# Mortality Improvement in the USA: Analysis, Projections and Extreme Scenarios

Joseph Lu Mortality Risk Actuary, Legal & General Assurance Society Ltd. Wun Wong, Ph.D. Research Scientist, Legal & General Assurance Society Ltd.

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### Abstract

Future mortality improvement is an important assumption for the pricing and valuation of annuity liabilities. The 1994 Group Annuity Mortality Table and 1994 Group Reserving Table published by the Society of Actuaries included a set of annual rates of mortality improvement by age and gender, known as Scale AA, with the recommendation that its continued use be reviewed around 2010. A task force was set up in 2010 to review the life tables for Group Annuity and the assumptions for mortality improvement associated with them.

This paper examines the trends of mortality improvement in the United States for the consideration of the task force, using more recent information from the Social Security Administration (SSA) life tables and data derived from the Human Mortality Database (HMD).

This paper seeks to:

- Compare the mortality improvement trends used in the development of Scale AA (1977 to 1993);
- Examine the historic performance of Scale AA since 1995 to its most recent period (subject to data availability); and
- 3. Using stochastic models, compare the projection of mortality improvement with figures from Scale AA (2010 to 2030).

We note that annuitants may experience different changes in mortality rates from that of the general (HMD) or SSA populations. However, our analyses should contribute to the debate over the continued implementation of Scale AA. We have found, for the recent past, the annual rates of improvement in mortality implied by SSA life tables to be higher than that of Scale AA. Historic and projected annual rates of improvement in mortality based upon the U.S. HMD population were also generally higher than those suggested by Scale AA. The analysis raises questions about the adequacy of Scale AA for use in the valuation of annuity liabilities.

### 1. Introduction

The Group Annuity Valuation Table Task Force of the Society of Actuaries (SOA) recommended in 1995 the adoption of a new Group Annuity Reserve Valuation Standard.<sup>1</sup> The standard included the use of generational mortality, allowing for the explicit recognition of future mortality improvement in the calculation of institutional capital reserves. The mortality improvement factors to be used within the new standard (beyond 1994) were incorporated into Projection Scale AA (or Scale AA) and represented the annual rates of improvement in mortality by age and gender. The age range provided was 1 to 80 years.

Scale AA is based upon a blend of mortality improvement trends among Civil Service Retirement System (CSRS) and Social Security Administration (SSA 107) participants between 1977 and 1993. It is the most current mortality improvement table produced by the SOA.<sup>2</sup> Since its release, the standard has been widely adopted by both public and private institutions within the United States. However, it was the recommendation of the task force that the standard be used only for 15 years after its adoption in 1994.

The future use of Scale AA is now under investigation. As the life tables for Group Annuity and assumptions for mortality improvement are being reviewed, we take the opportunity to re-examine the trends for mortality improvement and compare the published rates from Scale AA with the historic and projected annual rates of improvement in mortality in the United States.

Using published figures from the Social Security Administration (SSA) and data derived from the Human Mortality Database (HMD), we aim to:

• Compare the mortality improvement trends used in the development of Scale AA (1977-93);

- Examine the historic performance of Scale AA since 1995 to its most recent period (subject to data availability); and
- Using stochastic models, compare the projection of mortality improvement with figures from Scale AA (2010-30).

### 2. Data and methodology

Comparisons were made between the published figures from Scale AA, SSA and the historic/projected data from the U.S. dataset of the HMD. The periods of analysis were chosen to correspond with the development (1977-93), introduction/implementation (1995-2006) and 20-year projection (2010-30) of annual rates of improvement in mortality based upon more recent data.

Projections in mortality improvement within the United States were examined using a variety of data smoothing or stochastic projection techniques. Methods included two-dimensional P-Spline smoothing and two versions of the Lee-Carter family of models.

### 2.1. Data

### 2.1.1. Data from the Human Mortality Database

Data imported from the HMD website (<u>http://www.mortality.org</u>) was used to generate the outputs for this paper. The HMD contains death and exposure-to-risk data for various nations around the globe. The HMD figures for the United States used for this report include: 1) deaths, and 2) exposure to risk in a 1x1 (by age and year) data format.

For the United States, population estimates were derived from the population census conducted every 10 years (e.g., U.S. Census Bureau 2001). The census counts serve as a basis for the production of annual and monthly estimates for interand post-census periods. For the year 1959 to the most current period, deaths included within the dataset have been tabulated from individual death records disseminated by the National Center for Health Statistics (NCHS). Period death and exposure data (1970-2006) was selected by gender (male, female) and age (60 to 90 years).

### 2.1.2. Data from the Social Security Administration

Period life tables for the Social Security population for 1990<sup>3</sup>, 2000<sup>4</sup>, 2006<sup>5</sup>,  $2010^{6}$  $2030^{7}$ and were imported from Social Security Online (www.socialsecurity.gov). The Social Security population comprises: 1) residents of the 50 states including the District of Columbia (adjusted for net census undercount); 2) civilian residents of Puerto Rico, the Virgin Islands, Guam, American Samoa and the Northern Mariana Islands; 3) federal civilian employees and persons in the U.S. armed forces abroad and their dependents; 4) crew members of merchant vessels; and 5) all other U.S. citizens abroad. Annual tabulations of numbers of deaths (age and gender) are made by the NCHS based upon information supplied by states.

