

A CONVENIENT METHOD OF PROVIDING FOR  
MORTALITY IMPROVEMENT BASED ON  
THE *a*-1949 TABLE

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MESSRS. Fassel and Noback in *TSA* II, 279, showed that annuity values allowing for future mortality improvement could sometimes be approximated without special commutation functions by suitable adjustment of the ratable age. Specifically, they approximated the values produced by the *a*-1949 Table with Projection B, given by Messrs. Jenkins and Lew in *TSA* I, 386-389 and 425-426. For this purpose they produced a special mortality table which has been designated Progressive Annuity Table, to represent the mortality, in successive calendar years, of annuitants born in 1900. Female lives were assumed to have the same mortality rates as males four years younger. For annuitants born in other years the approximation consists in increasing or decreasing the age by .04 years for each year by which the year of birth was earlier than or later than 1900, as the case may be.

The purpose of this paper is to suggest that an even closer approximation, generally speaking, can be obtained by using the *a*-1949 Table itself as the basic table. That table is used to produce approximate annuity values for annuitants born in 1875 or earlier. The age of male lives is to be reduced by .075 years, and that of female lives by .06 years, for each year by which the year of birth was later than 1875.

No setting forward of ages for annuitants born before 1875 is proposed. It was found that keeping the true ages for such annuitants would produce a closer approximation on the whole than if the setback proposed for annuitants born after 1875 had been used negatively for those born earlier. The proportion of a company's business arising from annuitants born before 1875 will in most instances decrease rapidly.

Companies using this new method for valuation would probably use the nearest integral ratable age according to the table on the next page. Fractional ages are used, however, in the illustrative tables in this paper.

Doubtless the assumptions in Projection Scale B will have been reconsidered long before annuitants born after 1961 enter into our calculations.

The tables at the end of the paper compare various annuity values produced by this approximation with the exact values given by the *a*-1949 Table with Projection B and with those produced by the Fassel-Noback process. In those cases where approximate values by the method sug-

gested by Mr. Sternhell are given in his paper in *TSA* II, June, 30, they are also included for comparison. The approximation suggested herein does not purport to be as close as that produced by Mr. Sternhell's method but has the advantage of using only the ordinary annuity formulas without the addition of special functions.

The use of the *a*-1949 Table as the basic table avoids the artificial relationship between the mortality of male and female lives which is a feature of the Progressive Annuity Table, while at the same time it permits the approximation of projected joint life annuities by the same method as was suggested by Messrs. Jenkins and Lew for the unprojected joint life annuities (*TSA* I, 453).

YEAR OF BIRTH		SETBACK IN AGE ON <i>a</i> -1949 TABLE— YEARS
Male	Female	
1881 and earlier	1883 and earlier	0
1882-1894	1884-1899	1
1895-1908	1900-1916	2
1909-1921	1917-1933	3
1922-1934	1934-1949	4
1935-1948	1950-1966	5
1949-1961	1967-1983	6

The setback formulas for male and female lives were suggested respectively by Mr. Dickinson C. Duffield and Mr. William H. Kelton. The formulas are slightly more conservative as to base year and setback for each successive year than the values produced by a least squares calculation from a number of specific cases. For each of several representative points the number of years setback necessary to reproduce annuity values on Projection B was computed. A straight line was then fitted to the setbacks as a function of the calendar year of birth. They are chosen so that there will be few cases—but still a few—where the exact annuity value exceeds the approximation. A formula which would eliminate all deficient annuity values would probably produce such excessive values at some points that it would not be a good approximation for the aggregate of a company's business.

Obviously if the *a*-1949 Table is used as an approximation, for annuitants born in 1875, to the *a*-1949 Table with Projection B, there is a deficiency for such annuitants which can be seen at a glance from the table of projection factors in *TSA* I, 425-426. The greatest relative error in this age group appears to be about  $1\frac{1}{2}\%$ . The assumption in Projection B of no improvement after age 90 keeps this error small.

An approximation which gives fairly close results in most cases, but in which the excessive values predominate over the deficient values in a number of cases, is likely to produce adequate reserves even on a rather extreme distribution of business.

