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Coherent Mortality Modeling for a Group of Populations

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Abstract

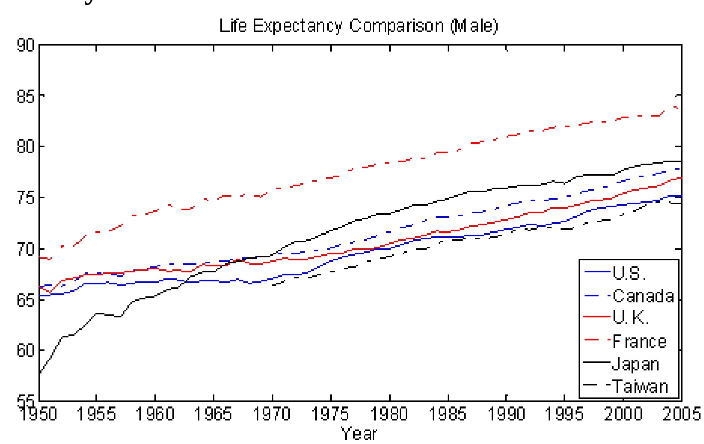
Men and women in a country or people in nearby countries share comparable living conditions and are likely to have similar mortality behaviors, i.e., period effect. Combining mortality experience for a group of populations with similar mortality behaviors might increase the stability of mortality modeling and solve the problem when mortality data is insufficient. In this research, we study the coherent mortality modeling by combining a group of populations with similar period effects. The group of populations shares the same period effect in projecting the future mortality rate. We employ the Lee-Carter model (Lee and Carter 1992) to illustrate the feasibility of coherent mortality modeling. We use U.S. and Canadian data from the Human Mortality Database (HMD) to evaluate whether the coherence exists and if it can increase the stability of mortality modeling. The goodness fits for evaluating the coherence also is based on Akaike information criterion (AIC) and Bayesian information criterion (BIC). We found that combining countries with the same gender has better results than combining genders in a country. In addition, we also use annuity products to evaluate the performance of the coherent forecast, by simulating the confidence intervals of the future dynamics of mortality and annuity price.

Keywords: Lee-Carter Model, Longevity Risk.

1. Introduction

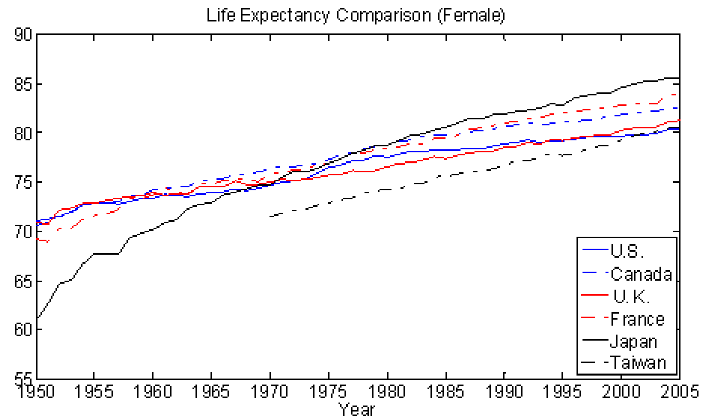
Because life expectancy has been increasing dramatically, the longevity risk becomes non-negligible and its influence is increasing gradually and globally. According to results derived from the Human Mortality Database (HMD), the human life span has increased substantially over the past 40 years. Human life expectancy has been increasing significantly since the start of the 20th century, though increments of life expectancy vary with country and gender. For example, in most Western countries, life expectancies at birth were approximately 66 and 70 years for men and women in 1950, but they increased to 75 and 80 years by 2005. The life span of an American male increased from 68.91 to 77.25 years, from 1970 to 2005, respectively, and the average life span of an American female increased from 75.21 to 81.43 years in the same period. Similar patterns appear in Canada, where the average life span from 1950 to 2005 increased from 66.16 and 70.59 to 77.85 and 82.53 for males and females, respectively. In Asia, life expectancies were much lower at the turn of 20th century, but the rate of mortality improvement has been much greater, such that life expectancies in Asia today are similar to those of Western countries. For example, life expectancies in 2005 for Japan are 79 and 86 years for men and women; in Taiwan, they are now approximately 75 and 81 years (see figures 1 and 2).

Figure 1
Life Expectancy at Birth for Males in Various Countries: 1950-2005[4]



4. The calculation of life expectancy is based on the period life tables. The mortality data for different countries is accessed from the Human Mortality Database in 2008 except for Taiwan's. The data for Taiwan is based on the Minister of Interior, Taiwanese government and data availability limits the Taiwanese data to 1971-2005.

