age $x+s$) for a person underwritten for life insurance or annuity at age *x* and defined as $\mu(u) = -S'(u)/S(u)$ where $S(u)$ is the survival function (probability that newborn is still alive at age u) and $S'(u) = \mathrm{d}S/\mathrm{d}u$ is its derivative.

As shown by MacMinn, et al (2006), changes in mortality have a similar impact on annuity value as changes in interest discount rates. We briefly recall these arguments.

We start with presenting an annuity as a series of pure endowments:

$$a_x = \sum_{n=0}^{\omega} {}_nE_x$$

where pure endowment ${}_nE_x$ can be calculated as:

$${}_nE_x = \exp\left[\int_0^n \left(\delta(s) + \mu_x(s)\right)\mathrm{d}s\right]$$

with $\delta = \ln(1+i)$ being the force of interest and μ_x is the force of mortality.

This shows that the effect of mortality and interest discount rates on the annuity value can be studied by analyzing the impact those variables have on a price of the pure endowment.

MacMinn and others examined the sensitivity of the pure endowment to the forces of interest and mortality. Under the assumption of the constant force of interest, they found:

$$\frac{\mathrm{d}}{\mathrm{d}\delta}\,{}_nE_x = \frac{\mathrm{d}}{\mathrm{d}\delta}\left(\mathrm{e}^{-n\delta}\exp\left[\int_0^n (\mu_x(s))ds\right]\right) = -n\mathrm{e}^{-n\delta}\exp\left[\int_0^n \left(\mu_x(s)\right)ds\right] = -n\,{}_nE_x$$

And under the assumption of constant force of mortality they have:

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\,{}_nE_x = \frac{\mathrm{d}}{\mathrm{d}\delta}\left(\mathrm{e}^{-n\mu}\exp\left[\int_0^n(\delta(s))ds\right]\right) = -n\mathrm{e}^{-n\mu}\exp\left[\int_0^n(\delta(s))ds\right] = -n\,{}_nE_x$$

This continuous model may not be practical for calculations, but clearly illustrates that changes in mortality impact the value of endowment the same way as changes in interest rates.

As a side remark, we want to note an interesting implication of the first formula: the *Macaulay Duration* of a pure endowment under the constant force of interest is simply the same as the length of the contract:

$$\mathrm{D_M} = -\left(\frac{\mathrm{d}}{\mathrm{d}\delta}\,{}_{\mathrm{n}}\mathrm{E_x}\right)\left({}_{\mathrm{n}}\mathrm{E_x}\right)^{-1} = \mathrm{n}$$

B. Risk Premium for Error around the Mean

In pricing the fair value of the pension liability, the expected CBO cash flows should be adjusted for the risk of variation around the mean, particularly for smaller plans.

As described in the report by the SOA Group Annuity Valuation Table Task Force (SOA 1995), group annuity mortality (GAM) tables had margins built in, to account for random variations in mortality rates among participants. Those margins were set to provide a two-standard-deviation margin (theoretical) for a 3,000-life block of business (SOA 1995). Margins in the 1983 GAM table were intended to account for future mortality improvements too. The 1994 GAM table has a separate margin built in, for variation around the expected rates, but also presented Projection Scale AA to reflected mortality improvements.

In other words, tables used in reserving for annuity products have, by design, some protection against random variations around the expected values.

There is no golden rule as to how to select the best method for hedging against adverse experience. Some might argue that stochastic simulations should be performed to address the risk of random variations around the mean. This however is rarely a feasible approach. Perhaps large statewide plans could afford such an approach but for thousands of local plans this would be cost-prohibitive.

On the other hand, the approach taken by the Group Annuity Valuation Table Task Force when designing 1994 GAM and 1994 GAR tables (SOA 1995) would be worth considering. We briefly recount this method as applied to future lifetime.

For a single life age x, we assume Y is a random variable representing the future lifetime (t). Y would have the following distribution:

Y	t=0	t=1	t=2	t=3	…
Pr(Y=t)	$1 - p_x$	${}_{1\|}q_x$ $[= p_x(1 - p_{x+1})]$	${}_{2\|}q_x$	${}_{3\|}q_x$	…

From here we can compute the mean, variance and the standard deviation of this distribution:

$$e_x = E[Y] = \sum_{t=0}^{\infty} t \cdot \Pr(Y = t)$$

$$E[Y^2] = \sum_{t=0}^{\infty} t^2 \cdot \Pr(Y = t)$$

$$\sigma = \sqrt{E[Y^2] - E[Y]^2}$$

For a distribution of total future lifetime for N lives, all age x, assumed to be independent, the mean, variance and standard deviation would be:

$$e_x^{(N)} = N \cdot e_x$$

$$\sigma^{(N)^2} = N \cdot \sigma^2$$

$$\sigma^{(N)} = \sqrt{N} \cdot \sigma$$

From here we obtain the standard deviation of the total future lifetime per retiree to be:

$$\sigma = \frac{\sigma^{(N)}}{N} = \frac{\sqrt{N} \cdot \sigma}{N} = \frac{\sigma}{\sqrt{N}}$$

The Task Force determined that a 5-percent margin in mortality rates is appropriate for 3000-lives blocks of business. Size of 3,000 was selected to ensure proper protection for more than 95 percent of companies. Since public sector pension plans vary in size, different margins are appropriate for different individual plans.

Figure 5

	Number of Retirees Expected (N)				
	1	10	100	1,000	10,000+
Mortality Rate Margin Multiplier	36.10%	70.30%	89.20%	96.40%	None
Resultant Life Expectancy Margin	44%	14%	5%	1%	None

Values in the table are based on the Combined Healthy Male RP2000 table for age 60. The user can adjust the Margin Multipliers up if the average age at retirement is expected to be below 60. Rather than apply a one-size-fits-all adjustment, we computed Margin Multipliers for select plan sizes to provide a better perspective.

The Margin Multiplier selected based on the expected number of retirees over the life of the plan should be multiplied by each mortality rate in the base unprojected table to achieve a two-standard-deviation margin in future lifetime.

These results are consistent with common sense. In measuring deviation around a given mean, larger plans enjoy the luxury of the "Law of Large Numbers" working to dampen such deviations. Sample error is diminished with large sample sizes. However, in order to build protection for a smaller plan against adverse deviation around the mean, as required for pricing margins in a fair value of the liabilities, we need to reduce our mortality rates to cover that risk. This table provides practitioners with simple Margin Multipliers.

For our case study plan, we multiplied the unprojected RP2000 rates by a 95 percent Margin Multiplier (to reflect the expected number of retirees and an average retirement age 60) to account for stochastic risks.

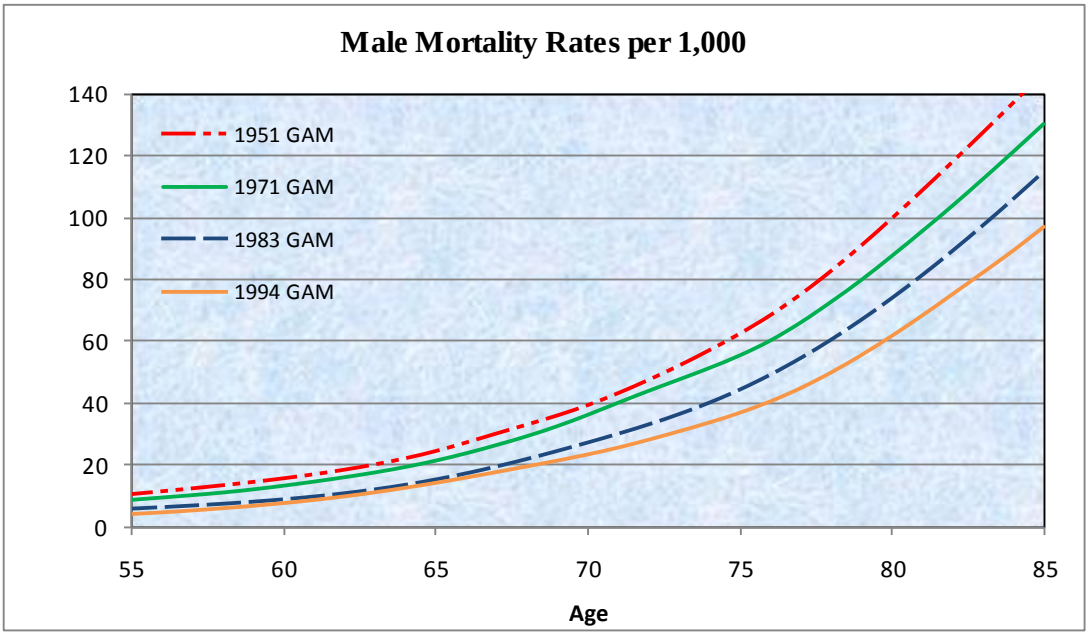
The margin illustrated above accounts only for error around the mean (stochastic risk). We believe that error in the mean (systemic risk) cannot be neglected either. This systemic risk is mostly reflected in broad mortality improvements beyond what we expect.

C. Mortality Tables and Life Expectancy

We now take a look at the improvement in mortality rates often used in reserve calculations for annuity contracts and pension valuations in the United States. Those are based on group annuity mortality tables that were updated in 1951, 1971, 1983[6], and most recently in 1994. On Figure 6, we graph mortality rates for male annuitants taken from those tables. We illustrate improvements in the mortality rates (for most ages in retirement) observed in the second half of the 20th century.

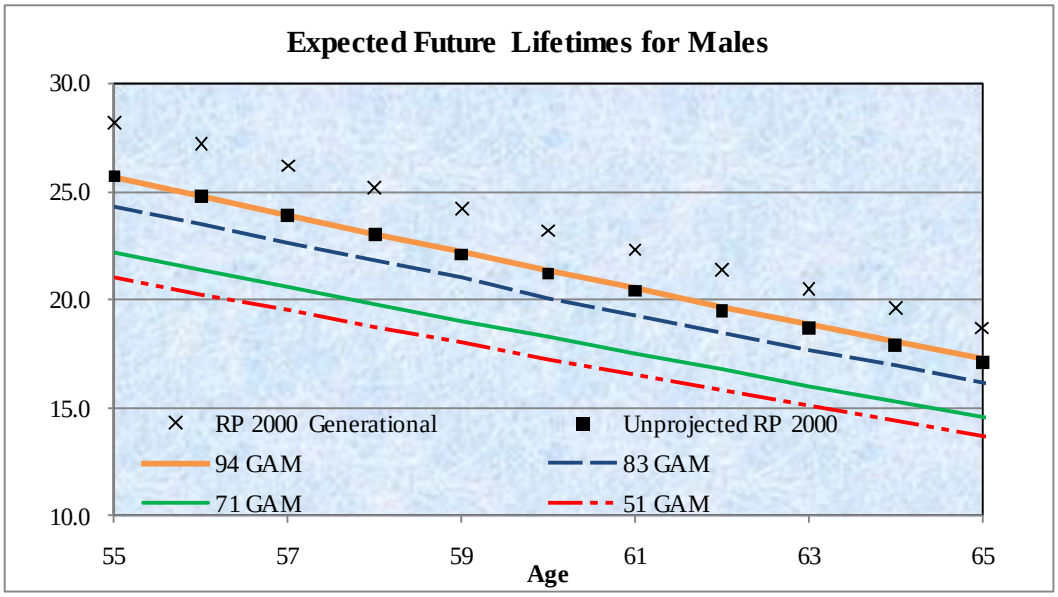
[6] According to the Committee on Annuities report on the "Development of the 1983 Group Annuity Mortality Table," there was insufficient data to develop a new table, but sufficient data to conclude that GAM 1971 with Projection Scale D was no longer valid. Therefore, a new projection scale was developed and applied to GAM 1971 to produce GAM 1983.

Figure 6



As an alternative to looking at raw mortality rates, it is common to examine the random variable of future lifetime for the research population, as the "choice of numeraire". Figure 7 plots the curtate future lifetimes for a typical range of retirement ages for the same set of mortality rates as well as expected lifetimes based on mortality rates from RP-2000 mortality tables (combined healthy issue).

Figure 7



Expected lifetimes for males commencing benefits at ages 55 through 65. Lines (solid, dashed and dotted) correspond to expected future lifetime derived from rates from GAM tables. Markers illustrate expected future lifetime derived from un-projected (squares) and generationally projected (crosses) rates from RP 2000 mortality tables. Mortality improvement rates used in projections are based on scale AA.

Figure 7 also presents expected lifetime as derived from projected (Generational) RP-2000 mortality tables. We graph expected lifetime corresponding to mortality rates with generational projections for annuity starting dates in 2009 (employees are assumed to retire in 2009 and the remaining lifetime is computed based on the age at retirement).

It should be noted that RP-2000 mortality tables were developed differently than GAM series. GAM tables were developed from the mortality experience collected by insurance companies providing annuity products, while RP-2000 is developed from the mortality experience of uninsured pension plans. In addition, unlike RP-2000 basic tables, GAM tables had margins built into their rates. It may be a little surprising that expected future lifetimes based on unprojected rates from RP-2000 mortality tables are nearly identical to those based on rates from 1994 GAM tables. The margins embedded in rates from 1994 GAM tables were designed to protect from random variations in mortality (error around the means). It is interesting to see that mortality improvements from 1994 to 2000 are fully offset by margins for variation and, potentially, differences stemming from different populations being used.

The concept of mortality improvements is not new to actuaries. In the insurance industry, reserves for annuity products are required to be set with recognition of future mortality improvements. For pension valuations, actuaries are advised to consider mortality improvements in calculating liabilities (ASOP 35).

A survey conducted by the Society of Actuaries (SOA 2003) among life insurance companies found that nearly 100 percent of responding firms use mortality improvements (either generational or durational) in pricing of the products.

Projection is performed using Scale AA, which was developed for use with the Group Annuity Reserving 1994 tables (1994 GAR) whose rates are projected from the rates in 1994 GAM tables. The Retirement Plans Experience Committee of the SOA recommends using Scale AA for projecting mortality rates beyond the year 2000 and encourages the use of generational mortality projection. In general, Scale AA had been based on a blend of Federal Civil Service and Social Security experience from 1977 through 1993, with certain adjustments.

Any actuarial assumption will always be wrong, in one direction or the other. When exit-pricing the fair value of pension liabilities, a risk margin needs to be built into the mortality tables in case the plan's retirees live longer than expected by the selected mortality table. Generationally projected mortality rates constitute the current preferred method for anticipating mortality improvements.

Some larger public sector pension plans develop their own tailored mortality table based on the plan's own experience; even if it is just applying a factor to an existing published table to match its own experience. Alternatively, for a given plan, an actuary might rely upon a tailored table for another plan if there is sufficient reason to believe that the experience of the given plan will be essentially the same as the other plan. Tailored tables, if based upon credible experience, constitute a reasonable choice of table within the relevant assumption universe. In the absence of reliance on a tailored table, an actuary's choice could be a published table (e.g., 94GAM or RP2000). However, as ASOP 35 recommends, we should consider using mortality improvements

in our calculations. To that end, Projection Scale AA would be a reasonable choice prior to any margins for error. As such, a generational mortality table can be treated expected values (means).

There has been a significant amount of research on random variations in mortality rates (error around the means) and methods of hedging against them (Bauer 2006). But the risk of the expected value differing from observed mean has not been addressed in the design of those tables. The Retirement Plans Experience Committee warns in their report describing the development and suggested use of RP-2000 mortality tables: "Even mortality tables that are specific for the collar type and industry of the plan are unlikely to match the true underlying mortality of the plan."

The authors also note that: "Statistically significant differences in mortality between plans were found in all four of the industries investigated. The majority of plans had mortality experience that differed from the average experience of plans of the same collar type in the same industry. Adjusting for differences in annuity size explained some of the variation, but statistically significant differences of about plus or minus 12 percent were still found even after this adjustment"

This strongly suggests that the proposed model for measuring the liability of the pension plan should include some measures of protection against adverse mortality experience whether it results from random variations around the mean or the error in the mean itself.

D. Risk Premium for Error in the Mean

We see two sources of error in the mean itself. One source results from a plan's population not being representative of the population used in the development of the table. The second source results from future mortality improvements being better than currently expected (Projection Scale AA was developed in early 1990s). One can load for both types risk by increasing the level of future mortality improvements assumed.

To build a risk premium into the cash flows for a possible error in the mean (system error), we develop a Scale Factor. This is multiplied by each of the improvement rates found in Projection Scale AA. In Figure 8 we present the Scale Factor that would result from various levels of life expectancy improvement desired. For example, if the actuary wishes to build sufficient risk margin into the fair value that will account for an extra 20 percent life expectancy improvement beyond the best estimate of mortality (presumed in our example to be RP2000 Generational), it would require a Scale Factor of 3.5 to be multiplied by all of the improvement rates found in Projection Scale AA.

Figure 8

	Additional Expected Future Lifetime Desired		
	10%	20%	30%
Scale Factor	2.1	3.5	4.95

Since the Scale Factor is multiplied by each entry in Projection Scale AA, this relationship presented above, between the additional expected future lifetime desired and the Scale Factor, is not very sensitive to the ages within a reasonable range of retirement.

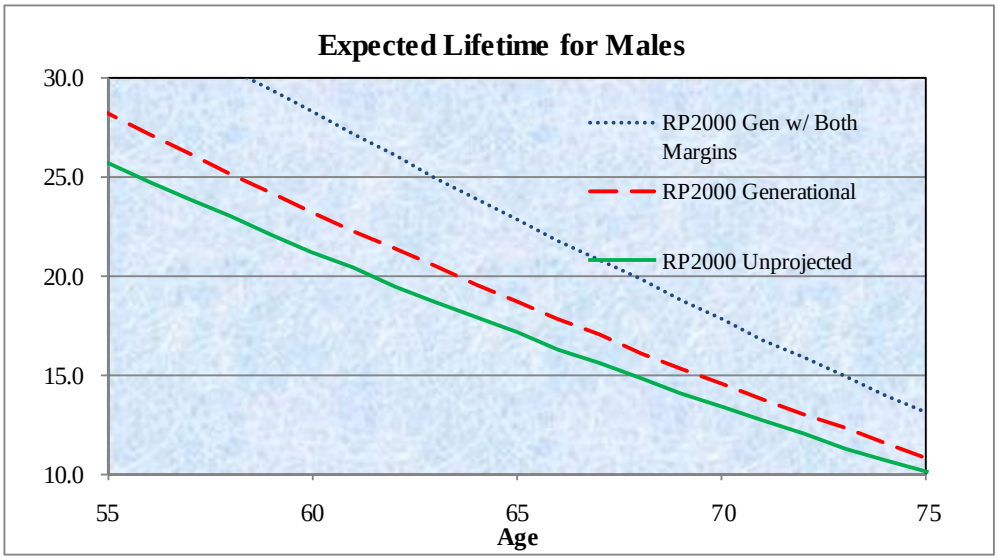
We have presented a simple algorithm for actuaries to build into their mortality rates to adjust for two types of longevity risks.

1. A Margin Multiplier (depending on the plan size) applied to the mortality rates themselves for stochastic risk, that is, to adjust for error around the mean.

2. A Scale Factor (depending on how much additional future lifetime to hedge against) applied to the Projection Scale AA rates for systemic risk, that is, to adjust for error in the mean.

In Figure 9 we illustrate the effect that these two margins would have on the expected lifetime.

Figure 9



Expected lifetime for male annuitants, retiring at ages 55 through 75 derived from mortality rates based on RP-2000 Mortality Tables with and without projections. Dashed line corresponds to generational projection of mortality rates. Dotted line illustrates expected lifetime derived after loading mortality rates with 5 percent margin (95 percent Margin Multiplier) and adding additional 20 percent improvement in future lifetime (Scale Factor 3.5).

Mortality improvements have become part of life for actuaries practicing in areas of life insurance and pension benefits. But there is very little done so far to hedge the longevity risk in pensions. There are wide differences in opinions whether the improvement rates will keep increasing as a result of advances in medicine or will taper off as a result of less healthy lifestyles.

