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A RETIREMENT SYSTEM GRANTING UNIT ANNUITIES AND INVESTING IN EQUITIES

ROBERT M. DUNCAN

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HOWARD H. HENNINGTON:

Mr. Duncan and T.I.A.A. are to be congratulated for this pioneering development in the field of pension planning. It is now taken for granted among individual investors that part of the individual's investments should be in bonds and part in equities. The counterpart in pension planning is for part of the plan to be a traditional plan in terms of promises of dollar benefits and part to be an equity plan where the benefits to employees vary with the experience of the fund. Mr. Duncan's statistical data show historically how the unit annuity system would have worked out advantageously in the past. I believe this will mark the beginning of a broad use of the equity variable-benefit principle in pension planning generally and perhaps later in the insurance field. In connection with insurance, it is interesting to refer to a paper by Mr. Dwight C. Rose entitled "The Policyholder's Interest in Equity Investments" in the *Proceedings* of the American Life Convention of October, 1939.

In order to use the equity variable-benefit principle, T.I.A.A. needed enabling legislation and this legislation applies only to C.R.E.F. The principle can also be used in an employer's pension plan by establishing a trusteed supplemental equity plan. Such a plan which is a supplement to a group annuity dollar benefit plan has recently been announced by the Long Island Lighting Company.

The retirement system outlined in Mr. Duncan's paper is a system patterned closely after the existing T.I.A.A. system. This, for example, dictated the use of a contributory money purchase plan with full death benefits based on employee and employer contributions prior to retirement date. In a supplemental trusteed plan established by an employer, the plan can be kept much simpler if it is set up on a noncontributory basis without any death benefits. On this basis a single unit will be sufficient, and it is not necessary to use two units, an accumulation unit and an annuity unit as described by Mr. Duncan. This use of a single unit of course distributes mortality gains and losses to all individuals both active and

retired. If a single unit is used, employees would be credited at the outset with annuity benefit units payable monthly at normal retirement age. Such a supplemental trusteed plan may be established on either a money purchase formula or on a unit benefit formula where the dollar contribution with respect to an employee is a function of his salary (which determines a benefit) and his attained age.

Many of Mr. Duncan's formulas are somewhat complicated because the contributions participate in the fund as of the end of the month in which they are received. In a noncontributory supplemental trusteed plan, these formulas can be greatly simplified if it is planned that employer contributions would be made once a year on each anniversary of the plan. The formulas are also simpler if dividends do not have to be distinguished from capital gains and if a single unit is used.

At some time sooner or later it seems inevitable that the assumptions of interest and mortality which were originally chosen become no longer suitable. Mr. Duncan has not discussed how this problem would be met. One solution seems to involve a revision of annuity units for the individuals receiving income and a new value of one annuity unit. The obvious disadvantage of this solution is that abrupt changes might be involved and that it would change the number of annuity units credited to individuals. An alternative solution might be to establish two parts to the Annuity Fund, one part which would be closed as of the date of change of assumptions and the other part which would be confined to individuals commencing income on and after the effective date of change of assumptions. This alternative, of course, would require two values of one annuity unit, one for each part of the Annuity Fund.

The acceptance of this new system will depend somewhat on careful explanation so that the individuals understand the arrangement and so that they are prepared for reductions as well as increases in benefits. Perhaps the individuals would not take the reductions quite so seriously if this variable benefit arrangement were on a noncontributory basis. Perhaps it will have to be emphasized repeatedly to the individuals concerned that the test of the success of the plan will rest on the combined effect of the fixed dollar benefit part of the system and the variable benefit part. If this combined benefit follows the cost of living index better than a plan solely in terms of dollar benefits, it can be considered that the unit annuity system has introduced an improvement.

We are indebted to Mr. Duncan for his paper giving us so many details of this new system. Since the principle has so many further applications, its use by C.R.E.F. will be watched with much interest.

ROBERT J. MYERS:

The new type of retirement plan inaugurated by the College Retirement Equities Fund has created great interest among all concerned with problems of old-age security. Mr. Duncan has made an extremely valuable contribution to actuarial literature by setting forth so clearly the actuarial and statistical bases of the program.

