Revisiting Pension Actuarial Science: A Five-Part Series

Part 2
Fair Value of the Liability – Risk-Adjusted CBO Cash Flows

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Abstract for the Series

The current financial model put forth as the market value of public sector pension benefit liabilities is simply the expected cash flows of the accumulated benefit obligation, as defined for current private sector financial reporting, discounted using a risk-free yield curve. This model is in serious need of an overhaul. It fails to faithfully represent the fair value of a currently accrued public sector pension benefit liability in three important ways:

1. Its use of the accumulated benefit obligation cash flows fails to accurately represent the terms of the employment contract which gives rise to the obligation being valued – a violation of labor economics principles.

2. Its use of expected cash flows as if they were fixed fails to recognize the risk premium load, which a fair exit price would include for the potential for adverse cash flow experience – a violation of actuarial finance and pricing principles.

3. Its use of risk-free discount rates fails to adequately reflect the observable and not-so-observable inputs from market participants' behavior – a violation of financial engineering principles.

Parts 1 through 3 in this series propose solutions to these three flaws.

Part 4, "The Residual Benefit Liability," presents an alternate approach to obtaining the fair value of the public sector employer's pension benefit liability. It approaches the task by modeling the real world operation of the pension fund, rather than approaching the task from the perspective of a theoretical construct. This alternate approach dares to model the long-term agency operation of the plan rather than ignoring it in favor of a pass-through approach. The current model ignores the effectiveness (even the existence) of the pension fund itself, while the alternate approach attempts to model the plan's operation in practice over time in order to determine the employer's residual asset or liability.

In spite of these three improvements and the alternate model, we believe the fair value of public sector post-employment benefit liabilities has little to no usefulness in most venues. There are legitimate roles which the market or fair value might play in valuing an individual member's personal wealth, a minor role in the context of certain discussions concerning risk measurement and risk management, and a major role in the context of plan terminations and freezes.

However, for purposes of advance funding, taxpayers, financial reporting, lenders and rating agencies, comparability, and the major part of risk measurement and analysis, the decision-usefulness of market or fair value is negligible, possibly even misleading. Other existing models and methods are far more suitable for these purposes, including conventional actuarial approaches and others that are less conventional or popular, but which should be considered in the actuarial toolbox and have higher decision utility.

Part 5 in this series, “Consider the Measurement Purpose,” addresses various purposes for measuring a public sector pension liability and which measures have the most practical usefulness.

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Fair Value of the Liability – Risk-Adjusted CBO Cash Flows

With respect to pension liabilities, David Wilcox (2008), an economist with the Federal Reserve Board, recently testified, "You'll need to study the behavior of participants and see whether the choices that they make are systematically related to actual market conditions. If they are not, then you can treat expected values as if they were known with certainty. And the reason for that is because those are so-called unpriced factors, if they're not systematically related to market conditions."

Whether the cash flows are or are not systematically related to actual (financial) market conditions is a narrow view of risk, often taken by non-actuaries to simplify their world. In pricing pension liabilities, there are a number of noninvestment risks that must be considered.

Developing the fair value of a pension benefit liability is about pricing. Understanding the noninvestment risks inherent in projected cash flows is fundamental to actuarial finance and pricing principles. It is patently wrong and dangerous to price an asset or liability by assuming that expected cash flows are certain. Pension valuation systems are built on models often relying on a number of assumptions derived from experience studies and each of these assumptions can be a source of a risk. Any fair value of an exit liability should include risk premiums for the possibility of adverse or worse-than-assumed experience.

We often divide pension and OPEB assumptions into three broad categories:

- Economic assumptions (rates of investment return, discount rates, price and wage inflation, salary scales, medical trend, Medicare payments, etc.),
- Demographic assumption (mortality rates, retirement and DROP rates, termination patterns, disability, marriage rates, etc.) and
- Behavioral assumptions (option election rates, retiree medical acceptance and lapse rates, etc.)

In this section we focus on two demographic risks – longevity and retirement rates. Risk premiums for these uncertainties will be analyzed and useful tools provided for inclusion in the pricing of fair values of pension liabilities.