The values in the accompanying tables which are headed "*a*-1949 with Projection B" were calculated by applying the projection factors to the *a*-1949 Table except for immediate and deferred nonrefund annuities issued in 1950 and 1960. The values in those instances are those labeled "Exact Value" by Mr. Sternhell in *TSA* II, June, 39 and 53. These are presumably the values derived from the *a*-1949 Table by applying the annual rates of mortality decrease given in *TSA* I, 417, from which exact values the projection factors were determined. These exact values cannot be derived from the paper of Jenkins and Lew without a laborious calculation, except that a few of them are given directly in the paper.

As compared with Projection B the method proposed herein generally produces values of  $q_x$  which are too low at the younger attained ages and too high at older ages. Consequently, if mortality exactly equal to the *a*-1949 Table with Projection B should be experienced, the use of the proposed method would produce spurious mortality gains in the Gain and Loss Exhibit among younger annuitants. In other words, the excess reserves produced at younger attained ages will decrease with the passage of time, up to a certain point. The error introduced into the Gain and Loss Exhibit will be less, however, than if the Progressive Annuity Table were used for reserve calculation. In any event, it has always been necessary to use the Gain and Loss figures with caution.

Comparison of the present and proposed methods of annuity valuation might be made as follows. Under the typical present method, a table is chosen with somewhat lower mortality rates than those current when the table is adopted. It is used for all annuitants, regardless of year of birth. Then, even if mortality improves about as expected, the table continues to be used without change until it is no longer conservative. At that time a new table is chosen similarly with regard to the mortality rates then current. Reserves on old business are strengthened to the new table as the company's financial position permits. In contrast, the method proposed by Messrs. Fassel and Noback, and adapted herein to fit more closely to Projection B, makes it convenient to substitute gradual for abrupt strengthening as long as mortality improvement approximates Projection B. Even if the volume in force, and the age and plan distribution, should remain stationary, aggregate reserves would increase slightly from year to year, just as if a small decrease in the average age had occurred.

TABLE 1

IMMEDIATE LIFE ANNUITIES ISSUED IN 1950—MALE—2½% INTEREST

AGE AT ISSUE	YEAR OF BIRTH	ANNUITY VALUE AT ISSUE				EXCESS OVER $a-1949$ WITH PROJECTION B		
		$a-1949$ with Projection B	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation
Nonrefund— $a_x$								
15 . . . . .	1935	30.917	30.924	31.242	31.018	.007	.325	.101
25 . . . . .	1925	28.296	28.368	28.725	28.370	.072	.429	.074
35 . . . . .	1915	24.962	25.108	25.548	25.005	.146	.586	.043
45 . . . . .	1905	20.849	21.050	21.641	20.867	.201	.792	.018
55 . . . . .	1895	16.330	16.482	17.049	16.336	.152	.719	.006
65 . . . . .	1885	11.744	11.823	12.077	11.744	.079	.333	.000
75 . . . . .	1875	7.396	7.323	7.354	7.395	-.073	-.042	-.001
85 . . . . .	1865	3.927	3.923	3.652	3.927	-.004	-.275	.000
With 10-Year Certain Period— $a_{10} +_{10} a_x$								
15 . . . . .	1935	30.944	30.948	31.246	31.044	.004	.302	.100
25 . . . . .	1925	28.337	28.403	28.740	28.410	.066	.403	.073
35 . . . . .	1915	25.042	25.173	25.593	25.084	.131	.551	.042
45 . . . . .	1905	21.082	21.238	21.770	21.099	.156	.688	.017
55 . . . . .	1895	16.912	17.020	17.426	16.916	.108	.514	.004
65 . . . . .	1885	12.979	13.023	13.126	12.979	.044	.147	.000
75 . . . . .	1875	10.055	10.016	9.979	10.055	-.039	-.076	.000
85 . . . . .	1865	8.882	8.881	8.838	8.881	-.001	-.044	-.001
With 20-Year Certain Period— $a_{20} +_{20} a_x$								
15 . . . . .	1935	31.011	31.011	31.266	31.111	.000	.255	.100
25 . . . . .	1925	28.456	28.506	28.799	28.526	.050	.343	.070
35 . . . . .	1915	25.331	25.413	25.766	25.371	.082	.435	.040
45 . . . . .	1905	21.860	21.949	22.270	21.873	.089	.410	.013
55 . . . . .	1895	18.603	18.673	18.790	18.602	.070	.187	-.001
65 . . . . .	1885	16.354	16.385	16.354	16.353	.031	.000	-.001
75 . . . . .	1875	15.632	15.631	15.621	15.632	-.001	-.011	.000