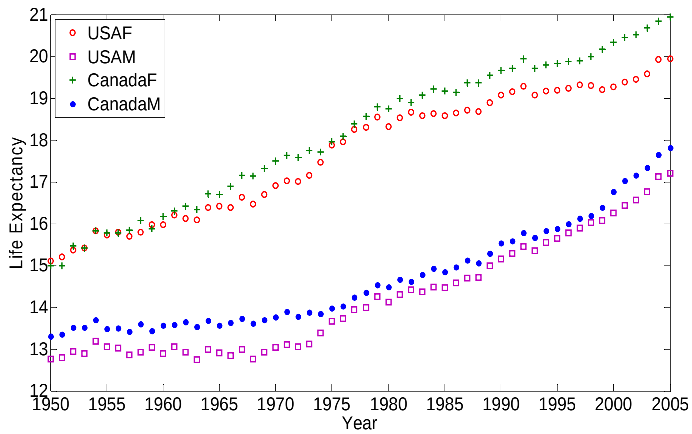
Figure 2
Life Expectancy at Birth for Females in Various Countries: 1950-2005



It seems that men and women in a country or people in nearby countries have similar mortality behaviors. For example, the Americans and Canadians, or the American/Canadian male and female, show similar patterns in the life expectancy. We use the life expectancies of age 65 for the Americans and Canadians as an example (Figure 3). The correlation of the life expectancy for males between Canada and the United States is 0.9922 and for females between Canada and the United States it is 0.9926. Some previous studies also found similar mortality phenomenon of convergence for a group of populations. Wilson (2001) pointed out a global convergence in mortality and mentioned it is improper to prepare mortality forecasts for individual nations in isolation. White (2002) found there is a faster increase in life expectancy for the countries with higher mortality such as developed countries. The phenomenon that the populations sharing similar mortality behaviors has not been taken into account for mortality modeling, except in Li and Lee (2005). Most of the mortality studies emphasize a single population[5] (Lee and Carter 1992; Renshaw and Haberman 2003; Cairns et al. 2006).

5. Which could be either one sex or two sexes combined.

Figure 3
Life Expectancy of Male and Female age 65 for Canada and the United States



The original Lee-Carter model (1992) has been widely applied to deal with single-population mortality modeling based on age and period effects. Many countries suggest using the Lee-Carter model as the base mortality model for their population projections in valuing annuity products. In addition to age and period effects, the cohort effect[6] represents an important risk factor that governs the dynamics of mortality (Renshaw and Haberman 2006; Cairns et al. 2009). Renshaw and Haberman (2006) incorporate cohort effects into the Lee-Carter model. Cairns et al. (2009) quantitatively compare eight stochastic mortality models and demonstrates the Cairns-Black-David(CBD) model (Cairns et al. 2006) that incorporates a cohort effect fits data about English and Welsh men best, and Renshaw and Haberman's (2006) extension of the Lee-Carter model that also allows for a cohort effect provides the best fit for data pertaining to U.S. men.

Applying the Lee-Carter model to a single population might produce unreasonable results; for example, the life expectancy of countries with similar mortality profiles may diverge. Tuljapurkar et al. (2000) studied the mortality among the G-7 countries separately and found the largest gap in the projection of life expectancy among these countries over a 50-year forecast horizon increases from about four to eight years, which is not plausible. To

6. Different rates of mortality improvement depend on the year in which the person was born.

overcome the divergence problem, Li and Lee (2005) proposed that the Lee-Carter method should not be used for a single country separately, and they applied the Lee-Carter model to a group of populations, forcing a common mortality change by age. They pointed out that the populations treated as a group have similar socio-economic conditions and close connections, and the similarity is expected to continue. Thus, using coherent mortality modeling can be important to understanding the dynamics of future mortality rates. In addition, employing a proper mortality model to price life annuity products is an important task for the actuary to deal with longevity risk. In practice, the challenge to price life annuity products for the elderly is that the data used to fit a mortality mode may not be sufficient. The credibility and the stability of the mortality model is then not satisfied. By combining the mortality data for a group of population, it can increase the stability of mortality modeling.

In this research, we extend the approach of Li and Lee (2005) to investigate the effect of coherent mortality modeling for a group of population in mortality forecasts. We employ the Lee-Carter model to illustrate the feasibility of coherent mortality modeling. The parameter estimates in the coherent mortality model are assessed using maximum likelihood estimation (MLE) method. Under the MLE estimation method, we can evaluate the selection of a group of population for a coherent group based on Akaike information criterion (AIC) and Bayesian information criterion (BIC). As Li and Lee (2005) point out, the populations may be treated as a group when they have similar socio-economic conditions and close connections and when these conditions are expected to continue in the future. Thus, we consider the neighbor countries of Canada and the United States in North America and illustrate whether it is more appropriate to model mortality for different gender or different country group in these two countries. The goodness of fit of the coherent Lee-Carter model is examined. In addition, our analysis focus is on the mortality rates at older population ages (65 and older). In addition, we also use the annuity products to evaluate the performance of the coherent forecast, by simulating the confidence intervals of the future dynamics of mortality and annuity price.

In the remainder of this study, we first introduce the Lee-Carter model and its coherent mortality model. The parameter estimation method of these models is introduced in Section 2. In Section 3, we make an empirical study to assess the model fitting and mortality forecast for the selection of groups for coherent mortality modeling based on the AIC and BIC. The patterns of mortality forecast for different selection of groups are analyzed in Section 4. The discussions and limitations of the study are given in Section 5.

2. The Coherent Mortality Modeling

2.1. The Mortality Model: Age-Period Model

The Lee-Carter model is an age-period model. It is assumed that the logarithm of central death rate $m_{x,t}$ for a person age *x* in time *t* satisfies the following equation:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \tag{2.1}$$

where the parameter α_x describes the average age-specific mortality rate, κ_t represents the general mortality rate level, β_x indicates the change rate of mortality at age *x*, and $\varepsilon_{x,t}$ denotes the deviation of the model or the error term. The parameters (α_x , β_x) represent the age effect and κ_t shows the period effect. There are two restricted formulas in the Lee-Carter model, which are $\sum_x \beta_x = 1$ and $\sum_t \kappa_t = 0$.

