Revisiting Pension Actuarial Science: A Five-Part Series

Introduction to the Series

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Abstract for the Series

The current financial model put forth as the market value of public sector pension benefit liabilities is simply the expected cash flows of the accumulated benefit obligation, as defined for current private sector financial reporting, discounted using a risk-free yield curve. This model is in serious need of an overhaul. It fails to faithfully represent the fair value of a currently accrued public sector pension benefit liability in three important ways:

1. Its use of the accumulated benefit obligation cash flows fails to accurately represent the terms of the employment contract which gives rise to the obligation being valued – a violation of labor economics principles.

2. Its use of expected cash flows as if they were fixed fails to recognize the risk premium load, which a fair exit price would include for the potential for adverse cash flow experience – a violation of actuarial finance and pricing principles.

3. Its use of risk-free discount rates fails to adequately reflect the observable and not-so-observable inputs from market participants' behavior – a violation of financial engineering principles.

Parts 1 through 3 in this series propose solutions to these three flaws.

Part 4,"The Residual Benefit Liability," presents an alternate approach to obtaining the fair value of the public sector employer's pension benefit liability. It approaches the task by modeling the real world operation of the pension fund, rather than approaching the task from the perspective of a theoretical construct. This alternate approach dares to model the long-term agency operation of the plan rather than ignoring it in favor of a pass-through approach. The current model ignores the effectiveness (even the existence) of the pension fund itself, while the alternate approach attempts to model the plan's operation in practice over time in order to determine the employer's residual asset or liability.

In spite of these three improvements and the alternate model, we believe the fair value of public sector post-employment benefit liabilities has little to no usefulness in most venues. There are legitimate roles which the market or fair value might play in valuing an individual member's personal wealth, a minor role in the context of certain discussions concerning risk measurement and risk management, and a major role in the context of plan terminations and freezes.

However, for purposes of advance funding, taxpayers, financial reporting, lenders and rating agencies, comparability, and the major part of risk measurement and analysis, the decision-usefulness of market or fair value is negligible, possibly even misleading. Other existing models and methods are far more suitable for these purposes, including conventional actuarial approaches and others that are less conventional or popular, but which should be considered in the actuarial toolbox and have higher decision utility.

Part 5 in this series, "Consider the Measurement Purpose," addresses various purposes for measuring a public sector pension liability and which measures have the most practical usefulness.

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Introduction to the Series

The market value of public sector pension benefit liabilities lacks substantive decision utility. Nevertheless, it is entirely possible that actuaries might be required to calculate a market or fair value of such liabilities. This requirement might be imposed upon actuaries and upon public sector employers and plans by the Governmental Accounting Standards Board or the U. S. Securities and Exchange Commission, the Actuarial Standards Board or, indirectly, by the American Academy of Actuaries. Even less likely, such a requirement could be imposed by public sector plans and employers themselves or by the marketplace.

The model commonly put forth as the market value of pension liabilities is the sum of the expected cash flows of the accumulated benefit obligation (per private sector financial reporting standards) discounted using a risk-free yield curve observed as of the measurement date. We believe this model and its resultant value of the pension liability have limited usefulness, if any. Even viewing this model from the perspectives of financial engineering and actuarial finance, it is a poor representation of the fair value price of the pension liability.

With respect to pension liabilities, David Wilcox (2008), an economist with the Federal Reserve Board, recently testified:

> "The economics of how cash flows with no credit risk should be discounted back to the present are completely unambiguous and utterly noncontroversial. Those cash flows should be discounted back to the present using interest rates that are derived from securities with no credit risk. Every first year MBA student, even as we speak, is having this simple point drilled into their head right now in an introductory finance class. The only factors that matter for the determination of the scale of these obligations are the size of the promised cash flow and their essential characteristic which is that they are free of risk. That's all you need to know. These are riskless cash flows. There's an unambiguous answer as to what their value today is. What I'm trying to suggest, over and over again, is that the analytics of valuing cash flows that have no credit risk in them – those analytics are very straightforward. There's no professional dispute associated with that question. These happen to be really simple cash flows to value. They're free of credit risk. There's only one conceptually right answer to how you discount those cash flows. You use discount rates that are free of credit risk. This is one of those things where it's just really is that simple."

This sounds so easy, even a caveman could do it. This testimony is exactly why we must revisit pension actuarial science. If actuaries will be required to calculate and report the market value of public sector pension benefit liabilities, we must give the current model described by Wilcox an overhaul. What he described is very simple; too simple for financial modeling and too simple for any actuary charged with determining the fair values of public sector pension liabilities.