We are not attempting to find a solution to that question here, but we want to point out that this risk is not being addressed in the current model of market value of liability measurement. Adding two types of longevity risk margins as discussed above is a reasonable way to load for a fair value price.

E. Risk Premium for Retirement Rate Risks

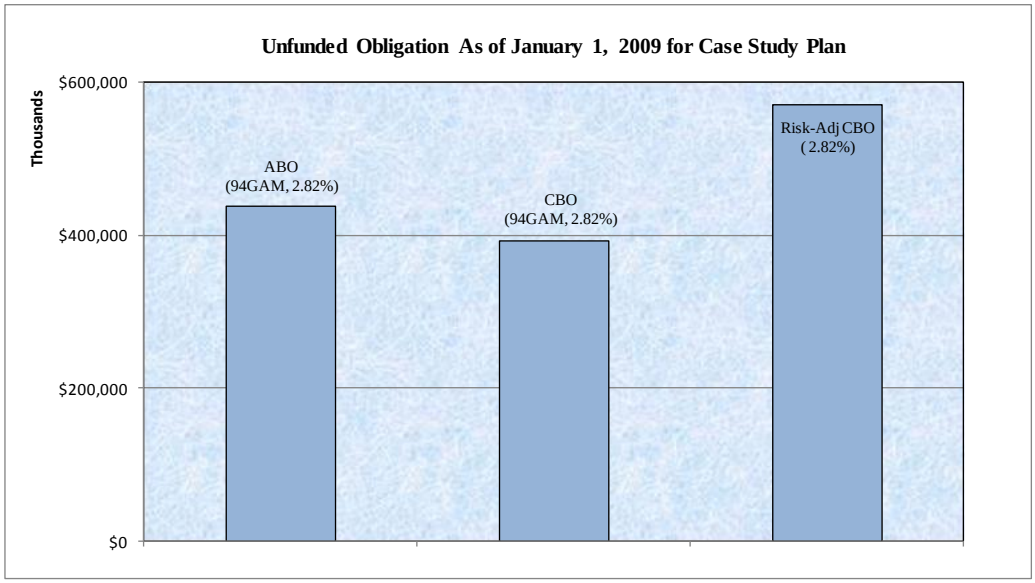
We now move to discussing risks associated with rates of retirement. Our Case Study Plan offers a normal retirement benefit to vested employees (five years of service) at age 60, or 30 years of service regardless of age. Employees are eligible for a reduced early retirement benefit at age 50 with at least 15 years of service. Reduction is 3 percent for each year the retirement precedes age 60.

Actuarial methods measure the liabilities by employing a pattern of retirement rates derived from observed and expected experience. However, no one can guarantee that future experience will closely follow such rates. When the early retirement benefits are subsidized, as with our case study plan, the cost of providing the benefit usually goes up as employees retire earlier in the early retirement period.

One way to avoid the risk is to calculate the liabilities assuming that employees would retire at the most valuable age, i.e., the retirement age resulting in the highest liability. For an employee who has accrued 15 years of service, most valuable age for our case study plan is age 50 (eligibility for early retirement with a reduced benefit), while for another employee who did not earn the right to this feature, the most valuable age is 60. We should note, that for employees eligible for early retirement with unreduced benefit (30 years of service), the most valuable age is age on the valuation date.

Figure 10 adds the unfunded obligation of the CBO after incorporating these risk adjustments to the chart from Part 1 in this series, "The Contractual Benefit Obligation."

Figure 10



F. Summary of Risk-Adjusted Pension Cash Flows

In fair value pricing, expected cash flows should not be treated as if they were fixed. Cash flows for a public sector pension fund may be free of default risk, but that does not make their expected amounts certain. There are certain risks in the level of the CBO cash flows that should be recognized in pricing the fair value of the public sector pension benefit liability.

Financial economists and financial engineers are fond of expressing all sorts of risks in terms of how many basis points should be added or subtracted from a given discount rate to account for such risks. We cannot let this happen in the loading for risks in the cash flows. We actuaries spent the better part of a couple decades changing our previous paradigm of implicit actuarial assumptions to one where we justify each assumption on its own, with explicit actuarial assumptions. Taking an actuarial shortcut by adding or subtracting basis points to the discount rate to recognize cash flows risks would be turning back the clock for our profession.

Therefore, making reasonable adjustments to the CBO cash flows for longevity and retirement rate risk is appropriate, even necessary for fair value calculations.

There are other risks that might be hedged. For example, some plans provide cost of living adjustments (COLAs) in benefits calculated as a function of the consumer price index. Not many, but others calculate them based on the wage increase granted to current employees in the compensation grade level from which the retiree retired. Both types of COLAs can easily emerge at levels far in excess of our deterministic assumptions. The proper loads on fair value exit-pricing would include premium margins for COLA risk. Another example arises in plans that contain gain-sharing provisions. Again, there are risks in the expected cash flows that should be hedged with risk premium loads when calculating a fair value exit price.

If risk margins for longevity risks, retirement rate risks, COLA, gain-sharing and other such cash flow risks are not built into the cash flows we are discounting, then we are not calculating a market or fair value of the pension liability.

G. Risk-Adjusted OPEB Cash Flows

If you thought that the CBO for OPEBs was a challenge as we discussed in Part 1 of this series, "The Contractual Benefit Obligation," imagine making risk premium adjustments for worse-than-expected future medical costs. It is no mystery why there is no market for single premium (other prepaid) group retiree medical coverage. No one in his right mind would sell such a policy. Imagine how high the price would need to be to protect the issuer against worse-than-expected cash flows.

This is just another reason why a strict fair value attribute model for postemployment (pensions and OPEBs) should be treated from hereafter evermore as a mere curiosity in a Ripley's Believe It or Not Museum; as an interesting idea that once garnered some support. Refer to Part 5 in this series, "Consider the Measurement Purpose," for much more appropriate measures of pension and OPEB liabilities for the most common purposes.

Part 3: Fair Value of the Liability – A Market-Related Discount Rate

A. Modigliani-Miller Friction

Whether or not an entity's financing method affects its value is a key subject of modern finance research. Modigliani and Miller (1958), in a seminal work affecting the subject, showed that, under specific conditions, the value of a company is invariant with respect to the leverage policy, or the method of financing of the company, in general. This irrelevance proposition rests on the assumptions of no taxes, no bankruptcy costs, and no agency costs. Consequently, if we observe the method of financing affecting the value of a company, it must be so because of taxes, bankruptcy costs, or agency costs.

While a pension plan is not a company, the general frame of reference applies. Taxes should not matter, as a qualified plan is not subject to them. However, for our purposes, pension "taxes" include any required payments to the government or regulators – even if, not termed taxes. Such a situation exists if the plan surplus cannot be fully returned to the plan sponsor, while a deficit is covered by the sponsor.

Most importantly, for financial institutions, such costs of benefits are imposed by regulatory capital requirements. As pointed out by the Casualty Actuarial Society Task Force on Fair Value Liabilities (2000), financial intermediaries face capital requirements. The capital held by industrial firms is primarily fixed assets, such as plants and equipment, and working capital used for operations. Efficient firms that produce with less capital drive out less efficient firms. In a competitive industry, the capital used is typically at the level of the capital needed. But in insurance and banking, capital serves to produce the product, but also to protect consumers. Capital is determined by statutory requirements or by rating agency measures. Statutory risk-based capital (RBC) formulas are minimum capital standards, and insurers usually hold more than the RBC requirements to avoid regulatory interventions or compete for discerning consumers seeking financially strong firms. Additionally, insurers base their target capital on the rating agencies' standards. The cost of holding capital includes the rate of return that the capital must earn investment restrictions, double taxation, as well as competitive and other friction costs. The relative importance of each cost depends on the industry. For example, for property-casualty insurers, double taxation is a substantial cost, while for pension plans that cost is not relevant. On the other hand, if banks hold 10 percent of deposits as non-interest-bearing deposits with the central bank, and the opportunity cost of capital (i.e., the available rate of return on alternative uses of the funds) is 5 percent, the economic cost of capital is 0.5 percent of the deposits per annum.

The pension plan's situation is quite different, as illustrated in comparison to an insurance company. Suppose an insurer has a liability to be paid in three years exactly, in the amount of $1,000,000.00. Assume that the risk-free interest rate is 3 percent per annum. The present value of the liability is $1,000,000.00 times 1.03^{-3}, or $915,141. 66. Suppose that the company is required by law and regulatory agencies to hold the amount of capital equal to 10 percent of the liabilities' market value, i.e., $91,514.17. As a consequence, the fair value of this liability is $915,141.66 + $91,514.17= $1,006,655.83, because any other entity assuming this liability must pay the liability, and simultaneously acquire appropriate capital to hold this liability. This

increase in the fair value of the liability, in relation to the equivalent market instrument paying the same amount at the same time, results from the fact that the equivalent market instrument cannot be legally represented to be an insurance policy. Only a firm organized as an insurance company and holding appropriate capital would be able to hold this liability -- then fair value would be established in the manner presented here.

In the case of a pension plan, the capital requirements are actually the exact opposite. Not only is a public pension plan not required to hold any form of risk-based capital, but it is specifically allowed, and encouraged by the regulation structure, to spread the funding of any surplus shortfall over an extended period of time. This means that the required regulatory capital for a public pension plan is, effectively, negative, and this results in a lower fair value of the liabilities. A financial economics approach calls for discounting of the pension plan liabilities cash flows at a risk-free rate appropriate for a given cash flow's maturity. The conventional actuarial approach uses the long-term expected rate of return on the asset portfolio. One can actually view the actuarial method of discounting as a substitute for the measurement of the reduction in the liability value due to delayed funding practice being encouraged in the reality of public pension plans.

Conventional actuarial practice is a reasonable approximation of the economic reality, while financial economics imposes complete prefunding, and inflexibility of funding, not applicable to the economic reality. The motivation of the financial economics approach is complete security of pension benefits, as it naturally and often imposes an investment strategy on a portfolio consisting entirely of risk-free bonds of appropriate maturities. In reality, pension plans are invested in a balanced portfolio of stocks and bonds.

To a degree, such a portfolio is imposed on pension plans by regulatory diversification requirements. Because of this situation, it is often argued (Gabriel, Roeder, Smith & Co., 2008) that financial economics imposes excessive funding requirements on present generations at the expense of future generations, because, historically, the actual realized rate of return has been consistently higher than the risk-free rate of return.

But we must also notice that the historical rate of return is subject to *survivorship bias:* only securities and securities markets that survived the period of measurement are included. Historical equity rates of return are higher than the risk-free rate, because there have been entire markets (e.g., the Russian stock market of 1900) or individual securities that did not survive the period of observation. If their rates of return are included in measurement, realized rates of return will be lower. Notably, for pension plans, historical averages that accurately represent experience should include what happened to pension plans that failed or required any form of emergency assistance from any form of government. We should also note survival of the pension plan itself, though the apparent goal of its funding and regulation is not assured. Thus the fair value of the pension liability that does not include a reduction allowing for the possibility of non-survival does not truly correspond to economic reality. Of course, such an adjustment is unlikely to be used in fair value accounting, as it would represent admission of funding regulatory failure. A survivorship bias adjustment applied to the asset side of the balance sheet without an equivalent adjustment to the liability side of the balance sheet represents a significant departure from the basic philosophy of fair value accounting principles -- and a rather pronounced inconsistency.

But one could argue that for a public pension plan, survival of the plan is assured by what one could call the *higher government backstop.* Imminent insolvency of a sole or agent plan and its government sponsor is very likely to bring about support from a higher level of government, or the Federal Government. This additional backstop can be represented in the balance sheet as an extra asset, or through a reduction in the actuarial liability. That reduction (or a part of it) can be—and in practice is—achieved by using a valuation rate which is higher than risk-free.

Let us also note that while the conventional actuarial approach uses a discount rate in excess of the risk-free rate, it also effectively incorporates the flexibility of delayed funding granted to plan sponsors and, possibly a portion of the adjustment for survivorship bias on the liabilities side. It, therefore, represents a pragmatic and realistic alternative to a rigid risk-free discounting, which does not incorporate regulatory costs and benefits, and survivorship bias on the liability side.

The second friction cost identified in the Modigliani-Miller irrelevancy proposition was the cost of bankruptcy,. It should not matter in the case of a pension plan. The very existence of the plan is a design created for the purpose of lowering the probability of plan insolvency. Insurance against that calamity is also often provided by some form of regulatory agency, such as Pension Benefits Guaranty Corporation for private plans in the United States. Public pension plans are not subject to such insurance. Instead, it is the good faith and credit of the plan sponsor that provides the secondary or residual guarantee, in addition to plan pre-funding. In fact, expected continuous (in fact, infinite) existence of public plans sponsors ties the solvency of the plan to the sponsor's solvency, and given the taxing power of government sponsors, the cost of bankruptcy is significantly reduced, probably even to zero.

Additionally, given the state and federal governments' history of supporting municipal governments, one could conclude that the cost of bankruptcy is negligible. It frees management of the pension fund to take reasonable risks to generate greater potential for rewards than just investing entirely in Treasuries.

The final friction cost identified by the Modigliani-Miller irrelevancy proposition is the agency cost: the cost of the relationship of delegating control over property from its owner (principal) to agents. Generally in an insurance company case, agency problems raise the cost of holding capital. Imagine a situation where shareholders wish the insurer to write profitable but risky business, but managers avoid risk or buy reinsurance in order to protect their jobs. The managers' actions lower rates of return to shareholders. Shareholders may also incur the cost of incentives to induce managers to take more risk. These costs are no different for insurers than for other companies. They may become large, and are difficult to measure.

In pensions, these costs have complicated structures. Investing public pension plan assets in a diversified portfolio of stocks and bonds, as opposed to risk-free bonds, may result in lower plan cost, and the benefits of such actions will accrue to taxpayers, and not plan beneficiaries. This has been in fact put forth as one of the arguments for financial economics approach. However, if the cost of possible lower rates of return due to, for example, stock market declines, are also borne by the plan sponsor, this structure does not impose any agency costs on plan beneficiaries. That is a more realistic picture of the case of public pension plans. One could,

however, argue that agency costs in the case of public pension plans are only imposed on the liabilities side: as the plan participants are a small group with homogeneous interests, if they organize successfully, they can become a powerful political lobby that can raise benefits for plan participants by imposing small costs spread among a large number of taxpayers.

This situation of concentrated benefits and dispersed costs is a standard model in Public Choice Theory (Mueller, 1989). The case for it is especially powerful when the costs of benefits can be transferred to future taxpayers, not yet alive or resident, through borrowing. This again, reinforces the theme of postponement of funding: it is actually that phenomenon that lowers the fair value of pension liability in the case of public pension plans, as we presented it in the analysis of tax and regulatory costs.

It should be noted that in the case of private pension plans, overfunded plans may be subject to a form of "pension arbitrage"; liabilities may be settled by purchasing annuities from an insurance firm. This can only be done for an overfunded plan, as the same liabilities for pension payouts will have different values when held by a pension plan or an insurance company. An insurance firm can function as an insurance firm only if it holds appropriate capital, and it charges the cost of that capital to liabilities, causing the value of liabilities to increase. Financial economics may argue that this is yet another argument for a risk-free bond funding of pension plans. While an extreme viewpoint, some believe that this is a proof that an overfunded pension plan loses its reason for existence as a pension plan. A pension plan is created to spread the funding over an extended period of time, and in the case of public pension plans, such extension can be quite substantial, due to the infinite horizon of existence of plan sponsors. The financial economics approach fails to incorporate these considerations.

Finally, the Modigliani-Miller irrelevance proposition is not free to be invoked whenever it is convenient or supports one's position. It is a very theoretical construct. It requires numerous conditions, including no taxes, no bankruptcy, no agency costs and a competitive and complete capital market, a "perfect" market (Myers, 2001). It requires a frictionless, pristine environment. Modigliani-Miller is instructive. We appreciate its value for pedagogical reasons for the insight it brings to understanding the theoretical principles underlying capital structures. However, all its conditions never really exist in the real world.

Furthermore, Modigliani-Miller is about corporate finance and raising capital for corporate operations and investments. It is about the irrelevance of a corporate entity's capital structure (internal cash flow, debt and corporate equity) on its *corporate stock valuations*. Extending this theoretical concept to claim the irrelevance of a government or bidding entity's funding/investment policy on its *pension liability valuation* is a stretch.

One might argue that the value of the pension liability should be determined as its market or fair value, as tradable in the marketplace. Such a market-driven value would be independent of how that current employer/plan has, in the past, invested the assets set aside for that purpose. And one might argue that the public sector pension liability should be valued using a risk-free return in order to match the default-free nature of the obligation. These other arguments are more "on-point" and form a better foundation for advocating market or risk-free discount rates. One would not need, and should not, appeal to Modigliani-Miller to support that position.

In this section of Part 3, we have accommodated the professional discussion of the application of Modigliani-Miller irrelevance proposition, in part, by drawing upon how private sector insurance companies' capital structures affect reserving requirements. In reality, however, the Modigliani-Miller irrelevance proposition is virtually irrelevant to public sector pension liabilities in the real world.

Thus a fair value model may use discount rates higher than risk-free to reflect market-based observables.

B. Return to Fair Value Definitions

There is some logic in connecting the dots between a public sector pension benefit's lack of default risk and use of default-free discount rates. However, the markets do not necessarily conform to that notion.

There is an apparent inconsistency between current thinking about risk-free discount rates and discount rates that reflect the manner in which exchange prices would be developed in a market for pension liabilities. Fair value is not about assigning an economic value in a vacuum; it is about pricing; about how pricing would operate in a real market.

Market participants regularly demonstrate their willingness to pay higher prices for loans (even pledging personal collateral) for ventures in which they are more confident of profitable outcomes, whether short or long term. The market participant's intentions for investing the proceeds (regulated or not) and its capital structure will affect the price it is willing to quote for settling an employer's pension obligation. An exit price must be developed with consideration given to the settlement rates available in the market.

There are, indeed, markets (some active some imagined) where public sector pension liabilities might be settled. Exit price bids would likely be lower (higher discount rates) than risk-free prices.

C. Single Premium Group Annuity Market

Thanks to the U. S. Department of Labor's Interpretive Bulletin 95-1, there are only approximately 10 or 12 market participants who pass the threshold for the U. S. single premium group annuity market for settling corporate pension plan obligations. The single premium annuity market is highly regulated, with statutory, GAAP and tax reserves to consider. U. S. and European regulatory environments differ. As such, differences in prices quoted to settle pension obligations may occur.

Insurance companies setting the exit prices for employers who are settling their pension obligations do not discount the expected cash flows with risk-free rates. It is not because they know they can go out of business and renege on their "guarantee." It is because they take into consideration their own current general account investment portfolios, their plans for investing the proceeds and the fixed income market conditions at the time of the quote. They consider their capital structure, their own externally imposed reserving requirements, and a host of other factors.

The Modigliani-Miller irrelevance proposition does not apply so nicely and neatly to pricing in the real world of single premium group annuity contracts. The actual market for settling corporate pension obligations utilizes discount rates higher than risk-free rates. That is an observable market input for our fair value purposes at hand. According to financial engineering principles, as many components of our fair value pricing model as possible must be calibrated to real market data.

Very few state and local government defined benefit pension plans have ever been settled with insurance company single premium group annuity contracts. But there is nothing preventing them from doing just that. There is no reason insurance companies would price public sector plan settlements and different than private sector plan settlements, as long as the contract provisions are basically the same.

Thus, the single premium group annuity market can be thought of as the principal market for public sector pension plan exit transactions. The exit prices paid for such would be based on discount rates of high grade corporate bonds and other such investments expected to back up the promise.

If a public sector pension fund or employer were to settle an accrued pension obligation in the single premium group annuity market (a genuine and likely market for relevant observables), there would be no more liability held in the books for the risk of the insurance company's insolvency or other default. There are also state insolvency guaranty funds to consider as a hedge against that possibility. That residual contingent liability is not appropriate for financial reporting.[7] More on the definition of liability for financial reporting can be found in Part 5 of this series, "Consider the Measurement Purpose." Furthermore, there is no need to recognize any residual contingent liability related to numerous other insurance transactions; none for long-term disability, health insurance, property/casualty insurance, general liability insurance. Such residual contingent liabilities are generally not measurable and do not qualify for reporting purposes.