Differences between the compilation of the SSA and HMD deaths and exposure figures can cause the two populations to be less than fully comparable. For example, the SSA includes the deaths of beneficiaries who have died abroad, whereas the NCHS does not. However, the focus of this paper is to compare the annual rates of improvement in mortality published in Scale AA with that derived from the most comprehensive sources of U.S. death and exposure data available.

### 2.2. Methodology

# 2.2.1. Derivation of Past Trends and Base Mortality qx for Comparison with U.S. Data From the Human Mortality Database

Annual rates of improvement in mortality were generated by age (60 to 90) and gender (male, female) using U.S. data from the HMD and compared with the published figures from Scale AA. Figures are presented in five-year age gaps.

# 2.2.2. Projection of Future Mortality Trends Using Data From the Human Mortality Database

A web-based stochastic projection modeling tool (Projections Toolkit 1.3) supplied by the consultancy Longevitas Ltd. (http://www.longevitas.co.uk/site/) was

used to fit the stochastic models onto the U.S. population. Designed for actuaries working on long-term liabilities, the web-based service offers point-and-click access to a variety of stochastic models (including the Lee-Carter method and its extensions) for mortality projections. For our purpose, future trends in mortality improvement were examined by fitting a penalized spline (P-Spline) regression model<sup>8</sup> and Lee-Carter model.<sup>9</sup> Values for mu(x) were derived from each model fit and the probability of dying within a year after attaining age x (qx) subsequently calculated. Annual rates of improvement in mortality were then generated by age (60 to 90) and gender (male, female).

Stochastic mortality projections were generated using the P-Spline or Lee-Carter methods and their extensions. Specifically, the following methods were used: 1) P-Spline (2-D age-period penalty), 2) Lee-Carter (1992), and 3) Lee-Carter Currie-Richards.<sup>10</sup> Other models used but not shown include the: 1) Lee-Carter Delwarde, Denuit and Eilers (DDE) time series projection, 2) Lee-Carter smoothed alpha-beta, 3) Lee-Carter Gompertz (time series projection), and 4) Lee-Carter fully smoothed (alpha, beta, kappa).

P-Splines comprise a subset of a class of (piecewise) polynomial functions. First described by Eilers and Marx (1996), the P-Spline method combines the use of B-splines and differences penalties (e.g., on the estimated coefficients of a generalized linear regression model) to smooth and provide projections of the data. For our purpose, a two-factor (2-D) P-Spline method<sup>\*</sup> was used to: 1) model and project the forces of mortality  $mu_{x,t}$  (x=age; t=period), and 2) understand its associated uncertainty.<sup>11</sup>

Data smoothing was performed by means of the penalized splines.<sup>8,12</sup> Within the model, a two-dimensional surface of the forces of mortality  $mu_{x,t}$  was both fitted over the (x, t)-plane of the data and the region of projection.<sup>13</sup> The surface fit was

The basic form is Log mu(x,t) =  $\sum_{l=1}^{L} \sum_{k=1}^{K} B_{l}^{a}$  (x)  $B_{k}^{t}$  (t)  $\theta_{1,k}$  where D (number of deaths) is assumed to have a Poisson

distribution with mean E x mu (E is the corresponding central exposure and mu is the force of mortality).  $B_1^a$  (x), ...,  $B_L^a$  (x) is the basis of L B-splines by age (x).  $B_1^t$  (t), ...,  $B_K^t$  (t) is the basis of K B-splines by year t.  $\theta_{1, k}$  is the set of regression coefficients. Projections Toolkit report, Longevitas Ltd.

determined by a combination of the data and the penalty applied. The provision of a penalty within the model ensures a "well behaved" projection.

In addition, the fitted surface and extrapolations of values  $\stackrel{^{\wedge}}{mu}_{x,t}$  was used to create a variance matrix of the estimated parameters and "standard deviation sheet" of values  $\stackrel{^{\wedge}}{s}_{x,t}$ . These measures have then provided us with information regarding parameter uncertainty. Further details on the P-Spline methodology can be found in the Continuous Mortality Investigation (2005) Working Paper 15, pp. 12-15, by the Faculty of Actuaries and Institute of Actuaries.

The original Lee-Carter (1992) method is a three-factor model incorporating: 1) a mortality effect by age, 2) a mortality effect by period (time), and 3) an agerelated modulation of the period mortality effect. Its relative simplicity (a single time indexed parameter) is a primary reason behind its use in forecasting mortality trends. However, output derived from the model may be subject to volatility (resulting in inconsistent results) if datasets with a small number of deaths are examined. Extensions to the original Lee-Carter model have since been developed to enable the smoothing of some or all of the model parameters. For example, the Lee-Carter Currie-Richards (CR) model is a recent extension of the Lee-Carter method. It was developed to allow for: 1) the smoothing of the beta and kappa coefficients, and 2) a penalty function that enables forecasting.<sup>10</sup>

We derived mu(x) and calculated qx for 2006 using the P-Spline method and applied the figures as a base for the projection of annual rates of improvement in mortality. The P-Spline regression method is a localized two-dimensional (age and period) smoothing mechanism. We applied these figures to ensure an equivalent starting qx for model comparison when generating life expectancies and annuity values.