**TABLE 2**  
**IMMEDIATE LIFE ANNUITIES ISSUED IN 1950—FEMALE—2½% INTEREST**

AGE AT ISSUE	YEAR OF BIRTH	ANNUITY VALUE AT ISSUE				EXCESS OVER $a-1949$ WITH PROJECTION B		
		$a-1949$ with Projection B	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation
Nonrefund— $a_x$								
15.....	1935	31.935	31.942	32.058	32.032	.007	.123	.097
25.....	1925	29.611	29.645	29.765	29.685	.034	.154	.074
35.....	1915	26.672	26.740	26.854	26.719	.068	.182	.047
45.....	1905	23.018	23.122	23.231	23.043	.104	.213	.025
55.....	1895	18.640	18.754	18.884	18.649	.114	.244	.009
65.....	1885	13.686	13.749	14.001	13.687	.063	.315	.001
75.....	1875	8.714	8.642	9.090	8.713	-.072	.376	-.001
85.....	1865	4.564	4.560	4.916	4.564	-.004	.352	.000
With 10-Year Certain Period— $a_{10} +_{10}a_x$								
15.....	1935	31.949	31.954	32.061	32.047	.005	.112	.098
25.....	1925	29.639	29.668	29.775	29.712	.029	.136	.073
35.....	1915	26.725	26.787	26.883	26.772	.062	.158	.047
45.....	1905	23.133	23.227	23.317	23.159	.094	.184	.026
55.....	1895	18.917	19.013	19.134	18.926	.096	.217	.009
65.....	1885	14.427	14.469	14.714	14.428	.042	.287	.001
75.....	1875	10.691	10.647	10.970	10.691	-.044	.279	.000
85.....	1865	8.961	8.961	9.061	8.961	.000	.100	.000
With 20-Year Certain Period— $a_{20} +_{20}a_x$								
15.....	1935	31.991	31.992	32.075	32.088	.001	.084	.097
25.....	1925	29.717	29.740	29.814	29.790	.023	.097	.073
35.....	1915	26.889	26.940	26.998	26.935	.051	.109	.046
45.....	1905	23.514	23.590	23.651	23.536	.076	.137	.022
55.....	1895	19.872	19.950	20.071	19.878	.078	.199	.006
65.....	1885	16.875	16.915	17.092	16.874	.040	.217	-.001
75.....	1875	15.678	15.676	15.739	15.678	-.002	.061	.000

TABLE 3

## IMMEDIATE LIFE ANNUITIES ISSUED IN 1960—MALE—2½% INTEREST

AGE AT ISSUE	YEAR OF BIRTH	ANNUITY VALUE AT ISSUE				EXCESS OVER $a-1949$ WITH PROJECTION B		
		$a-1949$ with Projection B	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation
Nonrefund— $a_x$								
25.....	1935	28.574	28.568	28.833	28.704	-.006	.259	.130
35.....	1925	25.307	25.360	25.684	25.401	.053	.377	.094
45.....	1915	21.263	21.359	21.804	21.319	.096	.541	.056
55.....	1905	16.759	16.804	17.236	16.785	.045	.477	.026
65.....	1895	12.092	12.150	12.268	12.100	.058	.176	.008
75.....	1885	7.588	7.617	7.522	7.590	.029	-.066	.002
85.....	1875	3.965	3.923	3.767	3.965	-.042	-.198	.000
With 10-Year Certain Period— $a_{10} +_{10}a_x$								
25.....	1935	28.610	28.601	28.847	28.739	-.009	.237	.129
35.....	1925	25.378	25.422	25.727	25.470	.044	.349	.092
45.....	1915	21.469	21.530	21.929	21.521	.061	.460	.052
55.....	1905	17.276	17.311	17.598	17.296	.035	.322	.020
65.....	1895	13.219	13.284	13.278	13.220	.065	.059	.001
75.....	1885	10.125	10.171	10.066	10.124	.046	-.059	-.001
85.....	1875	8.883	8.881	8.850	8.883	-.002	-.033	.000
With 20-Year Certain Period— $a_{20} +_{20}a_x$								
25.....	1935	28.713	28.701	28.904	28.839	-.012	.191	.126
35.....	1925	25.634	25.645	25.893	25.717	.011	.259	.083
45.....	1915	22.162	22.193	22.409	22.196	.031	.247	.034
55.....	1905	18.814	18.879	18.912	18.813	.065	.098	-.001
65.....	1895	16.418	16.496	16.415	16.414	.078	-.003	-.004
75.....	1885	15.635	15.646	15.628	15.634	.011	-.007	-.001

