Development of a Simulation-based Model to Quantify the Degree of a Bank's Liquidity Risk

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Abstract

This study investigates whether simulation-based models can be used to predict liquidity risk for banks, which must comply with Basel III by January 2015. The importance of holding more capital has become the main prerequisite for Basel III, which emphasizes a strong liquidity framework to support funding, but the aftermath of the global financial crisis that began in 2007 poses an even greater challenge to the banks that have to meet their obligation to remain solvent. In the current scenario, to achieve economic recovery globally, banks must manage their liquidity gap. The concept of liquidity risk has emerged as the new problem for such banks and must be measured and managed.

The paper looks at the process of developing a measurement framework using Black-Scholes and Merton's asset-based models to measure liquidity risk. The study shows how to apply these models in measuring liquidity risk, to determine the probability of reaching the stage of insolvency and to estimate the probability of being unable to meet payment obligations. This provides a foundation for implementing a solid liquidity framework required by banks to meet the Basel III standards.

The study showed that liquidity risk can be measured using static liquidity gaps from Monte Carlo simulation. The main findings from this study were: the models can be used further to provide the probability of a bank becoming insolvent within six months and quantify the probability of a bank's failure to meet its payment obligation.

The measurement of liquidity risk shown in this study provides a framework for managing liquidity in banks based on static measures of liquidity gaps. This serves the need for a global liquidity standard and an effective supervisory review process for Basel III.

Keywords and Their Definitions

Liquidity risk	Risk of not being able to meet payment obligations due to a shortage of liquidity or cash
LaR	Liquidity at risk
Liquidity gap	Difference between the total value of assets and liabilities for a bank
Static liquidity gap	Liquidity gap measured at constant marginal funding cost
BCBS	Basel Committee on Banking Supervision
Credit spreads	Difference between the yields of treasury and corporate bonds
Monte Carlo simulation	Computational algorithm that uses random sampling
B-S model	Black-Scholes model used for pricing stock options
$Vdef$	Value of assets at the point of callable liabilities
Kurtosis	Measure of the peakedness of the probability distribution

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1 Introduction

For Basel III implementation, the minimum common equity and Tier 1 requirements will be phased in between Jan. 1, 2013, and Jan. 1, 2015. The total minimum capital level will be raised to 10.5 percent of both Tier 1 and Tier 2. This will force banks to seek more funds in securing higher levels of equity and managing their liquidity effectively in order to achieve the Basel III targets. The Basel Committee also issued proposals for an "international framework for liquidity risk measurement, standards and monitoring." Under the current tight market scenario, where banks are operating under the lowest interest rates in 30 years and with profit margins declining, a real challenge stands on achieving stable liquidity. In such tough times, a strategy is required to ensure banks are able to manage their liquidity risk. If this cycle continues, it will be even more of a challenge to raise equity in the form of capital to meet Basel III targets.

"During the aftermath of the global financial crisis that began in 2007, banks throughout the world realized the importance of strengthening the resilience of the banking sector and international framework for liquidity risk measurement, standards and monitoring with Basel III."

Having successfully implemented Basel II, which enabled banks to take an advanced approach in reading their risk with greater accuracy, the challenge continues to cover other risks. During the aftermath of the global financial crisis that began in 2007, banks throughout the world realized the importance of strengthening the resilience of the banking sector and international framework for liquidity risk measurement, standards and monitoring with Basel III. As a result, a new accord was laid out that requires banks to have an effective framework which can manage and measure liquidity risk. The Basel III accord expects all the large banks to hold more capital, with the benchmark level being raised to 10.5 percent. This requires the banks to raise more capital and also to provide a global liquidity standard that can manage liquidity. This paper can be useful in understanding the requirements to manage and measure liquidity risk in pursuit of providing an effective liquidity risk framework for Basel III implementation.

The study investigates the quantification of liquidity risk using a stochastic model based on simulation and, later with the application of Black-Scholes and binomial models, can provide an estimate on the likelihood of a bank becoming insolvent due to liquidity shortage over a given time frame. Merton's asset-based model is also applied to evaluate the likelihood of a bank defaulting on its payment due to liquidity risk. The whole objective would be to provide a basis to identify the point of liquidity risk that later builds up as a liquidity framework. This enables the banks to take necessary actions before being impacted with liquidity risk. This involves a detailed quantitative approach involving various mathematical options that can be useful in predicting financial uncertainty in the capital markets. However, the work is limited to developing a stochastic model and not to justify the state of liquidity risk faced by any banks. Therefore, data being used to develop the model will not be actual data. This is done to prevent the final result from the model being misinterpreted by the readers.

2 Measuring Liquidity Risk and Basel III

On Dec. 17, 2009, the Basel Committee on Banking Supervision (BCBS) published its consultation paper on strengthening the resilience of the banking sector. The key focus was mainly to cover the definition of capital, counterparty credit risk, leverage ratio, systemic risk and countercyclical buffers. This was proposed as part of the Basel III accord with the main purpose to strengthen the financial sector after the financial crisis. According to BCBS:

> *"The Basel Committee proposals to strengthen global capital and liquidity regulations with the goal of promoting a more resilient banking sector."*

"The previous accord, Basel II, did not propose enough benchmarks within the liquidity framework and did not highlight the importance of holding extra capital in absorbing systemic risks."

The objective of the BCBS's reform package is mainly to improve the stability of the banking sector to absorb shocks arising from economic stress. The previous accord, Basel II, did not propose enough benchmarks within the liquidity framework and did not highlight the importance of holding extra capital in absorbing systemic risks. Basel III has been proposed to mitigate the risk of spillover from the financial sector to the real economy.