Effectively, such a plan or employer which settled its pension obligation in the single premium group annuity market has settled the obligation at a market-related discount rate. Therefore, those plans and employers which have not yet actually settled their accrued obligation should not be held to a higher liability than those which have actually settled theirs at a fair market rate.

Thus, fair value should use discount rates higher than risk-free, as observable in the marketplace.

[7] The City of West Palm Beach settled an accrued pension obligation for general employees with an insurance company (1997). The negotiated price was based on market conditions and fixed plan cash flows. Since then, the city has not recorded any residual contingent pension obligation on its books.

D. High Quality Corporate Bond Settlement Rates

The FASB deliberately chose to require the use of settlement rates for discounting cash flows, which has evolved into the common use of high quality (AA or better) corporate bond yield curve observed at the measurement dates, which is much different from a risk-free yield curve.

Based on a valuation of our case study plan as of January 1, 2009, equivalent single discount rates were derived from risk-free (Ryan Labs) and high quality corporate (CitiGroup) spot yield curves, on the basis of the plan's expected ABO benefit cash flows. These equivalent single discount rates were obtained using this method for the spot yield curve for each December 31 from 1995 to 2008. Thus, the same duration and convexity were used to derive the equivalent single discount rate for each year's curve. Yields for spots above 30 years were assumed to be the same as the 30-year spots. Refer to the section on funding in Part 5 of this series, "Consider the Measurement Purpose," for more details on the methodology and sources. Figure 11 below presents these single equivalent rates for each of the last 14 years.

Figure 11

Single Equivalent Discount Rates			
Dec 31	Based on Treasury STRIPS	Based on High Quality Corporate Spots	Spread of Corporates over STRIPS
1995	6.32%	6.71%	0.39%
1996	6.68%	7.43%	0.75%
1997	6.00%	6.75%	0.75%
1998	5.38%	6.46%	1.08%
1999	6.72%	7.93%	1.21%
2000	5.43%	7.18%	1.75%
2001	5.59%	6.85%	1.26%
2002	4.91%	5.88%	0.97%
2003	5.12%	5.86%	0.74%
2004	4.88%	5.58%	0.70%
2005	4.58%	5.50%	0.92%
2006	4.84%	5.85%	1.01%
2007	4.45%	6.41%	1.96%
2008	2.82%	6.07%	3.25%
Average of 14 years			1.20%
Average of middle 12 years			1.09%

Thus, a fair-value model may use discount rates at least up to high quality corporate spot rates to represent settlement rates in the marketplace. This gives consideration to real market observables available for settling similar obligations, a fundamental principle of financial engineering and fair-value modeling.

E. Liquidity Risk

Small-denominated bond issues trade at higher yields (and lower prices), than do large ones with identical terms at the same time for the same issuer, because they are more difficult and expensive to sell. Private equity transactions have a higher return expectations (and lower initial prices) because they are not liquid. Investment economics principles require a higher return for illiquid instruments.

Public sector pension obligations are likely among the most illiquid financial instruments in existence. If we are to be true to the fair value measurement attribute, the fair value of public sector pension liabilities must recognize their illiquidity when setting the discount rates.

Again, fair value should use discount rates higher than risk-free.

F. Low Percentile in the Capital Asset Pricing Model

The single premium group annuity market is highly regulated and, thus, not a truly free market laboratory for adopting discount rates for fair value calculations.

A more advantageous market for transfer of public sector pension liabilities, if one needed to be imagined, might be one in which the market participants were other public sector pension funds. In such a market, these funds would buy and hold or sell pension liabilities for gain, just as they buy and hold or sell assets for gain. The plan exiting the liability would not likely be able to settle its obligation in that market at the expected long-term return of those pension funds.

Although if the terms of the exchange transaction were to guarantee that the exiting plan would never have to retain any residual liability, there might actually be bidders who would quote the low price, based on their own expected returns. Usually, in other commercial markets, such low bidders are dismissed as not having the backing or solvency necessary to make good on the transfer. However, if the bidders are all viable public sector pension funds, with the ability to go back to their own respective employers to make good on the liability for the benefits of its own members or of those in the acquired block, then the exiting employer may not have the same concern about solvency as in other commercial markets.

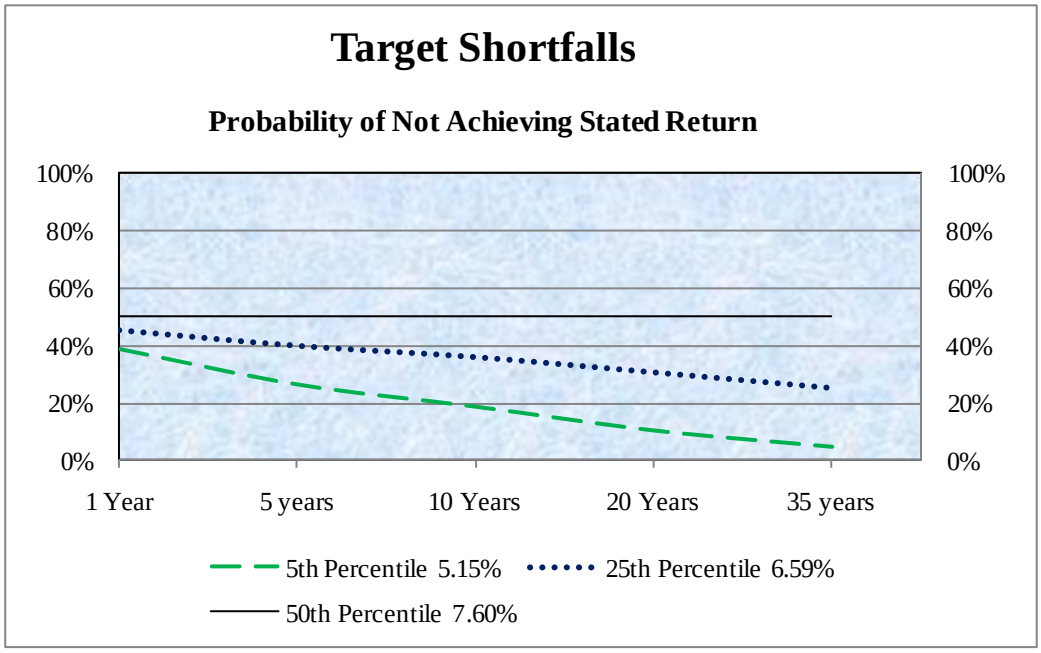
Nevertheless, the exit prices will more likely be set using discount rates below the expected long-term return on the underlying portfolio, based on fairly sophisticated assessments of risk. We imagine the pension funds would consult with their chief investment officer, investment consultants and actuarial consultants to assist in pricing the liability. Such analysis would surely include Monte Carlo simulations involving risk tolerance and stress-testing.

To simplify the analyses for illustration purposes, consider a pension fund whose investment asset allocation is 50 percent in domestic, large-cap stocks (split evenly between value and growth), 10 percent in international stocks, 35 percent in fixed income (split evenly between intermediate term government bonds and corporate bonds) and 5 percent in cash equivalents.

Based on current capital market assumptions for each asset class (SunGard, 2009) and assuming the alpha achieved equals the investment-related expenses, under a conventional capital asset pricing model, the mean return is 7.97 percent with standard deviation of 8.97 percent under a normal/lognormal distribution model. While interest in fat-tail distributions such as Paretian and other log-stable distributions is returning of late, we will use the currently accepted conventional analysis.

Because of the volatility drag, such a portfolio has a 50th percentile return of 7.60 percent, a 25th percentile return of 6.59 percent and a 5th percentile return of 5.15 percent over a 35-year period.

Figure 12

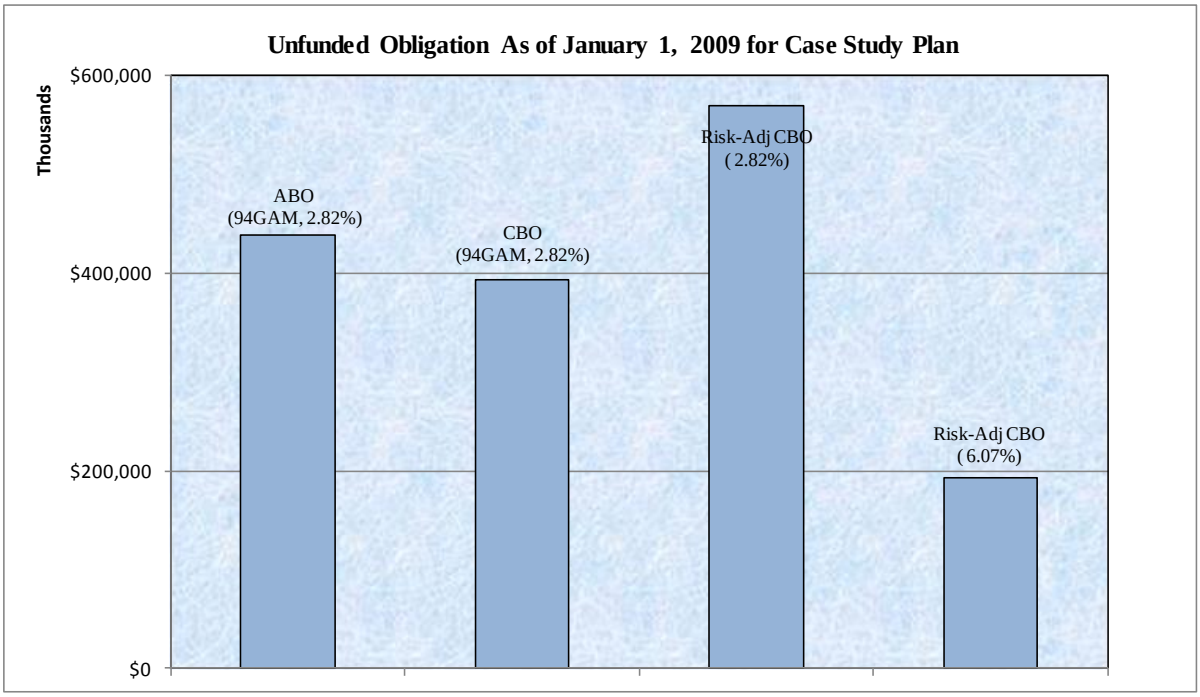


For a market in which the participants are other public sector pension funds, which do not have the reserving constraints that insurance companies in the single premium group annuity market have, it is reasonable to expect that these market participants can and would guarantee the payment of the transferred pension benefit obligation while quoting a price for risk-adjusted cash flows discounted at rates higher than risk-free but below the expected long-term return of their balanced portfolio. The rate might be at or near the 25th percentile (6.59 percent), but possibly down as low as the 5th percentile (5.15 percent).

In any event, the fair value of the exiting employer's pension benefit liability may be discounted using rates significantly higher than risk-free rates. The above analysis does not arrive at a single bright-line discount rate to use. Judgment is needed to set the discount rate in response to market inputs.

In Figure 13, the risk-adjusted CBO is discounted at a higher rate and added to the comparison chart. For illustration, we have used 6.07 percent, the single equivalent rate for the high quality corporate yield curve observed at Dec. 31, 2008.

Figure 13



G. Possible Resolution

Pension liabilities are another form of financial instrument. They are private issues, not marketable and not traded. This makes establishing their exact market value impossible. But financial theory does provide us with methods of calculating or approximating the values of privately traded and illiquid financial assets. This emerging methodology calls for stochastic modeling of future cash flows, and valuation based on either the risk-neutral model with risk-free discounting, or risk-adjusted model with real world probabilities and real world interest rates scenarios.

These approaches are represented by emerging regulatory methodologies of principles-based reserving, required cash flow and stress testing, and risk-based capital modeling using value-at-risk, or conditional tail expectation. We believe that a realistic pension liability valuation methodology may incorporate fully stochastic modeling of liability cash flows, using investment-risk-adjusted methodologies, essentially producing fair values of the public sector pension liability.

Part 4: Fair Value of the Liability – The Residual Benefit Liability

A. Modeling the Real World

Mathematical models, whether physics or financial models, are intended to simulate the operation of real world structures and, hopefully, present results that approximate real world outcomes. A fair value model for financial instruments should take care to simulate the operation of all moving parts in the structure and contract. Ignoring or glossing over major components can be disastrous.

Financial economists tend to group players into principals and agents. Among the qualities Warren Buffet looks for in a company, is solid management personnel whom he likes. Agents can add or deplete value for principals. As mentioned previously, one of the primary conditions for Modigliani-Miller's irrelevance proposition is that agency costs and benefits do not matter. That is a condition for the theoretical proposition—not a fact.

We emphasized in Part 1 of this series, "The Contractual Benefit Obligation," the importance of recognizing the employment contract terms between employer and employee when valuing the benefits for fair value purposes. Respect for this labor economics principle led us to revise the benefits valued from the accumulated benefits obligation (ABO) to the contractual benefits obligation (CBO).

Similarly, we must give the same respect to contract terms between the employer and the plan. The pension plan trustees and managers are agents standing between taxpayers and plan members, but the manner in which they discharge their duties directly affects the actual true cost to taxpayers of the benefits payable. Furthermore, the pension fund is not merely the employer's collateral for satisfying its direct contractual liability. That is not how it works.

Contractually speaking, the employer does not owe next year's pension benefit payments to plan members, nor the year after that, or the year after that. The pension plan owes them. If due to insolvency, the pension plan cannot pay the benefit, then the employer will step in to satisfy its original promise to plan members. While the public sector pension fund should not be deemed a mere pass through (more about this in the next section), the employer does retain a residual pension benefit liability in the event of the plan's default on its obligation to plan members.

In the spirit of modeling all moving parts carefully, we should consider an alternate model which simulates the operation of the pension fund over time, to see what liability or assets may remain for the employer to assume after the dust has settled. This alternate approach is truer to real world modeling.

B. The Pension Fund is Not a Pass Through

The public pension fund is the five-ton elephant in the room, which financial economics proponents ignore, in the name of "pass through."

There are inconsistencies in the arguments of those who advocate this pass through treatment. Pass through proponents never suggest that the pension fund assets be placed on the statement of net assets as a government asset in the name of pass through, like other collateral, such as cash with fiscal agents, or defeasance funds. Similarly, pass through proponents never suggest that the pension benefit liability itself be placed on the statement of assets as a long-term liability in the name of pass through. They insist that the *net* liability (market value of pension benefit liability minus market value of pension assets) be placed there. This feels much more like a residual liability, rather than pass through, but it is not modeled that way by financial economics proponents. Even the three improvements outlined above (CBO, risk-adjusted cash flows, and higher discount rates than risk-free) do not fix this flaw.

Let us turn our attention to the substantive ways in which the public sector pension fund is a material player (not to be ignored) to warrant specific treatment in our alternate fair model of pension benefit liabilities. This discussion applies to cost sharing employers as well as to sole and agent employers. While the arguments set forth in this section are stronger for cost sharing, multiple employer plans and their participating employers, they are equally applicable to sole and agent employers and their plans.

Multi-employer plans in the private sector have some of these same characteristics, which is why the pass-through concept is strained to the breaking point with private sector multi-employer plans. Private-sector, single-employer plans may have some of these characteristics, in theory only, which is why pass-through is not an unreasonable concept in that environment. However, these characteristics are much more apparent, exposed and exaggerated in the public sector.

The following qualities of public sector sole employer plans and agent multiple employer plans (in addition to cost-sharing, multiple-employer plans) drive this alternative model and our objection, in general, to the pass-through concept otherwise applicable to single-employer plans in the private sector environment. More on this topic and why public sector pension accounting is and should be different from the private sector can be found in the introduction to Part 5 of this series, "Consider the Measurement Purpose."

An Independent Entity. Typically, the public sector employer created the pension trust as a separate, independent entity. For example, Massachusetts' 106 public systems are independent from the municipalities by whom the members of the systems are employed[8], Michigan MERS is a statutory municipal employee plan that is a "public corporation" separate and apart from state government[9], and Missouri PSRS was created by state statute as a "body corporate." Often, the plan and its agent-staff are not even subject to the same administrative rules applicable to mere

[8] See Everett Retirement Bd. v. Board of Assessors of Everett, 19 Mass. App. Ct. 305, 473 N.E.2d 1162, Mass. App., 1985.

[9] Michigan Compiled Law 38.1536(1) and 38.1502c(3).

agencies of government. The pension fund is an independent trust and not taxpayers' money.[10] The financial managers of private sector companies are inextricably linked to the management of their single-employer pension plans. This is not true in the public sector.

Sue and be sued. A marker of independence is whether the plan is a jural entity that can sue and be sued on its own and in its own name. This is commonly a characteristic of public sector pension plans.[11]

Contract. There is a contract between the employer and the pension trust. In this contract, the employer has put the responsibility for pension *benefit* payments onto the pension plan in exchange for taking on a *funding* responsibility.

Enforcement of the exchange. There have been times when the employer has breached its agreement with the pension trust. Pension plans often have the authority to sue the employer for failure to fulfill its funding obligation under the contract, and as proof that this contract between the public sector employer and pension trust is a very real one, pension trusts have indeed exercised that authority and sued the employer for not contributing as scheduled in order to enforce the contract, for diverting funds and other breaches. Furthermore, employers have, at times, sued the plans.[12]. In fairness, there have been times in which the employer has reneged on its funding responsibility, and it was upheld in courts. However, the weight of common law is in favor of the employer's funding obligation enforced. This litigation seldom ever happens with single-employer plans in the private sector.

The Creditor. Clearly, the employer has a pension liability, which should be presented somewhere in its financial statements. But if the employer is a debtor for pensions, who is the debt owed to? If there is a debt to pay by the employer, who is the creditor? The transaction between the employer and the pension fund is a real one. The debt owed by the employer is not a benefit liability (benefits payable to plan members) but a funding liability (contributions payable to the pension trust). The employer owes payments to the pension fund, not to the employees. The pension fund is not a pass-through. It is the creditor.

The Payer. The pension trust is the benefit payer of first resort, while the employer is the benefit payer of last resort.

Recourse. To illustrate this exchange and its resultant benefit payment priorities, consider an employee who believes his pension was not calculated properly. In most situations, he will get nowhere appealing to or suing the employer. He must appeal to or sue the pension trust. The

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[10] Dadisman v. Moore, 384 S.E. 2d 816 (W.V. 1989); City of Miami v. Gates, 393 So. 2d 586 (Fla. 3rd DCA 1981).

[11] City of Houston HMEPS (Article 6243h, sec. 2(g), HMEPS v. Ferrel, Thayer v. HMEPS), City of St. Louis ERS (see State of Missouri, ex rel. Employees Retirement System of the City of St. Louis et al., v. Board of Estimate & Apportionment of the City of St. Louis, et al., 43 S.W.3d 887 Mo. App. E. D., 2001), Kentucky RS (KRS 61.645), Mississippi PERS (MCA Section 25-11-119(5)).

[12] City of St. Louis (refer to the previous citation and the related case of Neske v. City of St. Louis, 218 S.W.3d 417 (Mo. en banc. 2007), Illinois IMRF (given specific statutory authority to sue to enforce contributions, Section 7-172.1 of the Illinois Pension Code, 40 ILCS 5/1-101 *et seq*.); McDermott v. Regan, 624 N.E. 2d 985 (N.Y. 1993).

employer does not retain any responsibility for paying the benefits. If he loses with the pension trust, there is nowhere else to go.

Economic engine. Often the pension fund is larger than the employer itself, in terms of net assets. According to a recent study published by Boivie and Almeida (2009), in fiscal year 2005-06 expenditures from state and local pension benefits totaled $151.7 billion to 7.3 million pensioners and had a total economic impact of more than $358 billion, supported more than 2.5 million American jobs, and had a large multiplier effect with every taxpayer dollar invested in state and local pensions supporting $11.45 in total economic activity, while each dollar paid out in benefits supported $2.36 in economic activity.