We developed several scenarios using as a base the smoothed (P-Spline regression) qx from 2006. With age 65 as the specimen age, we then created scenarios for: 1) best estimates for life expectancy and unit annuity figures assuming no future improvements in qx; 2) best estimates for life expectancy and unit annuity

figures assuming further improvements in qx; and 3) estimates for life expectancy and unit annuity figures to assess the uncertainty of the projections at the 2.5<sup>th</sup> and 97.5<sup>th</sup> percentile. To avoid issues relating to the quality and reliability of data above age 90, life expectancies and annuity values were calculated using the qx between ages 65 and 90.

# 2.2.3. Calculation of Annual Rates of Improvement in Mortality Using Data From the Social Security Administration

Annualized rates of improvement in mortality were calculated from the qx of the life tables (1990, 2000, 2006) published by the SSA. In addition, we used the projected life tables for the years 2010, 2020 and 2030 to examine future views of mortality trends.

### 3. Results (Historic and Future Trends)

### 3.1. Historic Trends in Mortality Improvement (1977 to 1993)

We compared the annual rates of improvement in mortality between Scale AA and HMD 1977-93 by five-year age gaps for males and females (Table 1). For males, annual rates of improvement in mortality from HMD 1977-93 were broadly comparable to figures from Scale AA. For females, the figures from HMD 1977-93 were generally higher for ages between 60 and 90 years. A comparison by one-year age gap (males, females) is provided in Appendix 1.

TABLE 1Comparison of Scale AA with Annual Rates of Improvement in MortalityBetween 1977 and 1993 in U.S. Population Data Derived from the HMD

	Ма	le	Female		
Age	Scale AA	HMD 1977-93	Scale AA	HMD 1977-93	
60	1.60%	1.84%	0.50%	1.06%	
65	1.40%	1.63%	0.50%	0.56%	
70	1.50%	1.85%	0.50%	0.90%	
75	1.40%	1.37%	0.80%	0.80%	
80	1.00%	0.97%	0.70%	1.02%	
85	0.70%	0.43%	0.60%	0.73%	
90	0.40%	0.15%	0.30%	0.52%	

### 3.2. Historic Trends in Mortality Improvement (1995-2006)

Since 1995, rates of improvement in mortality from Scale AA have been recommended for use in the valuation of annuity related liabilities. It was therefore important to examine how more recent changes in improvement in mortality have compared with the recommended rates. To do this, we compared the annual rates of improvement in mortality from 1995 to 2006 at five-year age gaps for males (Table 2) and females (Table 3) with published figures from the SSA (1990-2006 and 2000-06) and Scale AA. We included comparative rates from the SSA as data from the SSA

was used, in part to develop Scale AA. To approximate the period under investigation, we examined SSA life tables derived from census data (1990, 2000, 2006). In addition, we included 1995 to 2006 data from the HMD.

For males, the annual rates of improvement in mortality from SSA 1990-2006 were generally higher than Scale AA. Rates of improvement in mortality from SSA 2000-06 were even greater, indicating a rapid and more recent acceleration in annual rates of improvements in mortality. Rates of improvement in mortality from HMD 1995-2006 were consistently higher than Scale AA.

For females, the annual rates of improvement in mortality from SSA 1990-2006 were in part higher than Scale AA. However, figures from SSA 2000-06 were even greater, indicating a more rapid and recent acceleration in annual rates of improvements in mortality. The annual rates of improvement in mortality from HMD 1995-2006 were consistently higher than Scale AA. A comparison by one-year age gap is provided in appendices 2 (males) and 3 (females).<sup>\*</sup>

TABLE 2
Comparison of Scale AA (Males) With Annual Rates of Improvement in
Mortality Between 1995 and the "Recent Past" in U.S. Population Data Derived
From the SSA and HMD

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		HMD	SSA	SSA
Age	Scale AA	1995-2006	1990-2006	2000-06
60	1.60%	2.54%	1.90%	1.47%
65	1.40%	2.07%	2.06%	2.33%
70	1.50%	2.50%	1.99%	2.89%
75	1.40%	2.10%	1.83%	2.52%
80	1.00%	1.58%	1.51%	2.21%
85	0.70%	1.97%	0.95%	2.12%
90	0.40%	2.91%	0.26%	1.34%

In addition to our primary analyses, we compared changes in the annual rates of improvement in mortality by ethnicity and examined recent data derived from the National Vital Statistics Reports for 2000 (58, no. 10) and 2006 (58, no. 21). We found that for both white males and females, the annual rates of improvement in mortality were comparable with the total SSA population and differed by less than 0.5 percent of the overall population. For black males and females, the annual rates of improvement in mortality differed from 0.03 percent to 0.98 percent (males) and 0.09 percent to 0.77 percent (females) from the total SSA population.