The various case histories indicate that the unit annuity based on investment in common stocks turns out better in practically all instances (and certainly on the average for all cases) than the fixed annuity arising from a fixed dollar fund. In actual practice, however, the new plan will at best allow a fifty-fifty approach. At first glance it would seem that if the unit annuity basis produces such superior results, it should be followed completely. Perhaps, after some period when experience warrants, this possibility will be offered, but at present there has been adopted the understandably conservative procedure of combining the two methods. As an additional reason there may also be the desire for a somewhat greater degree of stability in the annuity payments from year to year. Under a plan based completely on common stocks the annuity payable could be reduced as much as 75% in a short period (*e.g.*, 1933 vs. 1929).

Mr. Duncan argues that the combination method has a historical advantage over either of the other two methods in providing some hedge against both inflation and deflation. While the combination method does have some advantages in producing stability, the historical experience indicates that this was of relatively minor importance in comparison with the great advantage that the unit annuity would have had in keeping up with the cost of living. In Mr. Duncan's Tables 6 and 7, which make purchasing power comparisons, figures are given only for the fixed annuity and combined annuity bases. The superiority of the unit annuity basis in maintaining purchasing power, at least on the basis of past experience, would have been clearly indicated if data for it had been included. For the 30-year accumulations (used in Table 6) the figures for all periods are:

Basis	Under 90%	90% to 110%	Over 110%
Fixed Annuity.....	60	10	0
Combined Annuity...	24	33	13
Unit Annuity.....	15	11	44

As a minor technical point, in considering the examples given, it occurs to me that rather than having them based on the accumulation of a constant contribution of $100 per year, the contribution should have varied

as the cost of living varied. Contributions in pension plans generally are based on salary, and these vary over the course of years more or less with the cost of living (actually the variation is also related to general productivity). However, Mr. Duncan takes this factor into account, at least in part, by the use of "the adjusted cost of living annuity." Apparently, the principal difference between using varying contributions and considering resulting annuities with final pay adjusted by changes in pay level subsequent to retirement, as compared with Mr. Duncan's method, reflects only the effect of the interest element (as well as capital gains and losses).

A pension plan based either entirely or partly on investment in common stocks has two objections. First, the value of stocks fluctuates more violently than the cost of living. Second, the cost of living is not completely correlated with the stock market. The first might be overcome by the use of a dampening device in the equities portion of the plan. However, this would destroy one of the essential beauties of an equities plan, namely, come what may in the realm of mortality or investment experience, the fund is always solvent. Of course, the combination of a fixed dollar plan with an equities plan does serve as a partial dampening procedure.

Many types of dampening devices can be developed for the portion of the annuity based on equities investments. However, to be successful, all of them would require some additional financial sources to "backstop" the plan during times of low security prices. Based on past history, any such backstopping to meet the otherwise excess of liabilities over assets would quite likely not mean any cost to the backstopper in the long run because, on the whole, security prices have always recovered from depression troughs. However, in individual cases the choice of securities might be such that recovery would not occur.

One dampening method would be as follows: The initial annuity amount would be the same as that developed according to Mr. Duncan's unit annuity method except that it would be multiplied by the ratio of the common stock price index averaged over the last 10 years to the corresponding index for the year of retirement. The annuity in subsequent years would be determined by the combined effect of the movement of this ratio (which would change each year, being determined for the year of payment of the annuity) and of the variations between actual and expected mortality and interest. This method would result in good equity as between individuals retiring in various years and would prevent violent fluctuations in the resulting pension. In fact, perhaps a shorter period than 10 years should be used for determining the average past stock price index, so as to prevent too much stability (and thus failure to follow cost of living variations).

J. ARTHUR GREENWOOD:

This paper forms a welcome and necessary complement to the Informal Discussion of this subject at the Washington meeting. At that meeting actuaries discussed this system as a combination of economists, sociologists, and investment bankers; the present paper contains enough technical detail so that actuaries may attempt to treat varying annuity plans as actuaries. Mr. Lawson[1] stated in Washington that it would be difficult to establish a standard of good management, since unit annuity funds are always in balance. This is not the first example of an accounting system failing to reflect the fortunes of an annuity fund; D. P. Fackler[2] reported