TABLE 4  
IMMEDIATE LIFE ANNUITIES ISSUED IN 1960—FEMALE—2½% INTEREST

AGE AT ISSUE	YEAR OF BIRTH	ANNUITY VALUE AT ISSUE				EXCESS OVER $a-1949$ WITH PROJECTION B		
		$a-1949$ with Projection B	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation	$a-1949$ Using Proposed Setback	Progressive	Sternhell Approximation
Nonrefund— $a_x$								
25.....	1935	29.797	29.789	29.863	29.922	-.008	.066	.125
35.....	1925	26.906	26.924	26.979	26.999	.018	.073	.093
45.....	1915	23.299	23.347	23.384	23.360	.048	.085	.061
55.....	1905	18.943	19.021	19.063	18.978	.078	.120	.035
65.....	1895	13.963	14.043	14.194	13.976	.080	.231	.013
75.....	1885	8.883	8.919	8.919	8.886	.036	.388	.003
85.....	1875	4.599	4.560	5.053	4.599	-.039	.454	.000
With 10-Year Certain Period— $a_{10} +_{10} a_x$								
25.....	1935	29.822	29.812	29.873	29.946	-.010	.051	.124
35.....	1925	26.952	26.968	27.007	27.045	.016	.055	.093
45.....	1915	23.401	23.447	23.466	23.461	.046	.065	.060
55.....	1905	19.191	19.266	19.303	19.221	.075	.112	.030
65.....	1895	14.636	14.721	14.879	14.645	.085	.243	.009
75.....	1885	10.765	10.819	11.088	10.765	.054	.323	.000
85.....	1875	8.963	8.961	9.094	8.963	-.002	.131	.000
With 20-Year Certain Period— $a_{20} +_{20} a_x$								
25.....	1935	29.891	29.881	29.910	30.013	-.010	.019	.122
35.....	1925	27.097	27.114	27.117	27.185	.017	.020	.088
45.....	1915	23.737	23.792	23.787	23.788	.055	.050	.051
55.....	1905	20.056	20.153	20.205	20.070	.097	.149	.014
65.....	1895	16.946	17.045	17.181	16.944	.099	.235	-.002
75.....	1885	15.682	15.700	15.761	15.682	.018	.079	.000

TABLE 5

## LIFE INCOME SETTLEMENT OPTIONS COMMENCING IN 1965—2½% INTEREST

AGE OF PAYEE IN 1965	YEAR OF BIRTH	PRESENT VALUE OF 1 A YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER a-1949 WITH PROJECTION B	
		a-1949 with Projection B	a-1949 Using Proposed Setback	Progressive	a-1949 Using Proposed Setback	Progressive
Male—With 10-Year Certain Period— $\bar{a}_{10} +_{10} a_x$						
35.....	1930	26.531	26.534	26.784	.003	.253
45.....	1920	22.607	22.642	22.983	.035	.376
55.....	1910	18.367	18.364	18.613	-.003	.246
65.....	1900	14.142	14.211	14.165	.069	.023
75.....	1890	10.726	10.821	10.672	.095	-.054
85.....	1880	9.186	9.202	9.146	.016	-.040
Male—With 20-Year Certain Period— $\bar{a}_{20} +_{20} a_x$						
35.....	1930	26.739	26.728	26.931	-.011	.192
45.....	1920	23.228	23.224	23.409	-.004	.181
55.....	1910	19.715	19.779	19.789	.064	.074
65.....	1900	17.050	17.157	17.045	.107	-.005
75.....	1890	16.061	16.079	16.048	.018	-.013
Female—With 10-Year Certain Period— $\bar{a}_{10} +_{10} a_x$						
35.....	1930	28.059	28.049	28.063	-.010	.004
45.....	1920	24.519	24.537	24.525	.018	.006
55.....	1910	20.283	20.345	20.341	.062	.058
65.....	1900	15.614	15.720	15.831	.106	.217
75.....	1890	11.463	11.566	11.821	.103	.358
85.....	1880	9.302	9.324	9.490	.022	.188
Female—With 20-Year Certain Period— $\bar{a}_{20} +_{20} a_x$						
35.....	1930	28.180	28.180	28.160	.000	-.020
45.....	1920	24.802	24.844	24.808	.042	.006
55.....	1910	21.032	21.129	21.144	.097	.112
65.....	1900	17.668	17.802	17.917	.134	.249
75.....	1890	16.129	16.162	16.239	.033	.110