### TABLE 3

		HMD	SSA	SSA
Age	Scale AA	1995-2006	1990-2006	2000-06
60	0.50%	1.91%	1.31%	1.64%
65	0.50%	1.46%	1.20%	2.34%
70	0.50%	1.44%	0.88%	1.91%
75	0.80%	1.37%	0.72%	1.60%
80	0.70%	0.76%	0.56%	1.59%
85	0.60%	1.02%	0.27%	1.37%
90	0.30%	1.67%	-0.08%	1.01%

### Comparison of Scale AA (Females) With Annual Rates of Improvement in Mortality Between 1995 and the "Recent Past" in U.S. Population Data Derived From the SSA and HMD

### 3.3. Future Trends in Mortality Improvement (2010 to 2030)

### 3.3.1. Projections Using Data From the Human Mortality Database

We projected rates of improvement in mortality for the near future (2010-30) using the following methods: 1) Lee-Carter (1992), 2) Lee-Carter Currie-Richards, and 3) P-Spline (2-D age-period), and compared the annualized rates of improvement in mortality for men (Table 4) and women (Table 5) with Scale AA.

For males and females, figures from Scale AA are consistently lower (sometimes to a larger degree) than the projected rates of improvement in mortality derived from the Lee-Carter or P-Spline models.

Overall, projected annual rates of improvement in mortality (LC HMD 2010-30) for males declines by age. Projected annual rates of improvement in mortality for females remains relatively flat across ages. Absolute differences between outputs from LC HMD 2010-30 and figures from Scale AA ranges from 0.44 percent (age 75) to 0.72 percent (65) per annum for males and 0.45 percent (75) to 0.75 percent (60) per annum for females. Comparisons of the output from the Lee-Carter Currie-Richards (penalty projection) model or the P-Spline (2-D age-period) method with the figures from Scale AA shows even greater differences. A comparison by one-year age gap is provided in appendices 4 (males) and 5 (females).

### 3.3.2. Projections Using Data From the Social Security Administration

Comparisons between the published rates from the SSA and Scale AA showed smaller differences in projected annual rates of improvement in mortality, with absolute difference of 0.02 percent (85) to 0.65 percent (70) per annum for males and 0.05 percent (75) to 0.35 percent (60) for females. For example, an assumption from Scale AA would imply a reduction in  $q_{70}$  for males of about 26 percent between 2010 and 2030. The equivalent reduction in  $q_{70}$  implied by projection of HMD data using the Lee-Carter method would be about 34 percent.

TABLE 4 Comparison of Scale AA (Males) With Annualized Rates of Improvement in Mortality of Projected qx Between 2010 and 2030 in U.S. Population Data Derived From the SSA and HMD\*

			LC	LC Currie-		
			Original	Richards	P-Spline	
		SSA	HMD	HMD	HMD	
Age	Scale AA	2010-30	2010-30	2010-30	2010-30	_
60	1.60%	1.00%	2.28%	3.77%	4.35%	
65	1.40%	0.89%	2.12%	3.59%	3.86%	
70	1.50%	0.85%	2.09%	3.39%	3.39%	
75	1.40%	0.85%	1.84%	3.03%	3.04%	
80	1.00%	0.90%	1.53%	2.56%	2.81%	
85	0.70%	0.68%	1.24%	2.14%	2.47%	
90	0.40%	0.58%	1.01%	1.75%	1.85%	

\* Data from the HMD were projected using Lee-Carter (LC Original), Lee-Carter variant Currie-Richards method (LC Currie-Richards) and P-Spline.

### TABLE 5

### Comparison of Scale AA (Females) With Annualized Rates of Improvement in Mortality of Projected qx Between 2010 and 2030 in U.S. Population Data Derived From the SSA and HMD\*

			LC	LC Currie-	
			Original	Richards	P-Spline
		SSA	HMD	HMD	HMD
Age	Scale AA	2010-30	2010-30	2010-30	2010-30
60	0.50%	0.85%	1.25%	2.71%	3.35%
65	0.50%	0.77%	0.97%	2.17%	3.03%
70	0.50%	0.73%	1.15%	2.40%	2.73%
75	0.80%	0.75%	1.25%	2.71%	2.53%
80	0.70%	0.87%	1.23%	2.73%	2.50%
85	0.60%	0.68%	1.12%	2.65%	2.57%
90	0.30%	0.59%	0.96%	2.22%	2.45%

\* Data from the HMD were projected using Lee-Carter (LC Original), Lee-Carter variant Currie-Richards method (LC Currie-Richards) and P-Spline.

### 3.3.3. Life Expectancies and Annuity Values

We derived the 2006 forecast for an approximate complete temporary expectation of life<sup>\*</sup> at age 65 (surviving until age 90) for males (Figure 1) and females (Figure 2) and compared the results assuming both: 1) no improvements in mortality and 2) future improvements in mortality. Data from 2006 was smoothed using the P-Spline method and used as a basis for model comparison. To examine improvements in life expectancy (age 65 surviving to 90), we applied a P-Spline or one of several extensions to the Lee-Carter model to fit the HMD data. Best estimates (50<sup>th</sup> percentile) and 95 percent confidence intervals (2.5<sup>th</sup> and 97.5<sup>th</sup> percentile) are shown.