Although it is not specified, the Lee-Carter model usually is used to model the mortality rates of a single population, to avoid data inhomogeneity. Li and Lee (2005) modify the Lee-Carter model with coherent modeling, which imposes the same period parameter for a group of populations. The group of populations can be males or females in a country or countries in a region sharing similar properties. To take into account the coherent pattern of period effect, the original Lee-Carter model is extended to be

$$\ln(m_{i,x,t}^{(j)}) = \alpha_{i,x}^{(j)} + \beta_x^* \kappa_t^* + \varepsilon_{i,x,t}^{(j)} \tag{2.2}$$

where β_x^* and κ_t^* represents the coherent pattern of age and period effect for a group of populations and $\alpha_{i,x}^{(j)}$ represents the age effect for the *j*th country and *i*th gender. In other words, Equation (2.2) allows each population has its own age pattern and level of mortality but is assumed to share the same change rate of time.

2.2 Parameter Estimation

There are a variety of approaches in estimating the parameters in the original Lee-Carter model. The singular value decomposition (SVD) method is used by Lee and Carter (1992) and becomes the most common approach (Alho 2000; Koissi et al. 2006). However, the main drawback of using the SVD method is that the error terms are assumed to be homoscedastic. Such assumption doesn't conform to the actual situation because the numbers of populations often is a decreasing function of age. Also, the SVD cannot be used where there are missing values in certain age groups. Then, Lee and Carter suggested using the SVD approximation when the SVD cannot be applied. Wilmoth (1993) proposed two approaches to modify the SVD approximation for the parameter estimation in Lee-Carter model: weighted least squares estimation (WLSE) and maximum likelihood estimation (MLE).

For the parameter estimation in equations (2.2) and (2.3), we use the MLE method for estimation for the Lee-Carter model. The log-likelihood function for the coherent model can be expressed as

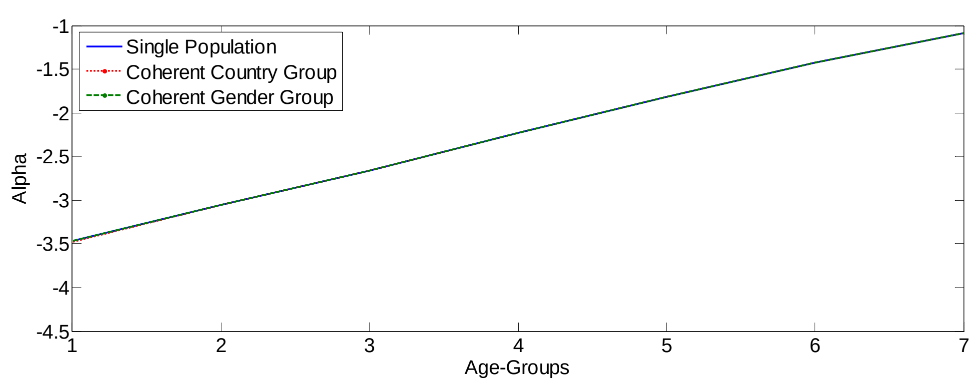
$$L\left(\theta^{*}\right)=\sum_{x,t}\left[D_{x,t}\ln(\lambda_{x,t})-\lambda_{x,t}-\ln(D_{x,t}!)\right] \tag{2.3}$$

where $D_{x,t}$ denotes the number of deaths at age x in year t. We assume the number of deaths follows Poisson distribution, or $D_{x,t}\sim \text{Poisson}(\lambda_{x,t})$, $\lambda_{x,t}$ is the expected number of deaths and is calculated as $\lambda_{x,t}=m_{x,t}\cdot E_{x,t}$, where $E_{x,t}$ denotes the exposure-to-risk at age *x* in time *t*. In the coherent Lee-Carter model, the parameter θ^{*} represents $\alpha_{i,x}^{(j)}, \beta_{x}^{*}$ and κ_{t}^{*}. To work out the parameter estimates we using the MLE method shown in Equation (2.3), we use the Newton method.

For illustration purpose, we consider the neighbor countries of Canada and the United States. We investigated whether it was more appropriate to model mortality for different gender or different country group in these two countries. The mortality data for men and women in these two countries are retrieved from the Human Mortality Database in the format of five-year cohorts from 1970 to 2005 for people age 65 to 99. There are two possible ways of combining data for the coherent model, i.e., combining either genders or countries. In a coherent country group, we calculate the coherent parameters for males and females in the same country. In a coherent gender group, we calculate the coherent parameters for the same gender in different countries.

Based on the age-period model, we investigated the differences of parameter estimates for the coherent Lee-Carter model with the original Lee-Carter model. It implies there is a coherent trend of period for mortality modeling in the same group. The corresponding parameters based on the Lee-Carter model are shown in Figure 4 for males and figure 5 for females. For a comparison purpose, we also illustrate the parameters obtained for a single population based on the Lee-Carter model. Figure 4 (and Figure 5) shows the parameter estimates based on age-period model for males (and females) in the United States for coherent group, coherent gender and single population separately. The parameters estimates for other illustrated gender and countries for Canada are shown in Appendix A. Note age group 1 is for ages 65 to 69, age group 2 is for ages 70 to 74, etc.