In his textbook Derivatives, Paul Wilmot (1998) provides sage advice "[E]very financial axiom I've ever seen is demonstrably wrong…The real question is how wrong is the theory, and

how useful is it regardless of its validity. Everything you read in any theoretical finance book, including this one, you must take with a generous pinch of salt."

Parts 1 through 3 in this series will revisit the current model of the market value of public sector pension benefit liabilities and propose three important and substantive improvements to better reflect the principles of labor economics, actuarial finance and pricing, and financial engineering and in a fair value pricing model. Part 4, "The Residual Benefit Liability," will also present an alternative model for the fair value of a public sector employer's pension benefit liability. Part 5 in this series, "Consider the Measurement Purpose," will explore the venues of usefulness for a fair value of public sector pension liabilities and propose more useful measures of the liability appropriate to various purposes.

A. Measurement Attribute Terminology

Recent literature and media coverage have made much of the notion that all of an employer's assets and liabilities should be valued and reported (in financial statements) at market, including its pension liabilities, and regardless of whether the employer is in the private sector or the public sector. While certainly an opinion held by more than a few, it is a fairly narrow and ideological position. There is a rich history and ongoing discussion among accountants and their financial-statement standards setters on measurement attribute models beyond just market value.

Measurement attribute models for the financial reporting of a public sector entity's assets and liabilities are currently under reconsideration. The Governmental Accounting Standards Board (GASB) is deliberating a conceptual framework project on recognition and measurement attributes. These may include initial transaction date-based measurement (initial value) and current financial statement date-based measurement (remeasured value) such as fair value, current acquisition, sale and/or settlement price, replacement costs, and value-in-use. These may vary depending on whether the assets or liabilities are used in the provision of services or not. There may also be an exception considered for assets that will be held to maturity. We are still some time away from having a final concept statement from GASB on recognition and measurement attributes. Even then, we should not be surprised if liabilities for postemployment benefits have a separate type of attribute for recognition and measurement when a new and final accounting standard on the topic is adopted.

Nevertheless, Parts 1 through 4 in this series will be limited primarily to discussions around market or fair values.

Usually and historically, proponents of a financial economics approach to public pension benefit liability measurement use the term, "market value" of liabilities. In this paper we will use the term "market value" of liabilities to refer to the current model proposed by those with a financial economics approach to public pension benefit liability measurement, primarily because that is the term they have used over the last few years. Unfortunately, the term market value is not the current term of preference in financial reporting circles. We should be using the term "fair value." This is significant because we are endeavoring to assign prices to liabilities that are not traded in any market.

We get most of what we know about the specifics of the terms "market value" and "fair value" from the worlds of financial reporting and pricing. The valuation of an employer's assets and liabilities is usually undertaken for the purposes of financial reporting or pricing (for financial transaction purposes). Hence, in developing a faithful model and definition for "fair value," actuaries should be looking to the world of financial reporting for its terminology; thus, our preference for "fair value."

The GASB sets generally accepted accounting principles (GAAP) and standards for governmental entities. One current project in the research phase addresses fair value measurement. GASB also has a major project being deliberated on postemployment benefit accounting and financial reporting. In short, it may be a long time before the GASB adopts amendments to existing standards for financial reporting of liabilities for public sector postemployment benefit obligations, including their measurement attribute, recognition and disclosure requirements.

The Financial Accounting Standards Board (FASB), which sets GAAP standards for private sector and not-for-profit entities, recently adopted a major standard on fair value measurements, Statement of Financial Accounting Standard (SFAS) No. 157.

Paragraph C50 of SFAS No. 157 states that FASB deliberately chose not to use the term "fair market value." Instead, FASB chose "fair value" for the purpose of financial reporting of certain assets and liabilities. SFAS No. 157 represents the most current authority on fair value for U. S. reporting entities. While it neither applies to postemployment benefit liabilities nor to certain other assets and liabilities, yet SFAS No. 157 is useful in guiding our opinions of a fair value model for pensions.

Paragraph 7 of SFAS No. 157 states, "A fair value measurement assumes that the asset or liability is exchanged in an orderly transaction between market participants to sell the asset or transfer the liability at the measurement date…..Therefore, the objective of a fair value measurement is to determine the price that would be received to sell the asset or paid to transfer the liability at the measurement date (an exit price)."