For males, a comparison of the smoothed data (no improvement) with the output from each method showed a range in life expectancy of 17.1 (4 percent) to 18.0 (10 percent) years between the methods. For females, a similar comparison

Defined as the expected number of years lived between age x and age x+n, the approximate complete temporary expectation of life is denoted as:  $\underset{0.5}{\overset{o}{e}} \cong (\sum_{t=1}^{2.5} t p_{t,5}) + 0.5$ 

indicated a range in life expectancy of 18.9 (2 percent) to 19.9 (7 percent) years between the methods.

Life expectancies assuming future annual rates of improvement in mortality derived from projections of HMD data are higher than those assuming Scale AA. The difference results in about 1 to 4 percent increase in values of annuities when various projections from HMD data are compared with Scale AA (Table 6).

Differences between models based upon either a time series or penalty projection approach were evident. We found models (P-Spline or Lee-Carter Currie-Richards) using a penalty projection approach showed the widest confidence interval (CI) for the calculation of life expectancy. Model output from the P-Spline and Lee-Carter Currie-Richards models showed large differences in estimates for life expectancy within 95 percent CI. For the Lee-Carter Currie-Richards model, the range of life expectancy estimates (individuals age 65 surviving to 90) were 2.5 years for males and 3.6 years for females. Estimates based upon the P-Spline (2-D age-period) model displayed confidence intervals of 3.6 years for males and 2.7 years for females.

The Lee-Carter (1992) method showed the smallest differences in estimates for life expectancy within 95 percent CI. The range of life expectancy estimates (individuals age 65 surviving to 90) were 1.0 year for males and 1.3 years for females. We have generated results using extensions of the Lee-Carter model such as Lee-Carter DDE, Lee-Carter Gompertz (time series), Lee-Carter Smoothed alpha/beta, and Lee-Carter Fully Smoothed (alpha, beta, kappa). Since the results from the runs were similar, with a less than 0.1 year difference in life expectancy at age 65, they are not shown here.

Actuaries may require assumptions that are prudent for various purposes. The stochastic models presented here provide a way of understanding uncertainty around the best estimate projection and the probabilities of the more prudent scenarios. The most prudent 2.5<sup>th</sup> percentile would give an increase in about 2 to 5 percent in the values of annuities at age 65 when compared with the best estimate of their respective models (Table 6).

Figure 1 2006 Best Estimate Forecast: U.S. Life Expectancies for Males at Age 65 (Surviving to 90)



Figure 2 2006 Best Estimate Forecast: U.S. Life Expectancies for Females at Age 65 (Surviving to 90)



# TABLE 62006 Best Estimate Forecast: U.S. Approximate Continuously Paid Temporary Annuity ValuesFrom Age 65 to 90 (5 Percent Interest)

	Male		Female	
Method	Unit Annuity:Best Estimate (% Increase From Base)	Unit Annuity:95% Cl (% Increase From Best Estimate)	Unit Annuity:Best Estimate (% Increase From Base)	Unit Annuity:95% CI (% Increase From Best Estimate)
Base (without future improvement in mortality)	10.63	N/A	11.63	N/A
P-Spline improvement	11.28 (6%)	10.50-11.87 (-7% to 5%)	12.13 (4%)	11.56-12.56 (-5% to 4%)
Lee-Carter Original (Time Series)	11.01 (4%)	10.78-11.23 (-2% to 2%)	11.87 (2%)	11.59-12.13 (-2% to 2%)
Lee-Carter Currie Richards (Penalty Projection)	11.26 (6%)	10.74-11.72 (-5% to 4%)	12.16 (5%)	11.36-12.71 (-7% to 4%)
AA Table	10.9 (3%)		11.77 (1%)	

### 4. Discussion

### 4.1. Comparison of Scale AA with Past Trends

We had observed that, between 1990 and the recent past, the annual rates of improvement in mortality implied by Scale AA were generally lower than those provided by either the HMD or the SSA populations. This suggests that Scale AA would not provide a prudent estimate if the future annual rates of improvement in mortality for annuitants were to mimic those of the overall population described by the recent HMD or SSA data.

### 4.2. Comparison of Scale AA with HMD Data Projections (2010 to 2030)

Our projections using HMD data have provided higher annual rates of improvement in mortality than those of Scale AA. The use of best estimate projected annual rates of improvement in mortality instead of Scale AA would result in about a 1 to 4 percent increase in the values of approximate continuously paid temporary annuities at age 65 (discount rate of 5 percent), depending on the methods of projection.

