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GROUP ANNUITY MORTALITY

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ELMER R. BENEDICT:

Mr. Peterson in this very fine work has done for us in the field of group annuities what Messrs. Jenkins and Lew have done in the field of individual annuities. Like them, he has provided us with a new mortality table and a set of projection factors, and has also suggested the feasibility of a method of calculating premiums and reserves providing for future improvement in mortality. For this, he and his colleagues certainly deserve the thanks of all of us and particularly those of us who are concerned with pensions.

The Group Annuity Table for 1951 is most welcome and especially so since it is the first published table based throughout upon group annuity mortality experience. The data on which this table is based, the methods of construction, and its general consistency with the results of other experiences on retired lives, are such as to engender confidence in this table as conservatively representative of current mortality experience under group annuity business. By preparing and publishing separate tables for males and females, and at the same time investigating the age adjustments required on the male table to reproduce corresponding values for female lives, he has given the actuary freedom of choice in selecting the method to be employed in connection with values for female lives. Personally, I am inclined to favor using the male table set back 5 years for determining values on female lives, since this seems to give conservative but satisfactory results even on those cases with a very large proportion of female lives. Certainly the convenience and simplicity afforded by a single table for both sexes outweigh the consequences of a small departure from theoretically correct values.

I am particularly pleased with the method of graduation used in connection with the data. The graduation formula does appear to graduate the data with, in Mr. Peterson's words, "scientific impartiality," giving a close fit where the data are heavy and introducing a strong Makeham influence where the data are light. This method also seems to be consistent with the graduation used on the a–1949 Table, projected rates of which were used for the younger ages of this table. Perhaps the Makeham influence in the resulting table is strong enough so that the law of uniform

seniority may be used for joint life values without appreciable loss of accuracy.

In addition to considering Projection Scale B, the author has suggested a new scale which he has designated Projection Scale C. If, as the author has pointed out, Projection Scale B is appropriate for individual annuities, it would seem that a higher scale of improvement should be used in connection with group annuities where there appears to be more room for mortality improvement. For this reason and also because I believe that the mortality basis should be self-sufficient so that the loading and interest rates may appear as realistic as possible to the layman, I am inclined to prefer Mr. Peterson's Projection Scale C in the group annuity area.

It is interesting to note that the male q_x curve of the Group Annuity Table for 1951 and that of the *a*–1949 Table projected one year on Projection Scale B or C cross just below age 89, the oldest age to which the projection scales are applicable. For ages 59 to 88 (the major area of difference between Scales B and C) the former table has the higher rates and thus the greater room for improvement, while above age 88 the *a*–1949 Table has the higher rates and hence the greater improvement potential.

Let us pass on now to the general question of the desirability and practicability of introducing into annuity premiums or reserves a provision for future improvement in mortality.

Ideally, a premium rate or reserve value should incorporate the actuary's best judgment with respect to all of the factors which influence the cost of a given benefit. Certainly one of these factors is the future change in the level of mortality rates. Some may argue that no specific provision should be made in premium rates or reserves on account of future improvement in mortality which is a function of attained age and calendar year, because of the difficulty of predicting such improvement. However, while such an estimate is admittedly subject to error, it would seem that the introduction of a reasonable estimate deduced from a careful analysis of a considerable volume of data (such as was done in the Jenkins-Lew paper and the Peterson paper) may be expected to yield results closer to the actual than would be the case if no such estimate were to be introduced. Certainly we would expect greater equity between ages even if there were no substantial differences between premium and reserve aggregates. Hence, while there may be differences of opinion as to the pattern and degree of future improvement in mortality, nevertheless the introduction of a reasonable estimate of this factor will give added flexibility which may be expected to result in values closer to the actual by attained age and calendar year.

The ultimate decision as to whether in a given area such provision

should be made will of course be the result of a careful weighing of the advantages to be gained by such a course against the additional complications consequent upon introducing such a provision. As between premiums on individual annuity business and those on group annuity business, it would seem more desirable to introduce such a provision on the individual annuity business where premium equity by age is more important. However, in the case of deposit administration and similar types of group annuity contracts, it would seem particularly desirable to provide for anticipated lower mortality because of the deferred application of rates and values in these contracts. With respect to reserves, on the other hand, it would seem relatively more desirable to introduce such a provision in connection with reserves under group annuity contracts since reserves are of fundamental importance in experience rating dividend formulas and rate adjustments. As to the relative practicability, it would seem easier to make such a provision in connection with premiums and reserves under individual annuity contracts because of their more limited variety and simpler technical structure.

There seem to have been five general methods advanced by which provision for future improvement in mortality can be made. These may be described as follows:

1. The use of a single table together with appropriate age adjustments. Messrs. Jenkins (*TASA* XLVII), Fassel and Noback (*TSA* II) have treated the specialized Gompertz case, and Mr. Hoskins, in his paper presented at this meeting, has given an illustration of an approximate method applied to the *a*–1949 Table.

2. The use of projection factors as illustrated by Messrs. Jenkins and Lew (*TSA* I) in their work in connection with the *a*–1949 Table.

3. The use of supplementary commutation functions to determine approximate values as shown in Mr. Sternhell's paper (*TSA* II).

4. The use of modern electronic equipment to obtain exact premium and reserve factors for each age and generation required, as suggested in Mr. Peterson's paper (*TSA* IV).

5. The use of a single table the mortality rates of which are derived from the mortality rates applicable to a particular calendar year by projecting the mortality rate for each attained age in that calendar year for the number of years which will on the average elapse before a group of new entrants at all ages will attain the given age. This method is described in the latest *Journal* of the Institute of Actuaries (*JIA* 78).

Now, if I may, I should like to comment briefly on each of these five methods, and then suggest a sixth possible method which, although approximate, gives results very close to the exact values.

With respect to the first method, I feel that Messrs. Fassel and Noback

have done as good a job as could be done in fitting a Gompertz curve to the *a*-1949 Table. Consequently, the magnitude of the resulting errors is a good index to the suitability of the method. A comparison of the errors produced by this method with those produced by some of the other methods leads me to believe that the choice of this method must rest primarily upon its simplicity of application. However, several other methods appear not only to be practicable but also to produce closer approximations and therefore seem preferable to this method in many areas. In the area of reserves for individual annuities and life income settlements (for which the Progressive Annuity Table was prepared) where relative equity among ages is not important as long as the aggregate reserve is proper, the Gompertz method would appear to be quite satisfactory.

The use of a single table with an age adjustment has, of course, much to commend it, if a satisfactory degree of accuracy can be achieved thereby. In this respect, therefore, the paper presented at this meeting by Mr. Hoskins is of great interest. With respect to the *a*-1949 Table with Projection B, his method gives values very much closer to the exact values than those yielded by the Progressive Annuity Table and yet it is as easy to apply. Consequently, as between these two methods of utilizing a single table together with an age adjustment, I feel the preference must lie with the Hoskins method, certainly with respect to the *a*-1949 Table. I have not had the opportunity since the receipt of the galley proofs to apply the method presented in Mr. Hoskins' paper to the new group annuity mortality basis presented in Mr. Peterson's paper. Perhaps another member has had such an opportunity and will give us the benefit of his results. I am sure many of us will be greatly interested in the results of such an application of this method.

The method of projection factors outlined by Messrs. Jenkins and Lew is, I believe, neither simple nor mechanical enough in operation to be employed in practice. Under the projection factor method, the procedure seems to require a considerable number of generation tables, the calculation thereon of projected values applicable to a given calendar year, the calculation of the ratio of these values to the corresponding values on the unprojected table, and then the application of these ratios (with or without intermediate interpolation) to a complete set of unprojected values. It seems to me that a simpler process can be evolved by concerning ourselves with pivotal values applicable to a selected pair of generations rather than with pivotal values applicable to a given calendar year. Since the latter part of this discussion is devoted to this, I shall defer further comments until then.

The method of special commutation functions suggested by Mr. Stern-

hell appears to be practicable of application to the simpler types of annuities, but very difficult of application in the case of more complicated annuity forms such as are found under group annuities. One of the most interesting consequences of his approach is the simple year-by-year adjustment of reserve aggregates to reflect the mortality expected to be experienced beyond the date of valuation. Although Mr. Sternhell's method is very appealing from the standpoint of the closeness of the approximations, I am dubious about its practicability in the group annuity area.

Mr. Peterson has suggested in his paper that with the aid of modern electronic equipment it may perhaps be practicable to determine exact values for any attained age and calendar year. As yet I have not delved deeply enough into the problems of such an approach and the solutions thereto to justify drawing any conclusions as to its practicability. Mr. Peterson has mentioned that one of his colleagues is preparing a paper on this subject, and I look forward to studying it.

The fifth method would seem to be an excellent approach if a single table is to be used without any age adjustment. It does seem to me, however, that among the various approximate methods which have been suggested there are some which will give in general closer results for each individual combination of age, calendar year and form of annuity than this method and which at the same time are not so onerous in application as to be impracticable. Certainly in the field of individual annuities there would appear to be such methods and possibly also in the area of group annuities.

It is worth noting that two annuity tables in current use, *viz.*, the 1937 Standard Annuity Table (and setbacks thereof) and the Prudential 1950 Group Annuity Valuation Table implicitly follow this fifth method. In both of these tables the percentage margin for improvement in mortality as measured against the Group Annuity Table for 1951 increases with age up to the very high ages. With respect to either Projection Scale B or C, this is equivalent to projecting mortality rates for periods which increase with increasing age.

I should now like to suggest a sixth possible method of determining premium rates and reserve factors which provide for future improvement in mortality. For purposes of reference herein, I shall call this method the "text" method. This text method, although approximate, will give results very close to the exact generation values. Only two, or at the most three, generation tables are required in place of the usual single table, and no special commutation symbols are required—the process in the main being the same as that now employed in connection with single static tables.

This approximate method is, I might say, a rather obvious corollary of Mr. Sternhell's excellent work.

To provide a background for understanding the nature of the approximation, I should like to refer to Mr. Sternhell's paper. In that paper he demonstrates that the value of any function with provision for future improvement in mortality can be represented very closely by the sum of three factors. The first of these factors is the value of the function on the base year static table; the second factor is the increment required to be added to the first factor to obtain the value in the base year of the function with provision for future improvement in mortality; and the third factor is the additional amount which must be added on account of each year which has elapsed from the base year to the year as of which the value of the function is to be determined.

Sternhell's formula tells us, therefore, that, for the type of mortality improvement so far assumed, the value for a given age of a function providing for improvement in mortality will vary in an approximately linear fashion from generation to generation, and also that the relative magnitude of this variation will be small. This suggests that, by linear interpolation between two exact values computed at the given attained age on two pivotal generation tables, it should be possible to obtain a good approximation to the value of a given function in any calendar year at that attained age together with the approximately constant yearly change in value, $^{\Delta}f_x$, without introducing any new commutation symbols or other complications in the regular formulas.

The practicability of an approximation such as this will depend on the maximum interval between the pivotal generation tables (in other words, the minimum number of such tables) consistent with interpolated and extrapolated results of the desired degree of accuracy. Since this method would appear to be a corollary of Mr. Sternhell's method, we should expect the magnitude of the errors introduced by this approximation to correspond to those indicated by his paper and therefore to be quite small. This indeed seems to be the case, as Tables 1, 2 and 3, discussed below, will indicate.

Before proceeding to discuss these tables, it should be mentioned that mortality and commutation functions pertaining to calendar years prior to the base year (the calendar year for which the basic unprojected table is applicable) would have to be computed for one or more of the pivotal generation tables in order that values pertaining to the reference year (the calendar year as of which projected values such as premiums are calculated) may be interpolated or extrapolated therefrom. The required mortality rates on the pivotal generation tables for years prior to the base

year will be obtained by a backward projection using the same projection scale as is used in the forward projection. This backward projection and the determination of values applicable to calendar years prior to the base year should not cause any concern since all we are interested in are the mathematical relationships which exist among the values on the different generation tables and which arise from the use of a given set of projection factors and a basic table. Any pivotal values which may be computed for years prior to the base year serve only as way stations to the desired result —always a value applicable to either the base year or a later one.

Since the generation tables calculated by Mr. Peterson in connection with his recent paper do not show values applicable to calendar years prior to 1951, it was not possible to obtain from his tables values for the year 1952 by the text method, or to make as complete tests of the method as might be desired. It would, of course, be a relatively simple matter to compute the missing section of a given generation table. However, since the available tables appear to permit sufficient tests to illustrate the use and accuracy of the method, they have been used for illustrative purposes.