Since there is no real market for public sector pension liabilities, there is no true mark-to-market concept. It is more of a mark-to-model concept. The accounting field, by way of FASB pronouncements gives some guidance on fair value, which provides some secular help as we develop a model for the fair value of the public pension benefit liability.

In the end, fair value is about pricing. In the absence of a market to observe, any acceptable model for fair value of public pension benefit liabilities must envision the market players and pricing principles they might likely employ. The field of financial engineering has developed models, not the least of which is the original Black-Sholes model and its variants, for pricing and valuation of financial instruments in the marketplace. This too will be useful as we seek to propose a true fair value model. Our job is to imagine the operation of a market (a gedanken experiment) where public pension benefit liabilities are bought and held or sold for gain, and apply financial engineering principles to develop a fair value model that describes the operation and prices in such an hypothetical market.

Particularly for uncharted or illiquid markets, Emanuel Derman (2004) observes, "So much of financial modeling is an exercise of the imagination…To estimate the value of an illiquid security, you find a set of similar liquid securities, with known market prices, whose payouts match those of illiquid security under all circumstances. The best estimate for the value of the illiquid security is then the value of the set of liquid securities with the same payout…Models are only models, not the thing itself. We cannot, therefore, expect them to be truly right. Models are better regarded as a collection of parallel thought universes you can explore. Each universe should be consistent, but the actual financial and human world, unlike the world of matter, is going to be infinitely more complex than any model we make of it….You must always ask: Does the model give you a set of plausible variables to describe the world…A little hubris is good. Catastrophes strike when people allow theories to take on a life of their own and hubris evolves into idolatry. Somewhere between these two extreme's a little north of common sense but still south of idolatry, lies the wise use of conceptual models. It takes judgment to draw the line.

It is instructive to examine different valuation techniques. Paragraph 18 of SFAS No. 157 requires "Valuation techniques consistent with the market approach, income approach, and/or cost approach shall be used to measure fair value." Even under the income approach (Life Practice Council, 2008), a risk-neutral approach has a number of apparently unrealistic properties, is merely one tool for valuing financial instruments, and may be relevant when the exit market consists of financial institutions other than insurance companies, many of which typically use risk-neutral methods to price their products.

However, in the case of exchange transactions to transfer a pension liability, the principal market is, arguably, the current single premium group annuity market where the players are limited to a handful of insurance companies, while the most advantageous market may be the other public sector pension funds. Imagine a market whose market participants are hundreds of public sector pension funds (including large statewide plans) which buy and hold or sell pension liabilities from each other for gain.

Finally, there is also the valuation premise to consider: whether the fair value should be based upon a value-in-exchange or a value-in-use premise. Part 5 of this series addresses the valuation premise.

Much more can and should be explored before all these relevant valuation parameters are chosen for fair valuation of public sector pension benefit liabilities. This is the job of standards setters.

Under a fair value model, the public sector pension benefit liability is viewed as if it were a financial instrument, with no market and whose fair value must be derived on a theoretical basis. In the private sector, the values placed on illiquid financial securities have a significant effect on the company's earnings, its stock price, and the bonuses of the traders that management them (Derman 2004). Over the last several months we have seen first-hand how these values have a significant effect even on the continued existence of the company itself. The implications in the public sector are no less serious. With so much riding on public sector postemployment benefit calculations, a dose of humility is in order. Such models of fair value are mere theoretical

contrivances. In terms of financial reporting criteria, the market or fair value fails in the categories of relevance, reliability and interperiod equity, especially considering their magnitude. Other measurement attributes (besides fair value) are more appropriate. Again, refer to Part 5 for more details.

A fair value attribute for public pension benefit liabilities was seriously considered by the GASB over 25 years ago, during the decade leading up to the adoption of Statement Nos. 25 and 27 (1994). GASB's board members chose to go in another direction. In current deliberations[1], the GASB has indicated it is open to the idea of a mixed attribute model for assets and liabilities reported in financial statements. In the event GASB chooses, again, not to apply a fair value attribute to pension liabilities and not to include such a value in disclosures, this issue should be dead on arrival.

However, it could go the other way with the GASB. Separately, the Actuarial Standards Board (ASB), through its actuarial standards of practice (ASOPs), could impose a requirement upon actuaries to calculate a marked-to-model fair value and to include such a calculation in relevant actuarial communications. Either of those two actions might keep the fair value attribute for public pension liabilities alive.