Based upon these findings, it may be desirable to use a more prudent assumption for the purpose of valuation. Methods such as the P-Spline or Lee-Carter model with its variants provide future scenarios with different degrees of prudence. Using the models described in this paper, the most prudent 2.5<sup>th</sup> percentile would give an increase of about 2 to 5 percent in the values of annuities at age 65 when compared with the model-derived best estimates. This highlights the financial consequences relating to the uncertainty surrounding the best estimate projections of various models.

### 4.3 Basis Risk

For life office portfolios, basis risk would occur when annuitants experience mortality trends dissimilar to that of the overall population. As a result, projections based upon the overall U.S. population data may not be relevant to the derivation of future mortality trends for their annuitants. For example, people under different socio-economic circumstances may experience differing annual rates of improvement in mortality. Studies within the United Kingdom have shown that the more affluent have experienced faster annual rates of improvement in mortality than the less affluent.<sup>14</sup>

In addition, census counts at very high ages may be overstated due to age exaggeration by the individuals interviewed.<sup>15</sup> In recent years, high age mortality rates have been derived by the NCHS from selected Medicare records in the preparation of their life tables. Medicare data are employed in the estimation of qx for ages 66 and older since death rates at the oldest ages are considered to be more accurate than those based upon vital statistics and census data.<sup>15</sup>

Within the United States, annuitants in life office portfolios would, on average, be more affluent and educated than the overall population. One could argue that the more affluent and educated would have better access to health care or information relating to health, and would as a result experience faster rates of improvement in mortality. However, people under less affluent circumstances can have a greater risk profile and thus have greater scope for lifestyle modification that would result in improvement in mortality. For example, a population in more deprived circumstances may have a higher proportion of smokers. They would be more susceptible to health policies targeted toward smoking reduction, which could in turn result in a reduction in the number of smokers and consequently death rates. Thus, a better understanding of trends in different segments of the population can be critical to financial decision-making.

### 4.4 Model Risk

With mortality projections there is no way of knowing if a model is the correct one to use. This problem is known as model risk and the impact can be considerable. For example, Richards and Currie (2009) compared two slightly different versions of the Lee-Carter model when applied to the same population data set in England and Wales.<sup>16</sup> The resulting projections were very different, with the entire 95 percent CI of one model fitting into just one half of the equivalent confidence interval of the other model. This is illustrated in Figure 3.



Figure 3 Different Projections From the Same Data Set Illustrating Model Risk

Different projections from the same data set illustrating model risk. Both models are part of the Lee-Carter family, with the only different being whether a time series or a penalty is used for projection. The data set is for males in England and Wales. (Richards and Currie 2009)

One natural point of reference is the model fit, as one might believe that a better-fitting model is likely to be a better projection model. Unfortunately, this is not the case—a closer-fitting model may nevertheless produce poor forecasts, whereas a poorer-fitting model may yield quite serviceable projections. The goodness of a

model's fit and its suitability for projections are two separate questions and in some circumstances may conflict.

Cairns et al. (2007) describe a series of quantitative measures to use when selecting a projection model. While this approach can be used to weed out poorer models, it does not guarantee that any remaining models are correct, so model risk remains.<sup>17</sup>

### 4.5 Conclusion

Scale AA has been widely adopted in the valuation of liabilities related to longevity risks. We have observed that the annual rates of improvement in mortality provided by Scale AA were generally lower than either: 1) recent trends derived from the SSA or HMD data, or 2) projections made from the HMD dataset. If the rates of improvement in mortality for annuitants were to mimic the recent trends or projections derived from the HMD population, then Scale AA would underestimate their improvement in mortality. These observations cast doubt on the prudence of using Scale AA for the valuation of liabilities related to longevity risks in annuity portfolios or defined benefit pension funds.

However, annuitants may experience different changes in rates of improvement in mortality than that of the HMD or SSA populations. Further work on more relevant data would therefore be useful to the process of assumption setting when projecting trends in the mortality rates for annuitants and pensioners.

# Acknowledgements

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# Comparison of Scale AA With Annual Rates of Improvement in Mortality Between 1977 and 1993 in U.S. Population Data Derived From the HMD (One-Year Age Gap)

	N	lale	Female	
	Scale	HMD	Scale HMD	
Age	AA	1977-93	AA	1977-93
60	1.60%	1.84%	0.50%	1.06%
61	1.50%	1.52%	0.50%	0.55%
62	1.50%	1.73%	0.50%	0.71%
63	1.40%	1.77%	0.50%	0.57%
64	1.40%	1.73%	0.50%	0.75%
65	1.40%	1.63%	0.50%	0.56%
66	1.30%	1.52%	0.50%	0.46%
67	1.30%	1.35%	0.50%	0.33%
68	1.40%	1.00%	0.50%	-0.04%
69	1.40%	1.41%	0.50%	0.49%
70	1.50%	1.85%	0.50%	0.90%
71	1.50%	1.38%	0.60%	0.64%
72	1.50%	1.70%	0.60%	0.65%
73	1.50%	1.32%	0.70%	0.58%
74	1.50%	1.60%	0.70%	0.91%
75	1.40%	1.37%	0.80%	0.80%
76	1.40%	1.36%	0.80%	0.98%
77	1.30%	1.15%	0.70%	0.76%
78	1.20%	0.57%	0.70%	0.37%
79	1.10%	1.03%	0.70%	0.90%
80	1.00%	0.97%	0.70%	1.02%
81	0.90%	0.82%	0.70%	0.86%
82	0.80%	0.68%	0.70%	0.74%
83	0.80%	0.71%	0.70%	0.81%
84	0.70%	0.69%	0.70%	1.03%
85	0.70%	0.43%	0.60%	0.73%
86	0.70%	0.38%	0.50%	0.66%
87	0.60%	0.36%	0.40%	0.66%
88	0.50%	0.29%	0.40%	0.58%
89	0.50%	0.07%	0.30%	0.46%
90	0.40%	0.15%	0.30%	0.52%