Table 1 illustrates the accuracy of the interpolation approximation if two tables 20 years apart are employed, while Table 2 illustrates the accuracy where the pivotal tables are 40 years apart. The illustrations are restricted, as mentioned before, by the truncation of the available generation tables. This is particularly noticeable in the 40 year interval where it was impossible on the basis of the truncated tables to obtain approximate values below attained age 50.

Application of the method is not confined to the area of interpolation bounded by the two diagonals representing the pivotal generation tables (see Chart 1 discussed below); very good results may also be obtained by extrapolation outside this area—thus further limiting the number of tables needed. Table 3 illustrates the accuracy of extrapolation at the higher ages for an extrapolation range of up to 20 years. Somewhat higher errors might be produced at the younger ages for the same extrapolation range, since mortality improvement has a relatively greater effect on annuities issued at the younger ages.

In studying these three tables and comparing the accuracy of the text method with that of other methods, it should be borne in mind that the values in these tables are based upon Projection Scale C, whereas the illustrations of other methods have generally been based upon Scale B. If the text method were to be employed in connection with Projection Scale B, we would expect even closer results. In any event the pivotal generation tables selected for use in any given area would be selected only after careful consideration of the desired accuracy, the permissible number of

TABLE 1

ILLUSTRATION OF THE TEXT METHOD WITH RESPECT TO INTERPOLATION USING PIVOTAL TABLES WITH A 20 YEAR INTERVAL

MALE LIVES

TYPE OF ANNUITY	AGE AT ISSUE (x)	AGE AT MATURITY (y)	EXACT PREMIUM FOR AGE AT ISSUE (x) ON GIVEN GENERATION TABLE a_n (WHERE n = AGE ATTAINED IN 1952)						APPROXIMATE PREMIUM FOR AGE AT ISSUE (x) FOR GENERATION TABLE a_n (INTERPOLATED FROM (1) AND (3))		ERROR (4)—(2)	PERCENTAGE ERROR (5)÷(2)	RATIO (1)÷(3)
			n	Exact Value (1)	n	Exact Value in 1972 (2)	n	Exact Value (3)	n	Approximate Value in 1972 (4)	(5)	(6)	(7)
Immediate Life Annuity...	35	.....	10	25.8782	15	25.7101	30	25.1681	15	25.7007	−.0094	−.04%	102.82%
	40	.....	10	24.1745	20	23.8052	30	23.4089	20	23.7917	−.0135	−.06	103.27
	60	.....	30	15.1610	40	14.7184	50	14.2508	40	14.7059	−.0125	−.08	106.39
Deferred Life Annuity Due*†	35	65	10	6.0874	15	5.9609	30	5.5593	15	5.9554	−.0055	−.09%	109.50%
	40	65	10	6.9261	20	6.6391	30	6.3355	20	6.6308	−.0083	−.13	109.32
	60	65	30	11.5029	40	11.0741	50	10.6220	40	11.0625	−.0116	−.10	108.29
	35	70	10	4.1509	15	4.0444	30	3.7093	15	4.0405	−.0039	−.10	111.91
	60	70	30	7.6749	40	7.2946	50	6.8970	40	7.2860	−.0086	−.12	111.28
Deferred Life Annuity Due† with 10 Years Certain...	35	65	10	6.3255	15	6.2115	30	5.8498	15	6.2066	−.0049	−.08%	108.13%
	40	65	10	7.1971	20	6.9391	30	6.6665	20	6.9318	−.0073	−.11	107.96
	60	65	30	12.1039	40	11.7442	50	11.3678	40	11.7359	−.0083	−.07	106.48
	35	70	10	4.4626	15	4.3679	30	4.0689	15	4.3642	−.0037	−.08	109.68
	60	70	30	8.4191	40	8.1000	50	7.7654	40	8.0923	−.0077	−.10	108.42

TABLE 1—*Continued*

Type of Annuity	Age at Issue (x)	Age at Maturity (y)	Exact Premium for Age at Issue (x) on Given Generation Table a_n (where n = age attained in 1952)						Approximate Premium for Age at Issue (x) for Generation Table a_n (Interpolated from (1) and (3))		Error (4)—(2)	Percentage Error (5)÷(2)	Ratio (1)÷(3)
			n	Exact Value (1)	n	Exact Value in 1972 (2)	n	Exact Value (3)	n	Approximate Value in 1972 (4)	(5)	(6)	(7)
Deferred Life Annuity Due*†—with Interest—Full Cash Refund.....	35	65	10	9.1727	15	9.1490	30	9.0813	15	9.1499	.0009	.01%	101.01%
	40	65	10	10.3680	20	10.3127	30	10.2598	20	10.3139	.0012	.01	101.05
	60	65	30	16.5092	40	16.3709	50	16.2277	40	16.3685	−.0024	−.01	101.73
	35	70	10	7.3626	15	7.3691	30	7.4107	15	7.3746	.0055	.07	99.35
	60	70	30	13.3372	40	13.3337	50	13.3526	40	13.3449	.0112	.08	99.88
Deferred Life Annuity Due† —with Interest—Modified Cash Refund (Employee contributes $2 for each $1 of annual annuity)	35	65	10	6.4491	15	6.3382	30	5.9885	15	6.3340	−.0042	−.07%	107.69%
	40	65	10	7.2849	20	7.0279	30	6.7578	20	7.0214	−.0065	−.09	107.80
	60	65	30	11.8369	40	11.4251	50	10.9922	40	11.4146	−.0105	−.09	107.68
	35	70	10	4.6343	15	4.5509	30	4.2923	15	4.5488	−.0021	−.05	107.97
	60	70	30	8.1238	40	7.7802	50	7.4232	40	7.7735	−.0067	−.09	109.44

NOTE.—The above values are all for annuities of 1 a year and are based upon the Group Annuity Table for 1951 with Projection C, and 2½% interest. Values are net values except in the case of the Full Cash Refund Annuity.

* Includes a loading of 8% of gross premium.

† Death benefit accumulation rate = premium interest rate = 2½%.

TABLE 2

ILLUSTRATION OF THE TEXT METHOD WITH RESPECT TO INTERPOLATION USING PIVOTAL TABLES WITH A 40 YEAR INTERVAL
MALE LIVES

TYPE OF ANNUITY	AGE AT ISSUE (x)	AGE AT MATURITY (y)	EXACT PREMIUM FOR AGE AT ISSUE (x) ON GIVEN GENERATION TABLE a_n (WHERE n = AGE ATTAINED IN 1952)						APPROXIMATE PREMIUM FOR AGE AT ISSUE (x) FOR GENERATION TABLE a_n (INTERPOLATED FROM (1) AND (3))		ERROR (4) − (2)	PERCENTAGE ERROR (5) ÷ (2)	RATIO (1) ÷ (3)
			n	Exact Value (1)	n	Exact Value in 1972 (2)	n	Exact Value (3)	n	Approximate Value in 1972 (4)	(5)	(6)	(7)
Immediate Life Annuity...	50	.....	10	20.3081	30	19.4638	50	18.5053	30	19.4067	−.0571	−.29%	109.74%
	60	.....	10	15.9718	40	14.7184	50	14.2508	40	14.6811	−.0373	−.25	112.08
	65	.....	10	13.5873	45	12.2263	50	12.0105	45	12.2076	−.0187	−.15	113.13
Deferred Life Annuity Due*†	50	65	10	9.0692	30	8.3498	50	7.5494	30	8.3093	−.0405	−.49%	120.13%
	60	65	10	12.2909	40	11.0741	50	10.6220	40	11.0392	−.0349	−.32	115.71
	50	70	10	6.1842	30	5.5711	50	4.9019	30	5.5431	−.0280	−.50	126.16
	60	70	10	8.3810	40	7.2946	50	6.8970	40	7.2680	−.0266	−.36	121.52
Deferred Life Annuity Due† with 10 Years Certain...	50	65	10	9.4240	30	8.7861	50	8.0794	30	8.7517	−.0344	−.39%	116.64%
	60	65	10	12.7717	40	11.7442	50	11.3678	40	11.7188	−.0254	−.22	112.35
	50	70	10	6.6485	30	6.1113	50	5.5191	30	6.0838	−.0275	−.45	120.46
	60	70	10	9.0102	40	8.1000	50	7.7654	40	8.0766	−.0234	−.29	116.03

TABLE 2—*Continued*

Type of Annuity	Age at Issue (x)	Age at Maturity (y)	Exact Premium for Age at Issue (x) on Given Generation Table a_n (where n = age attained in 1952)						Approximate Premium for Age at Issue (x) for Generation Table a_n (Interpolated from (1) and (3))		Error (4)−(2)	Percentage Error (5)÷(2)	Ratio (1)÷(3
			n	Exact Value (1)	n	Exact Value in 1972 (2)	n	Exact Value (3)	n	Approximate Value in 1972 (4)	(5)	(6)	(7)
Deferred Life Annuity Due*†—with Interest—Full Cash Refund.....	50	65	10	13.2209	30	13.0622	50	12.9206	30	13.0708	.0086	.07%	102.32%
	60	65	10	16.7691	40	16.3709	50	16.2277	40	16.3631	−.0078	−.05	103.34
	65	65	10	18.8417	45	18.2023	50	18.0976	45	18.1906	−.0117	−.06	104.11
	50	70	10	10.5921	30	10.6229	50	10.7543	30	10.6732	.0503	.47	98.49
	60	70	10	13.3862	40	13.3337	50	13.3526	40	13.3610	.0273	.20	100.25
Deferred Life Annuity Due† —with Interest—Modified Cash Refund (Employee contributes $2 for each $1 of annual annuity).....	50	65	10	9.4166	30	8.7493	50	8.0139	30	8.7153	−.0340	−.39%	117.50%
	60	65	10	12.5962	40	11.4251	50	10.9922	40	11.3932	−.0319	−.28	114.59
	65	65	10	14.8586	45	13.5172	50	13.3048	45	13.4990	−.0182	−.13	111.68
	50	70	10	6.6315	30	6.1033	50	5.5368	30	6.0842	−.0191	−.31	119.77
	60	70	10	8.7672	40	7.7802	50	7.4232	40	7.7592	−.0210	−.27	118.11

NOTE.—The above values are all for annuities of 1 a year and are based upon the Group Annuity Table for 1951 with Projection C, and 2½% interest. Values are net values except in the case of the Full Cash Refund annuity.

* Includes a loading of 8% of gross premium.

† Death benefit accumulation rate = premium interest rate = 2½%.

TABLE 3

ILLUSTRATION OF THE TEXT METHOD WITH RESPECT TO EXTRAPOLATION
PRESENT VALUE OF AN IMMEDIATE LIFE ANNUITY
MALE LIVES

AGE AT ISSUE (x)	EXACT PREMIUM FOR AGE AT ISSUE (x) ON GIVEN GENERATION TABLE a_n (WHERE n = AGE ATTAINED IN 1952)						APPROXIMATE PREMIUM FOR AGE AT ISSUE (x) FOR GENERATION TABLE a_n (EXTRAPOLATED FROM (1) AND (2))		NUMBER OF YEARS EXTRAPOLATED	ERROR (4) − (3)	PERCENTAGE ERROR (6) ÷ (3)	RATIO (1) ÷ (2)
	n	Exact Value (1)	n	Exact Value (2)	n	Exact Value in 1952 (3)	n	Approximate Value in 1952 (4)	(5)	(6)	(7)	(8)
	Using Pivotal Generation Tables with a 20 Year Interval											
70	45	9.9690	65	9.2343	70	9.0413	70	9.0506	5	.0093	.10%	107.96%
75	45	7.7534	65	7.2379	75	6.9685	75	6.9802	10	.0117	.17	107.12
80	45	5.7455	65	5.4514	80	5.2224	80	5.2308	15	.0084	.16	105.39
85	45	4.1327	65	4.0282	85	3.9205	85	3.9237	20	.0032	.08	102.59
	Using Pivotal Generation Tables with a 40 Year Interval											
70	25	10.6421	65	9.2343	70	9.0413	70	9.0583	5	.0170	.19%	115.25%
75	25	8.2362	65	7.2379	75	6.9685	75	6.9883	10	.0198	.28	113.79
80	25	6.0264	65	5.4514	80	5.2224	80	5.2358	15	.0134	.26	110.55
85	25	4.2340	65	4.0282	85	3.9205	85	3.9253	20	.0048	.12	105.11

NOTE.—The above values are all for annuities of 1 a year and are based upon the Group Annuity Table for 1951 with Projection C, and 2½% interest. All values are net.

tables, and the relative importance of various age ranges. In the case of group annuities, the generation tables selected as pivotal tables would probably be one of the following groupings (where the generation table is designated by the attained age in the reference year): 25 and 65; 30 and 60; 35 and 65; and 15, 45, and 75.

