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DISCUSSION OF PRECEDING PAPER

WILMER A. JENKINS:

This is an interesting, concise and practical paper—the kind we have learned to expect from Mr. Hoskins. With, I think, remarkable success he has adapted the so-called "generation" mortality theory to the *a*-1949 Table and Projection Scale B.

Mr. Hoskins' adaptation of this theory is that the *a*-1949 Table (without projection) is satisfactory for persons born in 1875 or before, and that this same table gives good results for persons born after 1875 if their ages are set back by a certain formula. This formula provides that a man's age is set back by $7\frac{1}{2}\%$ of a year for each calendar year by which his year of birth was later than 1875. For a woman this age setback is 6% of a year. These amount to an additional whole year of age setback for each succeeding generation born within a period of $13\frac{1}{3}$ years for males, and $16\frac{2}{3}$ years for females. It is interesting to compare these periods with those developed by other actuaries. Many years ago in connection with British annuity tables, Mr. Duncan Fraser arrived at a period of 10 years for both sexes. In developing the Progressive Annuity Table, Messrs. Fassel and Noback adopted a period of 25 years for both sexes.

The purpose of Mr. Hoskins' paper is to develop a method for making provision for future mortality improvement which is simpler and easier to use than other devices, such as the projecting commutation columns invented by Mr. Sternhell. Mr. Hoskins' figures show that his method produces annuity values which are close to those based on the *a*-1949 Table with Projection B—definitely close enough for practical purposes. Apparently he assumed that it is obvious that the age setback method is materially simpler than Mr. Sternhell's scheme, even when full use is made of modern machine methods. He did not say so, however, and I think it would be helpful if he would comment on this point in his reply to this discussion.

Mr. Hoskins seems to have devised his method solely, or primarily, for valuation, and in this connection his figures speak for themselves. If the object is to conform very closely to the *a*-1949 Table with Projection B in valuations, the use of this kind of method would, presumably, require test checks from time to time. This would reduce somewhat the advantages of the method. However, the *a*-1949 Table and Projection B were never intended to be precise goals—and time may show that they are not even

good goals—so I think a good case can be made for valuing by Mr. Hoskins' method with no periodic check, or with only rare checking. This is especially true at attained age 60 and over, where nearly all immediate annuity risks fall. Use of this method at younger ages presents a somewhat different question.

One of the author's statements needs clarification, I think. He implies that his method is one which involves gradual strengthening of reserves. Strictly speaking, this is not the case if we define reserve strengthening as a change from one fixed mortality table to another for a policy already in force. For each policy, Mr. Hoskins adopts a fixed mortality table at date of issue, and for this policy he never changes the table. He does, however, do two things which depart from general practice at this time: first, at any one time the mortality table differs by age at issue—the younger the age, the stronger the table; and, second, for each policy issued in a certain year the mortality table is stronger than for a policy issued at the same age in the preceding year. These departures follow the over-all pattern of Projection B. One result of these methods is that, at any particular time, question might be raised as to the adequacy of reserves at the longer durations. However, I do not think that this question can be a serious one, largely because Projection B assumes no mortality improvement at age 90 and over, and relatively small rates of improvement immediately preceding age 90. Moreover, with deaths occurring mainly at the older ages and presumably with a constant inflow of new lives, it is clear that the average level of a company's reserves is gradually strengthened by Mr. Hoskins' process. In this sense, reserve strengthening does occur.

The author mentions that his method can present spurious mortality figures if they are derived by the gain and loss exhibit method. To test the seriousness of such errors, I have compared the death rates implicit in his method with the death rates strictly according to the *a*-1949 Table with Projection B for male lives. The former expressed as percentages of the latter are shown in the accompanying table. This table indicates that at

Year of Birth	Year of Exposure				
	1950	1955	1960	1965	1970
1915	82%	84%	78%	86%	97%
1905	75	83	92	101	104
1895	88	95	99	102	102
1885	94	98	101	101	98
1875	100	103	103	100	100
1865	100	100	100	100	100

attained age 60 and over, where most immediate annuity risks fall, the death rates on these two bases are close together. The largest error shown at these ages is less than 10%, and most of them are less than 5%. However, at younger ages the departures are considerably larger, one being as much as 25%. As Mr. Hoskins says, if younger lives are involved, as under deferred annuities, there might well be spurious mortality results if they are derived by the gain and loss exhibit method or, in fact, by any method using the same tabular mortality.