TABLE 6

LIFE INCOME SETTLEMENT OPTIONS COMMENCING IN 1975—2½% INTEREST

AGE OF PAYEE IN 1975	YEAR OF BIRTH	PRESENT VALUE OF 1 A YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER <i>a</i> -1949 WITH PROJECTION B	
		<i>a</i> -1949 with Pro- jection B	<i>a</i> -1949 Using Proposed Setback	Progressive	<i>a</i> -1949 Using Proposed Setback	Progressive
		Male—With 10-Year Certain Period— $\overline{a}_{10} +_{10} a_x$				
35. ....	1940	26.832	26.778	26.917	-.054	.085
45. ....	1930	22.969	22.934	23.142	-.035	.173
55. ....	1920	18.713	18.662	18.788	-.051	.075
65. ....	1910	14.386	14.490	14.327	.104	-.059
75. ....	1900	10.810	11.009	10.771	.199	-.039
85. ....	1890	9.186	9.253	9.169	.067	-.017
Male—With 20-Year Certain Period— $\overline{a}_{20} +_{20} a_x$						
35. ....	1940	27.018	26.959	27.058	-.059	.040
45. ....	1930	23.515	23.475	23.551	-.040	.036
55. ....	1920	19.943	20.003	19.920	.060	-.023
65. ....	1910	17.134	17.296	17.122	.162	-.012
75. ....	1900	16.061	16.106	16.058	.045	-.003
Female—With 10-Year Certain Period— $\overline{a}_{10} +_{10} a_x$						
35. ....	1940	28.248	28.227	28.184	-.021	-.064
45. ....	1930	24.755	24.755	24.674	.000	-.081
55. ....	1920	20.536	20.598	20.512	.062	-.024
65. ....	1910	15.825	15.981	16.003	.156	.178
75. ....	1900	11.541	11.764	11.949	.223	.408
85. ....	1890	9.302	9.385	9.541	.083	.239
Female—With 20-Year Certain Period— $\overline{a}_{20} +_{20} a_x$						
35. ....	1940	28.371	28.352	28.278	-.019	-.093
45. ....	1930	25.017	25.047	24.946	.030	-.071
55. ....	1920	21.216	21.340	21.283	.124	.067
65. ....	1910	17.755	17.955	18.020	.200	.265
75. ....	1900	16.129	16.202	16.267	.073	.138

TABLE 7

RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES ISSUED  
IN 1950 WITH INCOME COMMENCING AT AGE 55—2½% INTEREST

AGE AT ISSUE OF ORIGINAL CONTRACT	YEAR OF BIRTH	PRESENT VALUE OF 1 A YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER <i>a</i> -1949 WITH PROJECTION B	
		<i>a</i> -1949 with Pro- jection B	<i>a</i> -1949 Using Proposed Setback	Progressive	<i>a</i> -1949 Using Proposed Setback	Progressive
Male—With 10-Year Certain Period— $\overline{d_{10}} +_{10} d_{55}$						
15.....	1935	19.203	19.110	19.049	-.093	-.154
25.....	1925	18.880	18.811	18.875	-.069	-.005
35.....	1915	18.539	18.513	18.701	-.026	.162
45.....	1905	18.181	18.215	18.526	.034	.345
Male—With 20-Year Certain Period— $\overline{d_{20}} +_{20} d_{55}$						
15.....	1935	20.267	20.344	20.119	.077	-.148
25.....	1925	20.058	20.115	19.985	.057	-.073
35.....	1915	19.829	19.890	19.855	.061	.026
45.....	1905	19.601	19.669	19.724	.068	.123
Female—With 10-Year Certain Period— $\overline{d_{10}} +_{10} d_{55}$						
15.....	1935	20.888	20.975	20.766	.087	-.122
25.....	1925	20.656	20.725	20.597	.069	-.059
35.....	1915	20.408	20.472	20.427	.064	.019
45.....	1905	20.144	20.218	20.256	.074	.112
Female—With 20-Year Certain Period— $\overline{d_{20}} +_{20} d_{55}$						
15.....	1935	21.461	21.658	21.494	.197	.033
25.....	1925	21.298	21.446	21.353	.148	.055
35.....	1915	21.114	21.235	21.213	.121	.099
45.....	1905	20.930	21.024	21.074	.094	.144