The Basel III framework is comprised of the following building blocks, agreed and issued by the committee between July 2009 and September 2010. They are as follow:

- To increase the quality of the capital to ensure banks are far more prudent in absorbing losses.
- To increase the risk coverage of the capital framework that relates to all the trading activities, securitizations and off-balance sheet assets arising from derivates and other engineered vehicles.
- To increase the minimum capital requirement that includes the minimum equity to be raised from 2 percent

to 4.5 percent. An additional capital conversion buffer of 2.5 percent is also included, which brings the total minimum equity to 7 percent.

- To introduce a leverage ratio that would prevent further build up of leverage and control the level of risk taking.

- To raise the standards for the supervisory review process with additional guidance on valuation, stress testing and liquidity risk management, and corporate governance.

- To introduce minimum global liquidity standards consisting of net stable funding ratio.

- To promote the build up of capital during good times so that it can be used during the period of stress and protect the bank from excessive credit growth.

Based on the committee's above proposition, strong capital requirements are a necessary condition for banking sector stability but by themselves are not sufficient. Equally important is the introduction of stronger bank liquidity (Bank for International Settlements, October 2010). Therefore, it's highly imperative for banks to develop a liquidity framework that would be used as a guide. This would enable banks to understand their liquidity and strategize accordingly. First, a clear measure of liquidity for banks is required before aiming for higher liquidity standards as per Basel III propositions shown above. The liquidity measure should not be treated any differently from other risk measures done previously by many risk managers/engineers. It involves developing a stochastic model that can be used to predict the level of current liquidity risk for a specific bank. Liquidity risk will be measured using a stochastic model developed through Monte Carlo simulation and later the point of liquidity crisis over a time frame is predicted using the binomial model and Merton's asset-based model.

Based on the new Basel III propositions, the key message for emerging banks is to improve capital and liquidity management. This paper investigates different mathematical options that can be used to predict liquidity risk, which enables banks to use it as a guideline in providing the type of liquidity management required by Basel III. The mathematical models give an estimate of a time frame.

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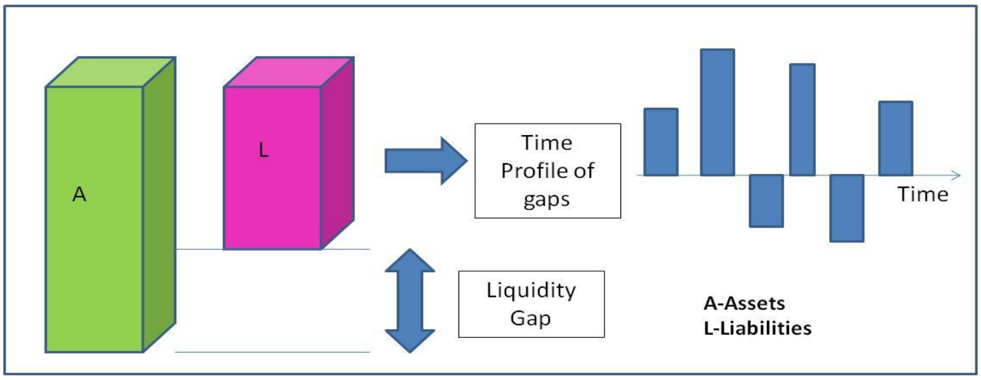
The technique applied in this paper to measure liquidity risk is based on the basic definition given by Bessis (2002) for liquidity gap in a bank.

According to Bessis, liquidity gaps are the differences between the outstanding balances of assets and liabilities, or between their changes over time, which can be depicted below in Figure 2.1.

Figure 2.1
Liquidity Gaps and Time Profile of Gaps



(Source: Bessis 2002, 137)

Marginal or incremental gaps are the differences between the changes in assets and liabilities. For any large bank, liquidity gaps are the differences, at all future dates, between assets and liabilities. Liquidity gaps generate liquidity risk, the risk of not being able to raise funds to meet the payment obligation.

When considering the time it takes assets to mature and the use of call option to bring in more cash, banks utilize their own risk appetite to set such parameters. These cannot be expected to reflect on the calculations because every bank will proceed at its own discretion based on its risk assessment and the level of risk appetite it can contain against the transactions. Also, banks have to decide to what extent they need to lock in their long-term funding against market dynamics. This has been stressed by Barfield and Venkat (2009), who clearly state that lending timescales will increase because banks will need to be more assertive when locking in their long-term funding. This decision to set lending timescales will be purely based on their own risk appetites and the necessity to meet near-term obligations once they have been measured.

"Quantification of liquidity risk continues to challenge banks because the value of the bank's liquid assets are always changing and therefore it's difficult to fit this change to any model that can predict future behavior."

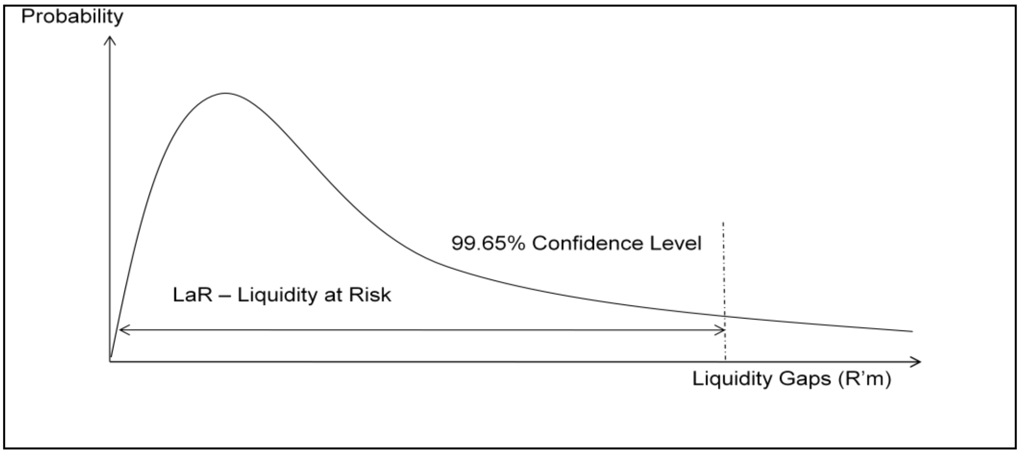
Quantification of liquidity risk continues to challenge banks because the value of the bank's liquid assets is always changing and therefore it's difficult to fit this change to any model that can predict future behavior. However, illiquid assets can be fitted to a financial model such as Merton's asset-based model because the movement is pro-cyclical.