Chart 1 provides a graphic illustration of the text method where two pivotal generation tables for ages 25 and 65 in the reference year are used. The y axis of the chart represents attained age, and the x axis represents calendar year. To keep the chart on a two dimensional basis, each point of the chart may be thought of as containing the value of the given function —for example, the value of the immediate life annuity—applicable to the attained age and calendar year represented by its coordinates. The two diagonal lines designated α and β connect the values for all ages pertaining to generations aged 65 and 25, respectively, in the reference year of 1957. (While any year could, of course, be taken as the reference year, the year 1957 would be the central year if a company were to expect to continue its group annuity rates unchanged for contracts issued in calendar years 1953 through 1957 inclusive with a 5 year guarantee of rates. Accordingly, this year has been taken as the reference year for illustrative purposes.) The areas of interpolation and extrapolation, the directions in which the extrapolations are performed, and the area of exact values (at ages 90 and over) can be readily seen from this chart.

The relationships among the exact values, the approximate values produced by Mr. Sternhell's formula, and those produced by the text method may perhaps be best illustrated by means of a graphical representation. Accordingly, Chart 2 has been prepared to illustrate this relationship with respect to the value of the immediate life annuity to a male life age 65. In order to emphasize the relatively small errors of these approximate methods compared with the major question of incorporating a margin for future improvement in mortality, the value of the annuity without projection is also shown.

On a graph such as Chart 2, Mr. Sternhell's formula would be represented by a straight line. As pointed out in his paper, the values given by his formula (at least with respect to the a–1949 Table with Projection B) are generally in excess of the exact value, and consequently the "Sternhell" line is usually either tangent to the curve in the base year or slightly above the curve in that year. In subsequent years the Sternhell and exact values will diverge. The broken line representing values on the Sternhell method has been estimated from the relationship, based upon the a–1949 Table with Projection B, between his approximate values and the exact values which were shown in his paper.

CHART 1

ILLUSTRATION OF TEXT METHOD USING TWO GENERATION TABLES WITH A 40 YEAR INTERVAL

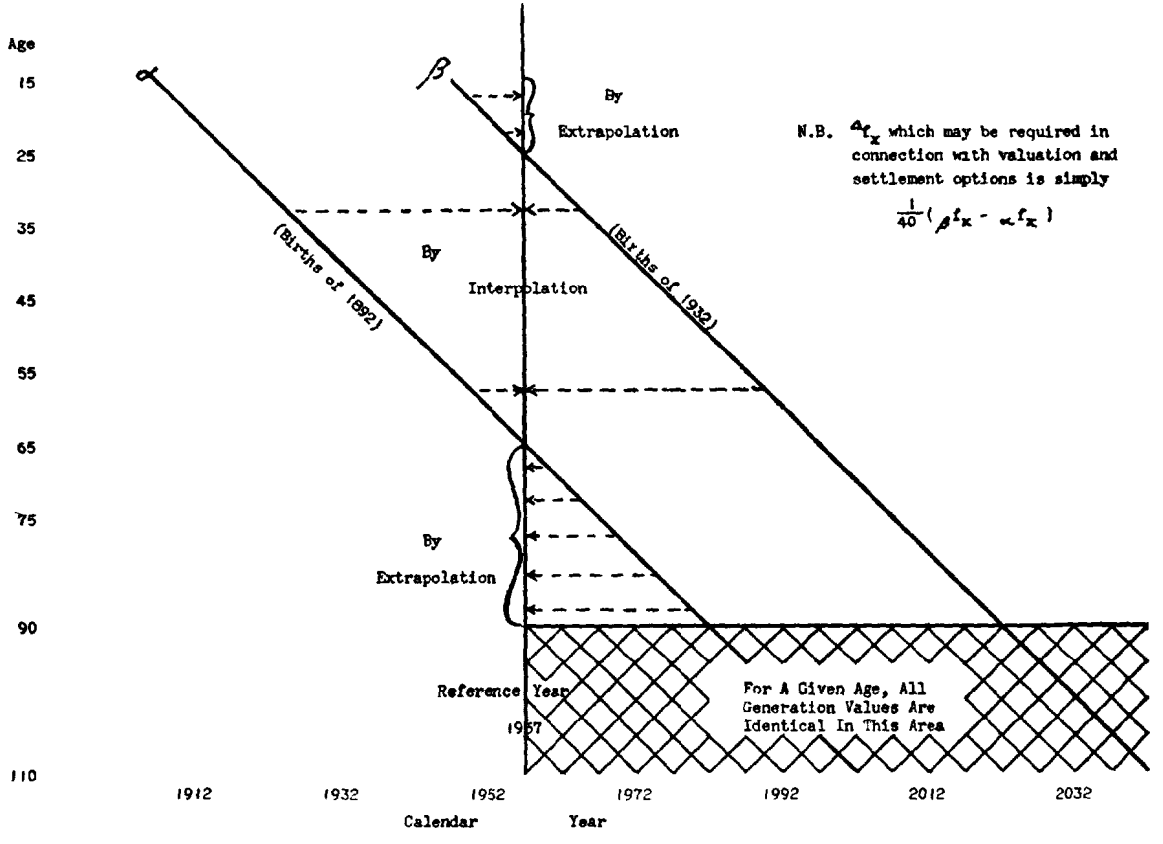
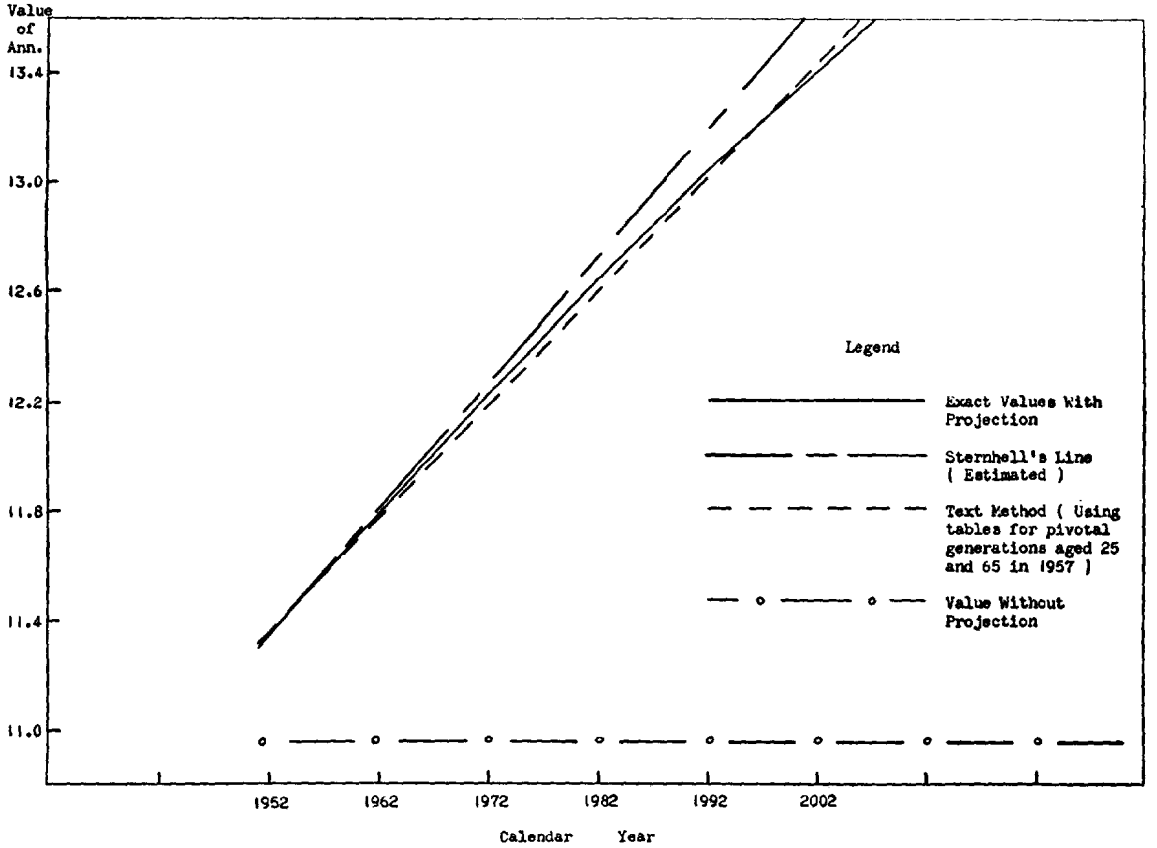


CHART 2

PRESENT VALUES OF AN IMMEDIATE ANNUITY OF 1 A YEAR PAYABLE TO A MALE AGE 65
(Group Annuity Table for 1951 with Projection C, 2½% Interest)



With respect to the text method, the values produced by this method would, on a graph such as Chart 2, be represented by a secant which intersects the curve of exact values in the calendar years when age 65 is attained by the two pivotal generations. On Chart 2 the broken line representing the text method is the one determined by the choice of pivotal generations aged 25 and 65 in the reference year of 1957. Accordingly, the years of intersection are 1957 and 1997. The choice of another pair of pivotal generations or another issue age would give slightly differing lines.

It is clear from the chart that, with respect to annuity values, we should expect this method to understate slightly the exact values in the interpolated area and to overstate slightly the exact values in the extrapolated area. Furthermore, as is borne out in Tables 1 and 2, we should expect the maximum error at a given age in the interpolated area to occur when interpolation is made about midway between the two pivotal generation tables.

To sketch briefly the *modus operandi* of the text method in the three important areas of premiums, reserves, and settlement options, we may say that, with respect to the calculation of premiums, two exact premiums would be calculated for each attained age on the pivotal generation tables applicable thereto by means of the usual formulas and commutation symbols. Then, by a simple linear combination (either interpolation or extrapolation) of each pair of exact values corresponding to a given attained age, the approximate premium for that attained age and any given calendar year may be obtained.

With respect to reserves, approximate valuation factors pertaining to any selected reference year can be readily determined by means of the simple linear combination outlined above for premiums. The approximately constant yearly increment, $^{\Delta}f_x$, corresponding to each reference year valuation factor can also be readily obtained as indicated on Chart 1. Since, therefore, the text method permits the relatively quick and simple determination of reference year values with provision for future improvement in mortality and also of the increments to these values for each elapsing year, the valuation procedure could be identical with that suggested by Mr. Sternhell on pages 31 and 42 of his paper. Accordingly, for each valuation factor on the usual single table basis there would instead be two factors, *viz.*, the value of the factor in the reference year with provision for future improvement in mortality and the value of the yearly change in the valuation factor. Two aggregate reserves, corresponding respectively to these two factors, would be determined, and that aggregate representing the yearly change in the first aggregate would then be multiplied by the number of years elapsed between the reference year and the

year of valuation. This product would then be added to the first aggregate to obtain the reserve as of the valuation date with provision for future improvement in mortality. As indicated by Mr. Sternhell, this simple procedure permits the reserve of each year to reflect the future mortality rates expected to be experienced from that year on, thus resulting in reserves closer to the theoretical and avoiding the discontinuities in reserves which would otherwise occur when new reserve factors were substituted after the expiration of a certain number of years.

Various other valuation procedures based upon the text method are, of course, possible. For example, rather than make yearly adjustments to the aggregate reserve as outlined above it might be considered desirable, particularly in view of the relatively small yearly variation in projected values, to use the same valuation factors for five years and then to change to a new set of valuation factors. In this case, only the usual valuation factors would be required and these would be derived for the reference year in the same manner as premiums. Another simple alternative procedure would be to use for a given attained age and generation the valuation factor at the same attained age on the nearest pivotal generation table.

With respect to settlement options, it would seem that a method such as this or Mr. Sternhell's could be adapted to present practice. Instead of a single column for each option giving the monthly payment for each age per $1,000 of proceeds, there would be two columns, the first giving the monthly payment applicable to an election in the reference year, and the second giving the decrease in such payment for each year which has elapsed between the reference year and the date the proceeds are applied under the option, *i.e.*, $^{\Delta}f_x$. The text would then state that the monthly payment under a particular option and at a given attained age for each $1,000 of proceeds would be equal to the monthly payment shown in the first column of the option at the given age reduced by the product of the corresponding entry in the second column and the number of years elapsed between the reference year and the date the proceeds are applied under the option. Where the settlement option is on the life of the insured, this would give very good results—the error corresponding in relative magnitude to that of the premium at issue since the interpolation or extrapolation range remains the same throughout for any given generation. Where, on the other hand, the settlement option is on the life of a beneficiary whose generation diagonal is very substantially to the right, for example, of the generation diagonal in Chart 1, the extrapolation implicit in this settlement option method may give monthly payments significantly below the theoretical payments. The number of elections in this area would un-

doubtedly be very few, and, since extrapolation may generally be expected to overstate an annuity value (or understate monthly annuity payments), the value will be on the conservative side. Consequently, if the understatement were significantly lower than the settlement option rates then in effect, the current scale could be used for the few elections in this area.