TABLE 8

RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES ISSUED  
IN 1955 WITH INCOME COMMENCING AT AGE 55—2½% INTEREST

AGE AT ISSUE OF ORIGINAL CONTRACT	YEAR OF BIRTH	PRESENT VALUE OF 1 A YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER a-1949 WITH PROJECTION B	
		a-1949 with Pro- jection B	a-1949 Using Proposed Setback	Progressive	a-1949 Using Proposed Setback	Progressive
Male—With 10-Year Certain Period— $\overline{a}_{10} + 10   \overline{a}_{55}$						
15.....	1940	19.354	19.259	19.135	-.095	-.219
25.....	1930	19.042	18.960	18.962	-.082	-.080
35.....	1920	18.713	18.662	18.788	-.051	.075
45.....	1910	18.367	18.364	18.613	-.003	.246
Male—With 20-Year Certain Period— $\overline{a}_{20} + 20   \overline{a}_{55}$						
15.....	1940	20.362	20.459	20.185	.097	-.177
25.....	1930	20.153	20.228	20.052	.075	-.101
35.....	1920	19.943	20.003	19.920	.060	-.023
45.....	1910	19.715	19.779	19.789	.064	.074
Female—With 10-Year Certain Period— $\overline{a}_{10} + 10   \overline{a}_{55}$						
15.....	1940	21.003	21.100	20.850	.097	-.153
25.....	1930	20.769	20.850	20.681	.081	-.088
35.....	1920	20.536	20.598	20.512	.062	-.024
45.....	1910	20.283	20.345	20.341	.062	.058
Female—With 20-Year Certain Period— $\overline{a}_{20} + 20   \overline{a}_{55}$						
15.....	1940	21.542	21.765	21.564	.223	.022
25.....	1930	21.379	21.552	21.423	.173	.044
35.....	1920	21.216	21.340	21.283	.124	.067
45.....	1910	21.032	21.129	21.144	.097	.112

TABLE 9

 RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES ISSUED IN  
 1950 WITH INCOME COMMENCING AT AGE 65—2½% INTEREST

AGE AT ISSUE OF ORIGINAL CONTRACT	YEAR OF BIRTH	PRESENT VALUE OF 1 YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER <i>a</i> -1949 WITH PROJECTION B	
		<i>a</i> -1949 with Pro- jection B	<i>a</i> -1949 Using Proposed Setback	Progressive	<i>a</i> -1949 Using Proposed Setback	Progressive
Male—With 10-Year Certain Period— $\ddot{a}_{10} +_{10} _{65}$						
15.....	1935	14.971	15.196	14.737	.225	-.234
25.....	1925	14.742	14.911	14.571	.169	-.171
35.....	1915	14.506	14.629	14.409	.123	-.097
45.....	1905	14.262	14.350	14.246	.088	-.016
Male—With 20-Year Certain Period— $\ddot{a}_{20} +_{20} _{65}$						
15.....	1935	17.337	17.675	17.321	.338	-.016
25.....	1925	17.252	17.518	17.237	.266	-.015
35.....	1915	17.168	17.366	17.160	.198	-.008
45.....	1905	17.084	17.226	17.083	.142	-.001
Female—With 10-Year Certain Period— $\ddot{a}_{10} +_{10} _{65}$						
15.....	1935	16.309	16.639	16.434	.330	.125
25.....	1925	16.121	16.375	16.261	.254	.140
35.....	1915	15.924	16.112	16.089	.188	.165
45.....	1905	15.721	15.850	15.917	.129	.196
Female—With 20-Year Certain Period— $\ddot{a}_{20} +_{20} _{65}$						
15.....	1935	17.947	18.366	18.284	.419	.337
25.....	1925	17.877	18.196	18.175	.319	.298
35.....	1915	17.790	18.035	18.072	.245	.282
45.....	1905	17.703	17.878	17.969	.175	.266