A. Longevity Risks

We begin with an overview of mortality (or one should say, longevity) risk. In this section we will address stochastic risks resulting from random deviations of experience from the expected best estimate (or mean) mortality rates, as well as systemic risks resulting from nonrepresentative sample subsets and from mortality improvements beyond our best estimates.

The arithmetic of annuity calculations when mortality rates are certain has been known for the better part of two centuries. One might even say that most actuarial students can calculate annuity factors when given the interest rate and the mortality rates. However, there is a lot more to computing pension plan liabilities than calculating the present value of expected cash flow associated with annuities. As a matter of fact, first year MBA finance students can calculate the present value of a well-defined and certain cash flow if given the discount rate. It is the modeling of that cash flow that presents actuarial challenges. Any such present value calculations thereafter seem trivial in comparison.

When comparing the effect of interest discount rates and mortality on the prices of annuities, it has been common to start with the observation that the price of a unit pure endowment, issued at age *x* for *n* years, is a function of the force of interest (i.e., continuously compounded forward interest rate) δ_x and the force of mortality $\mu_x(s)$ (representing the force of mortality at age $x+s$) for a person underwritten for life insurance or annuity at age *x* and defined as $\mu(u) = -S^{'}(u)/S(u)$ where $S(u)$ is the survival function (probability that newborn is still alive at age u) and $S^{'}(u) = \mathrm{d}S/\mathrm{d}u$ is its derivative.

As shown by MacMinn, et al (2006), changes in mortality have a similar impact on annuity value as changes in interest discount rates. We briefly recall these arguments.

We start with presenting an annuity as a series of pure endowments:

$$a_x = \sum_{n=0}^{\omega} {}_nE_x$$

where pure endowment ${}_{\mathrm{n}}\mathrm{E}_{\mathrm{x}}$ can be calculated as:

$${}_{\mathrm{n}}\mathrm{E}_{\mathrm{x}} = \exp\left[\int_0^{\mathrm{n}} \left(\delta(\mathrm{s}) + \mu_{\mathrm{x}}(\mathrm{s})\right)\mathrm{ds}\right]$$

with $\delta = \ln{(1+i)}$ being the force of interest and μ_x is the force of mortality.

This shows that the effect of mortality and interest discount rates on the annuity value can be studied by analyzing the impact those variables have on a price of the pure endowment.

MacMinn and others examined the sensitivity of the pure endowment to the forces of interest and mortality. Under the assumption of the constant force of interest, they found:

$$\frac{\mathrm{d}}{\mathrm{d}\delta}\, {}_nE_x = \frac{\mathrm{d}}{\mathrm{d}\delta}\left(\mathrm{e}^{-n\delta}\exp\left[\int_0^n \left(\mu_x(s)\right)ds\right]\right) = -n\mathrm{e}^{-n\delta}\exp\left[\int_0^n \left(\mu_x(s)\right)ds\right] = -n\, {}_nE_x$$

And under the assumption of constant force of mortality they have:

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\,{}_{n}E_x = \frac{\mathrm{d}}{\mathrm{d}\delta}\left(\mathrm{e}^{-n\mu}\exp\left[\int_0^n (\delta(s))ds\right]\right) = -n\mathrm{e}^{-n\mu}\exp\left[\int_0^n (\delta(s))ds\right] = -n\,{}_{n}E_x$$

This continuous model may not be practical for calculations, but clearly illustrates that changes in mortality impact the value of endowment the same way as changes in interest rates.

As a side remark, we want to note an interesting implication of the first formula: the *Macaulay Duration* of a pure endowment under the constant force of interest is simply the same as the length of the contract:

$$\mathrm{D_M} = -\left(\frac{\mathrm{d}}{\mathrm{d}\delta}\,{}_{\mathrm{n}}\mathrm{E_x}\right)\left({}_{\mathrm{n}}\mathrm{E_x}\right)^{-1} = \mathrm{n}$$

B. Risk Premium for Error around the Mean

In pricing the fair value of the pension liability, the expected CBO cash flows should be adjusted for the risk of variation around the mean, particularly for smaller plans.