Mr. Hoskins does not mention the possibility of using his method to determine premium rates for annuities and settlement options. I hope that, in his reply to this discussion, he will comment on this possibility. I think it can be assumed that, under any scheme providing for future mortality improvement, it is implicit that premium rates would have to be strengthened from time to time. And, under any such plan there are practical difficulties in arranging settlement options and retirement income policy maturity values in an acceptable way. These two comments apply to the author's method, the same as to other methods. Perhaps the only additional question as to premium rates raised by Mr. Hoskins' proposal is: Since premium rates aren't calculated very often, is the labor saved and the simplicity gained by his method sufficient to justify the use of his approximate values, particularly at the younger ages?

CHARLES F. PESTAL:

Mr. Hoskins' excellent paper has set forth a very interesting and practical method of valuing annuities under the *a*-1949 table with projection for improvement in future mortality. In our company we have considered valuing our annuities to provide for mortality improvement using Mr. Sternhell's supplementary commutation columns.

For valuing deferred annuities we have taken Mr. Sternhell's formula as given in *TSA* II, June, 54. To eliminate scheduling by modes of payment we have applied Mr. Lang's method as given in *TASA* XLVII, 304. Thus, in Mr. Sternhell's formula (89) we have used

$$(a)' = A^{(p)}N_{x+n+1} + \frac{r}{p} A^{(p)}D_{x+n} \qquad \text{for} \qquad (a),$$

and for (b) we have used

$$(b)' = (a)'[1 - G_{x+n} - (k+n)F_{x+n}] + A^{(p)}[J_{x+n} + (k+n)H_{x+n}]$$

where $A^{(p)}$ is the annual income payable in $p$thly instalments and r/p $A^{(p)}$ is the contingent payments received in year $1950 + k + n$.

The valuation constants (a)′ and (b)′ need be computed only once at the time the supplementary contract or annuity is issued and may be used

throughout the deferred period. These constants may be computed without undue labor by hand on prepared worksheets. Using a modern electronic calculator the constants may be calculated simply in a minimum amount of time. The valuation may then be made by attained age only without regard to mode of payment. Joint lives may be handled in an analogous manner.

Mr. Hoskins' method requires the use of age setbacks which must be made integral for practical use. In contrast, our card contains the actual office year of birth.

As an interesting sidelight, a problem came up in using Mr. Sternhell's method for temporary annuities resulting from the waiver of premium benefit on the recognized applicant. We have decided to use an annuity certain which gives a very slight overstatement in reserve.

HARRY WALKER:

Mr. Hoskins has presented an ingenious method for approximating to the *a*–1949 Table with Projection B by using the *a*–1949 Table itself with an appropriate age setback depending upon the year of birth of the annuitant. While the differences involved in the approximation, as compared with the exact use of the *a*–1949 Table with Projection B, appear to be relatively small in the case of an immediate life annuity with no certain period, I believe that the differences shown in the tables appearing in Mr. Hoskins' paper are deceptive in the case of life annuities with a certain period. Since no approximation is involved in the calculation of the annuity certain, it seems more appropriate to relate these differences to the value of the deferred annuity portion only of the annuity. When viewed in this light, the differences involved in Mr. Hoskins' approximation appear to me to be appreciable. If, for example, we refer to the figures shown in Table 6 for a male age 75, 10-year certain life annuity, the difference of .199 is about 11% of the value of the deferred annuity portion of the 10-year certain life annuity although it is only about 2% of the value of the entire annuity. In the same Table 6, the difference of .162 in the case of a male 65, 20-year certain life annuity, is about 14% of the value of the deferred annuity portion of the entire annuity although it is less than 1% of the value of the entire annuity.