Recently, the term "economic value" was used in high profile pronouncements[2] by the American Academy of Actuaries (AAA) in regard to a request to the ASB. This is an unfortunate shift in terminology because "economic value" has little if any historical authoritative or definitive basis for use. We are unaware of any standard setting body or other governing authority, which has defined economic value in any meaningful way. We think we know what the AAA meant (namely, the current market value model) since their statements were the culmination of a number of iterations and revisions among AAA staff and committees, which did use the term market value. Nevertheless, it appears to be a deliberate shift in terminology.

Fortunately, the ASB is a quasi-independent standard-setting body and has always been careful to define its own terms in sufficient detail for practicing actuaries. Whether and how the ASB will address this issue remains to be seen. Settling this will take time, and if a model with the label "economic value" is to have any contextual meaning, it must address the same three improvements and alternate model we propose in this series.

The use of the term economic value might be a convenient shift in order to deflect the reasons being posited, which the current model's failures in satisfying an honest measure of market or fair value. These failings have been raised in various literature and venues, and are set

[1] The Governmental Accounting Standards Board met on Nov. 4-6, 2008 to discuss, among other topics, the progress of its project on Conceptual Framework: Recognition and Measurement Attributes. According to Boaz (2009), the Board tentatively supported staff's recommendation as a path forward and as an approach that would not exclude a mixed-attribute model at this time. Each element would be evaluated as to the appropriate measurement attribute to use. Of course, this position of the Board could change.

[2] The Public Interest Committee (PIC) of the American Academy of Actuaries (AAA) issued a formal statement shortly after its meeting on Sept. 11, 2008. It stated that "it is in the public interest for retirement plans to disclose consistent measures of the economic value of plan assets and liabilities." Similarly, with input from the PIC, the AAA Board of Directors (BOD) asked the Actuarial Standards Board (ASB) "to develop standards for consistently measuring the economic value of pension plan assets and liabilities."

forth in substantive detail in Parts 1 through 4 of this series. In any event, for the purpose of this series, the term market value will continue to be used for the current model, and can serve as a surrogate tem for economic value because they both mean exactly the same thing in their actual usage by proponents.

One of the reasons market value and fair value of pension benefit liabilities have only limited utility in real world applications is that they are more of a theoretical construct, which interests only ideological purists. Nevertheless, it is possible that actuaries might be required to calculate the fair value of the public pension benefit liabilities.

Recognizing that possibility, we must be true to the term "fair value" in our model.

B. Improve the Current Model

The current model generally accepted as the market value of the public sector pension benefit liability as of a given measurement date is simply the present value of the expected cash flows of the accumulated benefit obligation (ABO per SFAS No. 87) as of the measurement date, discounted using a risk-free yield curve observed at the measurement date.

In financial engineering, we have learned that no model is perfect. In fact, some are found not to be even close. We must continually explore ways to improve, recalibrate and revisit our financial pricing models to ensure they fairly represent the fair values of the same or similar assets and liabilities under examination. Immature, erroneous or inappropriately applied models spell doom for financial and other institutions, which rely upon them and disclose them to various publics. There are serious unintended consequences for a company or government, even for a whole sector or the entire economy for wholesale reliance on flawed financial pricing models or, worse yet, on the wrong metric for the purpose at hand.

The current model for the market value of public sector pension benefit liabilities needs serious improvements, even an overhaul. The current model may be unambiguous, simple for first-year finance students to understand or, in its simplistic form, may be consistent with simple models used to price simple financial instruments. These are not reasons to cling to it. In fact, these qualities should be red flags signaling us to revisit the model.

Pension plans and other postemployment benefit (OPEB) plans have many moving parts, at least as many as collateralized mortgage obligations traded in foreign currencies, swaptions or weather derivatives. Both are highly complex structures with many economic, demographic and behavioral variables having separate and sometimes correlated distributions, with many complex contract terms in different contracts, and with many principals and agents. Adequately and honestly pricing the fair market value of public sector pension and OPEB benefit liabilities is far more complex than portrayed by the current market value of liabilities model.