To sum up briefly this somewhat lengthy discussion, it is my opinion that Mr. Peterson has provided us with some excellent tools with which to test the adequacy of our group annuity premiums and reserves in the light of both current and prospective mortality experience under this class of business. The Group Annuity Table for 1951 presented in his paper, in conjunction with one of the methods of providing for future improvement in mortality, may form a good and workable basis for future premiums and reserves under group annuity business. With regard to the approximate method described in this discussion under the designation "text" method, it would seem that this method represents a rather good combination of simplicity and accuracy.

In the latter part of my discussion, I have perhaps been rather unorthodox in going somewhat afield from Mr. Peterson's paper to review the whole problem of projection methods and to point out another possible method of providing for future improvement in mortality. However, since the subject of projection methods is inextricably involved in the determination of a proper basis for annuity premiums and reserves, I have taken advantage of Mr. Peterson's kind invitation to discuss these methods.

DONALD D. CODY:

As a former associate of Mr. Peterson, I have long admired his wisdom in the group annuity field. His current research into the mortality of annuitants is a major contribution to actuarial knowledge and will prove to be invaluable to actuaries in designing proper premium levels and valuation standards for group annuities. For many years, actuaries have been aware of the serious encroachment of improving mortality on our annuity funds, but now, thanks to the research of Messrs. Jenkins, Lew, and Peterson, we have available definitive measures of expected secular mortality trends. Of course, as further experience develops, these measures will have to be adjusted to reflect variations from currently expected trends.

In designing a proper actuarial basis for New York Life to use in entering the group annuity field we wished to use a mortality table which adequately and faithfully reflected future mortality for annuities purchased over the next 5 to 10 years. (It was immediately apparent that no simple

adjustment of the 1937 Standard Annuity Mortality Table could possibly be called faithful.) We did not want to use a deliberately deflated interest rate to bolster up an inadequate and unfaithful mortality table because of resultant distortions in costs of widely different benefits and because of pressures by policyholders for a full recognition of excess interest. In other words, we felt that proper funds could be built up more objectively by using a realistic mortality basis.

These criteria indicated the use of one of Mr. Peterson's tables with projection for secular trend, but we wished to use an already published table which would have greater acceptance for valuation purposes. Furthermore, the actual inclusion of projection factors presented mechanical difficulties which we could not face at this stage of our operations. We made some extensive tests in typical income and benefit distributions based on Mr. Peterson's mortality data with provision for secular trend and decided that the best choice for our purposes was the Male Annuity Table for 1949 with ages of males set back one year and the ages of females set back six years. It is our intention to keep our reserves adjusted in future years to the best available estimates of future mortality trends. The most desirable basis in my opinion is one into which projection is built so that adjustments in reserves will not require specific action.

We combined this mortality table with an annual interest rate of 2.5% and a loading of 5% of gross premium plus a policy charge of the excess, if any, of $750 per contract over 5% of gross premium (excluding the policy charge).

CHARLES A. SIEGFRIED:

This paper provides a lucid, extensive, and useful analysis of a matter of great import to the insurance companies underwriting annuity plans and to others who are interested in a sound appraisal of the mortality element in retirement plan costs. Aside from the well-merited praise with which this paper will be received, it is hoped that it will evoke a broad and searching discussion as it deals with a matter that involves highly subjective elements. The following comments are offered on just a few of the many interesting aspects of Mr. Peterson's study:

1. The paper provides us with the best available standard of measurement of current group annuity mortality rates which will be of much usefulness in appraising or determining a mortality basis for group annuity rates and reserves in the light of actual current experience. In developing this basis it seems that reasonable recognition has been given to such factors as the variability of mortality rates as between different groups with different underlying mortality, the effect of differing practices of

employers in retiring employees prior to normal retirement age, and the effect of ill-health withdrawals prior to retirement. Those factors contribute substantially to confuse and complicate analysis of mortality rates in connection with retirement plans, and there is still much that remains to be learned about their effect.

About all we can hope for in the present state of our knowledge is that reasonable recognition be given to this phase of the broad problem, and this Mr. Peterson seems to have done. While the G*a*-1951 Table produced by Mr. Peterson does not differ substantially from the Jenkins-Lew *a*-1949 Table, it will be very helpful to have available a basic table which does reflect some of the special characteristics of group annuities in a way which the *a*-1949 Table, of course, did not undertake to do. In brief, then, we are provided with a current measure of group annuity mortality which seems well designed and well founded as a basis by which to develop or judge actual rate and reserve bases.

2. The comparisons provided by the paper of the results produced by certain mortality and interest combinations now in use, based on so-called static mortality tables, with those produced by the new table with an allowance for further improvement in mortality are exceedingly interesting. In this regard, it seems to me that an effective demonstration is made that the 1937 Standard Annuity Table with ages set back one year is far from the excessively conservative table some critics of insurance company rates have maintained. The converse seems to be the better view. Considered by itself, it does not seem to be a satisfactorily conservative basis for use in many phases of current group annuity operations. On the other hand, when used in combination with a conservative interest rate, it still seems to retain considerable practicability as may be indicated by the following: At the present time, $2\frac{1}{2}\%$ seems to be a rather widely used assumed rate of interest for group annuity purposes. If this interest rate is accepted as a satisfactory level of the interest rate—considered by itself—then if this rate is combined with a mortality basis such as that developed by Mr. Peterson, the resultant values would seem to reflect what might be characterized as an up-to-date standard of group annuity costs. Now if we take the 1937 Table with ages set back one year and combined with a $2\frac{1}{4}\%$ interest rate, the combination still seems to be fairly closely in line in the aggregate with the up-to-date standard just referred to.

This combination is, of course, influenced by one's judgment as to the degree of accuracy that can be attained at any time in the measure of annuity costs. Over the last 20 years the measure of annuity costs as reflected by rates actually charged for the purchase of annuities has had a remarkably wide range—as much as 50% or more. Against this back-

ground, a deviation in reserve values of between 2% to 4% in 1952 and 3% to 5% forty years hence for a representative distribution of one year's issue as indicated by Mr. Peterson's Table 20 seems to indicate a rather narrow range of difference between the "Standard Annuity−1, 2¼% interest" basis and what would seem to be the most up-to-date standard. Twenty years hence, the difference in the aggregate values is less than 1%. These results would, of course, be altered if the distribution of the new business were different from that indicated by Mr. Peterson's sample, although variations which would alter these averages appreciably are probably unlikely.

3. The comparison just referred to is of particular interest in regard to the question of the desirability of using a static table or a projection table by a company now using the Standard Annuity Table with ages set back one year and an interest rate of 2¼%. The administrative and other problems normally created by a change in rates are such that changes are generally not made unless the existing basis is clearly unsatisfactory. Where a change involves as many new unknown difficulties as still seem to exist with respect to the use of a series of tables involving the projection principle, it would appear that the case for a change to such basis would have to be very compelling. Particularly in view of this consideration, I am not persuaded that the combination of the Standard Annuity Table with the one year age setback and, say, 2¼% interest is sufficiently different from a basis using the projection principle which might reasonably be adopted under present conditions, to warrant moving ahead in this direction until some of the collateral problems have been very carefully explored and satisfactory answers found.

4. There is a suggestion in the paper that considerations of equity favor the use of the projection principle—the thought being, I believe, that it will thereby be easier to achieve reasonable equity as between different contractholders than if static tables are used. That, of course, is a commendable and desired goal, but it is not yet entirely clear whether the desired results will be achieved any more simply and surely than by the methods heretofore employed. Does not this suggested advantage depend very largely on the fact that a table involving a projection element may continue to be used for a long period of years without change? Is it likely that this happy condition can be realized for any considerable period of years? There is admittedly considerable variation in thinking, not only as to the proper base rates of mortality to be used but also as to the rates of improvement. Is it not likely that with the passage of time many of these estimates will have to be revised? If changes in mortality occur at rates which differ from the assumed rates of change, will it not be more difficult

to defend the continuation of a particular table? Not only may the rate as of any particular year seem out of line with the emerging experience, but the rates of change may not be the same. If it is reasonable to expect that changes in the tables must be made from time to time as in the past, might it not be more cumbersome to give attention to matters of equity when projection tables are involved than when they are not?

It is not at all clear to me that in dealing with an element such as mortality rates that cannot be expected to move uniformly but are more likely to move irregularly, it is substantially better to deal with it on an automatic year-by-year basis as if it did so move than to deal with it at more extended intervals when there is greater knowledge of what changes are actually occurring. In any event, many companies already having substantial volumes of business in force may find it desirable to follow the more traditional approach with respect to such existing business, and thus would not realize much of the simplification which the projection method would normally provide.

This skepticism concerning the practicability of the use of the projection principle directly in mortality tables for group annuity rates and reserves, of course, is not mentioned as a criticism of what Mr. Peterson has done. Even though the principle may not be used directly by a company, it will be exceedingly helpful to have available, for purposes of comparison and analysis, the extensive material Mr. Peterson has produced. Also, it is recognized that the decision in a particular company may be influenced by such factors as the nature of the company's existing business, its current rate basis, and company policy with regard to modifications of reserve bases in the light of changing conditions. The important point is that the mortality rates which are indicated by current experience and by currently observable trends should, in one way or another, be taken into account realistically in all phases of group annuity operations. Mr. Peterson's paper contributes very substantially in emphasizing the importance of this consideration.

CHARLES D. RUTHERFORD:

Mr. Peterson has placed deeply in his debt all actuaries concerned in the everyday problems of group annuities. This is particularly the case where reserves on an experience basis are concerned. Where so much has been given it may seem ungracious to ask for more, but one would have liked to see the effect of the continued entry of new business. Perhaps if Table 22 had been calculated on the assumption that the amount in force was the same, and similarly distributed in each calendar year, it would have given some idea of the effect. Meanwhile, I have made some calculations on a

different basis, and it would appear that if a static basis is used which reproduces at the inception the experience reserves on active lives, without too much variation by age, then the true reserves on active lives will exceed the static basis thereafter by a nearly constant percentage of the order of $\frac{1}{2}\%$ for each year since the date at which the two sets of reserves were equivalent. This is a material amount. It means that if the original basis is not changed our required reserves will be understated in ten years by about 5%. The static basis cannot, of course, be the experience table without projection since that table will not allow for the initial effect of projection.

This leads on to another problem arising out of the recognition that mortality is steadily growing more unfavourable from the standpoint of annuities. Employing the technique of projection the table of premium rates used in group annuity contracts should change every year. It may be said that the change is too small to take account of so frequently, but even if a change is made only every five years it is still known in advance. Are we justified in issuing contracts which provide for a change in premium rates at will when we know that under normal circumstances an increase in rates of measurable amount will be called for at regular intervals? Should our contracts not stipulate the expected increases or should we not develop a new method for dealing with them?

Coming now to the mortality table itself, I find myself in disagreement with Mr. Peterson on matters of principle on a couple of points. However, in practice it is doubtful if these would make any real difference. There is one important point. Leaving aside comparisons of mortality rates, we have in Table 10, in Table 17 and in entries in Table 24 certain calculations stated to be on the G*a*–1951 Table without projection. Surely, if the table presented is the G*a*–1951 with Projection C, these figures have no real existence. The table cannot be separated into two component parts unless a new assumption is made. Either a table assumes a secular variation or it does not. If it does, the secular trend is an essential part of the calculations and the ordinary formulas for premiums and reserves do not apply unless the trend is taken into account. The calculation of the premium for a policy assumes that events occur to one group of lives with the same year of birth in successive years and not to a larger group at different ages in one year. We can assume, of course, either that there is no secular trend or that its omission in the calculations has a known effect which makes our result conservative. But we are not entitled to make the assumption where it is known that neither of these is the case.

It may be and probably is the case that the assumption of an exponential form for the secular trend is neither the most accurate nor the most

convenient we can make. But if it is made we ought surely to make the most of the facts at our disposal. Using all the published data both on group life insurance and on active lives under group annuity policies, I have found for active lives rates of mortality improvement varying from about 4½% per annum at age 15 to about 1½% per annum at age 65. Surely we should not assume rates for the future less than have appeared during the last 25 years unless we intend to make a different fundamental assumption, such as L. G. Starke's formula with six constants which he has applied over approximately 100 years. So far as mortality after normal retirement is concerned, I have used all the published data giving eight points on the curve for each age over a total period of 25 years to obtain the following results for male lives:

Age	Annual Rate of Decrease in Mortality	1,000 q_x 1951
63	0.37%	27.00
68	0.84	36.75
73	1.38	54.99
78	1.40	84.26
83	0.07	146.46
88	1.33	181.08
93	nil	.....