The percentage difference involved, when related to the deferred annuity element of the life income with a certain period, prompted me to consider how these differences measure up against the provision being made for mortality improvement under Projection B. For this purpose, I refer to Table 10 of Mr. Hoskins' paper which shows the maturity value to be accumulated under retirement income policies issued in 1955 with in-

come commencing at age 65, the maturity value varying with the age at issue of the policy and increasing as the age at issue decreases to provide for mortality improvement. It will be noted that for female issue age 45, to provide a life income 20 years certain at maturity, the difference of .200 in the maturity value (*i.e.*, the excess of 17.955 over 17.755) is just about equal to the increase in the maturity value to provide for 30 years' improvement in mortality. In other words, the difference of .200 is almost equal to the excess of the maturity value of 17.982 required for issue age 15 over the maturity value of 17.755 required for issue age 45. If a difference in the approximation of that magnitude is acceptable, it immediately raises the question of how meticulous we should be in making provision for mortality improvement, or alternatively, how meticulous we should be in varying the maturity value under a retirement income policy with the age at issue.

For rate making purposes, if *a*–1949 with Projection B is the objective, it would appear to me to be more acceptable to adopt the *a*–1949 Table with Projection B with no approximation other than the assumption that income commences in a given year, irrespective of the age at issue. To illustrate how this would work in the case of, say, retirement income policies issued in 1955, Table I below (corresponding to Table 10 in Mr. Hoskins' paper) compares the maturity value on the *a*–1949 Table with Projection B with (1) the maturity value using Mr. Hoskins' method, and (2) the maturity value based on the *a*–1949 Table with Projection B, assuming maturity in 1985. It will be noted that the latter approximation shows an average difference substantially less than the average difference under Mr. Hoskins' approximation, and has the added advantage of reproducing the Jenkins-Lew Projection B maturity value with a high degree of accuracy for the important issue ages around 35. Admittedly Mr. Hoskins has introduced an added element of conservatism that would not be present under the method I have suggested. The use of the *a*–1949 Table with Projection B, with the added assumption that life income settlements commence in a given calendar year, say 1985, as illustrated in Table I, would have the added advantage of avoiding a multiplicity of tables for life income settlements in the policy form. Moreover, as retirement income policy premium rates would assume commencement of income in the given calendar year, irrespective of issue age, no change in retirement income premiums would be required until a new assumed calendar year of income commencement is adopted for new issues, say ten years from now.

Mr. Hoskins has pointed out in his paper that, if the method which he has proposed is adopted, and "if mortality exactly equal to the *a*–1949

TABLE I

RETIREMENT INCOME INSURANCES AND DEFERRED ANNUITIES ISSUED IN 1955 WITH INCOME COMMENCING AT AGE 65—2½% INTEREST

AGE AT ISSUE OF ORIGINAL CONTRACT	YEAR OF BIRTH	PRESENT VALUE OF 1 A YEAR AT COMMENCEMENT OF INCOME			EXCESS OVER *a*-1949 WITH PROJECTION B	
		a-1949 with Projection B	Hoskins' Approximation	*a*-1949 with Proj. B Assuming Income Commences in 1985	Hoskins' Approximation	*a*-1949 with Proj. B Assuming Income Commences in 1985
		Male—with 10 Year Certain Period				
15.....	1940	15.076	15.338	14.629	.262	−.447
25.....	1930	14.859	15.053	14.629	.194	−.230
35.....	1920	14.629	14.770	14.629	.141	0
45.....	1910	14.386	14.490	14.629	.104	.243
					.175*	−.108*
		Male—with 20 Year Certain Period				
15.....	1940	17.370	17.755	17.219	.385	−.151
25.....	1930	17.286	17.595	17.219	.309	−.067
35.....	1920	17.219	17.442	17.219	.223	0
45.....	1910	17.134	17.296	17.219	.162	.085
					.270*	−.033*
		Female—with 10 Year Certain Period				
15.....	1940	16.397	16.771	16.020	.374	−.377
25.....	1930	16.216	16.507	16.020	.291	−.196
35.....	1920	16.020	16.244	16.020	.224	0
45.....	1910	15.825	15.981	16.020	.156	.195
					.261*	−.094*
		Female—with 20 Year Certain Period				
15.....	1940	17.982	18.451	17.825	.469	−.157
25.....	1930	17.912	18.281	17.825	.369	−.087
35.....	1920	17.825	18.116	17.825	.291	0
45.....	1910	17.755	17.955	17.825	.200	.070
					.332*	−.044*

* Average excess.