Figure 4
Parameter Estimates of Coherent Lee-Carter and Original Lee-Carter Models
(U.S. Male 65 and Older)



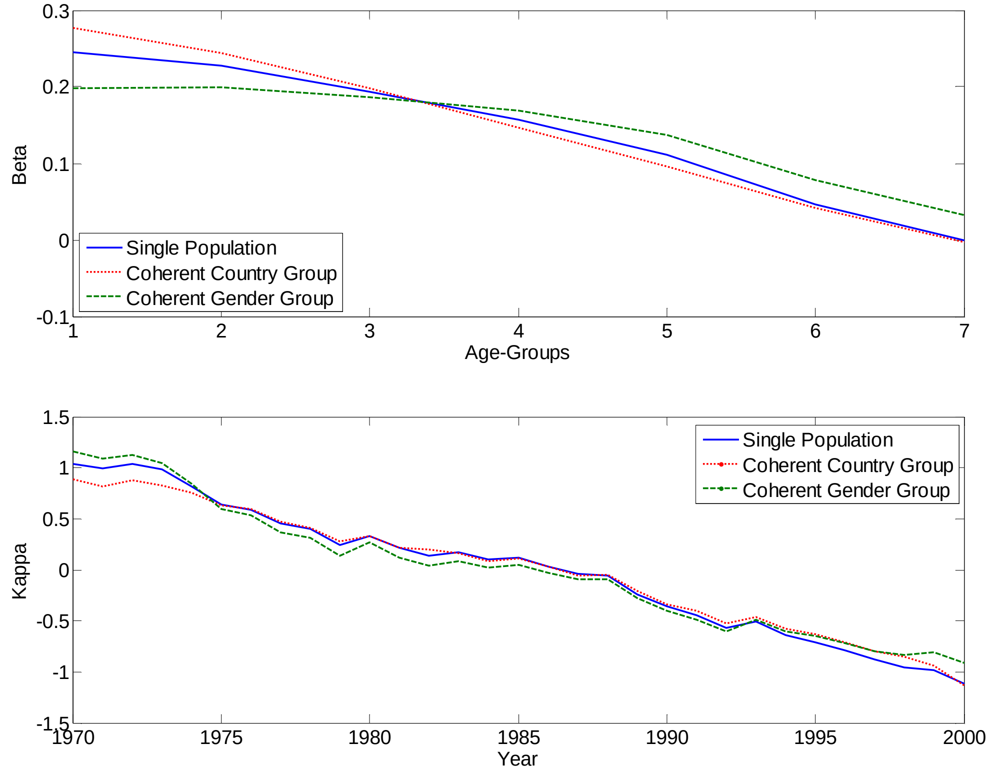
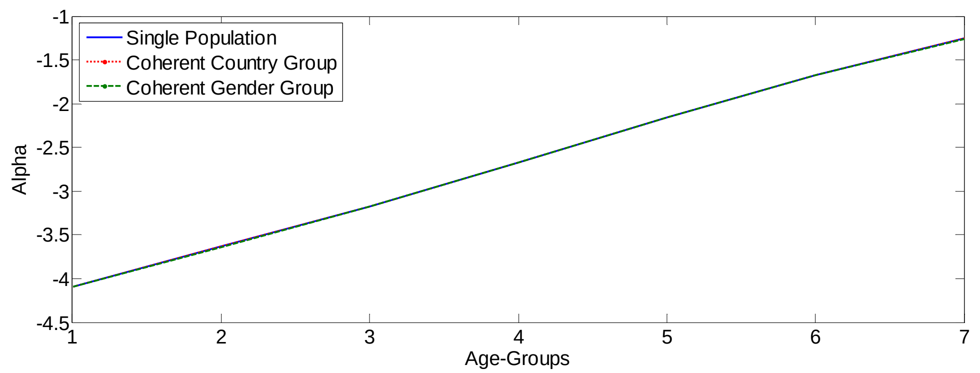
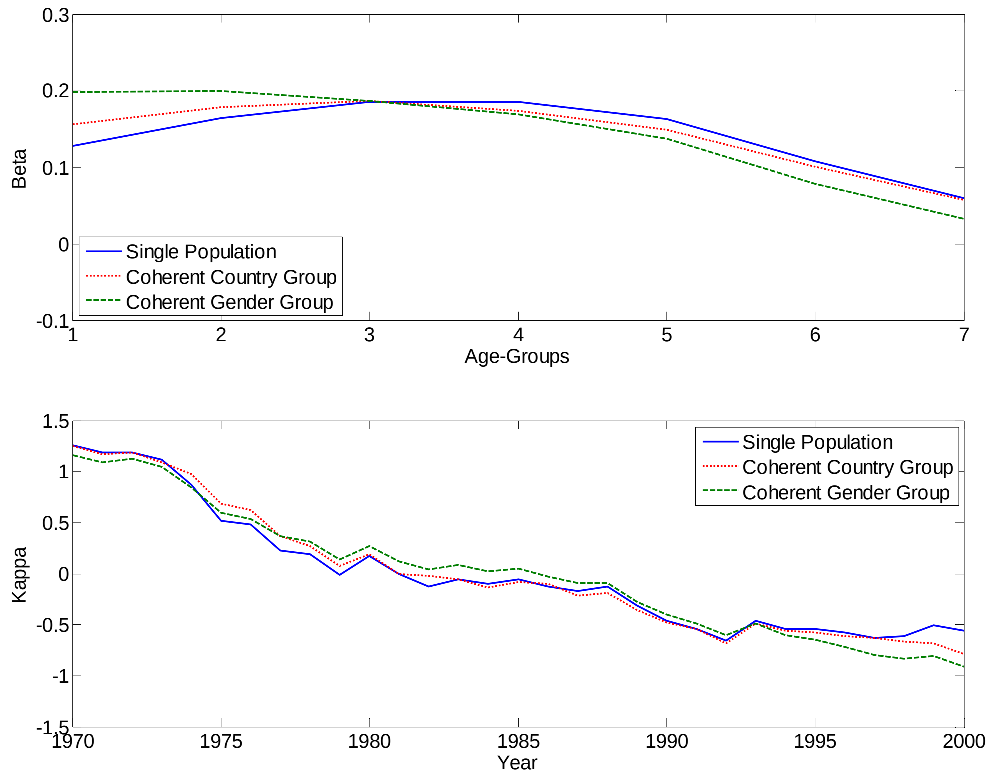