To graduate this in any satisfactory manner would require assumptions as to the form of both q_x and s_x since the two must be graduated simultaneously. However, it seems likely that Projection B may be stringent enough although the 1951 Table may not contain as large a margin as the 10% intended.

MILTON J. WOOD:

We are all greatly indebted to Mr. Peterson for his excellent paper which provides actuarial tools on group annuity mortality commensurate with those developed by Messrs. Jenkins and Lew for mortality on ordinary annuities. I particularly want to compliment Mr. Peterson for his very skillful presentation of the many special problems which are encountered in connection with group annuities and for his development of a modern mortality table with projection factors which should be of great help in meeting many of these problems.

One of the best features of the new G*a*–1951 Table is the safety factor which Mr. Peterson has introduced by discounting the experience mortality rates on retired lives by 10% for males and 12½% for females. It is the general practice on this business to carry large groups on a practically

self-supporting basis either through dividends or experience rating. This practice could lead to plenty of grief if the mortality table were based on average expected experience since there might then be no way of offsetting the losses on the cases which developed lighter than average mortality. This is a principle which could well be followed in developing group accident and sickness morbidity tables and we can say that it is already being followed on the mortality assumptions for group life.

I certainly agree with Mr. Peterson's suggestion that we would have a clearer insight on our group annuity business if we were to adopt realistic mortality and interest assumptions rather than to continue the practice of using a poorly fitted mortality table offset by an overconservative interest assumption. This is such an obvious improvement that I expect we will see a trend in this direction.

I should like to make one point concerning the table on page 251 which compares 1946–1950 mortality on group life insurance and group annuities. A first look at this table might give one the impression that the group life experience indicates the need for more conservative assumptions as to probable mortality above age 65. However, I think that the variation in these mortality ratios is largely due to the difference in the character of the lives covered in the two experiences. The group annuity exposure includes the whole group of lives which have passed normal retirement date, whereas under group life there are many cases where the insurance is discontinued at retirement so that the exposure contains a relatively large proportion of healthy lives who have continued in the working force beyond age 65.

RALPH H. MAGLATHLIN:

Mr. Peterson's paper on group annuity mortality is a most timely one. The problem of determining adequate group annuity rates, in view of the current and long term trends of improving mortality, faces all group companies today. His most excellent and comprehensive treatment of this subject should aid considerably in solving this problem.

In his concluding comments Mr. Peterson raises certain questions. In answer to these, I believe that my own thoughts coincide with his. In group annuity contracts, as opposed to ordinary annuities, we sell a great variety of forms of annuities and at greatly varying ages. The use of a static table, perhaps adjusted by a depressed interest rate, would definitely tend to distort equities between the various annuity forms and age distributions encountered. This is clearly shown by Mr. Peterson's figures in his Tables 18 and 19. Furthermore, those of us who have been in on selling negotiations know that, rightly or wrongly, the average layman

scrutinizes each of the various assumptions in our rate basis separately; and it is easier to justify and explain a premium rate where each element appears reasonable. In view of the modern machine processes available, I concur with Mr. Peterson that the use of multiple generation tables is not overly complicated. I believe, however, that it is not necessary or advisable to increase premium rates each year, but that rates should be used which will represent the average over a convenient period of years. On the other hand, it is important in dividend or experience rating calculations to take account each year of the assumed year-by-year decrease in mortality.

In conclusion, we in the group annuity business are most grateful to Mr. Peterson for having devoted so much time to the preparation and presentation of this most valuable material.

EDWARD H. WELLS:

This discussion is limited to the smoothing function of the graduator introduced in the paper as a variant of the usual Whittaker-Henderson B type. The function may be written as $G = \Sigma(r^{-1}\Delta q_{x+1} - r\Delta q_x)^2$. The question is whether such a function is expressible in mixed difference form, as analyzed in Mr. Spoerl's paper (*TASA* XLII, 292) and also as illustrated in Henderson's last edition of *Actuarial Study No. 4*, page 44.

It can indeed be so expressed, for

$$G' = \Sigma\,(\Delta^2 q_x)^2 + (r^{-1} - r)^2\,\Sigma\,(\Delta q_x)^2$$

produces the same set of partial derivatives with respect to q_x, for every value of x, except those involved in the extreme terms at which the summations are cut off. It is thus immaterial whether G or G' is employed as the smoothing function. The same graduation results from minimizing the expression of which either function is a part.

Going to the general case, it may be proved that if G is of the form:

$$G = \Sigma\,(a_0 u_x + a_1 u_{x+1} + \ldots + a_n u_{x+n})^2$$

where the a's are arbitrary coefficients, a set of c's can be determined such that

$$G' = c_n\Sigma\,(\Delta^n u_x)^2 + c_{n-1}\Sigma\,(\Delta^{n-1} u_x)^2 + \ldots + c_1\Sigma\,(\Delta u_x)^2 + c_0\Sigma u_x^2$$

has the same set of partial derivatives with respect to u_x, for all values of x, as G has. This is an interesting Pythagorean type of relationship between G and G'. It appears to be a special case of orthogonal polynomials (*cf.* Lidstone, *JIA* 64, p. 128).

The c's can be determined in terms of the given a's by taking the par-

tial derivative of G with respect to u_x. The resulting linear compound of the u's is symmetrical about u_x, so that it can be expressed in the form

$$2\sum_{t=0}^{n}(-1)^t c_t \delta^{2t} u_x .$$

Then, since

$$\frac{\delta}{\delta u_x}\Sigma(\Delta^n u_x)^2 = 2(-1)^n \delta^{2n} u_x ,$$

the expression G', using the c's so determined, must be equivalent to G.

The author of the present formula, Mr. Kingsland Camp, has made a distinct contribution to the theory of smoothing functions. By the use of this new approach it is possible to determine weights for the mixed difference case in such a way as to satisfy practically any set of basic conditions of constraint upon the graduated curve. It is even possible that this contribution will outlive the rest of the paper when group annuity mortality rates of the present time become a matter of history.

W. RULON WILLIAMSON:

Mr. Peterson's comprehensive paper on group annuity mortality has been meticulously prepared and presented. For the reader not persistently in this field, there is much to be known concerning the retirement plans as to the objectives of the system, the make-up of the coverage, the extent of the personal financial stake of the employee, the conditions of entrance and exit.

Four points seem to me worth my discussion:

1. The ending of the table at 110.
2. The doubts as to age veracity from 65 to 70.
3. Selectness of experience after retirement.
4. Contingency margin in the mortality table.

1. Dr. Greville adopted a graphic graduation at late ages in the U.S. Life Mortality Tables, 1939–41, where instead of closing out all lives at a given age, he makes the terminal age a function of the radix and the number surviving to the late ages. We do not *know* there is any terminal age. The crude data from the 1940 census showed 93 persons at 108, and 410 at 109 and later. The individual reports for the 410 were bunched into one age group, but suggest that 115 might have appeared. In 1950 not even the number of centenarians has been reported, and all are punched 100 and over, this time. I have a hunch there are some very high ages reported in this census. I prefer Dr. Greville's decision to that in this table.

2. In 1940, 600,000 persons seem to have migrated across the age 65

boundary, and in 1950 the migrants seem to number 1,000,000. These are in addition to those just growing 10 years older since the last census. Since this throws doubt on the accuracy of reported ages in the census, it also suggests doubt as to the age reporting to employers. But with this doubt, perhaps employers should add to the battery of modern machines an effective lie detector.

3. The general conviction that there is favorable selection—costwise—at retirement suggests the wisdom of developing select tables from retirement onwards. A seven year study at Cornell on the relation of work, health and death should be of interest here. It suggests another "customer" for the basic data.

4. Such tables have for so long been called *experience tables*, that there is one reason to place the provision for contingencies in the loading, not hidden in the major cost factor, mortality.

Somewhat fuller treatment of some of these points occurs in a paper in Best's *Insurance News* for October, after its presentation to the Gerontological Society in September, called respectively "Life after 65" and "The Lengthened Life-line Beyond 65." The complete gaps in our understanding are being closed.

ROBERT F. LINK:

As Mr. Peterson's paper grew, this writer watched the process at close range with mounting anxiety. Group annuities are tough enough without the introduction of another important variable. As one who would be most intimately concerned with the administration of a rate decision, I wondered, "Will forecast break the camel's back?"

Let us suppose that the actuary of a company looks with favor on the use of forecast mortality for group annuity premiums and reserves. What particular problems arise as a result of the forecast element? Can those problems be met without undue complication of operating procedures?

Group Annuity Rates

Of course, the principal immediate problem is that of computing the rates which are to be used in group annuity contracts. Since we traditionally install a rate table in the contract and use it for 5 years or more, we must start out with the concept of some initial compromise in the literal application of the forecast basis. Another way of stating the same thing is to say that we are not talking about a different method of using rates but merely of computing rates which will be more attractive to the actuary. An obvious device to get a scale of forecast rates is to compute the rates as

if the annuities were all to be purchased in a designated calendar year (such year to be central in the period during which the scale of rates is expected to be effective). When such a year has been chosen (along with a specified forecast mortality, interest and loading basis), an entire scale of rates is defined.

The next step is that of calculating these rates. The writer's company has not made any decisions along these lines, but it seems likely that such calculations, if done at all, will be done on a precise basis. Other procedures which might be used are:

a) Calculation of rates for quinquennial or decennial generations, followed by interpolation
b) Calculation of rates by Sternhell or Fassel-Noback methods
c) Modification of the forecast basis to fit the Jenkins-Frazer pattern (*TASA* XLVII, 265)

Mr. Peterson's table does not even approximate a Gompertz table, so the Jenkins-Frazer method does not seem too attractive. The Sternhell or Fassel-Noback methods enable one to derive basic functions for individual cases. However, group annuity rates, by their nature, are computed from derived functions considerably removed from basic commutation functions; therefore, these methods might result in very little saving of effort. Interpolation might save some work, but it would lead to later complications where individual tables of rates were required for certain contracts, or where individual rates were required for age changes or other purposes. It is an axiom of rate making that the method used for the basic rate scale should be followed as closely as possible in the calculation of any subsequent rates according to the same scale.

There is a lot of work involved in precise calculation, but our ability to do such calculations with amazing speed by the use of the IBM 604 Electronic Calculating Punch makes this work seem considerably less frightening than it would otherwise be. As a matter of fact, we have in our files a more or less complete plan for computing forecast rates by punch card methods. To describe the entire plan would be a rather lengthy process; however, a condensed description is given in an appendix to this discussion. We have actually programmed (but not yet tested) the steps for most of the calculating runs involved in this plan.

Contract Problems

Group annuity contracts typically contain tables of factors to be used in determining the annuity payable under optional forms or at optional

retirement ages. To be absolutely consistent with the basis of purchase, these factors should vary by calendar year. However, it turns out that in almost all cases the trend of the factors is upward with time. Therefore, if these tables are figured for some calendar year in the near future, they will be conservative in comparison to tables based on a literal application of the purchase basis. The company may take comfort in these margins or may adopt a practice of allowing the use of more favorable tables, as the tables initially installed become outmoded. This arrangement can be of particular advantage if it turns out that the next scale of rates introduced by the company is based on the same actuarial assumptions, but up-to-date in time; it might then be appropriate to use the same reduction factors for two scales of rates. Of course, some companies take a much more relaxed view of this entire problem of consistency.

The only cases in which reduction factors do not increase with time are cases where there is a decrease in the company's mortality risk (as where an employee is given the option of the life annuity instead of some form of refund annuity). An inspection of our own contracts indicates that there are almost no such options offered.

Somewhat the same type of problem arises with respect to deposit administration contracts, where the custom has been to put in the contract a table of immediate annuity rates which would be applicable to funds paid to the insurance company over a certain period of years stated in the contract, regardless of when those funds might be applied to "purchase" annuities. In order to use a forecast basis of purchase, it would be theoretically necessary to have these rate tables vary by calendar year. This can be quite a nuisance, but can probably be taken care of by having the tables vary according to 5 or 10 year calendar periods (or by retaining to the company the right to change the rates periodically upwards by stated percentages). If the latter device were used it might be possible to merge successive deposit administration funds rather than maintain separate funds with the resultant complication of the valuation of assets and liabilities under these contracts.