Table with Projection B should be experienced, the use of the proposed method would produce spurious mortality gains in the Gain and Loss Exhibit among younger annuitants." As I consider this a rather important point, I am illustrating in Table II the ratio of the rates of mortality on the Jenkins-Lew Projection B Table to the rates of mortality under Mr.

TABLE II

YEAR OF ISSUE 1950

Issue Age x	(1) Duration t	(2) q_{x+t} a-1949, Proj. B	(3) q_{x+t} Hoskins' Approximation	(4) Percentage (2)÷(3)
Male 45	0	.003625	.002723	133%
	9	.008661	.007848	110
	19	.017183	.017866	96
Male 55	0	.010565	.009261	114%
	9	.019231	.018932	102
	19	.042878	.043696	98
Male 65	0	.023066	.021729	106%
	9	.046417	.046678	99
	19	.115863	.114752	101
Male 75	0	.054501	.054501	100%
	9	.119396	.122669	97
	19	.292118	.292118	100
Female 45	0	.002019	.001744	116%
	9	.003856	.003705	104
	19	.009038	.009338	97
Female 55	0	.004705	.004239	111%
	9	.010116	.009918	102
	19	.027671	.028304	98
Female 65	0	.012406	.011679	106%
	9	.029955	.030204	99
	19	.088996	.088514	101
Female 75	0	.035829	.035829	100%
	9	.091710	.094224	97
	19	.261943	.261943	100

Hoskins' method. These ratios show how the ratio of actual to expected mortality for various attained ages would appear if Mr. Hoskins' method is adopted and if the mortality actually experienced follows the a–1949 Table with Projection B. The aggregate amount of the spurious mortality gains would, of course, depend upon the distribution of the business by age and type of annuity contract. I have intentionally elaborated on this

point as I feel that companies adopting approximations to the *a*-1949 Table with Projection B should expect such spurious mortality gains and should probably measure their expected mortality by the true *a*-1949 Table with Projection B in any study of their experience, particularly if the results of the mortality study are to be used in determining a formula for apportionment of dividends under life income settlements.

CLARK T. FOSTER:

Mr. Hoskins makes the point that his suggested method of taking account of mortality improvement makes it convenient to substitute gradual for abrupt reserve strengthening. Such a program of gradual strengthening is particularly convenient in the valuation of trusteed self-administered pension plans. Costs in a trusteed plan depend entirely on the actuarial assumptions chosen and the actual experience developed on the plan's own group of employees. The actuarial assumptions are usually such that moderate gains are expected from future years' experience with respect to interest earnings, employee turnover, and several other factors. Future costs must be reduced by crediting these gains against the gross contributions, and the recurring costs are, therefore, likely to be lower than the amount required in the first year.

The benefit formula is frequently fixed at whatever level can be supported by the contributions the company can afford. It is likely that the company would be willing to continue making contributions at the level of the first year if, by doing so, the actuarial gains could be used for some purpose which will benefit the pension program. It would seem reasonable to use each year's gains to guard against future losses due to the mortality improvement which has occurred during the last year or so, provided that this can be done in some convenient way such as the one suggested by Mr. Hoskins. If the originally chosen rates are in line with current mortality, gains in the first year might be used to strengthen the reserves for the lowest age group by rating the ages back one year. Future gains would extend this one-year rating to the older ages and then, as time goes by, a second year setback could be applied to the youngest age groups. This process could be continued until all reserves were held on a basis conservative enough to compensate for mortality improvement since the effective date and to anticipate the expected mortality improvement.

A similar program could, of course, be followed in strengthening the reserves of any plan which has been in force for a number of years on a reserve basis which is inadequate even as compared to current mortality experience.