Residual Assets and Liabilities. A clearer perspective of the employer's pension *benefit* liability would consider it a residual liability. After the pension fund has paid out all its assets on schedule with some remaining benefits yet to be paid, the residual obligation is an employer liability. On the other hand, if the last pensioner receives his last benefit and assets remains, some pension contracts say that such remainder may revert to the employer (after satisfaction of all liabilities), in which case the employer would reflect a residual asset. Other contracts may require that all assets be used for plan benefits of some sort or another, in which case there would be no residual asset. Nevertheless, it is difficult to imagine large, statewide, cost-sharing multiple-employer plans (as defined by GASB standards) running out of money with unpaid benefit obligations falling back onto each respective contributing employer ratably or otherwise. This model of the employer's residual benefit liability is more easily imagined for sole and agent employers.

The contract (including the enabling and operational documents) and the participating entity (the pension trust) are so very important in the delivery of public sector pension benefits, that their existence and operation must be considered when determining the fair value of the public sector pension benefit liability.

We propose an alternate approach to determine fair value of the employer *benefit* liability by modeling the operation of the public sector pension fund over time. This approach better represents the employer's risks and rewards, considering the true manner in which the pension obligation is defeased over time.

This alternate model is a fair value of the residual liability. The only *benefit* liability the employer has is a residual liability.

C. Modeling Residual Assets and Liabilities

If we modeled the operation of the plan's payment of the risk-adjusted CBO cash flows on schedule and the plan's investment return every year until the last pension benefit payment is due, we would have a model of the employer's residual asset or liability.

If there are benefits left to be paid after the pension fund comes to ruin, then the sum of their present values (discounted at a high quality corporate yield curve observed at the measurement date) would represent the employer's fair value of the residual pension benefit liability because the public sector employer would still be on the hook for their payment. On the other hand, there may be assets left after the last benefit payment is made. In that case, if the contract between the employer and the plan contains a reversion clause upon the satisfaction of all plan liabilities, then the present value of that remaining asset value (discounted at the same rate as the fund was assumed to have earned over the time period) would represent the employer's residual pension benefit asset.

Recall that there is a very real exchange agreement between the employer and plan in which the employer puts the pension *benefit* obligation to the pension fund, but accepts a substantial advance *funding* obligation pursuant to the pension fund's chosen actuarial cost method. The employer has no direct pension benefit liability until or unless the pension fund runs out of money before all pension benefits (contractually earned at the measurement date) are paid. Certainly, the employer may have a substantial *funding* obligation in accordance with conventional actuarial cost methods as of the measurement date, but its pension *benefit* obligation is the residual obligation described.

This leaves an actuarially interesting question. What assumptions and methods as to the investment rates of return should be used for simulating the operation of the pension fund, so that we may know what are the residual expected (or rather, risk-adjusted) benefit cash flows that constitute the employer's obligation?

Deterministic Modeling

Certainly, for a deterministic forecast, the 50^{th} percentile of range of expected rates of return for the investment policy in place on the measurement date is a reasonable candidate. Alternatively, the 25^{th} percentile might be used to build in a margin for error. Additional guidance for answering this question can be found a in previous discussion in Part 3 of this series, "A Market-Related Discount Rate," about how public sector pension funds might select a discount rate for pricing their exit liability in a market where they buy and hold or sell pension liabilities for gain. Furthermore, if the portfolio were invested entirely in intermediate government securities, an expected return of 5.0 percent is reasonable (SunGard, 2009), and for a corporate bond portfolio, 5.75 percent (SunGard, 2009). Finally, a sequence of consecutive annual returns was reverse engineered from the STRIPS spot yield curve for Dec. 31, 2008. This sequence of returns themselves can be used to model the operation of a pension fund if it were invested entirely in Treasury STRIPS, matching the cash flow, as some financial economics advocates suggest.

These are scenarios of the future investment performance of the pension fund and are treated as deterministic assumptions in each future year. Stochastic approaches provide more information including likelihood ranges, and will be explored below. However, initially, for simplicity and illustration, we are limiting the forecast to deterministic views of the future. To summarize the candidates:

Figure 14

	Deterministic ROR Assumption
Balanced Portfolio 50th Percentile	7.60%
Balanced Portfolio 25th Percentile	6.59%
Equivalent Single Rate for Dec08 AA Corporate Spots (*CitiGroup*)	6.07%
Corporate Bond Expected Return	5.75%
Intermediate Govt Bond Expected Return	5.00%
Equivalent Annual Returns for Dec08 STRIPS (*Ryan Labs*)	Sequence[1]

The following figure presents risk-adjusted cash flows and plan asset values at five-year intervals until the end, using the expected return (50^{th} percentile) of the balanced portfolio in a deterministic forecast of the plan operation.

In the same manner as the full run of risk-adjusted CBO cash flows were discounted using the CitiGroup Pension Discount Curve observed as of Dec. 31, 2008, in Part 3 of this series, "A Market-Related Discount Rate," we are discounting the residual benefit payments using the same assumptions.

Figure 15

Deterministic Simulation (7.60% ROR; 50th Percentile) of Plan Operation under Alternate Fair Value Model							
$ In Thousands	MVA (BOY)	Risk-Adjusted CBO Cash Flows	Assumed Investment Earnings	MVA (EOY)	Residual Employer-Paid Benefits Due	CitiGroup High Quality Corp Spot Rates	PV at 1/1/2009 of Residual Benefits
2009	$ 380,717	$ 28,476	$ 28,935	$ 381,175	$ -	4.90%	$ -
2014	371,567	33,622	28,239	366,185	-	5.62%	-
2019	331,009	37,320	25,157	318,845	-	6.59%	-
2024	253,675	40,175	19,279	232,779	-	7.14%	-
2029	127,936	41,726	9,723	95,934	-	6.99%	-
2034	-	41,245	-	-	41,245	5.84%	9,441
2039	-	38,020	-	-	38,020	5.03%	8,304
2044	-	31,653	-	-	31,653	5.03%	5,409
2049	-	22,865	-	-	22,865	5.03%	3,057
2054	-	14,077	-	-	14,077	5.03%	1,473
2059	-	7,379	-	-	7,379	5.03%	604
2064	-	3,430	-	-	3,430	5.03%	220
2069	-	1,578	-	-	1,578	5.03%	79
2074	-	760	-	-	760	5.03%	30
2079	-	315	-	-	315	5.03%	10
2084	-	88	-	-	88	5.03%	2
2089	-	17	-	-	17	5.03%	0
2094	-	2	-	-	2	5.03%	0
2099	-	0	-	-	0	5.03%	0
2104	-	0	-	-	0	5.03%	0
2109	-	0	-	-	0	5.03%	0
Total Present Value at January 1, 2009 of the Residual Employer-Paid Benefits							$ 137,497,517

In this model of the operation of the pension fund, no future employer or employee contributions are made following the measurement date and no new contractual benefits accrue thereafter either. No administrative expenses are assumed and benefit payments are assumed to be made at year end.

In this deterministic forecast of the employer's residual benefit liability the pension fund comes to ruin in the year 2033. At the end of that year, there are no more funds left in the pension trust to fulfill its obligations under the contract it has with the employer. Hence, the employer becomes the payer of last resort and must pay the benefits promised. The present value of that residual benefit liability, discounted at market-related discount rates, is the employer's unfunded benefit obligation at the measurement date. Under a deterministic forecast, this represents the fair value of the employer's residual benefit liability.

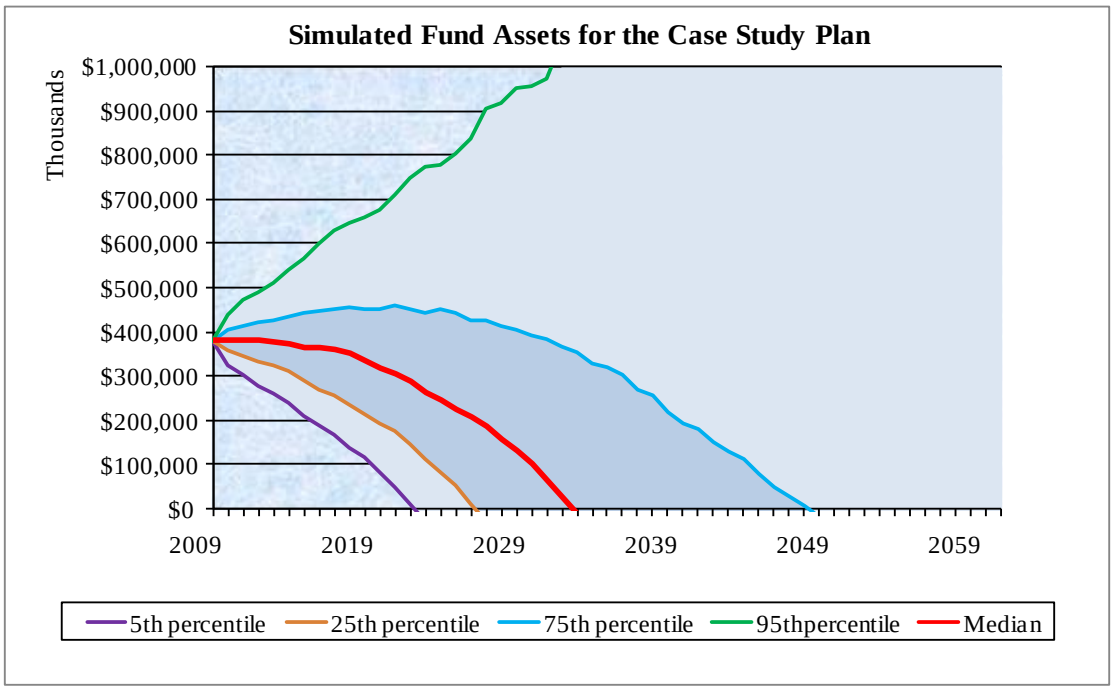
Stochastic Modeling

Rather than simply assume that the pension fund will earn exactly the expected return each year, more useful information can be extracted from the model by simulating, in a stochastic fashion, the pension fund's investment earnings in each future year.

We assumed that the future returns of the balanced portfolio will follow the normal/lognormal distribution curve. As mentioned previously, in Part 3 of this series, "A Market-Related Discount Rate," there is a renewed interest in fat-tail distributions such as Paretian and other log-stable distributions to more accurately reflect market swings in both tails of the distribution of returns. We seem to be having those once-in-a-century events every decade. But in the interest of simplicity and conventional practice, we will utilize the normal/lognormal distribution for modeling the pension fund returns, based on a mean of 7.97 percent and a standard deviation of 8.97 percent developed in Part 3 of this series. Again, these were developed using the capital asset pricing model and capital market assumptions from SunGard (2009).

A total of 500 trials were run (sufficient to stabilize the results), each producing returns for our case study plan for each of the next 100 years. The figure below tracks the value of plan assets over time.

Figure 16



Many plans had worse funded ratios than our case study plan as of Jan. 1, 2009. On the other hand, at times in the past (and hopefully in the future again), when the equity markets were not so depressed, many plans had much better funded ratios. In a not-so-depressed equity market, this model of the employer's residual liability would show a much more favorable picture. The distribution of the year of ruin (and resultant residual liability) is very sensitive to the beginning

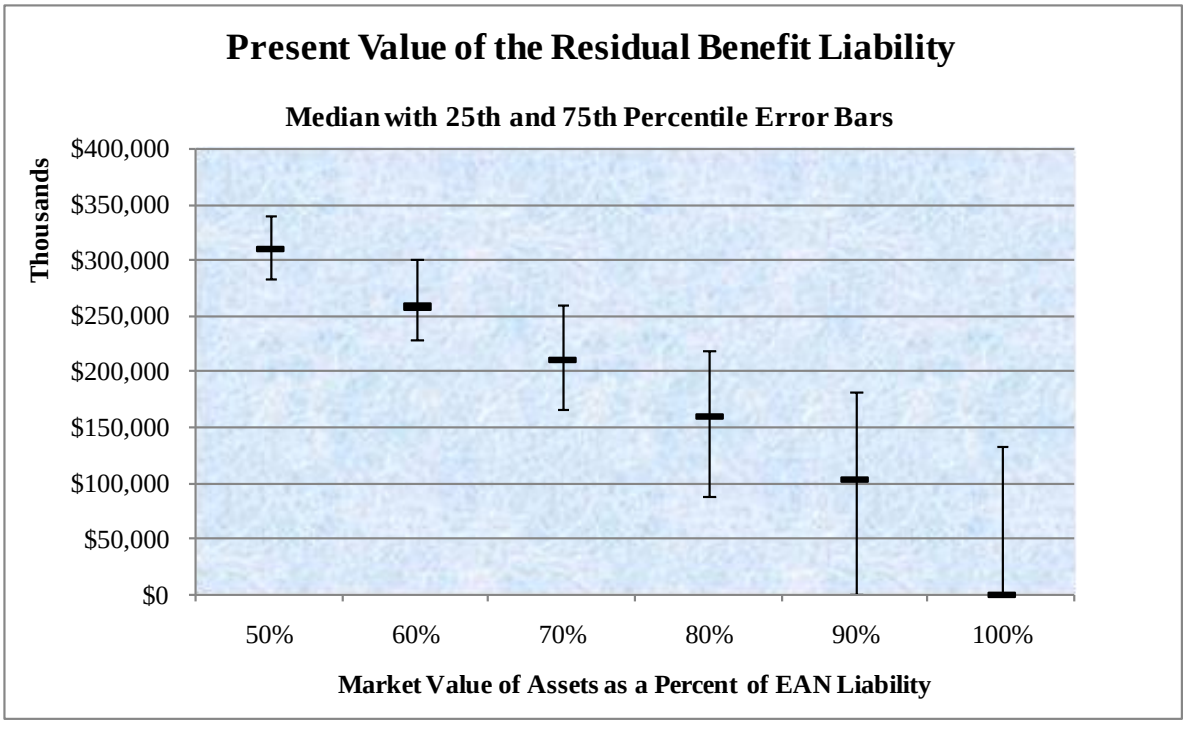
value of plan assets. In a better market, this model might easily show no ruin even at the 5th percentile.

With such volatility in the investment market and such volatility in the bond yields, the resultant volatility in the residual benefit liability is partly why the fair value of a public sector pension liability is of such limited utility for most purposes in a practical world where public sector plans seldom terminate or freeze.

Our case study plan had a beginning market value of assets equal to approximately 85 percent of the entry age actuarial accrued liability. We adjusted the beginning market value of assets up and down to illustrate the sensitivity of the resultant present value of the employer's residual benefit liability.

The figure below presents the median (50th percentile) value of the residual liability with error bars at the 25th and 75th percentiles, based on 500 trials of the same normally distributed returns, the same risk-adjusted CBO cash flows, and the same discounting of the residual benefit payments due using the CitiGroup Pension Discount Curve observed at Dec. 31, 2008.

Figure 17



Summary of Unfunded Obligations

Part 1 in this series, "The Contractual Benefit Obligation," proposed an improvement to the current model of the market value of liability, by revising the benefits valued to be more consistent with labor economics reflected in the contract between the employer and employee. We presented a bar chart comparing the present values of the ABO and CBO.

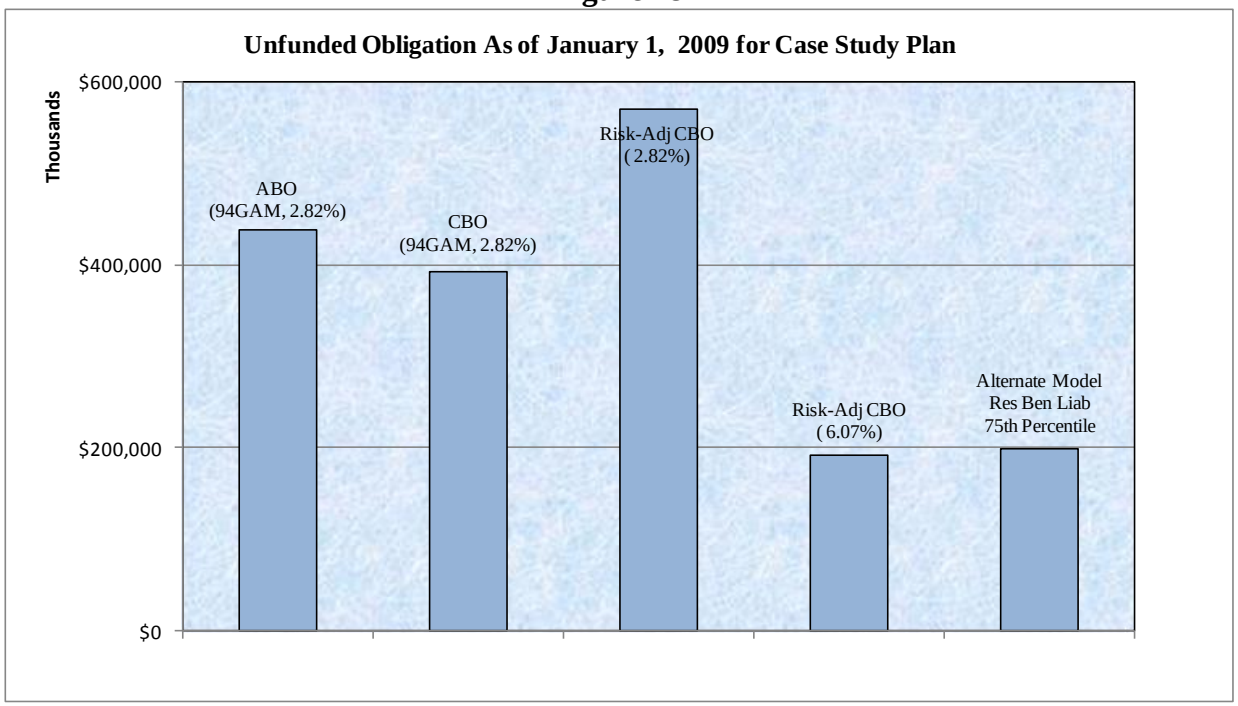
Part 2 in this series, "Risk-Adjusted CBO Cash Flows" proposed utilizing risk-adjusted cash flows instead of treating the expected cash flows as if they were fixed, to be more consistent with actuarial finance and pricing of a fair value. That part added the value of the risk-adjusted CBO cash flows to the bar chart for comparison.

Part 3 in this series, "A Market-Related Discount Rate," proposed the third improvement to the current model by suggesting a higher discount rate would be observable in the market, in order to be more consistent with financial engineering principles of fair value. That part added the fair value (using all three improvements) to the comparative bar chart.

Finally, this Part 4 of the series, "The Residual Benefit Liability," proposed an alternate model for measuring the fair value of the employer's pension benefit obligation, to reflect more consistently all the moving parts and the inherent contract between the employer and the pension plan.

For comparison purposes, the figure below adds the case study plan's employer residual liability expected at the 75th percentile (where percentiles above 50 are worse scenarios) to the bar chart of other unfunded pension obligations.

Figure 18



A great deal of effort has been expended in this series of papers proposing improvements and an alternate to the current model of market value of liability. As discussed previously, these models of employer liability have little usefulness in the real world.

But in the event that public sector actuaries, employers and plans are required to calculate and publish a fair value of the pension benefit liability, the current model needed a major overhaul. We wished to contribute to the body of knowledge for improving our methods to have more consistent integrity with the fair-value measurement attribute.

Part 5 of this series, "Consider the Measurement Purpose," explores eight common purposes for which pension costs and liabilities must be calculated, demonstrates the lack of decision utility inherent in a fair value model, and presents more suitable models for these common purposes.

Part 5

Introduction

The usefulness of actuarial or other types of calculations must take into account their environment, purpose and objectives. Without these considerations, the calculations are useless at best and misleading at worst. In particular, the measurement of costs and liabilities associated with pension and other post-employment benefit programs must consider these three elements.

The Environment

The environment in which public sector employers operate is very different from that of private sector employers: constituency power, governance, transparency, federalism, perpetuity, mission, management, budgeting, purchasing, revenue, financial reporting, taxation, finance, regulation, bankruptcy, and many other factors contribute to this distinction. Quite simply, public sector employers are governments with their own constitutions and their own laws.