If it is desired to use a forecast basis for the actuarial valuation of liabilities under a deposit administration contract (for the purpose of determining the desired amount of employer contributions), it would seem that severe approximations should be in order. For example, the forecast element might be confined to retired life mortality. After all, the assumption as to active life mortality is merged with (and to some extent outweighed by) the assumption as to turnover.

Valuation of Liabilities

In speaking of a forecast valuation basis, one does not have quite the same problems as would apply to the premium basis. It is theoretically possible to use a forecast basis literally, changing valuation factors each year and recognizing the true age for the annuities being valued. Once a forecast basis is installed, the problems of changing valuation factors each year should not be too formidable. One possible method is to put on punch cards the valuation factors for the beginning and end of a ten year period. Each year's set of current valuation factors can then be run off by interpolation. The initial calculation of such factors would probably proceed much along the same lines as that of premiums.

The introduction of a forecast basis for valuing existing annuities offers some difficulty. If one superimposes forecast on a present valuation basis, the immediate result would be a considerable strengthening of reserves. On the other hand, if one does not desire to do some immediate strengthening, this would indicate the introduction of a forecast reserve basis producing reserves of approximately the same aggregate level as the present basis. These reserves would be much higher for certain contracts and much lower for certain other contracts. It could be a messy thing to handle. However, we can take comfort in the fact that existing reserves based on versions of the 1937 Standard Annuity Table do not look too bad (in the aggregate at least) from the forecast point of view.

A minor annual statement problem arises in the use of a forecast premium basis and a forecast valuation basis due to the fact that the true ratio of net to gross would vary by calendar years, being low when the premium basis is first introduced and becoming progressively higher as long as premiums are paid on that basis. The rate of increase would be about .5% per year or less, depending on plan and age.

The actuary might feel dubious about reserves that creep upward regardless of whether the expected improvement actually takes place or not. If so, then he ought to be dubious about his annuity reserves based on the 1937 Standard Annuity Table. This table has a built-in forecast element due to the use of overconservative mortality rates for retired lives, balanced by overoptimistic mortality rates for active lives. A glance at Mr. Peterson's model offices shows this "forecast" element at work. The trouble is that the margins are misplaced, and the Standard Annuity reserves are therefore a little awry by plans and ages.

Actually it would seem that the actuary should take comfort rather than otherwise from the use of a reserve basis that has a built-in recognition of the mortality trend. If past history indicates anything, it indicates

that reserves must be strengthened every now and then. To the extent that this is due to improvement in mortality, it would seem likely that forecast reserves would require much less frequent strengthening. It would also appear that the same forecast mortality bases could be used for the issues of a large number of years, in contrast to our present arrangement of having several reserve bases in operation, all arising from the purchases of a mere ten or fifteen years. In certain respects, a forecast basis of valuation might be actually more convenient than a basis without forecast.

Conclusion

There can be no question that forecast will cause problems. However, I suspect that a company that wants it badly enough can solve the problems and still survive. There may even be a field here for some intercompany effort.

APPENDIX

A. *Formulas:*

Almost all formulas for group annuity (deferred) rates can be reduced to the following general form:

$A \cdot a + B \cdot b = C \cdot c$, where
a = employer payment;
b = employee payment; and
c = annual rate of annuity purchased.
A, B, and C are actuarial functions.

The assignment of the particular functions A, B, and C depends on the kind of annuities being purchased; the choice of values for a, b and c depends on the sort of *rate* being computed. To illustrate:

	a	b	c	Formula
Noncontributory Employer Rate for \$1 a Year	?	0	1	C/A
Employee Rate for \$1 a Year	0	?	1	C/B
Employer Share of Unit Plan Rate for \$1 a Year	?	r	1	$(C-rB)/A$
Annuity Purchased by \$10 Employee and \$10m Employer	$10m$	10	?	$(10mA+10B)/C$

A "family" of plans refers here to all plans with the same normal form of retirement annuity and the same death benefit interest rate before retirement. Noncontributory plans are always a special case of some contributory plan. Also, nothing is lost by assuming that there is no death

benefit on employer premiums as such. The functions A, B, and C can then be stated for various families:

Form of Annuity	A	B^*	C
Life	$\lambda_a D_a$	${}_aU_{x,0}$	${}_aH_{x,0}$
Modified Cash Refund	$\lambda_a D_a$	${}_aU_{x,n}$	${}_aG_{x,n}$
(This plan has a death benefit of the cash refund type, the initial value of which at retirement is the same as the death benefit which would have been paid immediately before retirement)	For modified cash refund plans, n is the highest integer in $b(1+j)^{x-a}\cdot 1/c$		
n-Year Certain Life	$\lambda_a D_a$	${}_aU_{x,0}$	${}_aH_{x,n}$
DCn	$\lambda_a D_a$	${}_aU_{x-10,0}$ or $\lambda_a D_a$ if $a \geqslant x-10$	${}_aK_{x,n}$
(This plan is like an n-year certain life plan except that if the employee dies within ten years of normal retirement, his beneficiary will receive an n-year certain annuity as if the employee had retired immediately prior to death)			

*These B's are for plans where employee premiums are returned with interest at death before retirement. For no-interest plans, use W-functions in place of U-functions.

The above formulas are taken mainly from Messrs. Miller and Coates' paper (*RAIA* XXX); the only changes were those required to attain a form more consistent with the conditions of the forecast problem.

B. *Definitions:*

These definitions involve familiar functions, with the possible exception of the last four items. These items were introduced to handle the problem of the DCn plans. The j^{DCn} series is a set of numbers such that any early retirement reduction factor under a DCn plan can be expressed as the ratio of j's for the normal and optional retirement dates. Thus, the income available at age x as a percent of the income at age z would be $j^{DCn}_x \div j^{DCn}_z$. Such a series exists for any conventional rate basis. Where forecast rates are involved, the retirement factors must be forced into a pattern which permits a j-series. Without a j-series, forecast rates for DCn plans would become virtually impossible to compute.

a = purchase age (age in base year)

x = attained age

z = retirement age

$n = x - z$

i = premium interest rate

$v^x = (1+i)^{-x}$

j = death benefit interest rate

$u^x = (1+j)^{-x}$

r = employee contribution ratio, unit plans

m = ratio of employer to employee contributions, money purchase plan

λ = ratio of net to gross

${}_aq_x$ = rate of mortality at age x for a person who was a in base year

${}_al_x = {}_al_{x-1} \cdot (1 - {}_aq_{x-1})$

${}_ad_x = {}_al_x - {}_al_{x+1}$

${}_a\mathrm{D}_x = v^x \cdot {}_al_x$

$${}_a\mathrm{N}_x = \sum_{y=x}^{\omega} {}_a\mathrm{D}_y$$

${}_a\mathrm{C}_x = v^{x+1} \cdot {}_ad_x$

$${}_a\mathrm{M}_x = \sum_{y=x}^{\omega} {}_a\mathrm{C}_y$$

${}_a\mathrm{D}'_x = (1+j)^x \cdot {}_a\mathrm{D}_x$

${}_a\mathrm{C}'_x = (1+j)^{x+1} \cdot {}_a\mathrm{C}_x$

$${}_a\mathrm{M}'_x = \sum_{y=x}^{\omega} {}_a\mathrm{C}'_y$$

${}_a\mathrm{N}_x^{(12)} = {}_a\mathrm{N}_x - \frac{11}{24} {}_a\mathrm{D}_x$

${}_a\mathrm{M}_x^{(12)} = {}_a\mathrm{M}_x - \frac{11}{24} {}_a\mathrm{C}_x$

$${}_a\mathrm{R}_x^{(12)} = \sum_{y=x}^{\omega} {}_a\mathrm{M}_y^{(12)}$$

${}_aH_{x,n} = {}_a\mathrm{D}_x \, \ddot{a}_{\overline{n}|}^{(12)} + {}_a\mathrm{N}_{x+n}^{(12)}$

${}_aG_{x,n} = {}_aH_{x,0} - {}_a\mathrm{R}_x^{(12)} + {}_a\mathrm{R}_x^{(12)} + n \, {}_a\mathrm{M}_x^{(12)}$

${}_aU_{x,0} = \lambda \, {}_a\mathrm{D}_a - u^a \, ({}_a\mathrm{M}'_a - {}_a\mathrm{M}'_x)$

${}_aU_{x,n} = \lambda \, {}_a\mathrm{D}_a - u^a \, ({}_a\mathrm{M}'_a - {}_a\mathrm{M}'_x) - (1+j)^{x-a} \, ({}_a\mathrm{M}_x - {}_a\mathrm{M}_x^{(12)})$

$$ {}_{a}W_{z,0} = \lambda_a D_a - ({}_{a}M_a - {}_{a}M_z) $$

$$ {}_{a}W_{z,n} = \lambda_a D_a - ({}_{a}M_a - {}_{a}M_z^{(12)}) $$

$$ {}_{a}K_{z,n} = {}_{a}Y_z^{DCn}\, \ddot{a}_{\overline{n}|}^{(12)} + {}_{a}H_{z,n} $$

$$ {}_{a}V_z^{DCn} = \frac{1}{j_z^{DCn}} ({}_{a}M_{z-10 \nless a}^{DCn} - {}_{a}M_z^{DCn}) $$

$$ {}_{a}M_x^{DCn} = \sum_{y=x}^{\omega} j_{y+1/2}^{DCn} \cdot {}_{a}C_y $$

j_y^{DCn} = a proportionality factor for DCn plans (see explanation above)

C. Calculation Program

The calculation program (up to the point of basic rate factor cards) is shown in Table I. A series of punch card files are designated as A, B_1, B_2, etc. Each file has its own identifying variables. The content of each card is indicated below the file and identification code. The letter code beside each actuarial function indicates how the function is put on the punch card. The first letter indicates the procedure (T = transfer, G = gang punch, C = calculate). Letters after the first indicate source of data for T, G, or C (A, B_1, B_2, etc., indicate A, B_1, B_2 cards, etc; M is special master card).

The end product of the steps indicated by Table I is a set of C cards and a set of E cards. The C cards are used to calculate rates where no cash refund type death benefit is involved. E cards are used where a cash refund death benefit is involved.

C cards are run five times. On each run all basic rates are computed for one family (Life, C5, C10, DC5, or DC10). The C cards may be run in any desired order; behind each C card are placed 7 or more completely blank cards. The 604 will read the C card and calculate and punch a basic employer rate, a basic employee rate and several employer rates under contributory plans with various ratios. Each rate will be punched, with proper identification, on one blank card.

E cards are trickier. Four runs are required, to compute rates with a "test for n" and rates where n is known in advance, for with-interest and no-interest cases. The "n known in advance" cases are the unit plans. If the E cards are run through in a,z groups and in descending order on n, the 604 will compute the value of n for a 6–1 plan, wait for the right card

TABLE I

Card	A		B_1		B_2		C		D		E	
Identification	a,x		a,x		a,x		a,z		a,z		a,z,n	
Function	${}_aq_x$	CA	${}_aD_x$	CB_1	${}_aC_x$	TB_1	$\lambda_a D_a$	CB_1	$\lambda_a D_a$	TC	${}_aM^{(12)}_{x+n}$	TB_1
	${}_ap_x$	CA	${}_aN_x$	TA	j^{DC5}_x	TM	${}_aU_{z,0}$	CB_1C	${}_aU_{z,0}$	TC	${}_aR^{(12)}_{x+n}$	TB_1
	${}_ad_x$	CA	${}_aC_x$	CB_1	j^{DC10}_x	TM	${}_aH_{z,0}$	TB_1	${}_aH_{z,0}$	TC	$\lambda_a D_a$	TD
	p^x	TM	${}_aM_x$	TA	${}_aM^{DC5}_x$	CB_2	${}_aD_z$	TB_1	${}_aR^{(12)}_z$	TB_1	$(1+j)^{z-a}$	TD
	${}_aD_x$	CA	$(1+j)^x$	TM	${}_aM^{DC10}_x$	CB_2			$(1+j)^{z-a}$	CB_1	${}_aG_{z,n}$	CDE
	${}_aN_x$	CA	u^x	TM	${}_aV^{DC5}_x$	CB_2			${}_aM_a$	TB_1	${}_aU_{z,n}$	CDE
	${}_aC_x$	CA	${}_aD_x$	CB_1	${}_aV^{DC10}_x$	CB_2	${}_aH_{z,5}$	CC	${}_aM_z$	TB_1	${}_aW_{z,n}$	CDE
	${}_aM_x$	CA	${}_aC'_x$	CB_1			${}_aH_{z,10}$	CC				
			${}_aM_x$	CB_1			${}_aU_{z-10,0}$ or $\lambda_a D_a$	CB_1C				
			${}_aN^{(12)}_x$	CB_1			${}_aK_{z,5}$	CB_2C				
			${}_aM^{(12)}_x$	CB_1			${}_aK_{z,10}$	CB_2C				
			${}_aR^{(12)}_x$	CB_1								

to arrive, compute the rate, punch it on the E card, *then* compute the n for a 5–1 plan, and so forth. At the end, we have the rate and plan code punched on *certain* E cards.