No matter what method of reserve strengthening is adopted, it is obvi-

ous that costs will be increased, but such a program as this is perhaps as painless as any. It has the advantage of avoiding the need for contingency reserves for mortality improvement, which have been frowned upon by the Internal Revenue Bureau.

CHARLES M. STERNHELL:

I must confess that I was very much surprised at the relatively close agreement between the approximate annuity values produced by Mr. Hoskins' proposed method and the exact annuity values based on the *a*-1949 Table with Projection B. While the idea of using the *a*-1949 Table with age setbacks varying with the year of birth may seem relatively simple in retrospect, I do not think that it was at all obvious that the use of a stationary mortality table with age setbacks would produce so much better results than the apparently more logical method of using a year-of-birth mortality table with age setbacks.

TABLE 1

RATIOS OF MORTALITY RATES UNDERLYING PROPOSED APPROXIMATE METHOD* TO EXACT MORTALITY RATES ON *a*-1949 TABLE WITH PROJECTION B—MALE LIVES

EXPERIENCE YEAR	YEAR OF BIRTH					
	1925	1915	1905	1895	1885	1875
1950	85%(25)	82%(35)	75%(45)	88%(55)	94% (65)	100% (75)
1955	87 (30)	84 (40)	83 (50)	95 (60)	98 (70)	103 (80)
1960	88 (35)	78 (45)	92 (55)	99 (65)	101 (75)	103 (85)
1965	91 (40)	86 (50)	101 (60)	102 (70)	101 (80)	100 (90)
1970	81 (45)	97 (55)	104 (65)	102 (75)	98 (85)	100 (95)
1975	89 (50)	108 (60)	105 (70)	99 (80)	94 (90)	100 (100)
1980	102 (55)	110 (65)	103 (75)	94 (85)	94 (95)	.....
1985	114 (60)	108 (70)	97 (80)	88 (90)	95 (100)	.....

NOTE.—Figures in parentheses indicate attained ages corresponding to indicated years of birth and experience years.

* Based on fractional ages.

The tables in the paper clearly demonstrate that the proposed approximate annuity values are generally much closer to the exact annuity values than the approximate annuity values produced by using the Progressive Annuity Table. In addition, as the author points out, by using the separate male and female values in the *a*-1949 Table, he avoids the artificial relationship between male and female mortality resulting from the 4-year age setback for female lives in the Progressive Annuity Table. It seems to me that Mr. Hoskins' proposed method retains practically all of the ad-

vantages of simplicity inherent in the method originally developed by Messrs. Fassel and Noback and achieves a much higher degree of accuracy than is possible by the use of the Progressive Annuity Table.

I would like to point out, however, that while the method proposed in this paper produces annuity values which are relatively close to those produced by the *a*-1949 Table with Projection B, the mortality rates underlying the proposed method may differ considerably from the exact mortality rates. This is suggested by the author when he indicates that the use of the proposed method may produce spurious mortality gains in the Gain and Loss Exhibit among younger annuitants.

TABLE 2

RATIOS OF MORTALITY RATES UNDERLYING PROPOSED APPROXIMATE METHOD* TO EXACT MORTALITY RATES ON *a*-1949 TABLE WITH PROJECTION B—FEMALE LIVES

EXPERIENCE YEAR	YEAR OF BIRTH					
	1925	1915	1905	1895	1885	1875
1950	84% (25)	85% (35)	86% (45)	90% (55)	94% (65)	100% (75)
1955	89 (30)	89 (40)	91 (50)	95 (60)	99 (70)	103 (80)
1960	93 (35)	93 (45)	97 (55)	99 (65)	101 (75)	103 (85)
1965	97 (40)	98 (50)	101 (60)	102 (70)	101 (80)	100 (90)
1970	101 (45)	104 (55)	104 (65)	102 (75)	99 (85)	100 (95)
1975	105 (50)	108 (60)	105 (70)	100 (80)	94 (90)	100 (100)
1980	112 (55)	109 (65)	103 (75)	95 (85)	95 (95)	.....
1985	115 (60)	109 (70)	98 (80)	89 (90)	95 (100)	.....