3 Development of Stochastic Model to Measure Liquidity Risk

LaR (Liquidity at Risk) Based on Monte Carlo Simulation

The first stage of the process was to quantify the liquidity gaps originating from the bank's financials. The data obtained was taken from the bank's financials published every six months starting in June 1998 and ending in June 2010. After every six-month period, liquidity gaps are tabulated and later put into a Monte Carlo simulation engine using @RISK Industrial software to obtain the distribution curve (details of Monte Carlo simulation are explained in section 4). The curve is similar to the loss curve but the axis is measured as a liquidity gap expressed as R'm (million dollars). The final distribution curve obtained through simulation is shown below over the fixed period of six months.

Figure 3.1
Probability Distribution Curve of Final Distribution Curve



The period of six months is based on the dates the financials are disclosed to the shareholders. This is the period taken by banks to justify their liquidity position to shareholders. The best fitting curve is found for the gap distribution and put through Monte Carlo simulation.

The results from Monte Carlo distribution provide the amount of liquidity required to avoid insolvency or failure to meet obligations.

This amount of required liquidity from Monte Carlo distribution is stated at a confidence level of 99.65 percent set for banks achieving a BBB rating set by Standard & Poors rating agency (Gene & Song, Empirical Examination of the Basel II Proposals for *Asset Securitization, May 2003).* The confidence level for BBB rating does not reflect the current rating of any banks in South Africa and is selected purely for the purpose of developing the model. The final distribution obtained from Monte Carlo simulation explains that at 99.65 percent confidence, the bank will not need more than the required liquidity over a six-month period in the event of facing insolvency. The bank would require that much liquidity in six months to cover its obligation. Similar to simulating loss data into Monte Carlo simulation, the final distribution states the total loss the bank will face at a specific confidence level will not exceed that amount. For example, if the Monte Carlo simulation gave the output of R5 billion of liquidity required at a confidence level of 99.65 percent, this result would be interpreted as saying there is a 99.65 percent confidence the bank will not require more than R5 billion worth of liquidity over a six-month period when facing insolvency. The final output from Monte Carlo stating the liquidity required will be termed as liquidity at risk (LaR) over six months at 99.65 percent confidence. This is because failure to provide the required liquidity of the value LaR in six months would result in the bank facing insolvency when impacted by liquidity risk. Also, if LaR increases over the next period starting from 2011, this would indicate the bank is facing greater liquidity risk and would require more liquidity within the six-month period to overcome the stage of insolvency. Therefore, higher LaR figures can be defined as higher liquidity risk.

Application of Binomial and Black-Scholes Models

The next stage of the process is to develop a predicting model that would allow banks to measure the likelihood of failing to meet their obligation under liquidity risk. This would provide an overview of what extent liquidity risk has on the bank's failure to meet its obligation. This model is directly related to liquidity risk being LaR, but uses the LaR values, i.e., required liquidity to predict the likelihood of facing bankruptcy. This still remains an estimate and not an exact measure because the amount of data available to reach maximum accuracy remains limited. The study investigates the common pricing formula being the Black-Scholes pricing formula, which states the value of entering into either a call or put option contract. Black-Scholes can be expressed in the following way:

$$V = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{3.1}$$

Where:

S = Stock price of the bank traded in the market.

K = Strike price of the stock at the point of exercising the option.

(T-t) = Period of exercising the option defined as a fraction of 12 months.

σ = Volatility of the stock price per annum.

r = Risk-free rate directly obtained from Bloomberg.

$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$.

$d_2 = d_1 - \sigma(T-t)^{0.5}$.

$N(d_1), N(d_2)$ are cumulative distribution functions for the normal distribution (directly calculated using the built-in function in MS Excel 2007).

The following assumptions are made while applying the above Black-Scholes model to value the stock option:

- Volatility of the stock price remains unchanged during the one-year period.

- The stock can be sold short.
- There are no costs associated with selling the stock, i.e., when exercising the put option.
- There is no risk of default.
- The strike price taken by the bank is greater than the stock by R1, i.e., point of arbitrage.
- Early exercise of the option is not permitted, i.e., the option is European.

In the event of being impacted by liquidity risk, banks will need to source enough funding equal to LaR calculated previously from simulation to remain operatable and avoid being liquidated. To have 100 percent guaranteed cash on hand, which will be enough to prevent insolvency, the bank must enter into a put contract, which gives it the right to sell the shares to a third party, securing enough cash as funding. The value of this put option based on Black-Scholes model can be expressed in the following way:

$$V = -SN(-d_1) + Ke^{-r(T-t)}N(-d_2) \tag{3.2}$$

For the bank to overcome liquidation and meet its obligation, it must secure cash equal to the value of the put option. Once the value of the put option is exercised, it provides the bank with the cash from the sale of its stock. The only risk is if the third party, i.e., the buyer to whom the put contract is entered with, are already liquidated and do not have any security to meet the contract requirement in providing the cash to purchase the stock. However, this scenario is not considered and it’s assumed that once the bank exercises its put option by selling all the shares it holds during the period stated on the data set, it is guaranteed to receive the value in return for selling the stocks.