As described in the report by the SOA Group Annuity Valuation Table Task Force (SOA 1995), group annuity mortality (GAM) tables had margins built in, to account for random variations in mortality rates among participants. Those margins were set to provide a two-standard-deviation margin (theoretical) for a 3,000-life block of business (SOA 1995). Margins in the 1983 GAM table were intended to account for future mortality improvements too. The 1994 GAM table has a separate margin built in, for variation around the expected rates, but also presented Projection Scale AA to reflected mortality improvements.

In other words, tables used in reserving for annuity products have, by design, some protection against random variations around the expected values.

There is no golden rule as to how to select the best method for hedging against adverse experience. Some might argue that stochastic simulations should be performed to address the risk of random variations around the mean. This however is rarely a feasible approach. Perhaps large statewide plans could afford such an approach but for thousands of local plans this would be cost-prohibitive.

On the other hand, the approach taken by the Group Annuity Valuation Table Task Force when designing 1994 GAM and 1994 GAR tables (SOA 1995) would be worth considering. We briefly recount this method as applied to future lifetime.

For a single life age x, we assume Y is a random variable representing the future lifetime (t). Y would have the following distribution:

Y	t=0	t=1	t=2	t=3	…
Pr(Y=t)	$1 - p_x$	${}_{1\|}q_x$ $[= p_x(1 - p_{x+1})]$	${}_{2\|}q_x$	${}_{3\|}q_x$	…

From here we can compute the mean, variance and the standard deviation of this distribution:

$$e_x = E[Y] = \sum_{t=0}^{\infty} t \cdot \Pr(Y = t)$$

$$E[Y^2] = \sum_{t=0}^{\infty} t^2 \cdot \Pr(Y = t)$$

$$\sigma = \sqrt{E[Y^2] - E[Y]^2}$$

For a distribution of total future lifetime for N lives, all age x, assumed to be independent, the mean, variance and standard deviation would be:

$$e_x^{(N)} = N \cdot e_x$$

$$\sigma^{(N)^2} = N \cdot \sigma^2$$

$$\sigma^{(N)} = \sqrt{N} \cdot \sigma$$

From here we obtain the standard deviation of the total future lifetime per retiree to be:

$$\sigma = \frac{\sigma^{(N)}}{N} = \frac{\sqrt{N} \cdot \sigma}{N} = \frac{\sigma}{\sqrt{N}}$$

The Task Force determined that a 5-percent margin in mortality rates is appropriate for 3000-lives blocks of business. Size of 3,000 was selected to ensure proper protection for more than 95 percent of companies. Since public sector pension plans vary in size, different margins are appropriate for different individual plans.

Figure 5

	Number of Retirees Expected (N)				
	1	10	100	1,000	10,000+
Mortality Rate Margin Multiplier	36.10%	70.30%	89.20%	96.40%	None
Resultant Life Expectancy Margin	44%	14%	5%	1%	None

Values in the table are based on the Combined Healthy Male RP2000 table for age 60. The user can adjust the Margin Multipliers up if the average age at retirement is expected to be below 60. Rather than apply a one-size-fits-all adjustment, we computed Margin Multipliers for select plan sizes to provide a better perspective.

The Margin Multiplier selected based on the expected number of retirees over the life of the plan should be multiplied by each mortality rate in the base unprojected table to achieve a two-standard-deviation margin in future lifetime.

These results are consistent with common sense. In measuring deviation around a given mean, larger plans enjoy the luxury of the "Law of Large Numbers" working to dampen such deviations. Sample error is diminished with large sample sizes. However, in order to build protection for a smaller plan against adverse deviation around the mean, as required for pricing margins in a fair value of the liabilities, we need to reduce our mortality rates to cover that risk. This table provides practitioners with simple Margin Multipliers.

For our case study plan, we multiplied the unprojected RP2000 rates by a 95 percent Margin Multiplier (to reflect the expected number of retirees and an average retirement age 60) to account for stochastic risks.