A duplicate E card file is run in a,z groups in descending order on n. Here, the 604 will compute first an employee cash refund rate, and then a series of money purchase modified cash refund rates, "testing for n" each time and passing to the next money purchase plan each time as the test is satisfied. The rates run in ascending order of ratio of employer to employee contributions and are punched with a plan code, on the particular E card for the n that tested.

After these E card runs are complete, the E cards which have rates punched are reproduced so as to get a clear card for further work. The remaining steps are those of obtaining derived rates from basic rates (monthly premiums, etc.). A rate book can be prepared by listing the desired rates on a tabulator and using an inexpensive photo-offset multilith process.

This description is deliberately sketchy, since the interested parties can probably be counted with something to spare on the heads of a two-headed rooster. It will be seen that the procedure as a whole depends upon the marvelous versatility of the 604 Electronic Calculating Punch—particularly its capacity to accept signals or make tests which guide it in a choice between alternate programs. Mr. James Attwood and Miss Felicitas Reich, both Fellows of the Society, contributed a good deal to the preparation of this plan, particularly in the solving of difficult technical problems and the planning of the many steps involved.

ABRAHAM M. NIESSEN:

The main reason I am here to discuss Mr. Peterson's paper is that Mr. Peterson was kind enough to make several references to studies put out by the Railroad Retirement Board, and yesterday he commented on the cost of our system. First of all, let me say that I was tremendously impressed by the amount of research which went into Mr. Peterson's paper, and I personally consider it as a kind of encyclopedia on recent trends in mortality under group annuity contracts.

Yesterday, I believe, reference was made to certain factors or actuarial assumptions which are made by the Railroad Retirement Board; and some of them Mr. Peterson considered not sufficiently conservative. He referred particularly to the recent mortality basis we are using. One of the reasons we do not incorporate very conservative mortality bases is that mortality is but one aspect of our problem. We cannot consider mortality alone, without retirement; and, if we were to introduce projections regard-

ing improvement in mortality, we might be asked to introduce projections regarding lower retirement rates. We have to consider these two problems side by side. The tendency now, at least among writers in the field, is to say that people will live longer, and will retire at later ages. Of course, if we want to weigh the two factors side by side, the conclusion is inescapable that lower retirement rates will more than offset improvements in mortality.

In an earlier paper which I presented before this Society, I covered the period from 1943 to 1946. At that time, we found ourselves almost in perfect agreement with the mortality experience of matured lives under group annuity contracts. From this point on, apparently, railroad retirement and group annuity experience started going in two different directions. Group annuity mortality has been improving and, judging from reports put out by the Committee and also from Mr. Peterson's paper, the improvement in recent years has been quite substantial. A railroad retirement study which covered the three-year period, 1946–49, and a new study which we just completed, covering the period 1947–50, have not shown any substantial improvement in mortality in the last 4 years. Why it is so, I do not know. I believe that probably there is a difference in the nature of the experience; we consider only mortality after actual retirement, whereas group mortality experience considers, I believe, mortality after normal retirement age. That may be one of the reasons.

I believe, also, that the whole picture of mortality at the older ages is not sufficiently clear as yet. Mr. Williamson made some reference to it today, and also discussed it in a pamphlet which he was kind enough to send me. There seems to be some confusion regarding the correct number of persons age 65 or over in the country. I myself was perplexed, just as Mr. Williamson was, by the differences in various estimates put out in this respect; estimates based on projections of the 1940 census give a significantly lower figure for the population age 65 and over than an estimate based on the enumerations of the 1950 census. I called this matter to the attention of Mr. Myers, and he has gone into that. We are trying to find out what is what. It seems entirely possible that a part of the apparent improvement in mortality which shows up at the older ages is due to some kind of error in ages themselves. Be that as it may, the problem of mortality improvement at the older ages requires a good deal of further study.

I would also like to mention briefly the remark Mr. Peterson made here yesterday that, if all factors were considered, the cost for the railroad retirement system would be closer to 18 percent than to 14 percent. If we were to consider only the difference between the mortality standards we

use and the bases which are appropriate for group annuity business, we could perhaps draw that conclusion. But let me point out again that we cannot regard mortality alone without retirement; and, if retirement rates can actually be expected to come down, the conclusion that our costs are seriously underestimated is not warranted.

I would also like to point out that the actuaries of the Railroad Retirement Board are not given directives or orders as to what cost figures to arrive at. We do have an Actuarial Advisory Committee, composed of three actuaries who are not connected with the Government or with the railroads. They are completely independent, and they pass on the reasonability of the major actuarial assumptions which we use in our calculations.

As a concluding remark I would like to say that I am very much pleased that our studies were apparently of some help to Mr. Peterson and other writers in the field. I believe that one of the great achievements of this Society, and one of the points which proves its great usefulness, is the fact that members of this Society exchange information from different sources, even though the organizations they represent do not exactly have the same aims and objectives.

REINHARD A. HOHAUS:

Without questioning in any way the merits of the paper as a major definitive work, I would like to add a historical footnote concerning the following sentence on page 247:

> In the light of the present day consciousness of the necessity of providing for mortality improvement at most ages, it was rather startling to the writer, as a "Monday morning quarterback," to reread the account of the construction of [the Combined Annuity Table].

The Varsity quarterbacks for that table were J. D. Craig and Robert Henderson. I was then (1928) on the freshman squad. While the following comments are based on memory, they are, I think, correct at least as to substance.

In the 1920's, group annuities were a new field requiring pioneering in many directions. One was the preparation of a mortality table because "few, or practically no annuities, had been issued at the younger ages and the table [American Annuitants' Select & Ultimate] was therefore not based upon actual experience, and the resulting annuity values at the younger ages were somewhat a matter of doubt" (*TASA* XXIX, 121). The phrase "somewhat a matter of doubt" was a decided understatement,

since the use of that table for the younger ages would have involved mortality rates higher than those then being experienced under ordinary and group life insurance.

Using again football analogy, the 1928 quarterbacks were backed up to their own goal line and their first concern was to advance to the middle of the field—*i.e.*, to have a mortality table that was at least adequate for current mortality conditions. As in the pioneer development of the forward pass by Dorais and Rockne, there was a paucity of theory and practice available as guideposts. Hence in both cases the pioneers proceeded largely on an empirical basis, without extensive traditional techniques but with a sagacious understanding of the underlying problems.

Apparently the actuarial quarterbacks recognized the need for margin for adverse mortality experience, since some provision therefor was provided in a variety of ways. One was that referred to by Mr. Peterson. There were also included in the actual gross rate calculations and terms of the group annuity contract provisions which indirectly would result in some margins. It was also anticipated (but not realized) that the high interest rates then being earned would continue and thereby give a margin in subsequent years. Moreover, such information as was then available concerning mortality trends among annuitants indicated that there had been little change in mortality from the American Annuitants' Table in the age range from 50 to 85.

A further consideration is that the 1928 quarterbacks had the very difficult task of explaining why the group annuity rates and provisions for options (*e.g.*, earlier commencement of annuity payments and survivorship annuity options) were higher or more restricted than those prevailing for Ordinary annuities and settlement options. That and other considerations placed very definite limitations on the extent to which the quarterbacks could plan to drive down the field at the outset.

I feel that the strategy and tactics devised by the 1928 quarterbacks in the light of the then conditions measured up to the best traditions of our profession, but readily concede that the account of the construction of the Combined Annuity Table may be "startling" to one not familiar with those conditions.

Incidentally, Mr. Peterson's statement that the new electronic punch card machines were extensively employed in the calculations for his paper recalled the visit to Mr. Henderson's office when I saw him using logarithm tables as the only tool for his calculations for the part (ages 35 and over) of the Combined Annuity Table he graduated.

(AUTHOR'S REVIEW OF DISCUSSION)

RAY M. PETERSON:

First of all, I wish to express my appreciation to those participating in the full discussion of my paper. I must also express my disappointment in not hearing the views of the considerable number of consulting actuaries in our Society who are active in the pension field. I can only assume that "silence gives consent."

It is very gratifying and encouraging to receive a general endorsement from Messrs. Benedict, Cody, Maglathlin, Siegfried and Wood of my interpretation and methods of adjustment of the basic data. There also seems general concurrence in the reasonableness of my assumptions as to future rates of mortality improvement, although Mr. Rutherford believes that I was not sufficiently conservative for the younger ages but that the less conservative Projection Scale B may be adequate for the older ages. It does seem clear that the purpose of the paper expressed in its introduction has been achieved, *i.e.*, to provide "actuarial tools, based upon group annuity experience, with which the actuary concerned with pension problems may shape or test the mortality basis which, in his judgment, is appropriate to such problems." This fact, of course, is bound to be a source of great satisfaction to the author of a paper.

I shall now comment on some of the individual discussions.

Mr. Benedict, in his comprehensive discussion, has added substantially to the fund of knowledge and material dealing with the problem of allowing for mortality improvement in our rate and reserve structures. He has summarized the methods previously considered and has offered a new method. Before writing this reply, we have not had sufficient time for a thorough study of his proposal in our office, but such study as we have given it makes clear that it deserves serious consideration. I am pleased to have such a fine discussion associated with my paper.

Mr. Cody has given us the benefit of the thinking of one company in arriving at its group annuity rate basis. Although a static table has been adopted, thus requiring, possibly, reserve adjustments in the future, Mr. Cody does endorse, as "most desirable," a reserve basis "into which projection is built so that adjustments in reserves will not require specific action." This is an objective that has been very prominent in the considerations in our office and has stimulated us to attempt to overcome the technical and administrative problems involved in using a "built-in" forecast. I think the satisfactory solution of this problem is fundamental to the successful management of the annuity business. However well-founded

the desirability and necessity for periodical reserve adjustments may be in the mind of the actuary, he must also convince lay persons in the management of a company that such changes should be made, but this may not always be easy or the occasion may not be opportune.

I am glad to have Mr. Hohaus' historical footnote. Since I was only a freshman in 1928 watching the freshman squad drill, I gather that Mr. Hohaus concedes that it is not surprising for me to have been "startled" by rereading of the construction of the Combined Annuity Table—as indeed, I was. I think actuaries must bear in mind that mortality tables constructed and used by them acquire a certain sanctification in the lay mind. The layman will not be aware of the qualifications surrounding their use, such as the necessity for concealed margins in the actuarial formulas for calculating rates, the gains on cancellation under "without interest" plans, actuarial equivalents for early retirements, etc. Indeed, the group annuity business these days operates in a goldfish bowl where all phases of its operations must be demonstrably reasonable, fair and justified. I sincerely hope that future actuaries will not be "startled" by our present labored efforts to deal with group annuity mortality.

Mr. Niessen quite properly warns us that we must be careful in interpreting mortality improvement statistics as they may be unreliable because of errors in age. (I will comment later on age reporting in connection with Mr. Williamson's discussion.) However, when you secure a general picture of improvement from several diverse areas as shown in Table 11 and as Mr. Rutherford has summarized in his discussion covering a period of 25 years, I believe you have conclusive evidence of significant mortality changes. We will watch with interest future reports on the railroad retirement experience as well as that from other areas.

Mr. Niessen has given me the unique opportunity, in reply to discussion of my paper, to amplify my discussion of another paper presented at the same meeting. In discussing Mr. Musher's paper as to costs of the Railroad Retirement Plan, I mentioned two factors which I believed would warrant a higher cost figure. The first one was the matter of the interest rate assumed. I should have made clear that I have no quarrel with the actuaries using a 3% interest rate since that is the rate the government guarantees. I did mean to indicate that the average taxpayer is now subsidizing that program to the extent that the government could borrow money for less than 3% interest. If that rate is $2\frac{1}{2}\%$, the extent of the subsidy may be in the neighborhood of $12\frac{1}{2}$ percent taken over the duration of funding. The other factor had to do with the mortality assumptions. If reasonable provision is made for mortality improvement (*i.e.*, more surviving to pension age and greater longevity of pensioners),

I indicated that calculations would produce estimated costs 10 to 15 percent greater than those based on the assumption that mortality continues at the current experience level. This assumes, of course, that there will be no advance in the average retirement age due to increased vitality and a longer work-life. Mr. Niessen is quite correct in pointing out that there is some relationship between improved mortality and a later retirement age. However, I do not believe that it is a necessarily automatic relationship. Much of the improved mortality observed in recent years and probably much of that to come may be due to the effective use of so-called "crutches" which lengthen the life of a person with a serious impairment. Improved surgery, better care and treatment of heart conditions, etc., are examples of these crutches. It does not necessarily follow that a large proportion of persons affected will have the increased vitality to work longer. Furthermore, the rate of retirement in the future will be influenced by the needs of the railroads for older workers. With increased mechanization and competition with other forms of transportation, who knows but what the average retirement age may be younger than it has been in the past? As another factor, the level of benefits has an important bearing upon willingness to retire. The history of plans vulnerable to political pressures such as the Railroad Retirement Plan does not encourage one to think that retirement benefits will be at a level to discourage retirements. If, in fact, an increased vitality does advance the average retirement age, the same increased vitality will result in a lighter mortality among those continuing active which can be easily overlooked in using experience tables of former years to measure mortality in the 50's and 60's. I am glad to have this opportunity given me by Mr. Niessen to expand my remarks bearing on Mr. Musher's paper. I still think it wise to allow for some mortality improvement in cost calculations for such an important public program as the Railroad Retirement Plan.