The Governmental Accounting Standards Board (GASB) issued a white paper: Why Governmental Accounting and Financial Reporting Is—and Should Be—Different. Certain corporate finance principles applicable in the private sector do, of course, also apply in the public sector environment. However, private sector corporate finance models must not be applied blindly in the public sector environment without careful scrutiny of the relevant environmental considerations.

It should be no surprise that the differences in these two employer environments extend to many aspects of the post-employment benefit plans that they sponsor.

Actuaries advising public sector plans and their sponsors (whether as employees or consultants) concerning pensions and other post-employment benefit programs must consider the environment in which the plan and sponsors function. Otherwise, they will find themselves (and their advice) irrelevant.

There are at least five characteristics of public sector employers and the post-employment benefit plans they sponsor, which are substantively different from private sector employers and their plans. These five environmental differences affect, in substantive ways, the proper choice of pension actuarial measurements.

1. *Perpetual existence.* Few public sector pension plans terminate (requiring plan settlements). Few public sector plans freeze benefits. Few public sector employers ever dissolve or merge with others (requiring plan settlements). Few public sector employers file for bankruptcy (requiring plan settlements). This experience is very different from private sector employers, where these events are relatively common.

2. *No stock value.* In the marketplace, when investors consider buying shares of a company, they want to know, “How much is it worth?” The public sector

employer has no financial shareholders. No one is asking that question. It is not for sale and there is no meaningful venue for a discussion of the market price of the public sector employer. Occasionally, the privatization of certain services raises the value proposition with respect to a specific unit within government. However, the fair value of a governmental entity itself is not a relevant issue.

3. *Unique funding and reporting*. Private sector employers have pension funding requirements imposed by the federal government, which are based upon the objective of achieving 100-percent funded status on a version of an accrued benefit settlement similar (although not identical) to market or fair value of the liability (IRC 430 and 436). Financial reporting standards applicable to private sector employers also impose actuarial calculations based upon another version of an accrued benefit settlement (FASB 87/132r/158). Neither of these is fully consistent with fair value as described in Parts 1 through 4 of this series. However, they are similar and close, numerically and conceptually, to a market or fair or settlement value.

   There are no such federal statutes or rules requiring that public sector pension plans be funded to any particular objective. State and local governments usually have their own funding requirements. In addition, public sector employers have their own GAAP financial reporting standards, which are designed to accommodate the unique characteristics of the public sector environment (GASB white paper).

4. *Budgets.* The budget process and decision-trees for an employer in the public sector is very different from the process and forces at work in the private sector. Public sector pension plans' funding requirements are generally designed to accommodate the employers' objectives for a funding budget that is level as a percent of pay. While seldom achieved exactly or completely, this funding pattern is a desirable objective for predictability and ensures that each generation of taxpayers is paying its fair share of an employee's deferred benefit cost.

   Private sector cash budgeting decisions are often driven by quarterly and annual financial reporting requirements and their effect on the company stock price and by volatile federal annual funding requirements, with less concern (although not entirely absent) for level percent of pay and intergenerational equity over the long term.

5. *Pension plan independence*. As outlined in some detail in Part 4, "Residual Benefit Liability," of this series, the public sector pension fund is very independent from the employer(s). The reader is encouraged to review the relevant sections of Part 4. Technically speaking, the private sector pension fund does have some independence. In law and practice, however, the public sector pension fund is far more independent from the employer than the private sector.

These five differences (among others) have profound effects on actuarial calculations if the intention is to be consistent with the environment. Many actuarial and economic concepts applicable to pensions in the private sector environment are rendered inapplicable in the public sector environment because of one or more of these five characteristics.

Concepts that are applicable in the private sector environment or of purely theoretical interest cannot be blindly ported over to the public sector environment.

The Purpose

There are various purposes for which pension values might be needed. Any time we make a calculation, it is important to ask, "What is the purpose?" In addition to the environment, the purpose of the calculation drives the methodology and assumptions employed. The methods and assumptions must result in useful and relevant numbers for the purpose at hand.

In this, Part 5 of the series, "Consider the Measurement Purpose," we will examine which measures of pension costs and liabilities are most appropriate in the public sector environment for various purposes, or venues of usefulness:

A. Advance funding
B. Taxpayers
C. Financial reporting
D. Lenders and rating agencies
E. Comparability
F. Risk measurement and analysis
G. Personal wealth
H. Plan terminations and freezes

The Objectives

Different objectives for a given environment and a given purpose also give rise to differences in calculations. As long as actuarial and intellectual integrity are maintained, alongside compliance with the Code of Professional Conduct and relevant Actuarial Standards of Practice, different objectives will legitimately influence actuarial calculations. We must consider the objectives.

Some declare that one size fits all; that there is only one true value of a pension obligation, namely, the market or fair value of the liability. However, in actuarial matters there is seldom one answer. Certainly, there cannot be one method for all environments and all purposes without regard to the objectives.

As we examine each of these purposes, giving full consideration to the environment and objectives, we will find that the market or fair value of public sector pension and OPEB liabilities have limited usefulness for most real life purposes, while other methods and assumptions are consistent with the public sector environment and satisfy common objectives for the purposes at hand.

Case Study Plan

The same case study plan utilized in Parts 1 through 4 of this series will be used here in Part 5, "Consider the Measurement Purpose" This section documents the plan provisions, valuation and asset information and methodology used.

Figure 19

Summary of Case Study Plan Provisions	
Normal (unreduced) Retirement Date (NRD) Eligibility	Age 60 with five years of service, or 30 years of service regardless of age. No DROP provisions.
Normal (unreduced) Retirement Date (NRD) Benefit	2 percent of final average pay; slightly different from the backloaded formula shown in Part 1.
Early (reduced) Retirement Eligibility	Age 50 with 15 years of service
Early (reduced) Retirement Reduction	3 percent for each year by which actual retirement precedes NRD
Vesting Eligibility	Five-year cliff vesting
Vesting Benefit	Accrued benefit payable at NRD, or a refund of contributions with interest
Nonduty Disability Eligibility	10 years of service
Nonduty Disability Benefit	The greater of accrued benefit or 25 percent of pay, payable immediately
Duty Disability Eligibility	From date of hire.
Duty Disability Benefit	The greater of accrued benefit or 42 percent of pay, payable immediately.
Nonduty Death Eligibility	10 years of service
Nonduty Death Benefit	Accrued benefit payable immediately to beneficiary.
Duty Death Eligibility	From date of hire.
Duty Death Benefit	The greater of accrued benefit or 50 percent of pay, payable immediately to beneficiary
Cost of Living Increase	Increase in Consumer Price Index, not to exceed 3 percent per year
Member Contributions	8.5 percent of pensionable pay

Figure 20

Summary of Relevant Valuation Information	
For Market Value of Liability Measures	
Discount Rate	The single discount rate equivalent to the Treasury STRIPS yield curve assumed to be observed on the valuation dates.
Mortality Table	RP2000 combined healthy table, with generational projections using Scale AA.
Retirement Rates	24 percent at age 50, then, 7 percent, 7 percent, 7 percent, 11 percent, 11 percent, 11 percent, 11 percent, 8 percent, 8 percent, then 60 percent at age 60, then 30 percent for each year through age 69, then 100 percent at age 70; also 100 percent at 35 years of service regardless of age
Benefits Valued	The accumulated benefit obligation (ABO; per FASB Statement No. 87) cash flows
For Fair Value of Liability Measures	
Discount Rate	The single discount rate equivalent to the CitiGroup Pension Discount Curve assumed to be observed on the valuation dates.
Mortality Table	RP2000 combined healthy table (with loads as per Part 2 of this series), with generational projections using Scale AA (with loads as per Part 2 of this series).
Retirement Rates	Risk-free rates; most valuable retirement age.
Benefits Valued	The risk-adjusted contractual benefit Obligation (R-A CBO; per Parts 1 and 2 of this series) cash flows
For Entry Age Normal Liability Measures	
Discount Rate	7.6 percent, long-term (50^{th} percentile) return expected on a balanced portfolio
Mortality Table	RP2000 combined healthy table, with generational projections using Scale AA.
Retirement Rates	24 percent at age 50, then, 7 percent, 7 percent, 7 percent, 11 percent, 11 percent, 11 percent, 11 percent, 8 percent, 8 percent, then 60 percent at age 60, then 30 percent for each year through age 69, then 100 percent at age 70; also 100 percent at 35 years of service regardless of age
Benefits Valued	Projected benefits expected at time of decrement, including salary increases
Common to All Three Liability Measures	
Turnover and Disability Rates	Based on a recent experience study
Price Inflation	3.0 percent per year compounded annually
Salary Increases	Service-based, from 14 percent to 4 percent annual increases
Pension Fund Annual Rate of Investment Return	7.6 percent each year.

Figure 21

Summary of Relevant Asset Information	
Asset Valuation Method for MVL and FVL	Market value of assets
Market Value of Plan Assets at 12/31/2008	$ 380,717,255
Actuarial Value of Assets for EAN at 12/31/08	$ 456,860,706
Asset Valuation Method for EAN	Five-year straight-line recognition of difference between actual and expected asset value, with a corridor of 20 percent around market value.

Valuation results presented herein are serial open group forecast valuations performed as of each future annual valuation date, i.e., each January 1 in the future.

These open group forecast valuations assume that the size of the active workforce in each future year is the same as the size on January 1, 2009, (with new hires replacing those exiting the group). The valuation horizon for our open group forecast valuations is 35 years. In other words, open group forecast valuations were produced for each of the next 35 years. This process takes a peek into the next 35 years of valuations, utilizing three different measures of the pension liability under study: the current so-called market value of liability model, the fair value of the liability as described in Parts 1 through 3 of this series, and the entry age normal liability method.

The market value of liability (MVL) method is basically the traditional (unprojected) unit credit cost method, using a different discount rate every year in each of the future forecasted valuations as described in the table above. The fair value of liability (FVL) method is a variation of the traditional unit credit cost method as outlined in Parts 1 through 3 of this series with risk-adjusted CBO cash flows discounted using a different discount rate every year in each of the future forecasted valuations as described in the table above and with ancillary benefits (death and disability) funded using one-year term costs. The entry age normal (EAN) liability method is the conventional method most commonly employed in public sector pension valuations, using a fixed discount rate every year in each of the future forecasted valuations as described in the table above.

Each set of 35 forecasted future valuations for each of the three methods use projected rates of investment return for the next 35 years to model the pension fund's growth.

Since the MVL and FVL methods vary their valuation discount rates every year in the future, the open group forecast valuations for these methods use projected yield curves and their single equivalent discount rates for each of the next 35 years. The EAN method uses a single actuarial valuation discount rate assumption of 7.6 percent per annum.

In our open group forecast valuations, we examine the next 35 years of forecasted employer contribution rates, unfunded actuarial accrued liabilities, and funded ratios, all under

the three different measures of the pension liability for our case study plan. The intent is to compare the three methods to assess which methods are most appropriate for the environment, purpose and objectives at hand.

Forecast valuations can be deterministic or stochastic. Deterministic open group forecast valuations assume a given set of assumptions used in each successive valuation, producing a single set of future results for each future year's valuation (35 in our case). Stochastic valuations provide much useful information about risk and the future. However, for our case study plan, we will present only the results of a deterministic valuation.

One form of deterministic forecasting is similar to back-testing, except in reverse. We assume the next 35 years will turn out the same as the last 35. We chose 35 years because it is long enough to include several business cycles, long enough to include the run up and down in spot yield curves, and the commencement point of the last 35 years (1974) is similar to 2009 in that it followed a year of significant stock market losses and was a loss year itself. Of course we are all hoping that 2009 is not another loss year.

The MVL and FVL measurement methods use spot yields observed as of each valuation date (actually the day before) to discount their respective cash flows. In our deterministic forecast valuations using these two methods, we assume the spot yield curves observed as of each future January 1 (beginning Jan. 1, 2010), are the same as those observed or estimated as of each prior January 1 (beginning Jan. 1, 1975).

For the MVL method, the risk-free spot yield curves for each of the prior 35 years (Dec. 31, 1974, through Dec. 31, 2008) were derived based on the actual Treasury STRIPS yield curves for each Dec. 31 from Dec. 31, 1989, through Dec. 31, 2008, obtained from Ryan Labs, Inc. Years 1989 through 2008 were used without change. For years from 1974 through 1988, spot yields for each maturity were approximated by adjusting and interpolating the Treasury Constant Maturity Yields for such years based on the relationship between such Yields and the Treasury STRIPS Yields for the years 1989 through 2008.

Equivalent single discount rates were obtained on the basis of the expected ABO benefit cash flow of the case study plan from the Jan. 1, 2009 valuation. The same 95-year cash flow was used to obtain the equivalent single discount rate for each year's risk-free spot yield curve. Thus, the same duration and convexity were used to derive the equivalent single discount rate for each year's curve. Risk-free spot yields for maturities above 30 were assumed to be the same as the 30-year risk-free spot yield. Equivalent single discount rates assumed for the Jan. 1, 2010 and 2011 valuations (4.00 percent and 5.50 percent) were revised from the original rates for Dec. 31, 1974, and 1975 (7.84 percent and 8.06 percent), respectively, in order to make for a better expected and autocorrelated fit with Dec. 31, 2008 (2.82 percent).

For the FVL method, the high-quality corporate spot yield curves for each of the prior 14 years (Dec. 31, 1995, through Dec. 31, 2008) were equated to the pension discount curves for those same years obtained from CitiGroup.

For Dec. 31, 2008, back to Dec. 31, 1995, equivalent single discount rates were obtained on the basis of the risk-adjusted CBO benefit cash flow of the case study plan from the Jan. 1, 2009, valuation. Again, the same 95-year cash flow was used to obtain the equivalent single discount rate for each year's high-quality corporate spot yield curve. For 1994 back to 1974, the equivalent single discount rate for FVL purposes was assumed to be approximately 109 basis points higher than the risk-free equivalent single discount rates for those years. This spread was based on observed average spreads between the equivalent single discount rates matching the actual Treasury STRIPS yield curve and the single rates matching the CitiGroup pension discount curve for their common years. High-quality corporate spot yields for maturities above 30 were assumed to be the same as the 30-year high-quality corporate spot yield. Rates assumed for the Jan. 1, 2010, and 2011 valuations (7.00 percent and 7.50 percent) were revised from the original rates for Dec. 31, 1974, and 1975 (9.00 percent and 9.18 percent), respectively, in order to make for a better expected and autocorrelated fit with Dec. 31, 2008 (6.07 percent).

While these two forecast valuations (for MVL and FVL) utilize only one possible outcome for yield curves for the future, they are indeed real outcomes which actually did occur during the last 35 years. So they are not just theoretical and hypothetical and are certainly unbiased.

Figure 22

Basic Economic Assumptions used in the Deterministic Forecast (MVL)								
Observed On December 31	Risk-free Spot Yields On Selected Maturities (%)						Equivalent Single Discount Rate (%)	Assumed Applicable To January 1 Valuation Dates
	1	3	5	10	20	30		
2008	0.27	0.66	1.66	2.99	3.19	2.62	2.83	2009
1974	7.43	7.45	7.48	7.81	8.26	8.05	4.00	2010
1975	6.23	7.21	7.62	8.19	8.37	8.00	5.50	2011
1976	4.91	5.79	6.23	7.19	7.47	7.09	7.08	2012
1977	7.05	7.47	7.67	8.21	8.31	7.94	8.08	2013
1978	10.68	9.74	9.47	9.65	9.36	8.86	9.38	2014
1979	11.82	10.78	10.55	10.90	10.58	10.00	10.61	2015
1980	14.01	13.05	12.80	13.11	12.59	11.85	12.76	2016
1981	13.49	14.20	14.20	14.75	14.62	13.50	14.42	2017
1982	8.77	9.89	10.26	10.93	11.06	10.31	10.70	2018
1983	10.19	11.30	11.76	12.47	12.47	11.74	12.19	2019
1984	9.32	10.68	11.26	12.19	12.18	11.41	11.87	2020
1985	7.68	8.35	8.63	9.50	9.89	9.17	9.37	2021
1986	6.01	6.66	6.92	7.63	7.69	7.41	7.50	2022
1987	7.18	8.16	8.47	9.32	9.32	8.85	9.09	2023
1988	9.12	9.32	9.27	9.22	9.22	8.90	9.15	2024
1989	7.91	7.94	7.89	8.12	8.04	7.65	7.99	2025
1990	7.03	7.55	7.89	8.39	8.51	7.48	8.12	2026
1991	4.28	5.37	6.24	7.29	7.89	7.32	7.47	2027
1992	3.89	5.04	6.25	7.23	7.89	7.15	7.38	2028
1993	3.79	5.30	5.32	6.17	6.96	6.99	6.75	2029
1994	7.06	5.57	7.82	7.95	8.11	6.82	7.55	2030
1995	6.39	5.83	5.40	5.77	6.28	6.66	6.34	2031
1996	5.68	6.09	6.18	6.56	6.90	6.46	6.68	2032
1997	5.59	5.67	5.72	5.91	6.08	5.98	6.00	2033
1998	4.57	4.68	4.68	5.05	5.63	5.44	5.40	2034
1999	6.08	6.40	6.58	6.81	6.87	6.61	6.72	2035
2000	4.91	5.04	5.08	5.34	5.75	5.08	5.45	2036
2001	2.28	3.87	4.60	5.58	6.04	5.25	5.62	2037
2002	1.04	2.12	2.94	4.33	5.34	5.14	4.95	2038
2003	1.22	2.51	3.40	4.62	5.55	5.30	5.16	2039
2004	2.73	3.21	3.71	4.50	5.15	5.04	4.91	2040
2005	4.28	4.35	4.35	4.53	4.68	4.56	4.59	2041
2006	5.01	4.71	4.24	4.80	4.95	4.76	4.84	2042
2007	3.22	3.12	3.45	4.30	4.66	4.46	4.47	2043
2008	0.27	0.66	1.66	2.99	3.19	2.62	2.83	2044

Figure 23

Basic Economic Assumptions used in the Deterministic Forecast (FVL)								
Observed On	High-Quality Corporate Spot Yields On Selected Maturities (%)						Equivalent Single Discount	Assumed Applicable To January 1
December 31	1	3	5	10	20	30	Rate (%)	Valuation Dates
2008	4.90	5.36	5.42	6.37	7.22	5.03	6.07	2009
1974							7.00	2010
1975							7.50	2011
1976							8.18	2012
1977							9.17	2013
1978							10.50	2014
1979							11.72	2015
1980							13.88	2016
1981							15.51	2017
1982							11.78	2018
1983							13.27	2019
1984							12.94	2020
1985							10.44	2021
1986							8.59	2022
1987							10.17	2023
1988							10.26	2024
1989							9.09	2025
1990							9.21	2026
1991							8.53	2027
1992							8.44	2028
1993							7.82	2029
1994							8.63	2030
1995	5.62	5.71	5.94	6.35	7.07	6.79	6.71	2031
1996	5.99	6.49	6.74	7.16	7.65	7.64	7.43	2032
1997	6.10	6.21	6.32	6.60	6.86	6.82	6.75	2033
1998	5.44	5.50	5.60	5.87	6.69	6.81	6.46	2034
1999	6.90	7.16	7.39	7.77	8.06	8.11	7.93	2035
2000	6.59	6.01	6.25	6.79	7.40	7.44	7.18	2036
2001	2.70	4.59	5.58	6.71	7.09	6.97	6.85	2037
2002	1.77	2.67	3.58	5.15	6.31	6.48	5.88	2038
2003	1.63	2.84	3.89	5.17	6.41	6.29	5.86	2039
2004	3.09	3.64	4.12	4.94	5.98	5.86	5.58	2040
2005	4.89	4.88	4.97	5.12	5.69	5.65	5.50	2041
2006	5.46	5.19	5.28	5.53	5.99	6.01	5.85	2042
2007	4.81	4.68	5.26	6.02	6.64	6.65	6.41	2043
2008	4.90	5.36	5.42	6.37	7.22	5.03	6.07	2044

The MVL, FVL and EAN methods were applied to our case study plan in open group deterministic forecast valuations over the next 35 years, using the assumptions and methods described above.