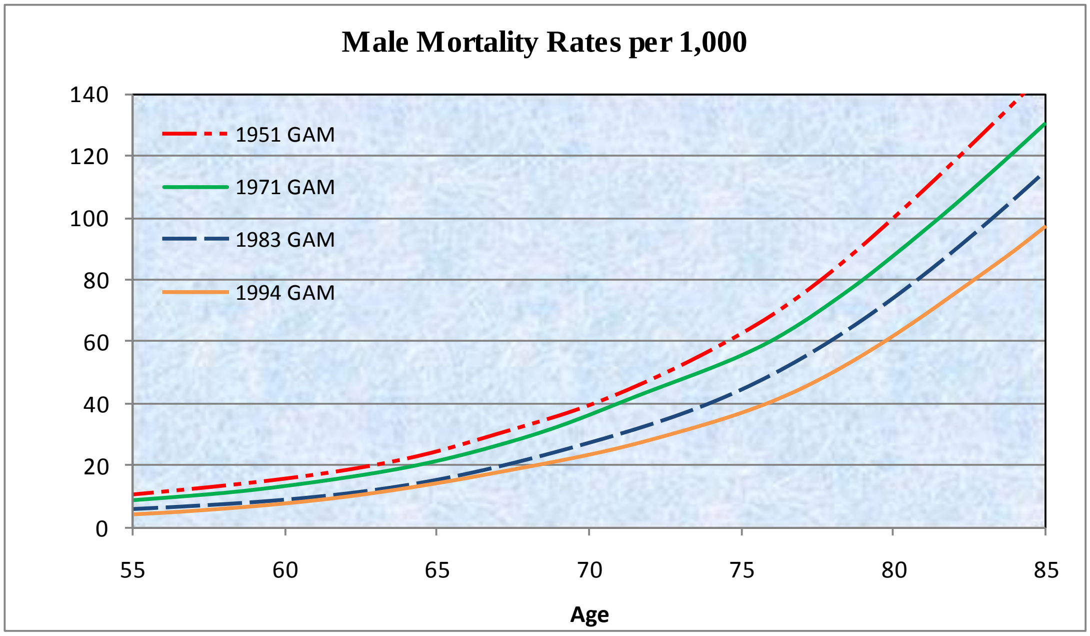
The margin illustrated above accounts only for error around the mean (stochastic risk). We believe that error in the mean (systemic risk) cannot be neglected either. This systemic risk is mostly reflected in broad mortality improvements beyond what we expect.

C. Mortality Tables and Life Expectancy

We now take a look at the improvement in mortality rates often used in reserve calculations for annuity contracts and pension valuations in the United States. Those are based on group annuity mortality tables that were updated in 1951, 1971, 1983[1], and most recently in 1994. On Figure 6, we graph mortality rates for male annuitants taken from those tables. We illustrate improvements in the mortality rates (for most ages in retirement) observed in the second half of the 20th century.