For the bank to remain solvent, i.e., to meet its liquidity requirements, the total value of the put option from the B-S model obtained by selling all the shares must either equal or exceed the calculated liquidity required (LaR) from Monte Carlo simulation. This requires the bank to obtain funds worth the amount equal or greater to LaR within six months. Failure to obtain this will result in insolvency. The boundary parameters can be set as follows:

If $V_{TOT} = \left(-SN(-d_1) + Ke^{-r(T-t)}N(-d_2)\right) * Total\ Stock \geq LaR$ in six months,

then the bank is solvent = S.

If $V_{TOT} = \left(-SN(-d_1) + Ke^{-r(T-t)}N(-d_2)\right) * Total\ Stock < LaR$ in six months,

then the bank is insolvent = IS.

The final outcome for the bank either going into IS or S is added for each period as shown in the table below.

Figure 3.2
Table Showing the Outcome of Bank Entering "S' or "IS"

Period	Vtot >= LaR	Outcome
June 1998	Yes	S
Dec 1998	Yes	S
Jan 1999	No	IS
June 1999	No	IS
Dec 2000	No	IS
Jan 2001	No	IS
June 2001	No	IS
Dec 2001	No	IS

From the table above, $N_{IS} = 6$ and $N_S = 2$.

Using the summary of the outcome above, the probability of experiencing the stage of insolvency, IS, will be calculated using the binomial distribution formula shown below:

$$\text{Probability(Insolvency} - \text{IS)} = \frac{n!}{N_{IS}!(n-N_{IS})!}(p^{N_{IS}})(1-p)^{(n-N_{IS})} \qquad (3.3)$$

Where n = the number of data period, i.e., June 1998 to June 2010,

$$p = \frac{N_{IS}}{(N_{IS}+N_S)}.$$

The probability of facing insolvency, IS, will show the likelihood of the bank not meeting its obligation within the six-month period to overcome liquidity risk and thus face the risk of declaring bankruptcy. Finally, this provides an overview of the liquidity risk based on LaR value and the probability of not being able to secure enough cash equal to LaR within six months and thus facing insolvency.

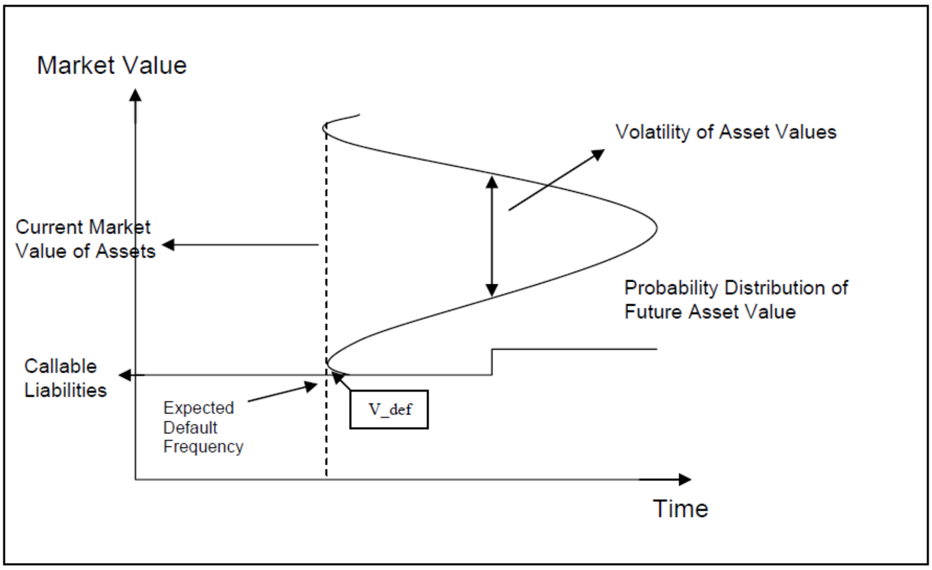
Application of Merton's Asset-based Model

Based on an earlier publication by the author ("Development of Stochastic Model to Evaluate Economic Capital for Concentration Risk," 2008), it was proved that Merton's asset-based theory can be used to build default correlations for measuring concentration risk. Similar to the binomial model, Merton's asset-based theory will be used in this paper to calculate the probability of facing insolvency and compared with the results from binomial distribution.

The point of liquidity risk is significant when the value of assets exceeds the value of liabilities as shown by Bessis (2006).

In Merton's asset-based model, it can be shown that when the value of a firm's assets approach the point of callable liabilities (as shown below), a minimum point of default is triggered.

Figure 3.3
Merton's Asset-based Model Showing the Point of Callable Liabilities



At the point of market value of the assets approaching callable liabilities (point of V_{def}), the firm begins to feel liquidity pressure because it has already defaulted on its first obligation, i.e., failed to make payment to its creditors. Therefore at this point, as highlighted by Merton's asset-based model, this is a perfect scenario of the bank's failure to meet its obligation and it can be shown that the probability of a bank defaulting its payment can be calculated using the approach in the later section.

The approach taken here is pure mathematics involving trial and error. First, a model is developed using the asset distribution of the bank over the data period obtained earlier. The distribution is fitted over the best fitting curve that is statistically possible using trial and error. The only curve the asset distribution is not fitted with would be the binomial distribution because this would create an overlap with the binomial model used previously.