In a practical sense, the fair value of a public sector pension benefit liability has limited utility. This will be explored in Part 5 of this series, "Consider the Measurement Purpose." Nevertheless, if actuaries are going to be required to calculate a fair value of public sector

pension benefit liabilities, then we must improve the model in ways that align it with best practices in financial engineering. We propose three areas of improvement:

1. Revise the benefits being valued to better reflect the employer's benefit obligation in its voluntary exchange transaction with its employees. Identifying the benefits that have been earned to date under the terms of the employment contract (whether implicit or explicit) is the first step toward assigning a fair value pursuant to proper financial engineering principles. The ABO does not reflect the contract being valued. Refer to Part 1 in this series, "The Contractual Benefit Obligation."

2. Build risk premiums into the fair value to better reflect the price required to protect market players from various non-investment-related risks. Pricing an obligation requires a fair and full recognition of the risks and risk premiums built into an exit price. Given the amount of dialogue and monologue that has transpired in recent years about recognition of risk in pension valuations, a great void has existed concerning demographic and other non-investment risks. These include longevity risks and retirement rate risks, as well as cost of living and other risks. Fair value pricing must not be built upon mere expected benefit cash flows, but must include premium margins for absorbing the material risks that the cash flows may very well exceed expected values. While there are various other risks that should be considered, we will address only longevity and retirement rate risk. Refer to Part 2 in this series, "Risk-Adjusted CBO Cash Flows."

3. Recognize market observables in setting the discount rates. This is a more controversial assumption, which must be addressed and improved for pricing a revised and risk-adjusted fair exit value of the liability. We present theoretical arguments as well as propose observables from the single premium group annuity markets, from high quality corporate bonds, and from the behavior and risk tolerance of public sector pension trustees. Refer to Part 3 in this series, "A Market-Related Discount Rate."

Again, while its utility is highly questionable, if actuaries will be required to calculate and publish the "fair value" of public sector pension benefit liabilities, then the current "market value" model must be overhauled in favor of one based on more careful rigor and integrity, and more faithful to current economics, actuarial and financial pricing principles.

C. Measure the Employer's Liability

The current model totally ignores the operation of the public sector pension fund, essentially treating it as if it did not exist. The current model values the pension liability the same, whether the obligation is funded or unfunded. But the public sector pension fund is the five-ton elephant in the room, which the current model ignores in the name of "pass through."

Part 4 in this series, "The Residual Benefit Liability," demonstrates that the public sector pension fund is too important to ignore. There is a very serious contract in place between the

public sector employer and the independent pension fund. The current model ignores this contract as well. A public sector employer owes only a residual pension benefit to employees, after the pension fund has paid all it can. Therefore, rather than price a first-dollar obligation as the current model does, an alternate model should be employed to price a fair value of the public sector employer's residual liability, as the payer of last resort. To do so, we must model the operation of the pension fund until its depletion, and then assign a fair value to the residual payment obligation.

The agency cost or benefit inherent in the pension fund's operation over time should be modeled before identifying the tail of the cash flow that must be settled. That tail is the employer's benefit liability.

D. Consider the Measurement Purpose

It is unrealistic to think that one measure of the liability should be used for all purposes; just as one calculation can never communicate useful risk information. Different purposes require different treatments.

Examples of this abound in the world all around us. The methods and degree of care employed in building a fence depends on the purpose of the fence. The rigor applied to composing music depends on the purpose of the end result. Why should we think that one measure of pension liabilities should ever apply to all purposes? As examples from the actuarial world, insurance company reserve calculations differ depending on the purpose; mortality and future lifetimes might be calculated differently for pension valuations of impaired lives as compared to personal injury litigation (which itself may differ depending on defendant or plaintiff); or methods and margins employed for calculating the per-member, per-month price charged by an HMO wanting to introduce a new product into market or capture market share might be different that HMO's methods and margins for en existing product in the same market, both of which might be different from those employed for reserving purposes.

Actuarial valuations of pension benefits are needed for numerous purposes, some of which should have the same measurement methods and some need to be different. The careful actuary will match the method with the purpose. Following are various purposes actuaries may encounter and which should force the actuary to consider carefully which methods are appropriate for the purpose at hand:

A. Advance funding
B. Taxpayers
C. Financial reporting
D. Lenders and rating agencies
E. Comparability
F. Risk measurement and analysis
G. Personal wealth
H. Plan terminations and freezes

The current model of the market value of the liability, the fair value with improvements and/or the alternate fair value model for residual employer benefit liability have some usefulness in the last two venues listed and a little usefulness in discussions of risk. However, for the other venues listed, market or fair value have little to no usefulness. These purposes will be examined in Part 5 of this series, "Consider the Measurement Purpose."

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