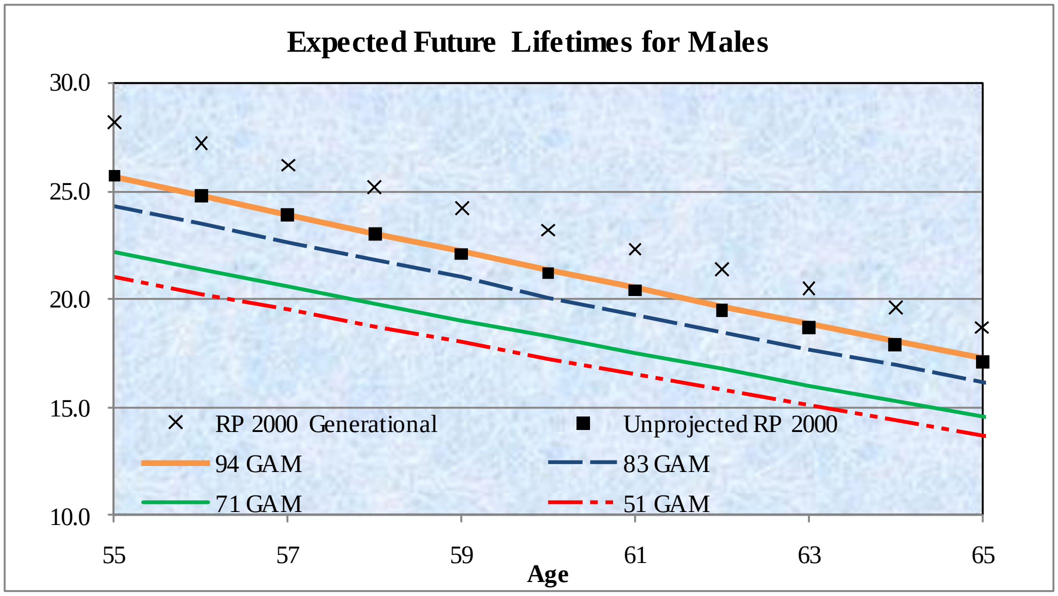
[1] According to the Committee on Annuities report on the "Development of the 1983 Group Annuity Mortality Table," there was insufficient data to develop a new table, but sufficient data to conclude that GAM 1971 with Projection Scale D was no longer valid. Therefore, a new projection scale was developed and applied to GAM 1971 to produce GAM 1983.

Figure 6



As an alternative to looking at raw mortality rates, it is common to examine the random variable of future lifetime for the research population, as the "choice of numeraire". Figure 7 plots the curtate future lifetimes for a typical range of retirement ages for the same set of mortality rates as well as expected lifetimes based on mortality rates from RP-2000 mortality tables (combined healthy issue).

Figure 7



Expected lifetimes for males commencing benefits at ages 55 through 65. Lines (solid, dashed and dotted) correspond to expected future lifetime derived from rates from GAM tables. Markers illustrate expected future lifetime derived from un-projected (squares) and generationally projected (crosses) rates from RP 2000 mortality tables. Mortality improvement rates used in projections are based on scale AA.

Figure 7 also presents expected lifetime as derived from projected (Generational) RP-2000 mortality tables. We graph expected lifetime corresponding to mortality rates with generational projections for annuity starting dates in 2009 (employees are assumed to retire in 2009 and the remaining lifetime is computed based on the age at retirement).

It should be noted that RP-2000 mortality tables were developed differently than GAM series. GAM tables were developed from the mortality experience collected by insurance companies providing annuity products, while RP-2000 is developed from the mortality experience of uninsured pension plans. In addition, unlike RP-2000 basic tables, GAM tables had margins built into their rates. It may be a little surprising that expected future lifetimes based on unprojected rates from RP-2000 mortality tables are nearly identical to those based on rates from 1994 GAM tables. The margins embedded in rates from 1994 GAM tables were designed to protect from random variations in mortality (error around the means). It is interesting to see that mortality improvements from 1994 to 2000 are fully offset by margins for variation and, potentially, differences stemming from different populations being used.

The concept of mortality improvements is not new to actuaries. In the insurance industry, reserves for annuity products are required to be set with recognition of future mortality improvements. For pension valuations, actuaries are advised to consider mortality improvements in calculating liabilities (ASOP 35).

A survey conducted by the Society of Actuaries (SOA 2003) among life insurance companies found that nearly 100 percent of responding firms use mortality improvements (either generational or durational) in pricing of the products.

Projection is performed using Scale AA, which was developed for use with the Group Annuity Reserving 1994 tables (1994 GAR) whose rates are projected from the rates in 1994 GAM tables. The Retirement Plans Experience Committee of the SOA recommends using Scale AA for projecting mortality rates beyond the year 2000 and encourages the use of generational mortality projection. In general, Scale AA had been based on a blend of Federal Civil Service and Social Security experience from 1977 through 1993, with certain adjustments.