NOTE.—Figures in parentheses indicate attained ages corresponding to indicated years of birth and experience years.

* Based on fractional ages.

I have prepared two tables which show, for males and females, the ratios of the proposed approximate mortality rates to the exact mortality rates in the *a*-1949 Table with Projection B. In addition to indicating the degree of distortion of mortality gains, these tables help to explain the relative accuracy of the annuity values produced by the proposed method and to indicate the areas in which the proposed method is likely to produce more or less accurate approximations.

These tables indicate that the general effect of the proposed age setback method may be described roughly as understating the mortality rates at attained ages below 60, overstating them at ages between 60 and 75, and then understating them again at ages over 75. The high degree of accuracy exhibited by the proposed approximate annuity values appears to reflect the fact that, for most annuity values, understated mortality rates

over parts of the age range will generally tend to be offset by overstated mortality rates over the remainder of the age range.

These tables also indicate that during the next five or ten years the proposed method will tend to produce spurious mortality gains over practically the whole range of ages under 75. For example, in the case of a male aged 50 in 1955, the proposed method produces a mortality rate of 5.10 per 1,000 instead of 6.16 per 1,000. The approximate mortality rate is about 17% less than the exact mortality rate. According to Projection Scale B, the approximate mortality rate represents the level of mortality at age 50 that would be attained in 1970 instead of in 1955.

The tables that I have prepared and the ones included in the paper are all based on fractional ages, while Mr. Hoskins indicates that companies using the proposed method for valuation would probably use the nearest integral age. It should be noted that the use of integral ages will tend to increase the differences between the exact and approximate values in some cases and reduce them in others. It will also introduce discontinuities in the annuity values at points where the age setback changes.

Mr. Hoskins indicates that the approximation he proposes does not purport to be as close as that produced by using projecting commutation columns but it has the advantage of using only the ordinary annuity formulas without the addition of special functions. While I do not exactly qualify as an impartial observer on this subject, I would like to point out that under the method described in my paper (*TSA* II, June, 30) the approximate annuity values would have to be calculated only at the time the method is adopted and then the same valuation factors would be used each year. The adjustment for improving mortality would have to be made only in the aggregate. Under the proposed method, some of the valuation factors would have to be changed each year, since the valuation factor would depend on both the year of birth and the attained age. The use of projecting commutation columns has an additional advantage in that it tends to reproduce the exact mortality rates during the next 20 years much more closely than the proposed method and thus avoids the distortion of mortality gains.

I think that the availability of the new electronic calculators raises some doubt as to the advisability of using any approximate method for calculating annuity values. Mr. Peterson, in his paper on Group Annuity Mortality, indicated that generation mortality tables were calculated on punch cards for all ages without undue time or labor. This indicates the necessity of carefully balancing the disadvantages of using an approximate method against the additional cost involved in using an exact method.

JOSEPH C. NOBACK:

In this paper Mr. Hoskins has provided an added starter in the Actuarial Sweepstakes which pits the annuity table greyhounds against the mechanical rabbit of constantly improving annuitant mortality.

The Hoskins entry is the result of carefully controlled breeding. Its maternal parentage (Jenkins-Lew *TSA* I, 369) is top-notch. Into this stock, Mr. Hoskins blended the Age Setback strain which has appeared earlier in certain actuarial mongrels. This cross-breeding, however, has been handled very expertly. The maternal characteristics are clearly dominant in the new offspring with the Age Setback strain merely adding a certain degree of adaptability to the noble lines of Jenkins-Lew. A champion has been produced and its sponsor is to be congratulated on the fine actuarial specimen which he has nurtured.

The author has compared the performance of his favorite with that of two other qualifiers. He has tried to demonstrate on paper the superiority of his pride. He emphasizes that his entry has all the blood-lines of its maternal ancestry and, by inference, is somewhat critical of the independent showing of *TSA* II, 279. This latter entry, you may recall, was by Gompertz out of Jenkins-Lew and has been referred to as Progressive Annuity Table. In this specimen, the Gompertz strain furnishes unexcelled maneuverability to the basic Jenkins-Lew stock. In the breeding, however, some of the maternal lines have been weakened with the result that no clearly dominant strain has emerged.