TABLE 10

RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES ISSUED IN  
1935 WITH INCOME COMMENCING AT AGE 65—2½% INTEREST

AGE AT ISSUE OF ORIGINAL CONTRACT	YEAR OF BIRTH	PRESENT VALUE OF 1 A YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER <i>a</i> -1949 WITH PROJECTION B	
		<i>a</i> -1949 with Pro- jection B	<i>a</i> -1949 Using Proposed Setback	Progressive	<i>a</i> -1949 Using Proposed Setback	Progressive
Male—With 10-Year Certain Period— $\ddot{a}_{\overline{10} } + {}_{10} \ddot{a}_{65}$						
15.....	1940	15.076	15.338	14.819	.262	-.257
25.....	1930	14.859	15.053	14.654	.194	-.205
35.....	1920	14.629	14.770	14.490	.141	-.139
45.....	1910	14.386	14.490	14.327	.104	-.059
Male—With 20-Year Certain Period— $\ddot{a}_{\overline{20} } + {}_{20} \ddot{a}_{65}$						
15.....	1940	17.370	17.755	17.363	.385	-.007
25.....	1930	17.286	17.595	17.279	.309	-.007
35.....	1920	17.219	17.442	17.199	.223	-.020
45.....	1910	17.134	17.296	17.122	.162	-.012
Female—With 10-Year Certain Period— $\ddot{a}_{\overline{10} } + {}_{10} \ddot{a}_{65}$						
15.....	1940	16.397	16.771	16.520	.374	.123
25.....	1930	16.216	16.507	16.347	.291	.131
35.....	1920	16.020	16.244	16.175	.224	.155
45.....	1910	15.825	15.981	16.003	.156	.178
Female—With 20-Year Certain Period— $\ddot{a}_{\overline{20} } + {}_{20} \ddot{a}_{65}$						
15.....	1940	17.982	18.451	18.339	.469	.357
25.....	1930	17.912	18.281	18.230	.369	.318
35.....	1920	17.825	18.116	18.123	.291	.298
45.....	1910	17.755	17.955	18.020	.200	.265

TABLE 11

DEFERRED LIFE ANNUITIES ISSUED IN 1950 WITH INCOME  
COMMENCING AT AGE 66—2½% INTEREST

AGE AT ISSUE	YEAR OF BIRTH	ANNUITY VALUE AT ISSUE				EXCESS OVER a-1949 WITH PROJECTION B		
		a-1949 with Projection B	a-1949 Using Proposed Setback	Progressive	Sternhell Approximation	a-1949 Using Proposed Setback	Progressive	Sternhell Approximation
Male—Nonrefund— $w_x   d_x$								
25.....	1925	4.078	4.084	4.185	4.117	.006	.107	.039
35.....	1915	5.036	5.077	5.269	5.061	.041	.233	.025
45.....	1905	6.271	6.361	6.682	6.284	.090	.411	.013
55.....	1895	8.160	8.271	8.676	8.164	.111	.516	.004
Female—Nonrefund— $w_x   d_x$								
25.....	1925	5.013	5.046	5.037	5.068	.033	.024	.055
35.....	1915	6.286	6.338	6.358	6.324	.052	.072	.038
45.....	1905	7.925	8.002	8.063	7.947	.077	.138	.022
55.....	1895	10.165	10.263	10.384	10.174	.098	.219	.009

TABLE 12

DEFERRED LIFE ANNUITIES ISSUED IN 1960 WITH INCOME  
COMMENCING AT AGE 66—2½% INTEREST

AGE AT ISSUE	YEAR OF BIRTH	ANNUITY VALUE AT ISSUE				EXCESS OVER a-1949 WITH PROJECTION B		
		a-1949 with Projection B	a-1949 Using Proposed Setback	Progressive	Sternhell Approximation	a-1949 Using Proposed Setback	Progressive	Sternhell Approximation
Male—Nonrefund— $w_x   d_x$								
25.....	1935	4.254	4.235	4.272	4.321	-.019	.018	.067
35.....	1925	5.269	5.271	5.377	5.320	.002	.108	.051
45.....	1915	6.573	6.604	6.822	6.607	.031	.249	.034
55.....	1905	8.524	8.561	8.847	8.544	.037	.323	.020
Female—Nonrefund— $w_x   d_x$								
25.....	1935	5.142	5.170	5.122	5.229	.028	-.020	.087
35.....	1925	6.458	6.495	6.464	6.528	.037	.006	.070
45.....	1915	8.149	8.205	8.200	8.199	.056	.051	.050
55.....	1905	10.439	10.516	10.554	10.469	.077	.115	.030