Any actuarial assumption will always be wrong, in one direction or the other. When exit-pricing the fair value of pension liabilities, a risk margin needs to be built into the mortality tables in case the plan’s retirees live longer than expected by the selected mortality table. Generationally projected mortality rates constitute the current preferred method for anticipating mortality improvements.

Some larger public sector pension plans develop their own tailored mortality table based on the plan’s own experience; even if it is just applying a factor to an existing published table to match its own experience. Alternatively, for a given plan, an actuary might rely upon a tailored table for another plan if there is sufficient reason to believe that the experience of the given plan will be essentially the same as the other plan. Tailored tables, if based upon credible experience, constitute a reasonable choice of table within the relevant assumption universe. In the absence of reliance on a tailored table, an actuary’s choice could be a published table (e.g., 94GAM or RP2000). However, as ASOP 35 recommends, we should consider using mortality improvements in our calculations. To that end, Projection Scale AA would be a reasonable choice prior to any margins for error. As such, a generational mortality table can be treated expected values (means).

There has been a significant amount of research on random variations in mortality rates (error around the means) and methods of hedging against them (Bauer 2006). But the risk of the expected value differing from observed mean has not been addressed in the design of those tables. The Retirement Plans Experience Committee warns in their report describing the development and suggested use of RP-2000 mortality tables: “Even mortality tables that are specific for the collar type and industry of the plan are unlikely to match the true underlying mortality of the plan.”

The authors also note that: “Statistically significant differences in mortality between plans were found in all four of the industries investigated. The majority of plans had mortality experience that differed from the average experience of plans of the same collar type in the same industry. Adjusting for differences in annuity size explained some of the variation, but statistically significant differences of about plus or minus 12 percent were still found even after this adjustment”

This strongly suggests that the proposed model for measuring the liability of the pension plan should include some measures of protection against adverse mortality experience whether it results from random variations around the mean or the error in the mean itself.

D. Risk Premium for Error in the Mean

We see two sources of error in the mean itself. One source results from a plan’s population not being representative of the population used in the development of the table. The second source results from future mortality improvements being better than currently expected (Projection Scale AA was developed in early 1990s). One can load for both types risk by increasing the level of future mortality improvements assumed.

To build a risk premium into the cash flows for a possible error in the mean (system error), we develop a Scale Factor. This is multiplied by each of the improvement rates found in Projection Scale AA. In Figure 8 we present the Scale Factor that would result from various levels of life expectancy improvement desired. For example, if the actuary wishes to build sufficient risk margin into the fair value that will account for an extra 20 percent life expectancy improvement beyond the best estimate of mortality (presumed in our example to be RP2000 Generational), it would require a Scale Factor of 3.5 to be multiplied by all of the improvement rates found in Projection Scale AA.

Figure 8

	Additional Expected Future Lifetime Desired		
	10%	20%	30%
Scale Factor	2.1	3.5	4.95

Since the Scale Factor is multiplied by each entry in Projection Scale AA, this relationship presented above, between the additional expected future lifetime desired and the Scale Factor, is not very sensitive to the ages within a reasonable range of retirement.

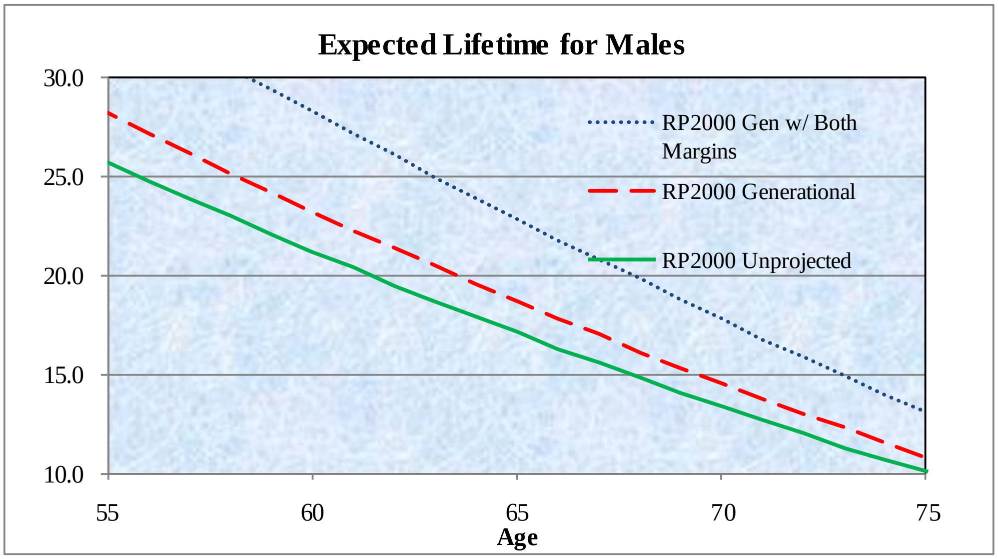
We have presented a simple algorithm for actuaries to build into their mortality rates to adjust for two types of longevity risks.