TABLE 1

PROJECTED RATES OF MORTALITY

AGE	MALES				FEMALES		
	q_x^{x+1}/q_x^x	q_x^{1948}	q_x^{1953}	q_x^{1964}	q_x^{1948}	q_x^{1953}	q_x^{1964}
63	.9886	23.264	21.967	19.364	12.404	11.712	10.325
64	.9888	25.137	23.760	20.991	12.901	12.194	10.773
65	.9890	27.449	25.972	22.996	13.859	13.113	11.611
66	.9893	30.168	28.588	25.397	15.297	14.495	12.878
67	.9896	33.261	31.567	28.137	17.258	16.379	14.599
68	.9899	36.698	34.881	31.196	19.799	18.819	16.830
69	.9902	40.446	38.502	34.549	22.976	21.872	19.626
70	.9905	44.472	42.399	38.173	26.829	25.578	23.029
71	.9909	48.745	46.567	42.112	31.361	29.959	27.093
72	.9913	53.234	50.958	46.288	36.504	34.943	31.741
73	.9917	57.905	55.541	50.675	42.094	40.375	36.839
74	.9921	62.727	60.288	55.251	47.834	45.974	42.133

in 1914: "One large company has, year after year, reported the expected annuity payments as exactly equal to the actual payments."

Mr. Duncan specifically restricts his fund illustrations to the case of no gain or loss from mortality; I wish to supplement his figures by another artificial example that errs in the other direction: continuously improving mortality and no fluctuation in the earned interest rate. It is instructive to note that under the assumption of no gain or loss from mortality and annual annuity payments, Mr. Duncan's system of annuity units reduces to the mathematical equivalent of the compound bonus plan of C. C. Ferguson.[3] The plans are not practically equivalent, because under Ferguson's plan a negative bonus for any year was considered undesirable, and a resultant annuity less than the original guarantee was impossible; both

[1] *TSA* IV, 162. [2] *TASA* XV, 46. [3] *TASA* XV, 355.

negative contributions to surplus and annuity incomes falling below their initial values are contemplated by C.R.E.F.

To illustrate the effect of progressively increasing vitality, the fund method of Kilgour[4] is suitable. The figures in the accompanying tables are illustrative only and are designed to point up possible inequities between age and sex groups in the unit annuity valuation method.

The mortality rates in Table 1 were formed by roughly graduating the 1946–50 group annuity experience,[5] treating these figures as applicable to the year 1948, and applying Projection Scale B[6] to them. The model office used is a simplified version of C.R.E.F., assuming:

1. Mortality year by year as in Table 1.
2. Earned interest rate less expenses 4%.
3. Valuation basis, 1937 Standard Annuity 3%.
4. Annuities without refund features, payable annually not in advance. Thus Mr. Duncan's formula (13) becomes $V_y^B = F_y^B/\Sigma N_x^{PB} a_x$.
5. Retirement age 63.
6. $V_{1953}^B = \$1.00$.

Table 2 represents the progress of a fund supported by a closed group of one million entrants of each sex with one annuity unit each at the beginning of 1953, calculated on the assumptions, (1) that the sexes are separated and (2) that they are combined, in the annual determination of unit values. Table 3 gives the same data for a fund supported by one million entrants of each sex with one annuity unit each annually.

Of the valuation systems suggested by Tables 2 and 3, Table 2 with the experience separated by age *and* sex comes nearest to being an extension of Ferguson's compound bonus to cover loss from vitality. Any of the four systems justifies Mr. Duncan's note[7] that "the value of annuity units will also be affected by the actual mortality experience of C.R.E.F. annuitants."

My object in presenting these comparisons is to ask whether throwing several ages and sexes into one mortality pot does justice to the individual payees. It is unfortunate that the actuaries have borrowed, from the jargon of the investment bankers, "equity capital" as a synonym for share capital, leaving us short of words in which to question the maintenance of individual equity. The tables suggest to me the conclusion that if a group is likely to be homogeneous with respect to age at retirement, so that each member has roughly the same chances of passing through the several ages of exposure to risk, rough justice is preserved by merging the vitality

[4] *TASA* XV, 391.
[5] *TSA* 1951 Reports, 111.
[6] *TSA* I, 417.
[7] *TSA* IV, 338.