## Comparison of Scale AA (Males) With Annual Rates of Improvement in Mortality Between 1995 and the "Recent Past" in U.S. Population Data Derived From the SSA and HMD (One-Year Age Gap)

Scale HM		HMD	SSA	SSA
Age	AA	1995-2006	1990-2006	2000-2006
60	1.60%	2.54%	1.90%	1.47%
61	1.50%	2.38%	1.94%	1.54%
62	1.50%	2.32%	1.97%	1.68%
63	1.40%	1.91%	2.01%	1.89%
64	1.40%	2.13%	2.04%	2.13%
65	1.40%	2.07%	2.06%	2.33%
66	1.30%	2.09%	2.06%	2.49%
67	1.30%	2.15%	2.06%	2.62%
68	1.40%	2.44%	2.05%	2.74%
69	1.40%	2.46%	2.03%	2.82%
70	1.50%	2.50%	1.99%	2.89%
71	1.50%	2.44%	1.95%	2.92%
72	1.50%	2.32%	1.92%	2.89%
73	1.50%	2.08%	1.90%	2.79%
74	1.50%	1.94%	1.87%	2.66%
75	1.40%	2.10%	1.83%	2.52%
76	1.40%	1.73%	1.77%	2.40%
77	1.30%	2.05%	1.71%	2.32%
78	1.20%	1.83%	1.65%	2.28%
79	1.10%	1.58%	1.59%	2.26%
80	1.00%	1.58%	1.51%	2.21%
81	0.90%	1.52%	1.41%	2.13%
82	0.80%	1.68%	1.30%	2.11%
83	0.80%	1.69%	1.19%	2.12%
84	0.70%	1.84%	1.08%	2.14%
85	0.70%	1.97%	0.95%	2.12%
86	0.70%	1.77%	0.82%	2.05%
87	0.60%	2.02%	0.68%	1.92%
88	0.50%	1.82%	0.54%	1.75%
89	0.50%	2.46%	0.40%	1.55%
90	0.40%	2.91%	0.26%	1.34%

## Comparison of Scale AA (Females) With Annual Rates of Improvement in Mortality Between 1995 and the "Recent Past" in U.S. Population Data Derived From the SSA and HMD (One-Year Age Gap)

	Scale	HMD	SSA	SSA
Age	AA	1995-2006	1990-2006	2000-2006
60	0.50%	1.91%	1.31%	1.64%
61	0.50%	1.80%	1.28%	1.59%
62	0.50%	1.92%	1.25%	1.67%
63	0.50%	1.33%	1.23%	1.86%
64	0.50%	1.40%	1.22%	2.11%
65	0.50%	1.46%	1.20%	2.34%
66	0.50%	1.43%	1.17%	2.49%
67	0.50%	1.31%	1.12%	2.49%
68	0.50%	1.54%	1.05%	2.35%
69	0.50%	1.59%	0.97%	2.13%
70	0.50%	1.44%	0.88%	1.91%
71	0.60%	1.40%	0.80%	1.74%
72	0.60%	1.57%	0.76%	1.64%
73	0.70%	1.21%	0.74%	1.60%
74	0.70%	1.00%	0.74%	1.61%
75	0.80%	1.37%	0.72%	1.60%
76	0.80%	1.07%	0.69%	1.57%
77	0.70%	1.07%	0.66%	1.57%
78	0.70%	0.94%	0.63%	1.58%
79	0.70%	0.92%	0.60%	1.59%
80	0.70%	0.76%	0.56%	1.59%
81	0.70%	0.86%	0.51%	1.57%
82	0.70%	0.83%	0.45%	1.54%
83	0.70%	0.84%	0.40%	1.49%
84	0.70%	0.92%	0.33%	1.43%
85	0.60%	1.02%	0.27%	1.37%
86	0.50%	0.79%	0.20%	1.30%
87	0.40%	1.09%	0.13%	1.23%
88	0.40%	0.90%	0.06%	1.16%
89	0.30%	1.37%	-0.01%	1.08%
90	0.30%	1.67%	-0.08%	1.01%

### Comparison of Scale AA (Males) With Annualized Rates of Improvement in Mortality of Projected qx Between 2010 and 2030 in U.S. Population Data Derived From the SSA and HMD (One-Year Age Gap)\*

