

**ACTUARIAL NOTE: VALUATION OF THE SHARES IN A
SHARE-AND-SHARE-ALIKE LAST SURVIVOR ANNUITY**

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SEE PAGE 128 OF THIS VOLUME

ANDREW DELANEY:

Mr. Rasor and Mr. Myers have given a simple and elegant proof of their theorem employing the Z notation. Perhaps some of our students would also like to see the theorem proved directly from first principles. We can begin with a restatement of the authors' equation 4:

$$V_w = \sum_{r=0}^m \frac{1}{r+1} \sum_{t=1}^{\infty} v^t {}_t p_w {}_t p_{\frac{[r]}{xyz \dots (m)}}$$

The m -life probability factor can be evaluated from first principles as follows:

It is the coefficient of c^r in the expression

$$(1 - {}_t p_x + c {}_t p_x) (1 - {}_t p_y + c {}_t p_y) \dots m \text{ terms,}$$

which may also be written as

$$[1 + (c - 1) {}_t p_x] [1 + (c - 1) {}_t p_y] \dots m \text{ terms.}$$

The coefficient of c^r is thus equal to $S_r - {}^{r+1}C_1 S_{r+1} + {}^{r+2}C_2 S_{r+2} \dots$ where S_r denotes the sum of terms of the form ${}_t p_{xy \dots (r)}$ for all possible combinations of r letters out of m .

Thus our expression for V_w may be written

$$V_w = \sum_{r=0}^m \frac{1}{r+1} \sum_{t=1}^{\infty} v^t {}_t p_w (S_r - {}^{r+1}C_1 S_{r+1} + {}^{r+2}C_2 S_{r+2} \dots)$$

Thus in our expression for V_w when any given number of lives n out of the group $xyz \dots (m)$ is involved, the coefficient will be

$$(-1)^n \left[{}^n C_n - \frac{1}{2} {}^n C_{n-1} + \frac{1}{3} {}^n C_{n-2} - \frac{1}{4} {}^n C_{n-3} + \dots (-1)^n \frac{1}{n+1} {}^n C_0 \right]$$

which equals $(-1)^n / (n + 1)$.

Thus

$$V_w = a_w - \frac{1}{2} (a_{wx} + a_{wy} \dots m \text{ terms}) + \frac{1}{3} (a_{wxy} + \dots {}^n C_2 \text{ terms}) - \dots$$

ROBERT F. LINK:

You can take these x 's and y 's, and so forth; and, recognizing that they are subscripts, you can chop them loose from their basic symbols, and manipulate them algebraically, just as if they were operators. It has something to do with status theory or class theory, which I do not understand.

The only point is, if you do it that way, results such as the ones in this paper come out more or less automatically, without any defining of special symbols, such as a Z which does something special with the particular life in question.