The asset distribution is fitted to the following probability distribution curves as recommended by "The Professional Risk Manager's Handbook: A Comprehensive Guide to Current Theory and Best Practices" (2006), which is commonly used by risk managers worldwide:

- Lognormal distribution
- Inverse Gaussian distribution
- Poisson distribution
- Uniform distribution
- Normal distribution

The asset distribution curve from the bank's data set over the entire period from 1999 to 2010 is tried on each of the probability distribution curves and checked for any abnormal kurtosis arising that can result in very thick tails. An abnormal kurtosis would result in assets being overstated and can increase the liquidity gaps, which would finally overstate the liquidity risk. After the trial-and-error process, the inverse Gaussian distribution gave the most normal and perfect fit for the asset distribution. Therefore, the asset distribution can be defined as an inverse Gaussian probability density function defined in the following way:

$$f(x,\mu,\lambda) = (\lambda|2\pi x^3)^{\frac{1}{2}} e^{((-\lambda(x-\mu)^2|2\mu^2 x)} \quad (3.4)$$

Where x = the value of the asset, i.e., the main variable,

λ = the shape parameter, and

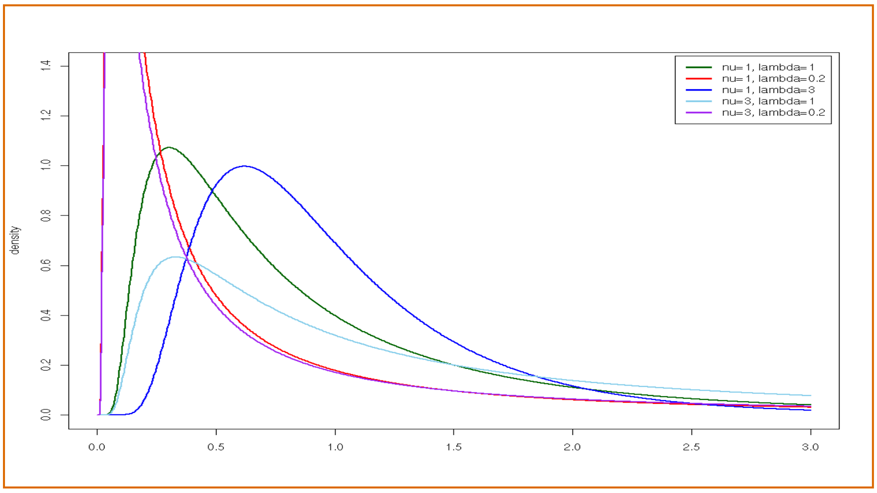
μ = the mean of the Gaussian distribution.

The probability of the bank reaching insolvency can be estimated by calculating the area under the above probability density function for $f(x,\mu,\lambda)$. This can be done by integrating the inverse Gaussian probability density function over the asset interval between 0 and V_{def} as shown below.

$$Probability = \int_0^{Vdef} (\lambda|2\pi x^3)^{\frac{1}{2}} e^{((-\lambda(x-\mu)^2|2\mu^2 x)} dx \tag{3.5}$$

The value λ is determined by looking at the different probability density function chart as shown below in Figure 3.4

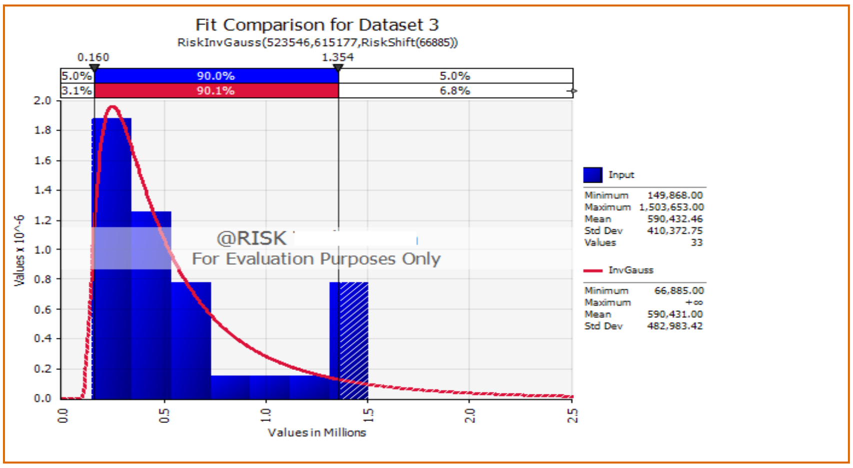
Figure 3.4
Chart for Various Curves Fitting Inverse Gauss Distribution



(Rausand, Marvin, and Arnljot Hoyland. *System Reliability Theory*. 247.)

Each Gaussian curve is compared to the fitting curve obtained from the distribution of the bank's assets done previously. The best fitting distribution of the assets, being inverse Gaussian distribution obtained using @RISK Industrial, is as follows.