Figure 5
Parameter Estimates of Coherent Lee-Carter and Original Lee-Carter Models
(U.S. Female 65 and Older)





It is obvious the coherent models share similar parameter estimates and are very different from the single-population parameter estimates in the original model. This indicates that, whether combining genders or countries, the coherent Lee-Carter model produces similar and/or more stable parameter estimates. The estimates of β_x for different coherent models have slightly larger differences, and the scales of different age groups have larger variations than those of the single population. Still, the time effect parameter κ_t for the single population shows a more obvious decreasing trend than those for the coherent models, and this is especially obvious for the male case (in both the United States and Canada).

3. Model Fitting for Different Coherent Group

3.1. Selection of Coherent Group

In this study, we consider two coherent groups, i.e., coherent country group and coherent gender group, and we want to know whether combining country or combining gender is more appropriate. We use the model fit and forecast to evaluate these two coherent models. To evaluate the model fitting and forecasting, the data are divided into the fitting period (in-sample) and prediction period (out-sample), where the period of 1970 to 2000 represents the fitting period and that of 2001 to 2005 is the prediction period.

Based on the age-period model, the fitting accuracy for two coherent groups is presented in Table 1. According the log-likelihood, AIC and BIC criteria, the coherent country group (i.e., combining country of the same gender) has a larger log-likelihood ratio, AIC and BIC. For example, when combining U.S. and Canadian males, the AIC is -6259.7 and the BIC is -6399.3. This means that combining the data of Canada and the United States is more appropriate than combining male and female data in either Canada or the United States.

TABLE 1
Fitting Accuracy for Different Coherent Groups of Older Ages: Lee-Carter Model

<table>
<tr><th rowspan="2">Coherent Group</th><th rowspan="2">Gender</th><th colspan="2">Log-likelihood</th><th colspan="2">AIC</th><th colspan="2">BIC</th></tr>
<tr><th>U.S.</th><th>Canada</th><th>U.S.</th><th>Canada</th><th>U.S.</th><th>Canada</th></tr>
<tr><td rowspan="2">Country</td><td>Male</td><td>-1,530.9</td><td>-1,711.2</td><td colspan="2">-6,259.7</td><td colspan="2">-6,399.3</td></tr>
<tr><td>Female</td><td>-1,692.6</td><td>-2,082.7</td><td colspan="2">-7,036.9</td><td colspan="2">-7,176.5</td></tr>
<tr><td rowspan="2">Gender</td><td>Male</td><td>-1,925.2</td><td>-2,008.2</td><td rowspan="2">-11,299.7</td><td rowspan="2">-18,061.6</td><td rowspan="2">-11,439.3</td><td rowspan="2">-18,201.2</td></tr>
<tr><td>Female</td><td>-2,314.8</td><td>-2,530.7</td></tr>
</table>

3.2 Forecasting Assessment

We analyze the coherent mortality forecast in this section. Another frequently used criterion for comparison is the mean absolute percentage error (MAPE), defined as

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|Y_i - \hat{Y}_i\right|}{Y_i}\times 100\%,$$

where Y_i and $\hat{Y}_i$ are the actual values and estimated (or predicted) values of mortality, and n is the number of observations. The MAPE will be used for comparing the coherent models and single-population model.

The prediction of future mortality rates are based on the fitted time series model for the period effect. The autoregressive integrated moving average (ARIMA) process is a common method for projecting the period effect for a single population under the Lee-Carter model (see, for example, Lee 1992; Renshaw and Haberman 2003; Koissi et. al. 2006). We extend the ARIMA(0,1,0) process to model the coherent period effect[7] and use the MAPE to measure the forecasting accuracy. For a comparison, we calculate the MAPE for coherent mortality forecast based on the Lee-Carter model in Table 2.