	_		LC Original	LC Currie-Richards	P-Spline
	Scale	SSA	HMD	HMD	HMD
Age	AA	2010-2030	2010-2030	2010-2030	2010-2030
60	1.60%	1.00%	2.28%	3.77%	4.35%
61	1.50%	0.98%	2.09%	3.73%	4.25%
62	1.50%	0.97%	2.14%	3.69%	4.16%
63	1.40%	0.95%	2.09%	3.66%	4.06%
64	1.40%	0.91%	2.06%	3.63%	3.96%
65	1.40%	0.89%	2.12%	3.59%	3.86%
66	1.30%	0.87%	2.03%	3.56%	3.76%
67	1.30%	0.86%	2.04%	3.52%	3.66%
68	1.40%	0.85%	1.97%	3.48%	3.57%
69	1.40%	0.85%	1.96%	3.44%	3.47%
70	1.50%	0.85%	2.09%	3.39%	3.39%
71	1.50%	0.85%	1.86%	3.33%	3.31%
72	1.50%	0.85%	1.98%	3.27%	3.23%
73	1.50%	0.85%	1.84%	3.19%	3.16%
74	1.50%	0.85%	1.85%	3.12%	3.10%
75	1.40%	0.85%	1.84%	3.03%	3.04%
76	1.40%	0.84%	1.72%	2.94%	2.99%
77	1.30%	0.85%	1.63%	2.85%	2.94%
78	1.20%	0.86%	1.48%	2.75%	2.90%
79	1.10%	0.88%	1.45%	2.65%	2.85%
80	1.00%	0.90%	1.53%	2.56%	2.81%
81	0.90%	0.90%	1.39%	2.47%	2.76%
82	0.80%	0.87%	1.37%	2.38%	2.70%
83	0.80%	0.81%	1.33%	2.30%	2.64%
84	0.70%	0.74%	1.32%	2.22%	2.56%
85	0.70%	0.68%	1.24%	2.14%	2.47%
86	0.70%	0.64%	1.18%	2.06%	2.37%
87	0.60%	0.61%	1.15%	1.98%	2.25%
88	0.50%	0.59%	1.08%	1.90%	2.12%
89	0.50%	0.58%	1.06%	1.82%	1.99%
90	0.40%	0.58%	1.01%	1.75%	1.85%

\* Data from the HMD were projected using Lee-Carter (LC Original), Lee-Carter variant Currie-Richards method (LC Currie-Richards) and P-Spline.

### Comparison of Scale AA (Females) With Annualized Rates of Improvement in Mortality of Projected qx Between 2010 and 2030 in U.S. Population Data Derived From the SSA and HMD (One-Year Age Gap)\*

			LC Original	LC Currie-Richards	P-Spline
	Scale	SSA	HMD	HMD	HMD
Age	AA	2010-2030	2010-2030	2010-2030	2010-2030
60	0.50%	0.85%	1.25%	2.71%	3.35%
61	0.50%	0.86%	1.00%	2.55%	3.29%
62	0.50%	0.84%	1.05%	2.41%	3.22%
63	0.50%	0.81%	0.90%	2.30%	3.16%
64	0.50%	0.79%	0.88%	2.21%	3.10%
65	0.50%	0.77%	0.97%	2.17%	3.03%
66	0.50%	0.74%	0.85%	2.16%	2.97%
67	0.50%	0.74%	0.90%	2.19%	2.91%
68	0.50%	0.73%	0.86%	2.25%	2.85%
69	0.50%	0.73%	0.97%	2.32%	2.79%
70	0.50%	0.73%	1.15%	2.40%	2.73%
71	0.60%	0.74%	0.96%	2.48%	2.68%
72	0.60%	0.74%	1.11%	2.56%	2.63%
73	0.70%	0.75%	1.07%	2.62%	2.59%
74	0.70%	0.76%	1.14%	2.68%	2.56%
75	0.80%	0.75%	1.25%	2.71%	2.53%
76	0.80%	0.75%	1.14%	2.74%	2.51%
77	0.70%	0.77%	1.08%	2.74%	2.50%
78	0.70%	0.80%	1.10%	2.74%	2.49%
79	0.70%	0.84%	1.13%	2.74%	2.49%
80	0.70%	0.87%	1.23%	2.73%	2.50%
81	0.70%	0.88%	1.08%	2.73%	2.52%
82	0.70%	0.86%	1.12%	2.72%	2.54%
83	0.70%	0.81%	1.16%	2.71%	2.55%
84	0.70%	0.74%	1.15%	2.69%	2.56%
85	0.60%	0.68%	1.12%	2.65%	2.57%
86	0.50%	0.63%	1.09%	2.59%	2.56%
87	0.40%	0.60%	1.04%	2.51%	2.55%
88	0.40%	0.59%	0.99%	2.42%	2.52%
89	0.30%	0.59%	0.95%	2.33%	2.49%
90	0.30%	0.59%	0.96%	2.22%	2.45%

\* Data from the HMD were projected using Lee-Carter (LC Original), Lee-Carter variant Currie-Richards method (LC Currie-Richards) and P-Spline.