TABLE 2

PROGRESS OF A CLOSED ANNUITY FUND

Year y	At Beginning of Year			At End of Year
	F_y^B	$\Sigma N_x^{PB} a_x$	V_y^B	Annuity Units Paid
	Male			
1953	11,283,810	11,283,810	1.0000	978,033
1954	10,757,129	10,677,440	1.0074	955,055
1955	10,225,291	10,078,657	1.0145	930,793
1956	9,690,013	9,486,000	1.0215	905,029
1957	9,153,126	8,898,887	1.0285	877,630
1958	8,616,608	8,317,773	1.0359	848,532
1959	8,082,278	7,743,872	1.0436	817,736
1960	7,552,179	7,178,937	1.0519	785,307
1961	7,028,201	6,625,211	1.0608	751,316
1962	6,512,333	6,084,780	1.0702	715,926
1963	6,006,642	5,560,217	1.0802	679,343
1964	5,513,081	5,054,135	1.0908	641,809
1965	5,033,518	4,569,012	1.1016	.....
	Female			
1953	13,136,770	13,136,770	1.0000	988,288
1954	12,673,952	12,615,269	1.0046	976,372
1955	12,200,046	12,100,266	1.0082	963,849
1956	11,716,295	11,587,498	1.0111	950,321
1957	11,224,077	11,073,368	1.0136	935,393
1958	10,724,925	10,554,797	1.0161	918,662
1959	10,220,469	10,029,271	1.0190	899,722
1960	9,712,471	9,494,730	1.0229	878,196
1961	9,202,663	8,949,967	1.0282	853,741
1962	8,692,953	8,394,587	1.0355	826,166
1963	8,185,176	7,830,021	1.0453	795,476
1964	7,681,071	7,259,673	1.0580	761,960
1965	7,182,160	6,689,277	1.0736	.....
	Male and Female Combined			
1953	24,420,580	24,420,580	1.0000	1,966,321
1954	23,431,082	23,292,709	1.0059	1,931,427
1955	22,425,502	22,178,923	1.0111	1,894,642
1956	21,406,849	21,073,498	1.0158	1,855,350
1957	20,378,458	19,972,255	1.0203	1,813,023
1958	19,343,768	18,872,570	1.0249	1,767,194
1959	18,306,321	17,773,143	1.0299	1,717,458
1960	17,269,763	16,673,667	1.0357	1,663,503
1961	16,237,663	15,575,178	1.0425	1,605,057
1962	15,213,897	14,479,367	1.0507	1,542,092
1963	14,202,176	13,390,238	1.0606	1,474,819
1964	13,206,070	12,313,808	1.0724	1,403,769
1965	12,228,910	11,258,289	1.0862	.....

TABLE 3

PROGRESS OF AN ANNUITY FUND SUPPORTED BY ANNUAL RETIREMENTS

Year y	At Beginning of Year			At End of Year
	F_y^B	$\Sigma N_z^{PB} a_z$	V_y^B	Annuity Units Paid
	Male			
1953....	11,283,810	11,283,810	1.0000	978,033
1954....	22,124,439	21,961,250	1.0074	1,933,338
1955....	32,509,197	32,042,636	1.0145	2,864,880
1956....	42,428,427	41,536,631	1.0214	3,771,406
1957....	51,875,463	50,451,131	1.0282	4,651,527
1958....	60,844,268	58,794,281	1.0348	5,503,790
1959....	69,333,676	66,575,265	1.0414	6,326,738
1960....	77,342,662	73,804,797	1.0479	7,118,968
1961....	84,872,922	80,495,572	1.0543	7,879,145
1962....	91,929,593	86,662,196	1.0607	8,606,074
1963....	98,519,267	92,861,500	1.0671	9,298,755
1964....	104,649,377	97,492,131	1.0734	9,956,397
1965....	110,329,028	102,197,231	1.0795	.....
	Female			
1953....	13,136,770	13,136,770	1.0000	988,288
1954....	25,871,151	25,752,039	1.0046	1,964,793
1955....	38,181,912	37,854,002	1.0086	2,929,039
1956....	50,051,998	49,446,469	1.0122	3,880,151
1957....	61,465,665	60,529,540	1.0154	4,816,866
1958....	72,411,733	71,100,225	1.0184	5,737,524
1959....	82,881,691	81,152,976	1.0213	6,640,062
1960....	92,868,829	90,680,122	1.0241	7,522,053
1961....	102,371,711	99,672,804	1.0270	8,380,742
1962....	111,393,057	108,121,815	1.0302	9,213,191
1963....	119,936,829	116,019,331	1.0337	10,016,462
1964....	128,012,311	123,360,741	1.0377	10,787,917
1965....	135,628,009	130,147,162	1.0421	.....
	Male and Female Combined			
1953....	24,420,580	24,420,580	1.0000	1,966,321
1954....	47,995,743	47,713,289	1.0059	3,898,131
1955....	70,690,975	69,896,638	1.0113	5,793,919
1956....	92,480,301	90,983,100	1.0164	7,651,557
1957....	113,340,766	110,980,671	1.0212	9,468,393
1958....	133,255,904	129,894,506	1.0258	11,241,214
1959....	152,215,426	147,728,241	1.0303	12,966,800
1960....	170,214,765	164,484,919	1.0348	14,641,021
1961....	187,253,135	180,168,376	1.0393	16,259,887
1962....	203,337,003	194,784,011	1.0439	17,819,265
1963....	218,476,372	208,340,831	1.0486	19,315,217
1964....	232,688,571	220,853,472	1.0535	20,744,314
1965....	245,996,047	232,344,393	1.0587	.....