TABLE 2

A Comparison of Mortality Forecast MAPE for Higher Ages (65 to 99)

Model	**Sex**	**MAPEs**	
		U.S.	**Canada**
Lee-Carter Coherent Gender	Male	0.0719	0.0304
	Female	0.0259	0.0547
Lee-Carter Coherent Country	Male	0.0886	0.0224
	Female	0.0461	0.0503

7. We investigated ARIMA(p,d,q) process and found ARIMA(1,1,1) performed better in mortality forecast.

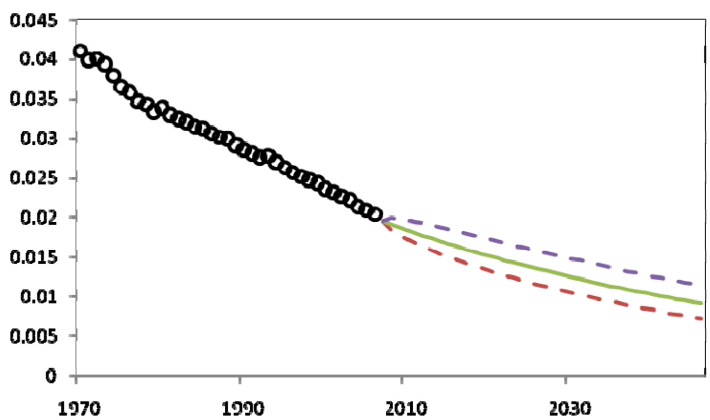
For the U.S. males, the coherent country group gives the better forecasting result because the MAPE is smaller, but, for the U.S. females, the coherent gender group gives better forecasting results.

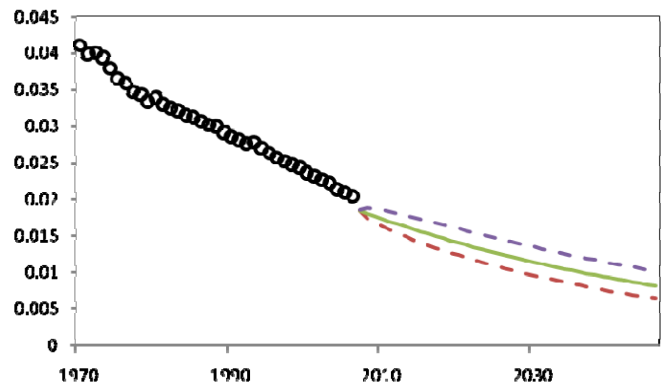
4. Pricing Life Annuity Using a Coherent Model

4.1 Mortality Projections

In addition to evaluating the forecasting errors of coherent and single-population models, we will compare their differences in confidence intervals. Because the coherent country model performs better in the forecasting, we will only focus on this model. We first consider the impact of coherent mortality on projecting future mortality rates of age 65, i.e., q(65, *t*). We shall use the mortality data from 1970 to 2005 to simulate the future mortality rates for the male age 65. The simulation is also repeated for the single-population model. The 95 percent confidence intervals (CIs) are shown in Figure 6. In general, the 95 percent CIs are slightly narrower than that simulated using the single-population forecast. This is because coherent mortality modeling can benefit from combining more mortality data and reach more stable parameter estimates (or mortality convergence).

Figure 6
The 95 Percent Confidence Interval of Simulated Mortality (U.S. Male Age 65): Lee-Carter Model. Top: Coherent Country Group; Bottom: Single Population

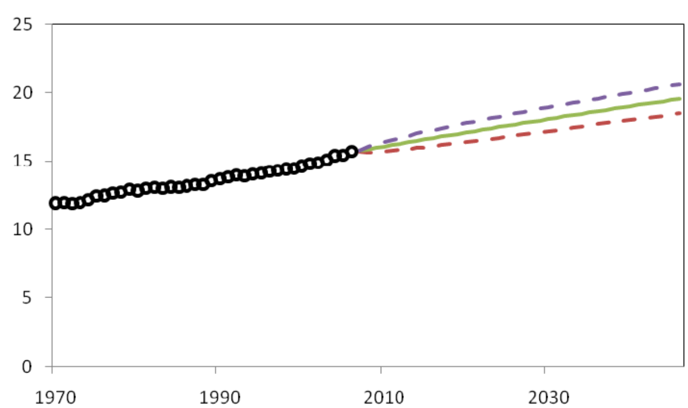




4.2 Projecting Annuity Values

To investigate longevity risk for annuity products, we conduct simulation for forecasting the value of a whole life annuity payable to a male age 65 in 2005. The 95 percent CIs of the forecast value of the annuity price from coherent country model and single-population mortality are shown in figure 7 based on the Lee-Carter model. It is clear the CIs of the forecast value of annuity from the coherent country model are smaller than those from the single mortality model. It implies the actuary can get more stable annuity prices if the coherent mortality model is applied to the group of population.[8]

Figure 7
The 95 Percent Confidence Interval of Simulated Annuity Value (U.S. Male Age 65)
Lee-Carter Model. Top: Single Population; Bottom: Coherent Country Group



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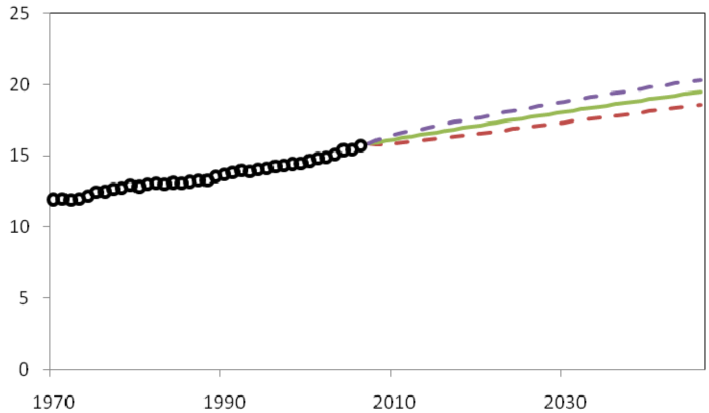
8. The coherent mortality model doesn't apply to every group of population.