Mr. Rutherford has shown a particular interest in reserves on an experience basis and would like an expansion of the figures in the paper to show the effect of the continued entry of new business. We did not include any such expanded calculation since we believed the reader's interest would be confined, as ours was, to existing blocks of business.

Mr. Rutherford has pointed out that, based upon the assumptions used as to future mortality improvement, pension costs for a plan with a fixed retirement age of 65 will advance at a rate of $\frac{1}{2}\%$ yearly solely by reason of mortality improvement. This is a striking point that should be made clear when discussing probable pension costs and proposed rate changes with employer clients. Mr. Rutherford suggests that periodical rate changes may be limited to a stated amount. While the mortality aspect of

rates may be adequately covered in this manner, we cannot overlook the need of retaining some latitude to recognize changes in prospective interest rates and also in expenses. I do think that it is feasible to introduce a scale of immediate annuity rates under deposit administration contracts which varies with calendar year or quinquennial periods although limiting the guarantee to accumulated contributions made during a limited period such as five years.

Mr. Rutherford does well to emphasize the limited significance of the G*a*–1951 Table. The warnings in the paper are intended to indicate that the table is not put forward as suitable, by itself alone, for either premium rates, pension cost calculation or group annuity reserves. The table or tables presented in the paper consist of two two-dimensional tables of q's for each sex, one based on Projection Scale B and the other based on Projection Scale C. The q's, of course, are a function of year of birth and calendar year of the future. The G*a*–1951 Table is merely the starting point in that array of q's.

However, this initial table does have uses for the actuary. It affords a conservative picture of current group annuity mortality levels which may be compared with other current tables such as population tables and the *a*–1949 Table, in the latter case showing how individual annuity experience differs from group annuity. Also, Tables 9 and 10 reveal the inadequacies of certain tables in use for pension purposes based upon current mortality levels, to say nothing of their failure to provide for mortality improvement. The publication of certain derived functions based on the G*a*–1951 Table gives actuaries an opportunity to study the use of Sternhell's functions or to examine the type of adaptation which Mr. Hoskins has just made of the *a*–1949 Table. However, any actuary who uses the table, by itself, to project the probable cost of pensions does so without the author's blessing and, indeed, with a stiff warning against such a course.

In considering what rates of improvement I should use for ages under 60, I found no good reason for departing from the Projection Scale B of Jenkins and Lew. I do not believe we should automatically project the improvement rates found in the past. At the younger ages, there have been dramatic changes during the last 20 or 25 years which can be attributed, in good part, to the control and treatment of the infectious diseases. There is a limit to what can be done in this area. The general improvement in standard of living, *i.e.*, better sanitation, housing, food, etc., has also been an important factor. Can we expect the same gain in the future from these causes? Prof. Ginzberg, Professor of Economics at Columbia University, asserted at a symposium at Mount Sinai Hospital

(as reported in the *New York Times*, November 30, 1952) "that we are already past the optimum point in health gains and that increases in economic wealth may no longer improve mortality and morbidity rates. . . . The advance in health of the West was owing mainly to improved sanitation, nutrition and housing, and a vast reduction in dangerous work." It therefore seems reasonable to assume mortality improvement rates at the younger ages at a more moderate pace than has been the experience of the past. Furthermore, the projected mortality rates assumed at ages under 40 are only .1% to .2% a year and even a 50 percent error would require no more than .1% excess interest earning to offset such understatement. Also, the proportion of funds accumulated during the ages under 35 or 40 is small and in actual experience a large proportion of deferred annuities are canceled. The important area is from age 45 or 50 upwards. As stated in the paper, "At ages over 40, the important area for group annuities, the figures in Table 8 indicate that the Group Annuity Table for 1951 may have a welcome inherent margin of conservatism."

We are all aware of the concentration today upon the so-called degenerative diseases. Dr. Dublin has pointed out that the population mortality rates of this country at ages 45 and over compare quite unfavorably with those of many foreign countries. He suggests that our higher standard of living, or, better, our high living, accounts for some of this difference. It is clear that much lower mortality rates at the older ages can be achieved. With the increased attention to detection and treatment of cancer, improved methods of treating the cardiovascular-renal diseases, the avoidance of excessive weight, etc., we should be prepared for material reductions in mortality rates at the older ages. My preference is for Projection Scale C. The reasons are indicated in the paper and have been repeated in the discussions of the paper. I am grateful to Mr. Rutherford for his stimulating comments coming as they do from a long-time student of mortality trends whose authoritative work has frequently found a place in our *Transactions*.

Mr. Siegfried, taking the Ga–1951 Table with Projection and 2½% interest as an "up-to-date standard of group annuity costs," observes that the 1937 Standard Annuity Table with ages set back one year with 2¼% interest "seems to be fairly closely in line in the aggregate with" that up-to-date standard. I agree with that general observation but it applies principally to deferred annuities without death benefit and, of course, is only an "on-the-average" statement. When we deal with age distributions that depart from the average or with contracts involving no mortality contingency for the insurance company prior to retirement (such as deferred annuities with death benefit and deposit administration plans), we

must be concerned with the distortions produced by using a static table and an artificially low rate of interest to make up for the deficiency in the mortality table itself. It is only a happy coincidence that we find the relationship shown in Table 20. In establishing premium rates for group annuity contracts, it should be the actuary's objective and obligation to determine a rate for each class of benefits that is as nearly appropriate as practical considerations will permit and that is consistent with rates for other classes of benefits in the light of differing characteristics. Rates should be adequate for each class and margins should be placed where they belong.

As to reserves, Mr. Siegfried seems to prefer a periodical change of a static table reserve basis guided by an examination of current trends and developments. Whether one is using a static table or a projected table, the actuary is predicting future mortality rates. Surely he should employ his best skill and judgment in that prediction. I think the paper demonstrates that the 1937 Standard Annuity Table, age by age, does a very poor job. By the use of a projected table, the necessity of future changes in reserve basis should be greatly minimized. I believe that future generations of actuaries will be grateful to us if we do the best job we can now in establishing reserve bases with built-in provision for mortality improvement. Adjustments may be required but the task should be much easier.

Mr. Williamson has stressed the importance of accuracy of age. Insurance companies are traditionally aware of the importance of this matter in dealing with annuities. Although practical considerations do not make it feasible to follow, in the group annuity field, the strict practices that are used for individual annuities, I believe that we are getting reasonably reliable age information. For deferred annuity plans, the dates of birth on the employer's records are usually accepted initially. For deposit administration plans, joint annuitants and immediate annuity purchases at the outset of a plan, reliable age evidence is secured by our company. In general, age evidence is also required when significant age changes are reported. We had some sweeping age change reports after the advent of Social Security and the increases usually were well-balanced by decreases. For large annuities, we check with available independent sources before commencing payments.

Although it would be interesting to study retired life experience on a select basis, as Mr. Williamson suggests, the results would have no value for the purpose of establishing rates and reserves for deferred annuities commencing at a fixed age. Also, while it may appear reasonable to use immediate annuity rates under a deposit administration plan which take account of mortality in the early years of retirement, the possibility of

adverse selection by the purchaser inclines one to be a bit cautious. However, a study of experience by duration for early retirements is desirable and needed in order to establish fair actuarial equivalents.

As to "experience tables," Mr. Williamson will observe that the margin introduced in the G*a*-1951 Table is not only a margin for contingencies but, of equal or greater importance, is also to provide an adequate basis

INTERCOMPANY GROUP ANNUITY MORTALITY MATURED LIFE EXPERIENCE FOR THE YEAR 1951

RETIREMENT ON OR AFTER NORMAL RETIREMENT DATE

ATTAINED AGE	NUMBER OF DEATHS	BY GROUP ANNUITY TABLE FOR 1951	
		Lives	Amount of Annuity Income
	Men		
61–65	308	107%	122%
66–70	1413	110	101
71–75	958	110	107
76–80	484	103	95
81–85	226	104	121
86–90	68	106	88
91–95	15	141	60
Total	3472	109%	105%
	Women		
61–65	57	129%	149%
66–70	91	111	131
71–75	63	108	103
76–80	37	94	97
81–85	26	136	147
Total	274	113%	121%

for the groups which have an inherent mortality level lower or lighter than the average. *A priori*, the insurance company or the consulting actuary has no knowledge as to whether a particular group of employees will develop a mortality level above or below the average. Mr. Wood, in his discussion, has underlined the importance and need of such a built-in provision for variations between groups. One of the weaknesses of the Com-

bined Annuity Table and the 1937 Standard Annuity Table, as to the younger ages, is that they were "experience" tables to start with.

For the actuary who wishes to use an "experience" table, the paper indicates (p. 256) that a one-year set forward of ages of the G*a*-1951 Table produces approximately that result.

In constructing a so-called 1951 Table, it will be recalled that the 1946–50 experience was projected to 1951. The actual 1951 mortality experience is now available and is shown in the table above in terms of the G*a*-1951 Table.

Having in mind the 10% margin in the male table and the 12½% margin in the female table, these results would confirm one in saying that the G*a*-1951 Table is truly reflective of 1951 experience.

There are two typographical errors in the paper. In the deferred annuity symbol in Table 10, the subscript, x, has been omitted. In the minimizing formula on page 288, the age subscript for the final q should be $x - \frac{1}{2}$.

Mr. Kingsland Camp is making a separate reply to the discussion of the graduation of the G*a*-1951 Table.

The prediction of future mortality rates for annuity purposes requires both humility as to one's prescience and courage to tackle a problem bristling with difficulties. My pleasure at the reception of this paper does not blind me to the fact that we still have much to learn. I will look forward to contributions from other actuaries in the future.

(REPLY TO REMARKS ON GRADUATION PROCESS)

KINGSLAND CAMP:

It is very gratifying to have a newly tried process approved by those who took the trouble to comment on it. The algebraic form of the smoothing function that was used is not of itself a new idea, as Mr. Wells correctly points out in his illuminating discussion generalizing the mixed-difference case of the Whittaker-Henderson processes. However, it seems to have had no previous practical trial, although I illustrated it in some specimen A-formula graduations in a manual published in 1950.

The problem presented by this mortality experience may justify some remarks on the philosophy of graduation. The purpose of graduation is to approximate, from the irregular series of ratios necessarily characteristic of a limited experience, the smooth series of ratios that we expect would characterize an infinite body of data of which the given experience is a random sample. While a heavily weighted area of the experience should of course be little changed by the process, areas of light data should be

constrained toward the main trend, and the whole job should satisfy established statistical criteria of fit—hence the accepted statistical weighting formula for W_x rather than Mr. Henderson's original use of the exposed. Thus, we need to consider the general shape of this "main trend," and the best available evidence at the higher ages in mortality experiences seems rather strongly to indicate that it is geometric or very nearly so. (If there were no such evidence, then minimizing some assigned difference-order would likely be the most scientifically impartial thing to do.)

The choice of graduation formula for the basis of the G*a*–1951 Table was not made to facilitate joint life calculations but rather to recognize the usual geometric trend evidenced by the fact that Gompertz and Makeham curves have so often acceptably represented mortality in the past, and at higher ages of life even in the recent past. Although the computation of joint life functions on the G*a*–1951 Table, assuming Makeham's Law to apply, may work pretty well, as mentioned in the discussion, this is not true for Projections by the B and C scales which will be the form of table to be used in practice for rate and reserve calculations. The projection scales, with different rates for different attained ages, (or anything resembling them) render equal ages computed on the geometric basis rather inaccurate, although (see page 295) it is possible enough to frame contracts so as to use conservative conversion factors.

I thank those who discussed my part in this paper.