Figure 3.5
Best Fitting Curve With Inverse Gauss Distribution



The best fitting curve in Figure 3.5 is very similar to the red curve in Figure 3.4 because both curves are positively skewed with the same height at the peak points, therefore the value of λ = 0.2. The integration of inverse Gaussian function is very complex and therefore the trapezium rule is used as an approximation to estimate the probability function. The trapezium rule can be used in the following way:

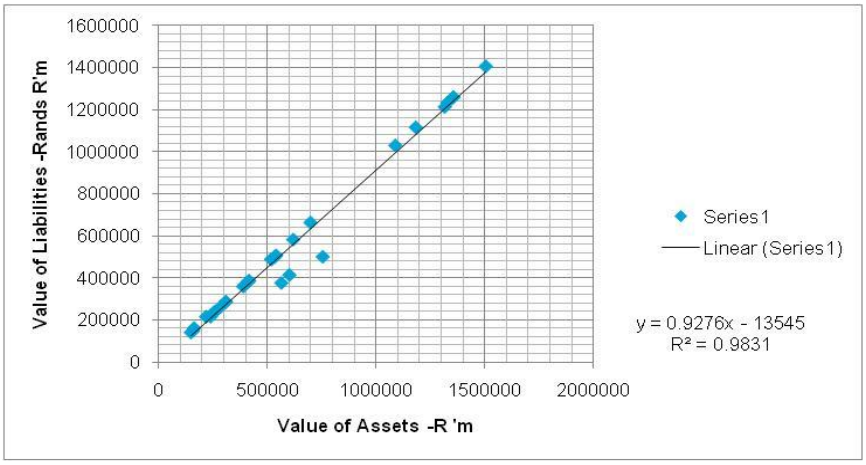
$$\int_0^{V\_def} f(x,\mu,\lambda)dx \approx \left(\frac{h}{2}\right)\left((y_o + y_n) + 2(y_1 + y_2 + \cdots + y_{n-1})\right) \qquad (3.6)$$

Where $h = \frac{V_{def}}{n}$ with n being the number of widths between the intervals in the trapezium. The value of h is determined by iterating the width into the above formula until the final probability converges.

The value of $V_{\_def}$ is calculated by determining the best polynomial curve over a data set plotted between the bank's total assets and liabilities over the period June 1998 to June 2010 as shown below. The degree of fitting is measured by the best R^2 (R-squared) achieved using the best curve fitting option in MS Excel.

Based on the best R-squared achieved, being 98.55 percent, the best fitting curve is found to be expressed in the following way.

Figure 3.6
Graph Showing the Best Fitting Linear Curve



The best fitting curve can be expressed as follows:

$$y = 0.9276x - 13545$$

where y is value of liabilities and x is value of assets.

To determine V_{def}, which would complete the integration in equation 3.5, the point of callable liabilities from Merton's asset-based model in Figure 3.3 is taken as the point when liquidity risk begins to enter into the bank. This is the first stage when the bank fails to meet its obligation, i.e., defaults on its payment due to liquidity shortage. At the point of callable liabilities, V_{def} can be defined in the following way:

V_{def} = point of callable liabilities = value of liabilities.

Therefore, the asset value V_{def} at the point of callable liabilities is equal to liabilities, and

$$x = y = V_{def}.$$

The best fitting polynomial can be found as follows by solving for x, which equals to V_{def},

$$x = V_{def} = 0.9276x - 13545.$$

Finally the value of the bank's asset at the point of callable liabilities V_{def} where it fails to secure enough liability to cover its obligation can be equated and calculated in the following way using the formula below,

$$V_{def} = 0.9276V_{def} - 13545.$$

Therefore,

V_{def} = 7027 in R'm.

The value of asset as at V_{def} is equal to R 7 billion and can be used to calculate the integral in equation 3.5 and thus the final measure, being the probability of a bank defaulting its obligation, can be calculated by completing the trapezium formula in equation 3.6.

4 Monte Carlo Simulation

Monte Carlo simulation was pioneered for derivative valuation in 1977 by Phelim Boyle. The process of simulation involves statistical sampling, a method that randomly selects a specific number between the given ranges and executes the selection based on uncertainty. This level of uncertainty is covered by the Monte Carlo method viewing the maximum possible scenarios and the number or the selected output is matched with these scenarios. For the simulation to be accurate, it's recommended to provide 10,000 scenarios or more. The more scenarios are utilized by the random generator, the greater the accuracy of the simulation. Other sampling techniques are also available, such as Latin hypercube sampling. In this study, Monte Carlo is used because it's the most common and effective tool for financial risk modeling purpose.

For measuring liquidity risk in this paper, LaR was calculated using Monte Carlo simulation. The first step taken was in finding the best statistical fitting curve for the liquidity gap data distribution shown below.

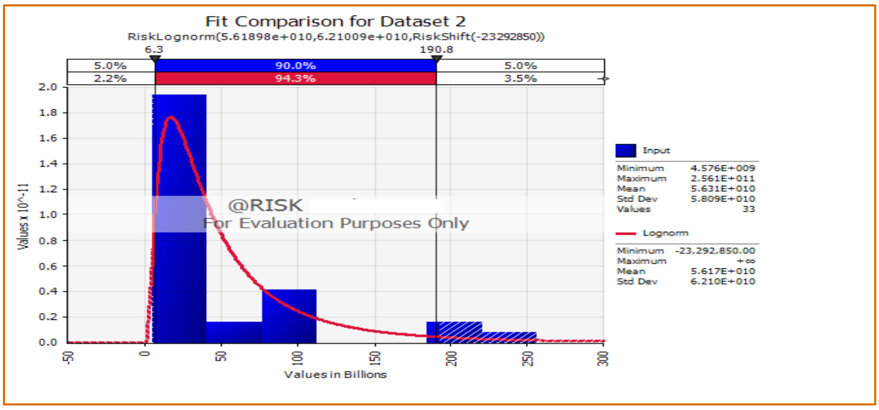
Figure 4.1
Data Set for Liquidity Gaps

Period	Liquidity Gap (R'm)
Jan 1998	156000
June1998	146000
Jan 1999	234000
June199	176456
Jan 2000	196874
June 2000	347250
Jan 2001	689220

(Note: These figures are not actual.)

The data distribution for the liquidity gap above is fitted with the best fitting curve as shown below.