It would appear to this handicapper that, in the Mortality Standard and in the Gross Premium Classes (where speed and simplicity of line are not essential), the original Jenkins-Lew entry is still the front runner. It has a quality and a bearing that surpasses any of its offspring.

It is felt, however, that in the Reserve Liability and Guaranteed Settlement Option Field Trials, the siblings will outperform their parent. Here flexibility and sleekness are indispensable. The Jenkins-Lew entry carries too much weight and it is difficult to see how it can negotiate the course with all its obstacles in sufficient time to make a sustained bid.

In the Reserve Liability trial, the Connecticut entry gains points because it hugs its mother's style. However, Progressive Annuity Table's independence gives it a maximum of speed and flexibility especially at the Joint and Survivor hurdles. It is too early to pick a winner. Competition will be very keen.

In the opinion of this observer, there have been very few serious contenders up to the present time in the Guaranteed Options Marathon. Jenkins-Lew appears to be too cumbersome. Hoskins, similarly, carries

the double burden of unrelated male and female tables and will be severely put to it to make a good showing. In addition, its life expectancy has for some reason been limited. This, however, will not call for disqualification since extension is always possible. In fact, it may be noted at this point that entrants in this event of the Sweepstakes are notorious for their longevity. We now can see greyhounds, that have outlived their usefulness long since and have been old dogs for many years, still negotiating the course. In fact (if we may modify the figure of speech slightly), this observer has gained the impression that the older the mongrel the higher the frequency of progeny in the form of life income options coming into being.

In the Guaranteed Option Marathon, a reasonable projection of future improvement in mortality must be combined with utter simplicity of presentation. No claims have been made along these lines for the Hoskins Table. However, in the case of Progressive Annuity Table, the results of one running are known to all the handicappers and it has been reliably reported that, in a recent time trial, the original showing was bettered by the use of a short two-column age conversion table.

WALTER G. BOWERMAN:

The relatively simple suggestion which Mr. Hoskins has made in this paper impresses one as a distinct improvement. It reminds me of Mr. Valentine Howell's proposal in 1920 (*TASA* XXI) as to graduating the American Experience table. He said, don't force it into the Makeham mold, but determine a value for log c, and then proceed to use the original smoothed table for joint lives just as though it were Makehamized. It is the famous doctrine of "as if." This applies equally to any table. The vast and long drawn out furor in England about the Perks curves for the law of mortality could have been avoided if Mr. Howell's proposal had been widely accepted. Even though W. P. Elderton and A. E. King (*JIA* 44, 293–301) had expressed the same notion in 1910, it did not "percolate." I hope that Mr. Hoskins will be more successful!

(AUTHOR'S REVIEW OF DISCUSSION)

JAMES E. HOSKINS:

I am grateful to the several members who have taken the time to discuss the paper, especially since it deals only with a tool and not with important facts or principles. I am particularly grateful to Mr. Sternhell for analyzing the reason why the approximation works as well as it does.

I believe Mr. Sternhell is mistaken in saying that under the proposed method some of the valuation factors would have to be changed each year.

If, for example, a female annuitant born in 1925 enters in 1952 at the true age of 27, she would be recorded for valuation purposes as if she had been born in 1928 and were now aged 24. This status would continue as long as the policy remains in force or until some complete change in valuation basis is made.

Mr. Jenkins urges me to comment on the usefulness of my suggested method in certain situations. The paper intended merely to call attention to a method of approximation. Whether for a given purpose it is preferable to other approximations, or to exact calculation, is a matter of individual opinion.

Mr. Jenkins is right in saying that when this method is used, reserve strengthening does not literally occur. I referred to reserve strengthening only in the looser sense described at the end of his fifth paragraph.

As some of the speakers have observed, the method was derived primarily with an eye to valuation, and the setback constants have therefore been taken slightly on the conservative side. The principle of the method could be used in such a way that positive and negative errors would more nearly balance.

If the paper should achieve no other result than that of evoking Mr. Noback's brilliant addition to the literature of actuarial humor, I feel that it will have been worth while.