25
20
15
10
5
0
1970
1990
2010
2030

5. Conclusions and Discussions

It is likely that people in a country or in nearby countries share comparable living conditions and thus have similar mortality behaviors. It would be reasonable to combine data with similar characters to increase the stability of mortality models, especially if there are insufficient data. Li and Lee (2005) proposed a coherent mortality model that combines male and female data in a country to reach a more stable mortality forecast, based on the Lee-Carter model.

We use the maximum likelihood estimation (MLE) for parameter estimates in the Lee-Carter model for a single population and a group of populations. The MLE can be used in model selections, such as a better model between combining two genders in a country or the same gender in two countries. Applying the log-likelihood, AIC and BIC, to the data of Canada and the United States, we found that it improves the fitting accuracy of the coherent mortality modeling for both coherent country and gender group. In addition, in these two countries, the coherent country model performs better than the coherent gender model. This might suggest the gender difference is likely to be larger than the country difference for Canada and the United States.

We also applied the coherent mortality model to obtain the confidence intervals for simulated mortality rates and annuity values. We found the confidence intervals obtained by the coherent model are narrower than those by the single population. This is because the coherent model assumes mortality convergence exists, i.e., different populations share the same changing mortality rate by age. This is also the case in Li and Lee (2005). In other words, if the combined populations considered share similar mortality characters, the coherent mortality model can provide a more stable estimate.

However, the proposed coherent model needs to be handled with care. We also applied the proposed model to other countries (or genders) and found diverse results. Neither combining genders nor combining countries dominates the other. For example, combining genders would get a better result for the case of Canada and the U.S. The distance between two countries can’t guarantee similarities in mortality profiles. It seems the correlation coefficient of life expectancy is a possible measure and it can be used as a preliminary tool for grouping populations with similar mortality characters.

In addition to grouping populations with different mortality risks, we are also working on using the MLE for further model selection. Like choosing a better model between a reduced model and a full model, we think the idea of sequential sum of squares can be used in the coherent model. If the tests and analysis procedure in regression can be applied, we can use the MLE to obtain a more reliable mortality forecast.

References

Alho, J.M. 2000. Discussion (on Lee, R. "The Lee-Carter Method for Forecasting Mortality, with Various Extensions and Applications.") *North American Actuarial Journal* 4: 91-93.

Brillinger, D.R. 1986. "The Natural Variability of Vital Rates and Associated Statistics." *Biometrics* 42: 693-734.

Brouhns, N., M. Denuit, and J.K. Vermunt. 2002. "A Poisson Log-Bilinear Regression Approach to the Construction of Projected Lifetables." *Insurance: Mathematics and Economics* 31: 373-93.

Brown, M.B., and A.B. Forsythe. 1974. "The Small Sample Behavior of Some Statistics Which Test the Equality of Several Means." *Technometrics* 16: 129-32.

Booth, H., J. Maindonald, and L. Smith. 2002. "Applying Lee-Carter Under Conditions of Variable Mortality Decline." *Population Studies* 56: 325-36.

Cairns, A.J.G., D. Blake, and K. Dowd. 2006. "A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration." *Journal of Risk and Insurance* 73: 687-718.

Cairns, A.J.G., D. Blake, K. Dowd, G.D. Coughlan, D. Epstein, A. Ong, and I. Balevich. 2007. "A Quantitative Comparison of Stochastic Mortality Models Using Data from England & Wales and the United States." *North American Actuarial Journal* 13, no. 1: 1-35.

Delwarde, A., M. Denuit, and C. Partrat. 2007. "Negative Binomial Version of the Lee-Carter Model for Mortality Forecasting." *Applied Stochastic Models in Business and Industry* 23: 385-401.

Efron, B. 1979. "Bootstrap Methods: Another Look at the Jackknife." *The Annals of Statistics* 7: 1–26.

England, P., and R. Verrall. 1999. "Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving." *Insurance: Mathematics and Economics* 25: 281–93.

Goodman, L.A. 1979. "Simple Models for the Analysis of Association in Cross-Classifications Having Ordered Categories." *Journal of the American Statistical Association* 74: 537-52.

Hoedemakers, T., J. Beirlant, M. Goovaerts, and J. Dhaene. 2003. “Confidence Bounds for Discounted Loss Reserves.” *Insurance: Mathematics and Economics* 33: 297–316.

Human Mortality Database. 2006. http://www.mortality.org.

Koissi, M.C., A.F. Shapiro, and G. Högnäs. 2006. “Evaluating and Extending the Lee–Carter Model for Mortality Forecasting: Bootstrap Confidence Interval.” *Insurance: Mathematics and Economics* 26: 1-20.