Figure 4.2
Data Distribution of Liquidity Gap



The best fitting distribution, being the lognormal distribution above, is then simulated using the Monte Carlo method by applying 10,000 iterations of various uncertain or risk scenarios generated within the program. The software used is @RISK Industrial and gives the final Monte Carlo results (shown in section 5).

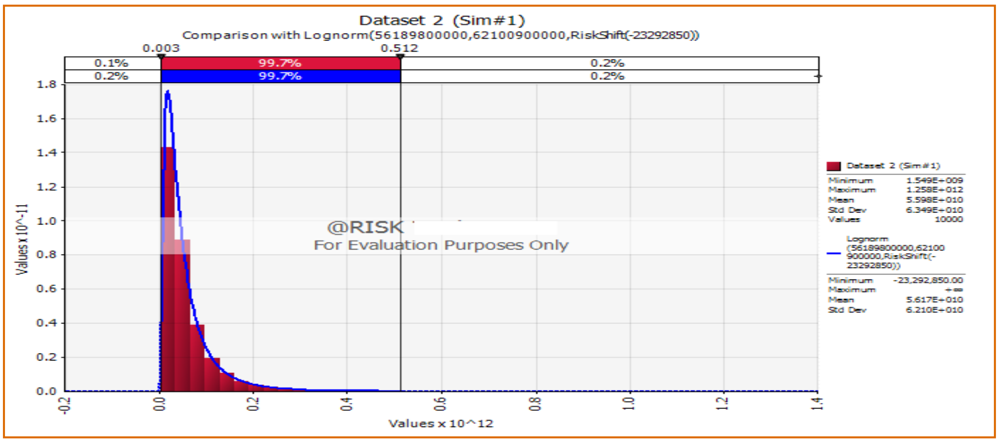
5 Results

Monte Carlo Simulation Output Measuring Liquidity Risk

Using @RISK Industrial, the liquidity gaps from the data set underwent Monte Carlo simulation to calculate the liquidity requirement within a confidence level of 99.65 percent over a six-month period.

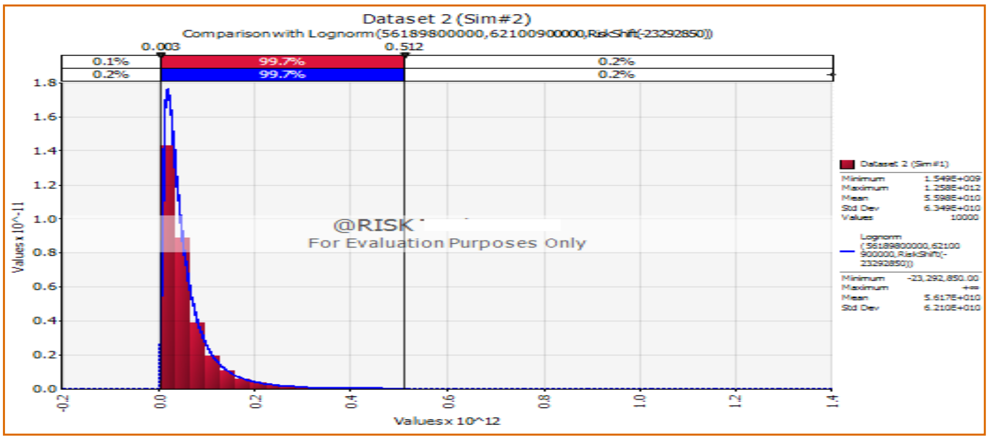
The simulation results are as follows.

Figure 5.1
Monte Carlo Simulation 1



A second simulation was run using the same data set to ensure that Monte Carlo is consistent.

Figure 5.2
Monte Carlo Simulation 2



Both the simulation converged to the exact LaR value of 0.512 x10^12, which equates to R512 billion worth of liquidity required by the bank within six months to remain solvent. The results also show there is 99.65 percent confidence level that the required liquidity within six months will not be more than R512 billion inclusive of liquidity buffer. This provides a measure of liquidity risk for the bank.

Probability of Insolvency Using Binomial and Black-Scholes Models

Based on the Black-Scholes model, the total value of the put option is calculated by multiplying the value of the put option by the total shares held over the specific period in the data set.

Figure 5.3
Stock Value Using B-S Model and Total Value of Put Compared With LaR

Period	Black-Scholes Value Put Option i.e V(Rands)	V_tot (R'm)	LaR (R'm)	IS or S
Jun-98	2.01	2428.58	512000	IS
Jun-99	1.09	1310.85	512000	IS
Dec-99	1.11	1520.71	512000	IS
Jan-00	1.97	2704.15	512000	IS
Jun-00	1.13	1549.25	512000	IS
Dec-00	1.19	1664.45	512000	IS
Jan-01	2.15	3031.34	512000	IS
Jun-01	1.13	1577.38	512000	IS
Dec-01	1.49	2120.59	512000	IS
Jan-02	3.20	4239.46	512000	IS
Jun-02	1.47	1948.09	512000	IS
Dec-02	1.95	2595.61	512000	IS
Jan-03	3.61	4807.39	512000	IS
Jun-03	1.97	2618.09	512000	IS
Dec-03	1.84	2204.71	512000	IS
Jan-04	3.58	4753.03	512000	IS
Jun-04	1.85	2462.64	512000	IS
Dec-04	2.00	2679.93	512000	IS
Jan-07	2.51	3345.97	512000	IS
Jun-07	3.10	4175.83	512000	IS
Jun-08	4.48	6087.93	512000	IS
Dec-08	9.46	562917.46	512000	S
Dec-07	6.22	1001169.00	512000	S
Jun-09	5.16	7881.37	512000	IS
Jun-10	5.30	7936.60	512000	IS

Application of the binomial model gives the final probability of IS as shown below in the table.