One area lacking in the literature on MVL is how its proponents might suggest accounting and funding treatments for the initial unfunded liability and for the annual actuarial gains and losses. MVL argues that the value of the benefit accruing for each individual for the year should be expensed and funded in that very same year (or the next year at the latest) in order to achieve a pure and true matching of the value received (services) from the employee and the value expensed/paid by the employer.

This implies that the entire amount of each year's actuarial gains or losses should be recognized along with the normal cost, all in one year, essentially a one-year amortization of gains and losses.

Actuarial gains and losses arise from three broad sources. All three methods have demographic and asset sources. Another source of gains and losses unique to MVL and FVL arise from each year's change in the valuation discount rate required by these two methods. To make matters worse, MVL and FVL also argue for the use of the market value of assets, with no smoothing, to determine the amount of the unfunded liability. EAN methods typically smooth the assets and amortize any actuarial gains or losses over time (often 30 years).

In our comparison of forecasted results under the three methods, we assume the emerging demographic and asset experience over time matches the assumptions exactly. There are no actuarial gains or losses arising from those two sources under any of the three methods. The purpose of this is to isolate the qualities of MVL and FVL that differentiate them from EAN. All three are treated the same with respect to those two sources of gain and loss. Thus, there is no bias in this comparison. When it is shown that MVL and FVL are unsuitable, it will not be on account of a bias in the comparison; it will be on account of the inherent nature of MVL and FVL themselves.

If we were to build a one year amortization of each year's gain or loss arising from the annual change in valuation discount rates for MVL and FVL into our comparison (as their proponents advocate), it would make MVL and FVL look even worse. That would be the most unbiased of comparisons.

However, we will give the MVL and FVL methods a handicap in the comparison. We will build into the annual employer contribution rate calculation a 30-year amortization of MVL and FVL's gain or loss arising from the annual change in valuation discount rate. This is, essentially, putting our thumb on the scale to help MVL and FVL in the comparison of their resulting contribution rates to EAN's rates. Furthermore, we will also amortize the initial, unfunded liability of all three methods over 30 years. These amortizations will all be calculated as a level percent of pay (a 4-percent payroll growth rate is assumed), as is permitted under GAAP standards for public sector pension and OPEB accounting.

With all these preliminaries and valuation details out of the way, the following are various purposes for which actuarial calculations are required for public sector pension liabilities.

A. Advance Funding

Advance funding is mostly about budgeting and intergenerational equity. Indeed, there are a host of other serious funding implications such as sustainability, collective bargaining, moral hazard, accountability, taxation, bond ratings, benefit security, etc. Still, the two important and rudimentary considerations for advance funding are budgeting and intergenerational equity. We will address these two considerations in this section.

Budgeting

Budgeting is among the most important activities undertaken by governments. The National Advisory Council on State and Local Budgeting[13] states that a good budget process incorporates a long-term perspective and that "the budget process is not simply an exercise in balancing revenues and expenditures one year at a time, but is strategic in nature, encompassing a multi- year financial and operating plan that allocates resources on the basis of identified goals." This perspective on public sector budgeting derives from the perpetual existence of state and local governments. This long-term perspective is particularly important in the public sector environment.

Predictability is an ever-present objective in budgeting. Funding with the intent of achieving a level percent of pay contribution rate is a worthy, even necessary, objective for budgeting and sustainability. Given the nature of a defined benefit promise with its actuarial gain and losses, it is not possible to actually achieve truly level percents of pay over time. However, the methods and assumptions should be designed to achieve that goal.

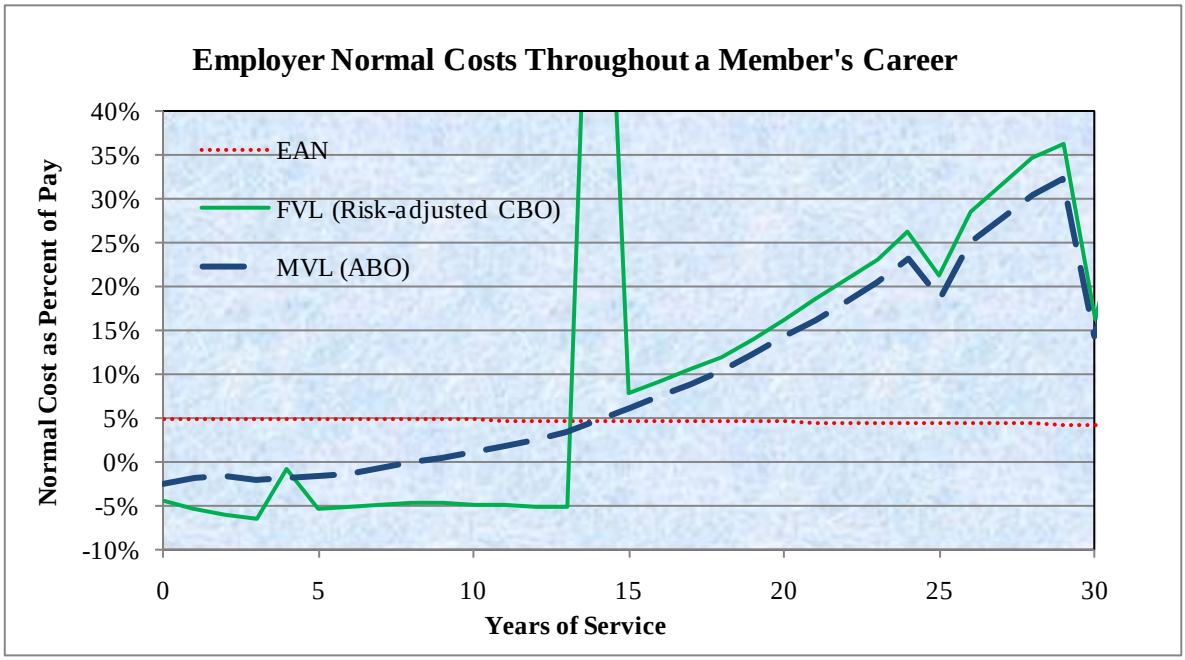
Government budget directors are used to operating under a level percent of pay for all pay-related benefits and taxes, including retirement, unemployment taxes, workers compensation, and Social Security. Even Medicare contributions are designed to be level as a percent of pay, in spite of the fact that its benefits are not pay-related. Defined contribution plans with a flat percent of pay or matching contribution are also designed with contributions that are level as a percent of pay.

From the perspective of an individual employee, the EAN method does a better job of allocating costs that are level as a percentage of pay than does the MVL or the FVL methods. The EAN cost method and several other conventional cost methods (including frozen entry age normal and aggregate) are specifically designed to allocate costs as a level percent of pay over time.

[13] Recommended Budget Practices, published by the Government Finance Officers Association.

The figure below presents the EAN, FVL and MVL normal costs, as forecasted throughout an individual employee's career (from age 25 to retirement at age 55) and expressed as a percent of pay. In order not to obscure the focus of this graph, normal costs for all three methods were calculated based on an interest discount rate assumption of 7.6 percent. These are kept constant over time to remove the white noise of the discount rate volatility inherent in MVL and FVL.

Figure 24



It is interesting to note that under the MVL and FVL, the employer's normal costs in the first several years of the employee's career are zero (less than zero actually). This occurs because the employee's contribution to the plan is level as a percent of pay, but the total normal cost under MVL and FVL are not. This creates a mismatch, resulting in negative normal cost for the employer. This is, in itself, an undesirable feature of MVL and FVL. For an employee's own personal budget, it is a desirable objective to have pension contributions that are level as a percent of pay. So it is with the public sector employer's budget as well.

The most obvious message of this graph is that the EAN method produces an employer-normal cost, which is level percent of pay for an individual, while MVL and FVL have grossly backloaded normal cost patterns.

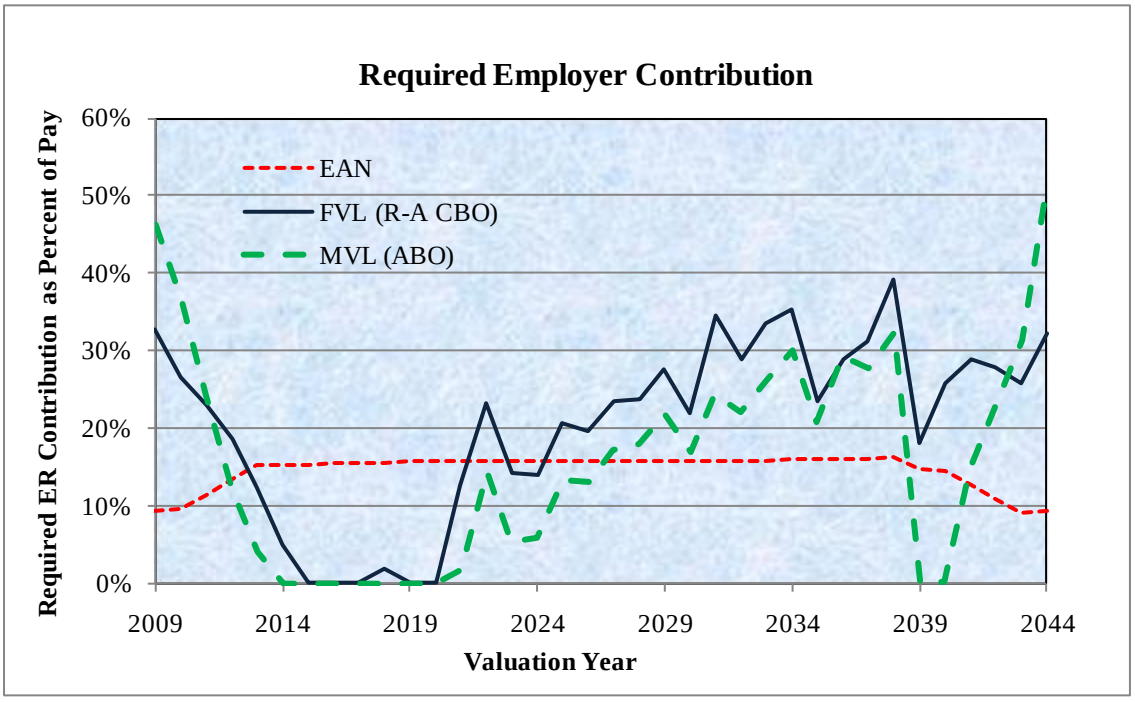
This failure of MVL and FVL to produce employer-normal costs that are level percents of pay for individuals arises from the very nature of the traditional unit credit cost method, upon which both MVL and FVL are based. EAN produces employer normal costs that are *designed* to be level as a percent of pay for the individual employee.

From the perspective of the *group*, the disparity is equally wide. MVL and FVL insist on valuing the benefits using a different discount rate every year, depending on the fixed income

yields in the market at each measurement/valuation date. This creates employer-normal costs and contribution rates which are volatile, unpredictable and certainly not level as a percent of pay.

To illustrate the combined effect of these two features of MVL and FVL, while comparing them with EAN, we performed an open group forecast valuation in accordance with the assumptions and methods described previously.

Figure 25[14]



If you were the budget director of a state or local government entity, which cost method would you prefer? Imagine how much more volatile the MVL and FVL graph would look if we required one year amortization of actuarial gains and losses, like MVL advocates say should be done.

These two features of MVL and FVL (backloaded traditional unit credit cost method and volatile valuation discount rates) disqualify MVL and FVL from serving the budgeting and funding purposes of public sector pension funds. They fail to satisfy the basic budgeting and funding objective of a level percent of pay.

Certainly, the EAN method is demonstrated herein to be superior in satisfying the budgeting and funding objective of a level percent of pay, on an individual basis and on a group

[14] The required employer contributions under EAN are expected to rise over the next few years because of some legacy losses from prior years working their way through the asset-smoothing method. These actuarial losses are amortized over 30 years each. Toward the end of the forecast, the required employer contributions drop back down after each 30-year amortization base is paid off.

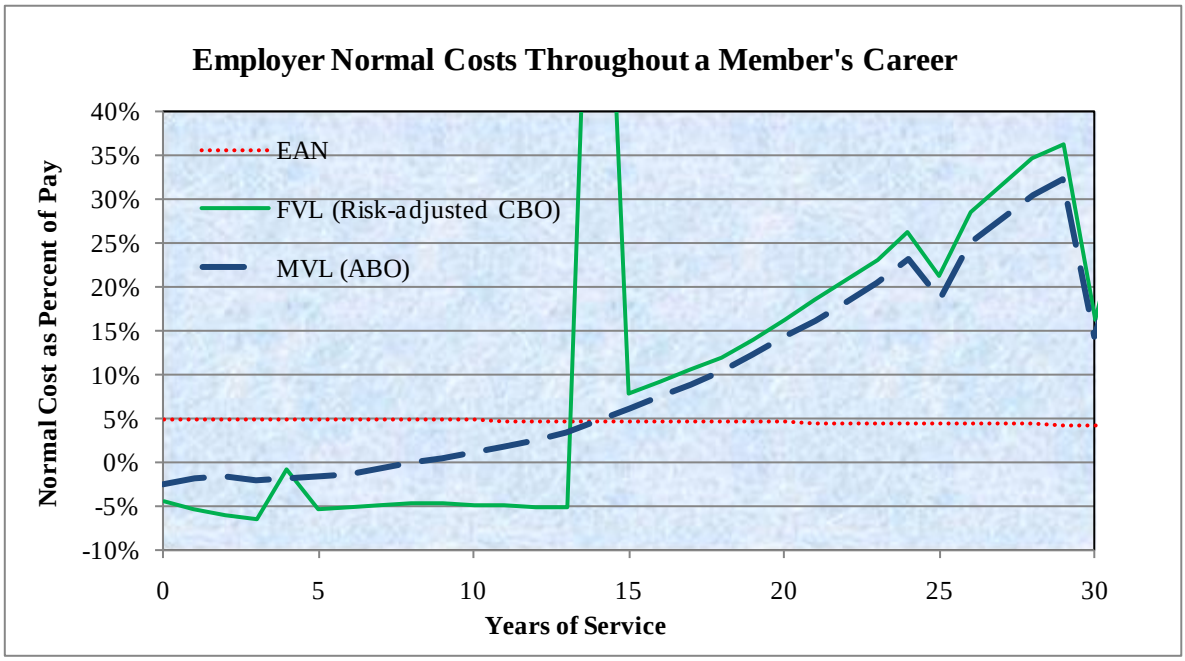
basis. Other conventional actuarial cost methods also satisfy this objective, e.g., the aggregate cost method and frozen entry age normal cost method.

Intergenerational Equity

A level percent of pay EAN approach does a better job of approaching intergenerational equity than do MVL or FVL, particularly for final pay plans. No method will ever produce complete intergenerational equity exactly for any defined benefit pension plan. But the EAN method produces closer results.

Let us return to the graph presented earlier in this section, illustrating the pattern of an individual's employer normal cost over his career. Again, the discount rates for these employer normal cost patterns are kept constant over time (7.6 percent). This focuses our attention on the nature of the unit credit cost method and how it fails to approximate intergenerational equity.

Figure 26 (same as Figure 24)



Under MVL and FVL the taxpayers served by this employee during his last 10 years of employment pay far more than the taxpayers in his first 10 years, for essentially the same 20 percent of final pay of benefits earned. Taxpayers at the end of his career should not be required to finance such a disproportionate amount of his pension as compared to taxpayers at the beginning of his career.

This graph demonstrates that the MVL and FVL methods (based upon variants of the traditional unit credit normal cost method), by their very nature, have a backloaded valuation pattern which saddles later taxpayers with a disproportionate share of the cost of services rendered. Again, the same level percent of pay objective for budgeting also serves to approximate intergenerational equity.

To conclude the matter of funding, while MVL and FVL may be of theoretical interest to some, they are poor candidates for real life funding purposes as compared to EAN or other such conventional methods designed to produce employer contribution rates which are level as a percent of pay.

B. Taxpayers

Taxpayers' interests include services provided and the costs thereof (taxes and user fees paid). Almost all of us are taxpayers, of one sort or another. A variety of state and local taxes accumulate to significant amounts paid out of our personal budgets (directly and indirectly).

While the cost of services does not always equal the taxes collected, there is a strong connection between the two. Thus, taxpayers have a keen interest in how government officials spend the taxes they collect. In other words, the cost of services is important to taxpayers. This obvious point is relevant to our discussion of what measure of pension benefit liability is appropriate for taxpayers' interests.

Price vs. Cost

MVL and FVL do not measure actual long-term costs to the taxpayers, or actual short-term costs for that matter. However, they do attempt to include in their calculations the "cost of investment risk." The market or fair value of a pension liability is about point-in-time pricing, not about the long-term costs collected from taxpayers to finance the long-term pension promise.

Under MVL and FVL the value of the pension benefit liability is measured on the basis of what the market price would be to discharge or settle the pension benefit liability, which had been accrued or earned by the employees (and retirees) as of a given measurement date. Never mind that there is no actual intention of settling the liability in the marketplace. Never mind that there is no marketplace in which to settle the pension liability. An MVL or FVL is a theoretical construct to calculate a reasonable price at which the employer *could* settle the liability *if* it wanted to and *if* there were a marketplace for settlements. As discussed in the introduction to this series, this is a fair-value measurement attribute.

The feature that makes MVL and FVL focus on *price* rather than *cost* is their insistence on discounting the future benefit cash flow using risk-free or market-related yield curves as observed in the market on the measurement date.

This method for selecting the discount rate(s) bears little to no relationship to what taxpayers of the future will actually have to pay for financing the pension promise. On any given measurement date, the values of the MVL and FVL are not affected by whether the pension or OPEB promise is unfunded or advance-funded. Their values are not affected by how the pension fund is invested.

If a pension fund were invested entirely in 90-day Treasuries for the entirety of its existence, its MVL and FVL as of every measurement date throughout that existence would not be any different than if it were invested in a balanced portfolio with 60 percent in equities and 40

percent in intermediate bonds. MVL and FVL values are unaffected by such matters. They "price" the liability (i.e., discount their respective cash flows) based on the fixed income yield curve observed in the marketplace on each measurement date, regardless of actual or expected long-term costs to the taxpayers. However, there is not much dispute that a plan invested entirely in 90-day Treasuries will cost the taxpayers more over the long term than if it were invested in a 60-40 balanced portfolio (at least matching the index performances).

MVL and FVL treat the pension fund as if it were a mere pass-through, completely ignoring the investment policy and operation of the fund. Again, MVL and FVL are pricing the settlement value (or something similar) as a market value or fair value price.

Taxpayers are more practical than that. Taxpayers are seldom interested in the prior year's government-wide financial statements, although many would argue they should be. More often, those taxpayers who take any interest at all are more interested in last year's government funds financial statement and in the next year's budget because those relate to costs. And costs drive taxes.

Past and Future Costs

Taxpayers are interested in knowing how their elected officials and management spent their taxes in the past, i.e., what it actually cost to provide last year's services. This is generally found in the governmental funds financial statement. In addition, taxpayers are interested in knowing how their elected officials and management intend to spend future taxes, i.e., how much services will cost in the future. This is generally found in the budget.

Taxpayers are practical. Among other matters relating to the government's pension plan (such as comparability), they want to know how much the government pension plan will cost them over time, i.e., how much in taxes they will have to pay over time to finance the pension promise. If there is no talk of settling the pension liability, it is of little to no interest to them what the market or fair value price would be. Serious consideration of settling the liability, however, would make the dialogue move from a theoretical price to a real cost.

The actual amount taxpayers will pay in pension costs (through taxes) over time depends in large part on how much investment return is generated over time. Pension promises are being advance-funded for a few reasons. A major one is that it costs taxpayers less over the long term than if the promise were satisfied by mere pay-as-you-go. The investment return helps pay the promised benefits, instead of the taxpayers footing the entire bill.

Any measure of the liability that would be useful to taxpayers would recognize the long-term cost expected to be borne by them. A market or fair value *price* in today's market might have some curiosity interest, but is not something that would interest taxpayers.

If a current *benefit* liability were to be of interest to taxpayers, it would be one that tells the expected long-term *cost* in a present value. This can be accomplished by discounting the expected CBO cash flow using the average annual rate of return expected from the pension fund.