Lee, R.D., and L.R. Carter. 1992. “Modeling and Forecasting U.S. Mortality.” *Journal of the American Statistical Association* 87: 659-71.

Levene, H. 1960. In *Probability and Statistics: Essays in Honor of Harold Hotelling* I. Olkin et al., eds., 278-292. Stanford University Press.

Li, N., and R. Lee. 2005. “Coherent Mortality Forecasts for a Group of Populations: An Extension of the Lee-Carter Method.” *Population Association of America* 42, no. 3: 575-94.

Marques de Sá, J.P. 2003. *Applied Statistics Using SPSS, Statistica and Matlab.* Springer.

Renshaw, A.E., and S. Haberman. 2003a. “On the Forecasting of Mortality Reduction Factors.” *Insurance: Mathematics and Economics* 32, no. 3: 379-401.

———. 2003b. “Lee-Carter Mortality Forecasting with Age Specific Enhancement.” *Insurance: Mathematics and Economics* 33: 255-72.

Renshaw, A.E., and S. Haberman. 2006. “A Cohort-based Extension to the Lee-Carter Model for Mortality Reduction Factors.” *Insurance: Mathematics and Economics* 38, no. 3: 556-70.

Wilmoth, J.R. 1993. “Computational Methods for Fitting and Extrapolating the Lee-Carter Model of Mortality Change.” technical report, Department of Demography, University of California, Berkeley.

Appendix A.

Figure A1
Parameter Estimates of Coherent Lee-Carter and Original Lee-Carter Models
(Canada Male 65 and Older)

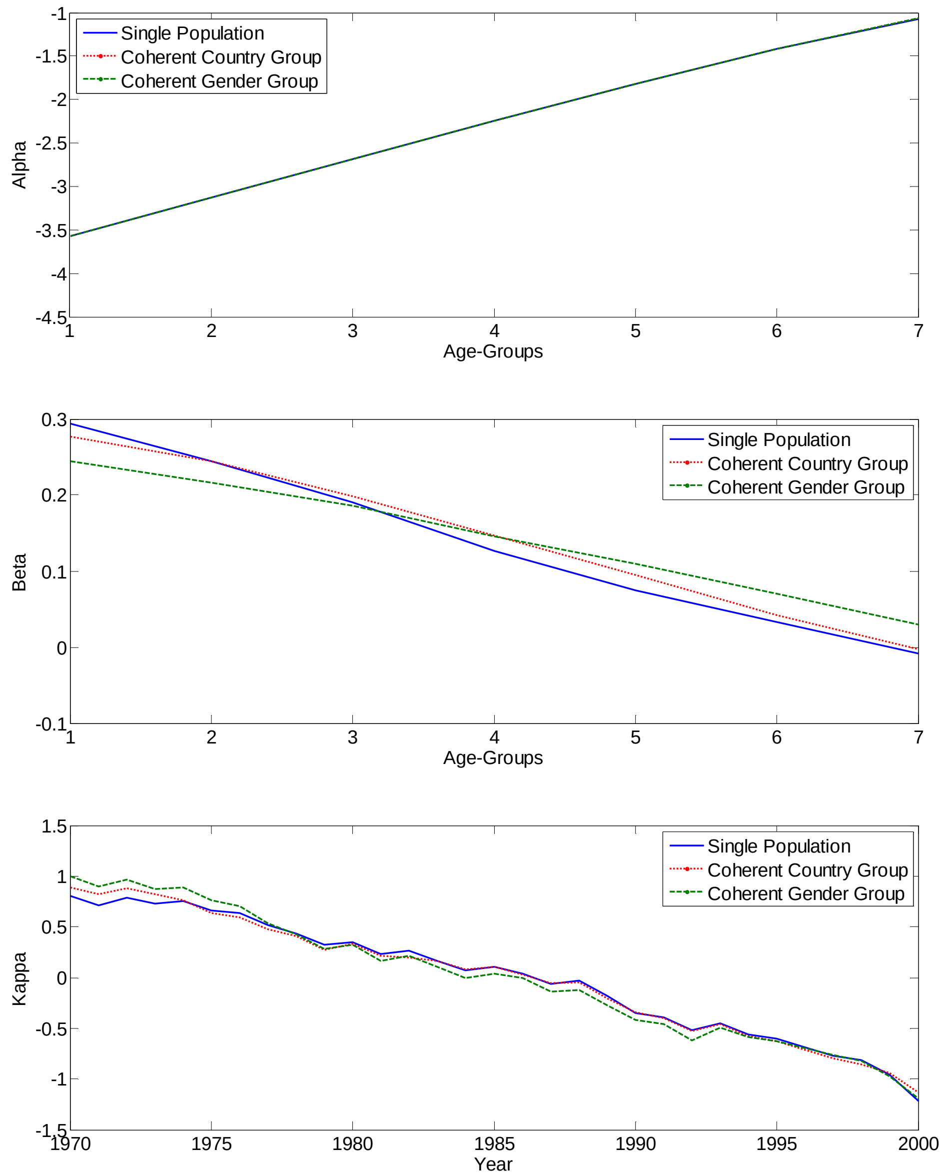


Figure A2
Parameter Estimates of Coherent Lee-Carter and Original Lee-Carter Models
(Canada Female 65 and Older)