Figure 5.4
Table Showing the Results From the Binomial Model

Volatility in %	9.51
N_S	2
N_IS	23
p_IS	0.92
N	25
(N-N_IS)	2
Probability of IS	**60.53%**

Based on Black-Scholes and binomial models, the probability of the bank becoming insolvent, i.e., failing to meet the liquidity requirement of LaR within six months, is equal to 60.53 percent.

Probability of Bank Defaulting on Payments Using Merton's Asset-based Model

Based on Merton's asset model, the point of callable liabilities $V_{\_def}$ is R7 billion.

Applying the trapezium rule with the width h converging to R 0.8 billion between the interval R 1 billion and R 7 billion.

Figure 5.5
Table Showing the Mean Asset Value Using the Trapezium Rule

Mean Asset Value -R' billion	590
Lambda	0.2
V_def	7
h	0.8

Figure 5.6
Calculation of Equation 3.6 Using the Trapezium Rule

x	fxn 1	fxn 2	exp(-fxn2)	InvGauss function
1	0.178	0.100	0.905	0.161
1.8	0.074	0.055	0.946	0.070
2.6	0.043	0.038	0.963	0.041
3.4	0.028	0.029	0.971	0.028
4.2	0.021	0.023	0.977	0.020
5	0.016	0.020	0.981	0.016
5.8	0.013	0.017	0.983	0.013
6.6	0.011	0.015	0.985	0.010

The final probability based on Merton's asset-based model can be calculated using the results from the trapezium rule. (All calculations are done in MS Excel 2007.)

Figure 5.7
Table Showing the Final Probability

Final Probability	
Trapezium rule	25.04%

Based on Merton's asset-based model, the probability of the bank defaulting, i.e., failing to meet its payment obligation over the next six months, is 25 percent.

(Note: Data used are not real and do not originate from a particular bank. Results from the model should not be used to draw any conclusion on the state of liquidity risk for any bank.)

6 Conclusion

The study shows that measuring liquidity risk for an emerging bank in South Africa is possible based on the measurement of liquidity gaps during a six-month cycle and simulating them using the Monte Carlo method. Also, the results show that the risk of facing insolvency can be measured using the Black-Scholes model, which allows banks to estimate their liquidity requirement over a six-month period.

"To improve the accuracy of this model, it's important to measure dynamic liquidity gaps."

The results have proved that using Merton's asset-based model, the probability of this bank not being able to meet its payment obligation can be estimated. These estimates, however, do not take into account the volatility of the stock prices, which can later have an impact on the decision to enter either into a put contract or retain the shares until stock prices increase. The models used in this study have shown that a simulation-based method can provide a measure of liquidity risk on those assets that are pro-cyclical and show a behavior that can fit into a model. In considering the liquid assets that can be converted to cash and provide immediate liquidity, banks must include various scenario-building processes into their measurement framework. The measurement framework provides a single factor scenario that is purely based on the bank's liquidity gap. The type of liquidity gaps used in this study for measuring liquidity risk is a static liquidity gap that results from existing assets and liabilities only. To improve the accuracy of this model, it's important to measure dynamic liquidity gaps. Liquidity gaps that are dynamic add the projected new credits and new deposits to the amortization profiles of existing assets. For this to be implemented, forecasting techniques need to be applied that can project the future movement of these assets and liabilities. For the results to be more accurate, the idiosyncratic factors driven by a macro economic cycle should be factored into Merton's asset-based model using a multifactor correlation matrix that gives a covariance factor. However, the challenge still remains with the Black-Scholes model as this formula is bounded to pricing of stocks and risk-free return from the bank's risk profile.

In conclusion, the models used in this study provide a static framework for measuring liquidity risk. However, the challenge for banks would be in the following areas when using these models:

- Integrating this model with the centralized database that can form a measurement framework per Basel III requirements.
- Developing the measurement process that uses the stochastic model into a supervisory review requiring continuous monitoring as per Basel III.
- Differentiating the final results from nonstatic dynamic liquid gaps as opposed to using data directly from static liquidity gaps.
- Inclusion of third-party risk coverage that are involved in buying the stocks from the bank into the model to define a more accurate liquidity risk measure within a six-month holding period.
- Measuring the ability of the banks to market its stocks prior to entering the put contract. This is necessary to raise public confidence prior to selling the shares and must be taken into account when estimating the bank's liquidity position.
- The current model does not take account of withdrawal rate and, for liquidity position to be measured more accurately, the withdrawal rate of funds from customers must also be inclusive.

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Farooqui qualified as an engineer and worked for various multinational companies offering technical service. During the course of his career, he developed a great deal of interest in financial risk modeling, which later became his area of expertise. After completing his MBA, he joined Standard Bank group in South Africa as a senior risk analyst where he developed behavioral scorecard monitoring frameworks for retail portfolios and was involved in the validation of Probability of Default (PD), Exposure at Default (EAD) and Loss Given Default (LGD) for the Basel II project. He extended his modeling skills at Riyad Bank in Saudi Arabia where he developed the first simulation-based economic capital model for credit concentration risk under Basel II. This achievement enabled him to enter the 2008 Canadian Actuarial Symposium where his paper on the VaR-based credit concentration risk model was selected as an international publication. Since then, he has been providing consulting service on Basel II, credit risk modeling and enterprise risk management. Currently, he is working at PwC in South Africa as an enterprise risk consultant and is fully associated with Professional Risk Manager's International Association (PRMIA) and Global Association of Risk Professionals (GARP).

In his free time, he enjoys reading, going to the gym and playing competitive squash.