It would represent the expected future cost of benefits earned to date in present value form. However, the government will not be paying the benefits. The pension fund will be paying them.

An expression of the *funding* liability may speak more about the future cost paid by taxpayers. The taxes paid to the government for the next 20 years will provide the government with the resources needed to pay its cash *funding* obligation to the pension fund for the next 20 years. The taxes paid to the government over the next 20 years will not be used to pay the pension *benefits* over the next 20 years. With rare exceptions, the employer's pension contribution cash flow (not benefits paid cash flow) is financed by taxes collected. MVL and FVL are poor representations of the cost to taxpayers.

The EAN's unfunded actuarial accrued liability (UAAL) is a good measure of the *funding* liability scheduled for payoff over time, with taxes collected. Whichever actuarial cost method is selected by the plan and agreed upon by the employer is an appropriate basis for measuring the cost to taxpayers. After all, with respect to the transaction between the employer and the plan, the restructured debt owed by the employer to the plan (converted from the initial debt owed by the employer to the member) represents the true basis on which the employer will need to go to the taxpayers to fund the contributions demands made upon it by the plan. MVL and FVL have never been used by the plan to require contributions from the employer (and therefore taxes from taxpayers).

While a homeowner might have some interest in the market price of his house (an asset), he has little to no interest in the market value of his mortgage (a liability). He is interested in the long-term *cost*, not the current market *price* of the liability. The outstanding balance and the scheduled amortization payments are real costs to the homeowner.

This discussion of taxpayers' interests is framed mostly for sole and agent government employers (as defined by GASB standards) and those who pay taxes to them. Treatment of this topic in the context of cost-sharing employers and their taxpayers would be slightly different due to the pooling nature of cost-sharing plans. The pension costs paid by cost-sharing employers cannot be traced directly back to their own employees and retirees for the services they rendered to that cost-sharing employer.

C. Financial Reporting

Not a Pass-Through

Cash with a fiscal agent and other such constructs are properly treated as pass-through, even if they are held as trusts for specific purposes and even if they are irrevocable. However, pension funds are different. We spent a good amount of time in Part 4 of this series, "The Residual Benefit Liability," demonstrating that the public sector pension fund is a separate legal entity and sufficiently independent to be treated as such in financial reporting. The reader is directed to Part 4 to review the case made for not treating the public sector pension fund as a mere pass-through. While the arguments were presented Part 4 in the context of legal purposes and financial pricing purposes, they are equally applicable for the purpose of financial reporting.

As rare an event as it may be, when a government files for bankruptcy under Chapter 9, the pension fund that had assumed all responsibilities for paying pension benefits for the government's employees and retirees continues to operate without itself filing for bankruptcy. The pension fund is usually far more solvent than its creator. While the government's employees and retirees may earn status as a creditor in the process, pension assets remain the property of the pension trust, separate from the employer.

Objectives

GASB's Concepts Statement 1, *Objectives of Financial Reporting*, identifies accountability, decision usefulness and interperiod equity as worthy objectives. The standards-setters for governmental GAAP will judge what measures of pension liabilities are most appropriate for the purposes of governmental financial statements—not actuaries. Nevertheless, we offer our opinions and judgment.

Accountability. The expense, liability and other pension information reported in the basic financial statements and in the notes and RSI of governmental employers should be measured and presented in a manner that holds elected officials and management accountable for their funding of this long-term obligation. The pension information provided in the government's financial statements should serve as a benchmark for performance with respect to funding. How people are measured often determines their behavior. Pension fund management and government management will be held accountable for their pension funding in large part based upon the funded ratios presented in the respective plan's and government's financial statements.

Having wide movement in funded ratios and contribution requirements caused by MVL and FVL is a poor benchmark for performance. If management were held accountable for its performance based on an MVL or FVL benchmark, they might well take serious and irrevocable actions based on volatile and temporary swings in liability measures, in asset measures, in funded ratios and in resulting expense numbers. Such a benchmark for the public sector could realistically result in benefit improvements while funded ratios are temporarily high and contributions low, or result in mass plan terminations or freezes while funded ratios are temporarily low and contributions high -- just like we have seen in the private sector.

The EAN or other actuarial cost method with asset smoothing serves the accountability objective more reasonably. The following figure compares the unfunded actuarial accrued liabilities of the EAN, MVL and FVL for our open group forecast valuations for the next 35 years. The succeeding graph compares the funded ratios. Keep in mind that these two graphs did not even introduce any volatility in asset returns; both assumed a constant 7.6-percent investment return each year. Even holding that variable constant in all three methods, the volatility of MVL and FVL make them poor benchmarks for accountability.

Figure 27

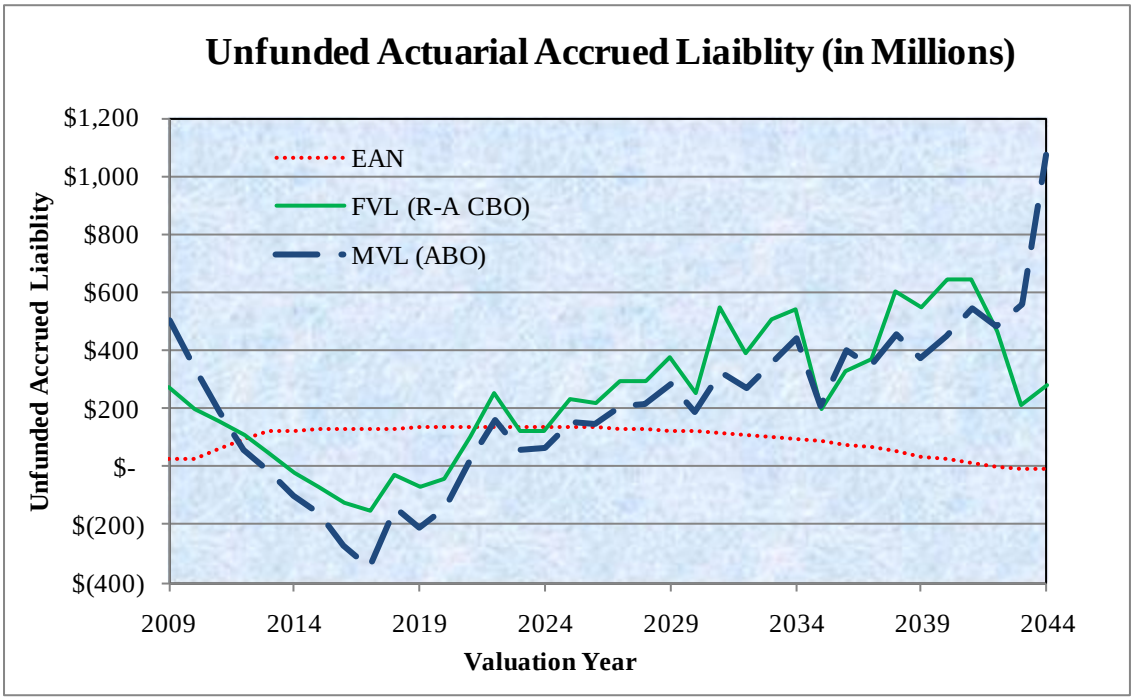


Figure 28

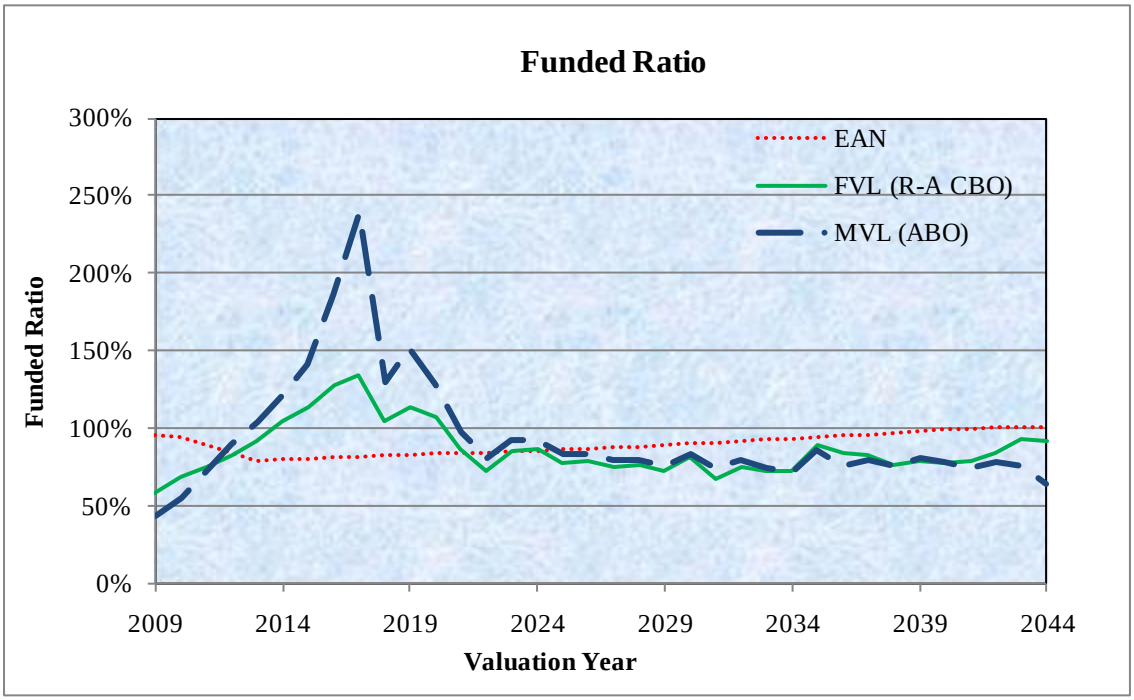
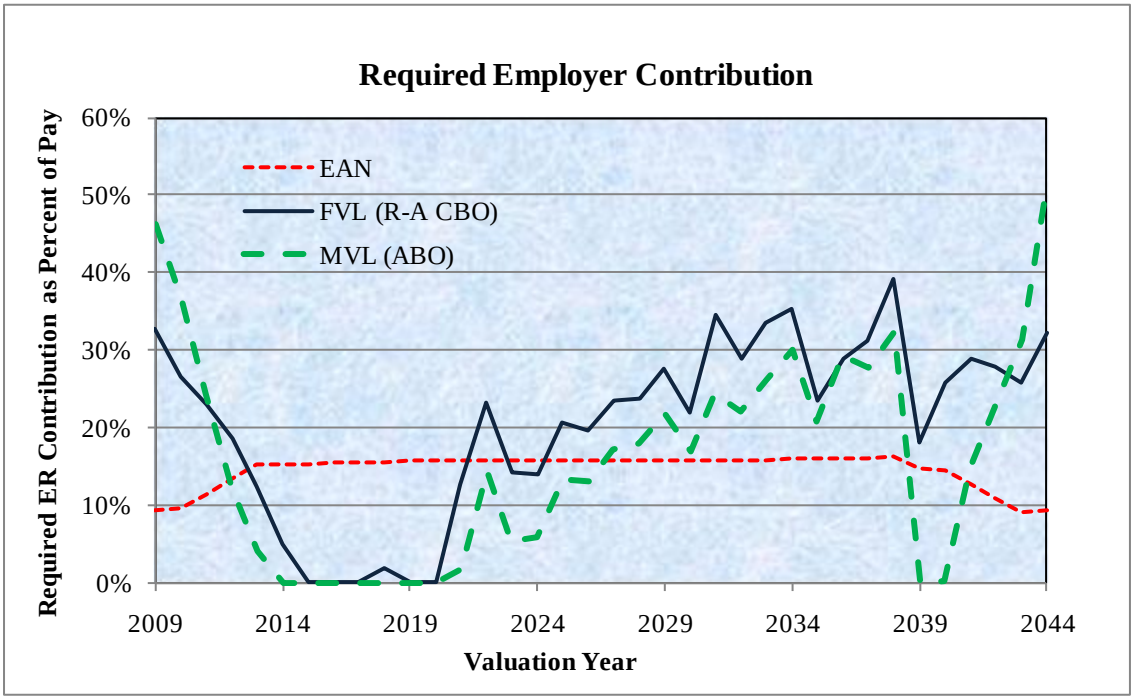


Figure 29 (same as Figure 25)



These graphs demonstrate that MVL and FVL make poor benchmarks for *accountability* and poor guides for behavior. Stakeholders would not want their plan and government management to be held to the standards of these two methods. EAN makes a much better benchmark for *accountability*.

Decision-Usefulness. MVL and FVL are totally bereft of any decision-usefulness for the purposes of annual financial reporting (unless settlement of the plan liabilities is under serious consideration). MVL and FVL are more prone to moral hazard than EAN because, with their greater volatility, there would be a greater tendency to rush to benefit increases during short periods of high funded ratios and low expense or funding levels.

If a balance sheet liability means anything, it should reflect the present value of future costs to taxpayers. Measuring the pension benefit liability at different risk-free or market-related fixed income discount rates every year is not reflective of what taxpayers are expected to pay for the obligation. They will pay a cost that is offset (in large part) by investment earnings of the trust fund. That net cost is not reflected if risk-free or market-related fixed income discount rates are used That would overstate the true cost to taxpayers, making MVL and FVL poor representations of what should appear in the government's financial statement.

MVL and FVL reporting would be tantamount to adopting a fair value measurement attribute, which is remeasured every year. The employer has no means of benefiting from the remeasurement gains (or vice versa), for a pension liability because it is not for sale or exchange.

EAN expenses and liabilities are already in wide use and have gained broad acceptance among governmental employers and their plans. Changes in actuarial assumptions, changes in

benefits, and actuarial gains or losses can all be reported separately under EAN. Funded ratios can be reported easily. Some advocate the use of EAN for note disclosures regardless of the method employed for expense and liability reporting so that plans and employers can compare their funded ratios more easily. Refer to the section on comparability for more on this topic.

Interperiod Equity. As mentioned earlier, interperiod equity cannot be completely and exactly achieved with defined benefit pension and OPEB plans. However, as demonstrated in the funding objective of intergenerational equity, MVL and FVL fail because of their two disqualifying features. They fail the test of interperiod equity because of their reliance on the traditional unit credit cost method, which grossly backloads the costs for any given individual. They also fail the test of interperiod equity because their reliance on varying annual discount rates fails to achieve anything close to level percent of pay cost patterns.

The EAN method is designed for interperiod equity, with a level percent of pay objective.

Definition of a Liability

GASB's Concepts Statement No. 4, "Elements of Financial Statements" states in paragraph 17, "Liabilities are present obligations to sacrifice resources that the government has little or no discretion to avoid." Paragraph 18 expands on that definition, "An obligation is a social, legal or moral requirement, such as a duty, contract, or promise that compels one to follow or avoid a particular course of action.[15] A present obligation that is a liability is a duty or responsibility to sacrifice resources that the government has little or no discretion to avoid. The reason that many liabilities cannot be avoided is that they are legally enforceable, meaning that a court could compel the government to fulfill the obligation. Generally, legally enforceable liabilities arise from legislation of other levels of government or contractual relationships, which may be written or oral.

The terms (whether implicit or explicit) of the contract between an employer and the plan have a "put" of the pension *benefit* obligation from the employer to the plan and an retaining of a *funding* obligation upon the employer to the plan. As demonstrated at length in Part 4 of this series, "*The Residual Benefit Liability*," the employer has no practical or legal *benefit* liability, but a serious *funding* liability. This exchange between the employer and plan exists regardless of whether the employer is a cost-sharing employer of a sole or agent employer. This recognition of contract law is another subject, which was developed thoroughly in Part 4.

Clearly, the employer has some sort of pension liability that should be presented somewhere in its financial statements. If the employer is a debtor for pensions, who is the debt owed to? If there is a debt to pay by the employer, who is the creditor? The transaction between the employer and the pension fund is a real one. The debt owed by the employer is not a *benefit* liability (owed to plan members) but a *funding* liability (owed to the pension trust). The employer owes payments to the pension fund, not to the employees (except for a potential residual benefit liability in the unlikely event of plan insolvency). The reader is encouraged to

[15] Quoted by GASB from *The American Heritage Dictionary of the English Language*. 4th ed. New York: Houghton Mifflin Company, 2000.

review the relevant sections of Part 4 of this series, relating to this exchange transaction and the independence of the plan.

Whether the pension funding liability should appear on the government-wide statement of net assets or solely in the notes and required supplementary information is a matter for GASB standards-setters to decide. In our opinion that decision turns on the degree of consistency in reporting standards, which GASB board members would require to exist between cost-sharing employers and sole and agent employers. More on this topic follows.

In any event, viewing the employer's pension liability as a *funding* liability is truer to the nature of the exchange transactions and the contracts in place. The only *benefit* liability that the employer owes is a residual benefit liability, discussed in Part 4. However, the employer (especially a sole or agent employer) does owe a measureable *funding* liability to the plan.

OPEB obligations (or components thereof) may be considered nonexchange transactions, to the extent that the employer reserves the right to change the terms of the benefits. But that notion would need to be balanced against the concept of the substantantive plan in place on the measurement date.

Consistency Among Reporting Entities

The *funding* obligation that a cost-sharing employer has to the cost-sharing multiple employer plan is fundamentally different from the one that the sole or agent employer has with their respective plans. The pooling arrangement is, essentially, an agreement with all other contributing employers in the cost-sharing plan to share the *funding* obligations ratably.

The pooling nature of the cost-sharing arrangement makes it impossible to measure the *funding* obligation borne by each individual cost-sharing employer as derived from the "put" of the *benefit* obligation upon the plan. The demographics of the individual cost-sharing employer's covered membership bear no relationship to the funding requirements imposed upon it by the cost-sharing plan. This, of course, is not true of sole or agent employer plans.

Therefore, the cost-sharing employer cannot measure a *funding* obligation of its own for its own financial reporting purposes. Other useful information concerning the cost-sharing plan should be included in the notes and RSI, but nothing that is unique to that employer's own *funding* liability can be measured. No value of assets held for its retirees and employees. No actuarial accrued liability for amortization *funding*. Certainly, such an employer cannot report a *benefit* liability for its own retirees and employees since that liability has been "put" to the cost-sharing plan.

Cost-sharing employers should continue to expense the contractually required contribution. If they continue to pay that amount each year, there should be no balance sheet liability remaining.

The contract between the sole or agent employer and its respective plan is just as real. Sole and agent employers have "put" their long-term *benefit* liability to their plan in exchange

for a long-term *funding* liability. The pooling nature of the cost-sharing plan simply rendered the *funding* obligation unmeasureable for the individual cost-sharing employer. However, the *funding* liability is indeed measureable for sole and agent employers.

At a minimum, a *funding* liability for sole and agent employers should be reported in the notes and RSI, along with other useful information – more than is being currently reported (but that is beyond the scope of this discussion of measurement).

GASB standards-setters will need to address how important it is for the statement of net assets to reflect consistent reporting between cost-sharing employers and sole/agent employers. Currently, cost-sharing employers' balance sheet pension liability is consistent with that of sole and agent employers. In a simplified sense, the balance sheet pension liability for all three types of employers is the cumulative difference between the amount the employer should have paid to the plan for sound actuarial financing (as judged by the plan and its actuary) and the amount that the employer actually paid. This creates consistent reporting among all three reporting types.

The downside, in the minds of some, is that there is no *benefit* or *funding* liability appearing on the statement of net assets of any of these employers. If consistency is more important, then the *funding* liabilities of sole and agent employers should not be recognized on the balance sheet because cost-sharing employers cannot measure their own *funding* liability for balance sheet recognition.

There is a choice to be made by GASB standards-setters for sole and agent employers. It is a choice between consistency on the one hand and balance sheet recognition on the other.

1. *Consistency and comparability* are important concepts in accounting and financial reporting. As long as sole and agent employers report their *funding* liability (and other information such as changes therein) in their notes and RSI and as long as their plans report a *benefit* liability (and other information such as changes therein), some may argue that consistency and comparability between reporting expenses and balance sheet liabilities for cost-sharing employers and sole/agent employers have been preserved. This way, the total actuarial-accrued funding liability for sole and agent employers would not appear on their statement of net assets. They may argue that the notes and RSI are included in the financials and are available for all to read; that analysts, rating agencies and others interested in such matters know where to find the information.