1. A Margin Multiplier (depending on the plan size) applied to the mortality rates themselves for stochastic risk, that is, to adjust for error around the mean.

2. A Scale Factor (depending on how much additional future lifetime to hedge against) applied to the Projection Scale AA rates for systemic risk, that is, to adjust for error in the mean.

In Figure 9 we illustrate the effect that these two margins would have on the expected lifetime.

Figure 9



Expected lifetime for male annuitants, retiring at ages 55 through 75 derived from mortality rates based on RP-2000 Mortality Tables with and without projections. Dashed line corresponds to generational projection of mortality rates. Dotted line illustrates expected lifetime derived after loading mortality rates with 5 percent margin (95 percent Margin Multiplier) and adding additional 20 percent improvement in future lifetime (Scale Factor 3.5).

Mortality improvements have become part of life for actuaries practicing in areas of life insurance and pension benefits. But there is very little done so far to hedge the longevity risk in pensions. There are wide differences in opinions whether the improvement rates will keep increasing as a result of advances in medicine or will taper off as a result of less healthy lifestyles.

We are not attempting to find a solution to that question here, but we want to point out that this risk is not being addressed in the current model of market value of liability measurement. Adding two types of longevity risk margins as discussed above is a reasonable way to load for a fair value price.

E. Risk Premium for Retirement Rate Risks

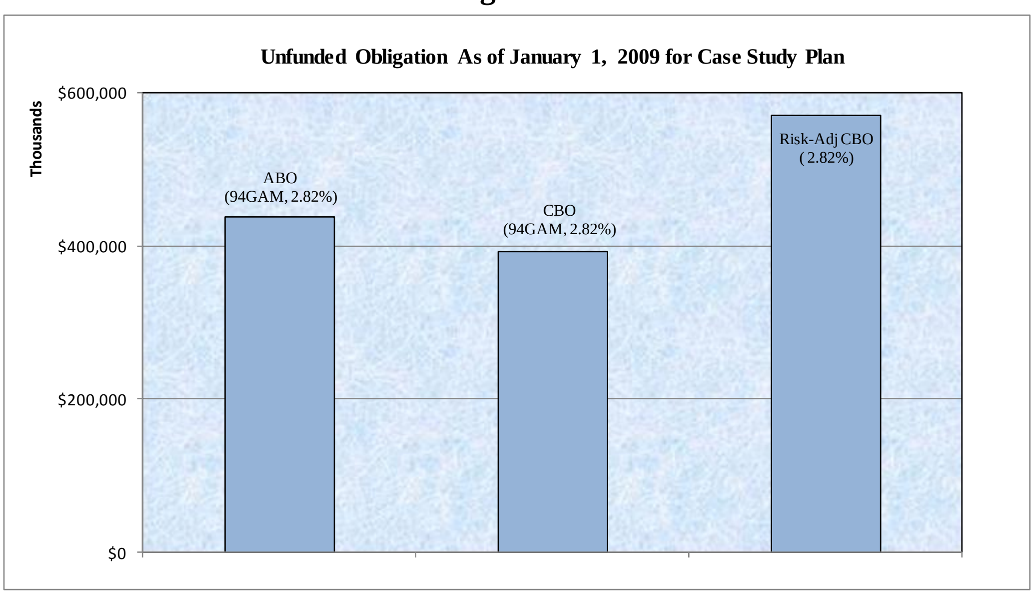
We now move to discussing risks associated with rates of retirement. Our Case Study Plan offers a normal retirement benefit to vested employees (five years of service) at age 60, or 30 years of service regardless of age. Employees are eligible for a reduced early retirement benefit at age 50 with at least 15 years of service. Reduction is 3 percent for each year the retirement precedes age 60.