2. *Transparency and balance sheet recognition* of liabilities are also important concepts in accounting and financial reporting. Even though cost-sharing employers cannot measure their funding liabilities, that fact may not be sufficient reason for sole and agent employers not to do so. Some may believe that transparency in balance sheet reporting for sole and agent employers trumps consistency with cost-sharing employers. In this case, a sole or agent employer should record its entire unfunded actuarial accrued liability on its statement of net assets. Exactly how it expenses and reconciles from one year to the next is an accounting matter beyond the scope of this paper.

The measure of that liability, however, is the subject of this paper. As demonstrated previously, MVL and FVL are poor measures for accountability, have little to no usefulness, and fail in approximating intergenerational equity. Besides that, the MVL and FVL measure the *benefit* liability, which is not a liability of the employer, but a liability of the plan.

EAN is already a widely used and accepted cost method. It measures the *funding* liability, not the *benefit* liability. It seems to serve well as a benchmark for accountability, provides useful information including the change in funding liability for retroactive benefit improvements, and is actuarially designed to produce level percent of pay contributions for approximating interperiod equity.

However, not all plans use the EAN method for funding. We believe that, for comparability (more on this later) all plans should report their unfunded actuarial accrued liability using the EAN method, regardless of the cost method used for developing the actual funding requirements.

The unfunded actuarial accrued liability (UAAL) under the EAN method should be understood as the current value of the debt that the employer currently owes to the pension fund. Under choice two above, it appears on the sole or agent employer's statement of net assets as a long-term liability.

Value-in-Use

Consider the valuation premise. Is it better to think of the pension liability as a value-in-exchange or as a value-in-use? There are several re-measurement attributes that can be used for assets and liabilities. One such approach is value-in-use. The employer's pension liability to the pension trust can be thought of as the cost of maintaining its resources—its human resource. There are two agreements at work in tandem: the voluntary exchange transaction between employee and employer and the contract between employer and pension trust.

The cost associated with these agreements is a cost of maintaining an intact workforce for an employer. The workforce can be thought of as an asset-in-use and pension contributions are on one of the costs of maintaining that asset in use. Employing a value-in-use attribute is a reasonable approach to valuing this debt, which the employer owes to the pension trust in satisfaction of these tandem agreements. This implies that the MVL and FVL are not the best attribute models for pension liabilities, with their market-driven discount rates. The value-in-use is best described with the unfunded actuarial accrued liability under the funding method employed by and required by the pension trust, or simply the EAN method for consistency.

Accounting vs. Funding

As discussed in the section titled "Comparability," it is a worthwhile goal to require a consistent recognition and disclosure method for public sector financial reporting. Using the EAN method along with the plan's long-term expected rate of return and other assumptions should be the basis for financial disclosure. This may be different from the plan's funding method, but it is worth the additional work and explanations which will inevitably be required.

Plan Financial Reporting

Since the *benefit* liability is assumed by the plan, some may argue that its own financial statements (if issued separately) should present its current benefit liability. If such a liability were on the pension fund's books, it should be valued at the long-term rate of return expected by the pension fund. It should be the present value of the expected contractual benefit liability (PVCBO), discounted to the measurement date using that expected rate of return. We suggest the expected long-term return (instead of a market-based return as MVL and FVL do), because plan liabilities are not generally being considered for settlement. They have virtually perpetual existence. If settlement were under serious consideration, that could change this treatment. Thus, a market-based discount rate would be inconsistent with that ongoing perspective.

Furthermore, a full-orbed treatment of the *benefit-funding* exchange transaction between sole/agent employers and their respective plans might suggest that the plan include an asset on its books equal to the amount of the *funding* liability held on the employer's books (the EAN UAAL under the second choice above). Development of this notion is beyond the scope of this paper.

By calculating and reporting this version of a *benefit* liability in the plan's financial statement, many might confuse the UAAL appearing in the employer's financial statement (the *funding* liability owed by the employer to the plan) with the PVCBO appearing in the plan's financial statement (the *benefit* liability owed by the plan to the members).

Many might even be tempted to calculate a *plan funded ratio* as the market (or actuarial) value of assets divided by this PVCBO. Whether the asset figure in the numerator should or should not include the UAAL (if it were to be included as an asset in the plan's financial statement) is also beyond this paper's scope. In any event, this may present some confusion between two measures of funded status — an *employer funded ratio* and a *plan funded ratio*.

D. Lenders and Rating Agencies

Rating agencies (and lenders who do their own research) obtain most of their information about a government's pension and OPEB obligations directly from the government's financial statements. They can also obtain their information from actuarial communications and from the government's or plan's staff as necessary.

Rating agencies are not asking for the MVL. Nor do they generally utilize unfunded actuarial accrued liabilities (under the reported method) the same as bonded debt in their evaluations. They are not interested in the expected or risk-adjusted future benefit cash flows because they are paid by the plan, not the employer. They are more interested in expected future employer contribution cash flows. Pension contribution demands on the employer create competition for scarce resources to service debt. Standard & Poor's Public Finance Criteria 2007 states (on page 64) that, *"The historical and forecast trends in pension funding are as important, if not more so, than the specific liability level at a single point in time."*

MVL's volatility makes it a poor indicator for use by rating agencies. It may create pressure to downgrade or upgrade ratings during temporary aberrations in the fixed income market, which do not actually affect the cash pension demands upon an employer's resources.

A very useful tool for users of financial statements (including lenders and rating agencies) would be an open group forecast of future employer contribution rates under an expected scenario, under alternate scenarios, and even under a stochastic approach.

Truly comparable measures, such as those described in the next section, "Comparability," would also be useful to lenders and rating agencies.

E. Comparability

Comparing one pension liability calculation to another is a worthy and useful purpose. Comparisons can be made of an individual plan's funded ratios over time, revealing useful progress trends. Furthermore, comparisons can be made of one plan's funded ratio to another plan's funded ratio at a point in time, or over time.

In addition to funded ratios, there are other statistics that are useful to compare (one plan with itself or with other plans), such as unfunded actuarial accrued liabilities (UAAL) as a percent of payroll, employer contribution rates and benefit levels.

Common Liability Types and Methods

In order to achieve comparability, we believe that all plans should calculate the same *type* of liability. We cannot compare *funding* liabilities to *benefit* liabilities. *Funding* liabilities are employer liabilities, while *benefit* liabilities are plan liabilities. We should be comparing employer *funding* liabilities to employer *funding* liabilities, and plan *benefit* liabilities to plan *benefit* liabilities.

Furthermore, except for employer contribution rates, of course, the actuarial methods for comparing *funding* liabilities should be the same. We cannot compare the funded ratio or the UAAL as a percent of payroll for frozen entry age to those of the entry age normal method. The aggregate cost method does not even have a UAAL.

It may be more difficult to settle on a single method of asset valuation for comparisons of funded ratios and UAALs as a percent of pay. Market value of assets is certainly convenient and consistent. However, it carries with it all the same volatile qualities of MVL and FVL. We suggest that calculating some or all comparative statistics using both market value of assets and smoothed value (whichever smoothing method is employed by the plan) is better than calculating only one or only the other.

Calculating additional actuarial statistics based on the EAN may require additional work, time and expense, especially if the plan is not already using the EAN method. However, the goal of improving comparability is worth it.

It may be useful to compare a plan's *benefit* liabilities to itself over time (and forecasted into the future) and to other plans' benefit liabilities at a point in time and over time.

Relevant Assumptions

Those who advocate MVL as the best or only measure of pension liabilities argue strongly that all employers should calculate and publish their benefit liabilities using the same discount rate. This is argued partly on the basis of comparability. The implication is that liabilities calculated using different discount rates are not comparable.

We believe this opinion is formed through a mistaken perspective on comparability. Many factors are at play within the employer and the plan that need to be captured for a comparison to reveal relevant similarities and differences. The long-term rate of investment return expected by the pool of assets, from which the benefit will be paid, is just as important in comparisons as the rates of rates turnover, retirement, salary increases and mortality.

A plan that invests solely in 90-day Treasuries is at a significant disadvantage in comparison to other plans which invest in a balanced portfolio. As stated previously, there is not much dispute that a plan invested entirely in 90-day Treasuries is expected to cost taxpayers more over the long term than if it were invested in a 60-40 balanced portfolio (at least matching the index performances). A pay-as-you-go OPEB program is expected to cost taxpayers much more over the long term than if it were advance funded with a balanced investment policy. Such disadvantage should be reflected in the comparative statistics. By discounting the plan's obligation at a designated rate, we lose that critical piece of comparative information. Funded ratios are *less* comparable (not more) when calculated without regard to their respective differences in investment policies and return expectations.

Similarly, a plan with employees who hardly ever terminate, who retire immediately upon eligibility, and who live longer is also at a disadvantage in comparison to other plans that have average levels of decrements. Such disadvantage should be reflected in the comparative statistics. That plan's liability (denominator in the ratio) needs to be larger to reflect its population's expected decrement pattern.

The discount rate should not be singled out as the one actuarial assumption that should be the same among all plans. By requiring the same discount rate (or the same demographic assumptions), valuable and useful comparative information about the long-term costs is lost.

We recognize that different actuaries and plans may adopt different investment discount rate assumptions, even if the investment policies and investments are identical. This does introduce some corruption in the comparability, but nothing is perfect. Different actuaries and plans are also likely to adopt different assumptions as to rates of turnover, retirement, disability, salary increases and mortality — even for the same plan. Forcing all comparative statistics to use the same discount rate introduces a worse corruption than using the actuaries' and plans' best estimates of all assumptions.

Advocates of MVL also strongly argue that the common discount rate that must be used is a yield curve that varies every year with the market, resulting in volatile liability valuations. Not only does this lose the essential characteristic of plans' investment policies and expectations, it causes comparability over time to be impossible. If the funded ratios and UAALs as a percent of payroll are volatile solely due to fixed income market conditions from one year to another, it makes discerning plan trends impossible. Assessing the progress of the funded status over time or the employer's ability to service the UAAL debt over time is obscured by all the variation in the liability calculations which have nothing to do with funding progress.

MVL and FVL make for a poor comparator method for comparing a plan to itself over time, and for comparing different plans to each other at a point in time and over time.

Employer Contribution Rates

An employer's contribution rates are routinely compared to prior years. They are also compared to the employer contribution rates of neighboring states and local jurisdictions, and nationally. If not statutorily fixed, these contribution rates are routinely calculated using different discount rates, other actuarial assumptions which are different, and even using different actuarial cost methods. While the comparisons, over time and with other entities, are imperfect, they provide useful information.

Employer-Funded Ratios

A sole or agent employer's *employer-funded ratio* should be understood as a measure of how close the employer is to being "paid up" on the debt it owes to the plan.

Such an employer-funded ratio should, therefore, be defined as actuarial value of assets (AVA) divided by the unfunded actuarial accrued liability calculated under the common entry age normal cost method (EAN UAAL). Some may want to supplement that with the market value of assets (MVA) divided by the EAN UAAL. Each year the EAN method establishes an allocation of annual funding requirements (for each employee since his respective entry age. This cost allocation creates an accumulated amount which the employer should have paid over time to be "paid up." The costs of all benefit improvements and all actuarial losses (and gains) are rolled into what should have been paid every year, in hindsight. It is called the actuarial accrued liability (AAL). The shortfall between the AAL and the current measure of assets is the UAAL.

Comparing an employer's own *employer-funded ratios* over time is useful in tracking the employer's funding progress, as long as they are comparable. Comparing an employer's current funded ratio with those of other entities is also useful, as long as they are comparable.

As set forth in this section on financial reporting, the EAN method should be used as the common actuarial cost method for calculating the actuarial accrued liability for the reasons described. In addition, an actuarially smoothed value of assets should be permitted as long as it complies with Actuarial Standard of Practice No. 44 (*Selection and Use of Asset Valuation Methods for Pension Valuations*).

UAAL as Percent of Payroll

A good measure of the employer's ability to service the UAAL debt the size of the UAAL compared to the payroll. Again, the EAN method and asset smoothing should be used for this purpose.

Plan-Funded Ratios

The last part of the "Financial Reporting" section discusses plan financial reporting. It mentions a *plan-funded ratio*. This is not to be confused with the *employer-funded ratio*, which measures how close the employer is to paying off the debt it owes to the plan. The *plan funded ratio* is a measure of benefit security. It is not nearly as useful for coming to reasonable conclusions as the *employer funded ratio*.

Benefit Levels

Often stakeholders want to compare the pension benefit levels of one public sector employer to another. To isolate solely the level of benefits, the best comparative statistic is a ranking of the present values of future benefits for employees hired at selected ages from, say, 25 to 50. These ranking statistics are useful for union negotiations, setting benefits policy, and designing overall compensation packages.

Present value calculations for these hypothetical employees for this purpose should be calculated with the current (and proposed) benefit structure for the target employer. The same hypothetical employees should also have present value calculations made for all other employers in the comparative universe. All actuarial assumptions should be identical for all such employees and all such employers. This commonality of assumptions will isolate the benefit level for ranking and comparison purposes.

F. Risk Measurement and Analysis

A lot can be said about pension and OPEB risk measurement and analysis. Other papers and resources provide a wealth of useful metrics, techniques, objectives, processes and communications. This section will be limited to a brief discussion of the practical usefulness of MVL and FVL for discussions concerning risk measurement and analysis with plan and employer officials.

The MVL is only one number. The FVL is one other number. Useful information and discussions concerning pension and OPEB risk measurement and analysis cannot be captured in a single number.

As a starting point for such discussions, a public sector pension fund (and/or the public sector employer) might be interested in knowing how much it would cost to remove all risks for benefits earned to date. If the price were acceptable, a transaction would be consummated with a third party to assume all responsibilities for paying the specified benefits when due. The third party would most likely be an insurance company active and reputable in the single premium

group annuity market, although other governmental retirement systems might have an interest in bidding for the transaction (as described in Part 3 of this series).

FVL is a better proxy for a settlement liability than MVL. An actual settlement of the liability would be found in the single premium annuity market, likely using the private sector standards and criteria imposed by the U.S. Department of Labor. FVL would be a closer representation of the benefits actually earned at the measurement date (CBO) as described in Part 1 of this series. It is a closer representation of the risk-adjustments that the marketplace would charge for assuming the responsibility (risk-adjusted CBO), as described in Part 2 of this series. Finally, it is a closer representation of the discount rate which the marketplace would utilize in setting the exit price, as described in Part 3 of this series.

In any event, it would be merely a starting point for the risk discussion. Some sensitivity modeling around the best estimate FVL might be helpful.

When fiduciaries consider portfolios invested entirely in bonds, the notion usually has its origin either retreating from equity volatility or embracing MVL as the measure of pension liabilities. When MVL is embraced, it naturally leads to serious discussions of pension portfolios invested entirely in bonds.

Liability-driven investments in the private sector are motivated because funding and financial reporting standards have adopted MVL/FVL-like liability measurements. The private sector environment (funding and reporting) is particularly well-suited for discussions about portfolios designed to match the liability behavior over time. Since pension liability measures in the private sector are all discounted using fixed income yields, for funding and accounting purposes, there is a certain amount of logic in designing a portfolio whose market value of assets would behave in lock-step with the market value of liabilities. Surplus optimization makes the most sense when the liability measure varies with fixed income yields, as MVL and FVL do.

However, public sector liabilities are not required to be valued at varying discount rates for funding or reporting, as are private sector liabilities. Furthermore, the primary purpose of this paper is to demonstrate that public sector pension liabilities should *not* be measured that way. Hence, there is little interest in the public sector for liability-driven investing, dedicated bond portfolios, liability matching portfolios or any other such system of investments designed to vary in tandem with the liability measure.

Therefore, public sector discussions of risk measurement and analysis would involve forecast valuations optimizations, asset allocation, fat-tail distributions, stress testing, sensitivity testing, recovery testing, risk tolerance assessments, stochastic analyses, and very little about MVL or FVL.

G. Personal Wealth

Personal Asset Allocations

Much work has been undertaken to develop models and methods for assisting individuals to adopt asset allocation policies for their personal portfolios, which reflect their personal goals and risk tolerances. Mean-variance optimization models have been used for many years to set asset allocation policies among different asset classes.

A major challenge to the typical optimization model occurs when an individual has or will have a defined benefit pension stream of income. How to incorporate the value of that stream of income payments into the overall optimization model has not been fully developed.

One possible way to do so would be to calculate the market value (discounted at risk-free or market rates) of such benefit stream and consider that as an asset class alongside the other asset classes. Expected returns, standards deviations, correlations and constraints for the pension asset could be developed along with the other asset classes' capital market assumptions for an integrated optimization.

This technique would not be without its own challenges. It may result in some odd answers and strain a client's confidence. Without developing this notion further, we present it herein as possibly some practical application for MVL. Certainly, the presence of an individual's pension stream of income should affect his liquid asset allocation in some fashion.

Plaintiff Advocacy

MVL may play a role in qualified domestic relations orders (and any governmental plan equivalents). Negotiations in divorce settlement often involve the value of pension benefits earned. Consider an employee or retiree who is a member of a public sector pension plan. If such plan member were to find himself in divorce negotiations over property values, his actuary may wish to keep the value of such pension as low as legitimately possible, while the alternate payee's actuary may wish to keep the value as high as legitimately possible. Her actuary may choose to argue that the value of the member-spouse's public sector pension should be discounted at risk-free rates, rather than IRC 417(e) rates, plan rates or some other rates suggested by his actuary or the plan's actuary.

While MVL may be of no practical use for those public sector pension purposes identified previously in this Part 5, it may indeed have usefulness in plaintiff litigation. Similar actuarial positioning may also occur in wrongful death or disability litigation and negotiations.

H. Plan Terminations and Freezes

MVL and FVL have legitimate uses as proxies for plan benefit liability settlement. As mentioned in the "Risk Measurement and Analysis" section of this Part 5, if a plan or employer were seriously considering a plan termination, calculation of the MVL and FVL might provide a reasonable estimate of what an insurance company might bid to assume all responsibility for paying the benefits earned to date when they fall due. Nothing replaces and actual request for bids, but these may provide some estimate, along with some sensitivity testing on the discount rate.

If a plan or employer were seriously considering a plan freeze, the approach described in Part 4 of this series, "The Residual Benefit Liability," would be useful in modeling the operation of the frozen plan.

I. Summary of the Series

The current model of the market value of public sector pension liabilities fails in three important ways in its attempt to serve as a fair value model. It fails certain labor economics principles by its use of the accumulated benefit obligation cash flow, in that the ABO does not represent the contractual benefit obligation between the employer and employee. It fails certain actuarial finance and pricing principles by treating expected cash flows as fixed. Finally, it fails certain financial engineering principles by not discounting cash flows with market-related rates, observable in real world markets.

Depending on standards-setting bodies or other forces, actuaries might be required to calculate and communicate the fair value of public sector pension liabilities. If so, the authors wanted to make a constructive contribution by proposing three *improvements* to the current model in order to produce a better model of fair value. These improvements include the use of the contractual benefit obligation (CBO), risk-adjusted CBO cash flows, and discount rates higher than risk-free to better reflect market-related rates.

An alternative model for fair value of the employer's benefit liability, one which is more faithful to the contract terms that exist between the public sector employer and the public sector pension fund, is presented as the residual benefit liability.

In spite of these improvements to the current so-called "market value of liability" model to offer a better representation of the fair value of the liability, these measures of the benefit liability have little usefulness for real world purposes. They are merely theoretical constructs with little to no decision utility.

Conventional and some not-so-conventional methods for measuring public sector pension and OPEB liabilities are much more appropriate for funding purposes, taxpayers' purposes, financial reporting purposes, lenders' and rating agencies' purposes, and for comparability purposes. Nevertheless, market and fair value of public sector liabilities may have some limited usefulness for certain personal wealth purposes, certainly including plan termination purposes, and might be worth a passing comment within risk measurement and analyses among more sophisticated and useful techniques.

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