Actuarial methods measure the liabilities by employing a pattern of retirement rates derived from observed and expected experience. However, no one can guarantee that future experience will closely follow such rates. When the early retirement benefits are subsidized, as with our case study plan, the cost of providing the benefit usually goes up as employees retire earlier in the early retirement period.

One way to avoid the risk is to calculate the liabilities assuming that employees would retire at the most valuable age, i.e., the retirement age resulting in the highest liability. For an employee who has accrued 15 years of service, most valuable age for our case study plan is age 50 (eligibility for early retirement with a reduced benefit), while for another employee who did not earn the right to this feature, the most valuable age is 60. We should note, that for employees eligible for early retirement with unreduced benefit (30 years of service), the most valuable age is age on the valuation date.

Figure 10 adds the unfunded obligation of the CBO after incorporating these risk adjustments to the chart from Part 1 in this series, "The Contractual Benefit Obligation."

Figure 10



F. Summary of Risk-Adjusted Pension Cash Flows

In fair value pricing, expected cash flows should not be treated as if they were fixed. Cash flows for a public sector pension fund may be free of default risk, but that does not make their expected amounts certain. There are certain risks in the level of the CBO cash flows that should be recognized in pricing the fair value of the public sector pension benefit liability.

Financial economists and financial engineers are fond of expressing all sorts of risks in terms of how many basis points should be added or subtracted from a given discount rate to account for such risks. We cannot let this happen in the loading for risks in the cash flows. We actuaries spent the better part of a couple decades changing our previous paradigm of implicit actuarial assumptions to one where we justify each assumption on its own, with explicit actuarial assumptions. Taking an actuarial shortcut by adding or subtracting basis points to the discount rate to recognize cash flows risks would be turning back the clock for our profession.

Therefore, making reasonable adjustments to the CBO cash flows for longevity and retirement rate risk is appropriate, even necessary for fair value calculations.

There are other risks that might be hedged. For example, some plans provide cost of living adjustments (COLAs) in benefits calculated as a function of the consumer price index. Not many, but others calculate them based on the wage increase granted to current employees in the compensation grade level from which the retiree retired. Both types of COLAs can easily emerge at levels far in excess of our deterministic assumptions. The proper loads on fair value exit-pricing would include premium margins for COLA risk. Another example arises in plans that contain gain-sharing provisions. Again, there are risks in the expected cash flows that should be hedged with risk premium loads when calculating a fair value exit price.

If risk margins for longevity risks, retirement rate risks, COLA, gain-sharing and other such cash flow risks are not built into the cash flows we are discounting, then we are not calculating a market or fair value of the pension liability.

G. Risk-Adjusted OPEB Cash Flows

If you thought that the CBO for OPEBs was a challenge as we discussed in Part 1 of this series, “The Contractual Benefit Obligation,” imagine making risk premium adjustments for worse-than-expected future medical costs. It is no mystery why there is no market for single premium (other prepaid) group retiree medical coverage. No one in his right mind would sell such a policy. Imagine how high the price would need to be to protect the issuer against worse-than-expected cash flows.

This is just another reason why a strict fair value attribute model for postemployment (pensions and OPEBs) should be treated from hereafter evermore as a mere curiosity in a Ripley’s Believe It or Not Museum; as an interesting idea that once garnered some support. Refer to Part 5 in this series, “Consider the Measurement Purpose,” for much more appropriate measures of pension and OPEB liabilities for the most common purposes.

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Special Thanks

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