losses and profits by ages as in Table 3. With sex it is another matter. In the present state of knowledge of annuitant mortality we have little confidence in the parallelism by sex either of the experience mortality rates, or of the deviation of these rates from the valuation mortality rates, or of the rates of increase in vitality. Therefore, although cogent nonactuarial reasons for aggregation may be adduced, I favor the separation of the sexes, effectively setting up *two* divergent annuity unit funds.[8]

When the valuation basis for outstanding annuity units becomes improper in the light of experience, what steps consistent with preserving equity would Mr. Duncan take to strengthen the basis?

CECIL J. NESBITT:

A retirement system based on mutual investment fund principles to retirement date and upon variable annuities after retirement is an interesting experiment with substantial promise of success. The Teachers Insurance and Annuity Association deserves credit for developing the Equities Fund as the practical means for such a system. Mr. Duncan has contributed greatly to the development by setting up actuarial procedures for the Fund, and by his clear exposition of them.

At present our faculty members are individually deciding whether they will participate in the Equities Fund and, if so, to what extent. A great deal of interest has been shown in the Fund and, to say the least, there will be no need to encourage participation.

In regard to specific details of the paper I have one question to raise. Formula (11) of the paper provides one expression for ΣN^{PA}_{y+1}, while summation of N^{PA}_{y+1}, determined from formula (12), would give another. My question is: How consistent are the two formulas? In other words, how close will be the relation between

$$(\Sigma' N^{PA}_{y})\ V^{A}_{y+1} + \Sigma B^{P}_{y+1} + \gamma^{A}_{y+1} - \Sigma T^{PA}_{y+1}$$

(Σ' denotes sum at end of year y)

and

$$(\Sigma N^{PA}_{y})\ V^{A}_{y+1} + \Sigma a^{P}_{y+1} + \Sigma N^{PA}_{y} \cdot V^{A}_{y} \times \left[\prod_{n=1}^{12} (1 + g_n + d_n) - \prod_{n=1}^{12} (1 + g_n) \right] ?$$

If the relation is not exact, it may be necessary to give a final readjustment to the value of V^{A}_{y+1} in order that formula (22) (written for $y + 1$) shall hold.

[8] It has been suggested that a third fund, covering man-and-wife last survivor annuities, is desirable.

As a more general question, I should be interested to know about any study of devices for dampening fluctuations in the values of the annuity units.

JOSEPH W. MORAN:

The two objectives of the proposed Equities Fund are an improvement in investment yields and an equalization of the purchasing power of premiums and benefits. I should like to discuss some possible devices which may aid in attaining the latter.

Mr. Duncan's formula (13) defines the dollar value of the annuity unit, V_v^B, by a method which does not recognize the level of the cost of living. Thus the purchasing power of the annuity unit may be altered in a year in which expectations as to expenses, mortality and investment return are all precisely realized. In other words, the change in purchasing power in any year will not precisely reflect actual experience of the Fund as much as it reflects changes in the relationship between the cost of living and market prices of common stocks. The problem is the elimination of these troublesome year-to-year fluctuations which mask the true long-term consistency between the two indexes.

The requirement that all members of the Fund participate equally in a parallel program of fixed-dollar annuities will tend to dampen some of the undesirable year-to-year fluctuations in purchasing power of the annuity unit. However, the effect of this device on the purchasing power of benefits will be unstabilizing in atypical periods such as 1946–48, when stock prices and the cost of living are moving in opposite directions.

Mr. Duncan also mentions that consideration has been given to the use of a moving-average asset-valuation method, but that the method was not found to produce desirable results. It might have turned out to be suitable if the inconsistencies which appear between the level of stock prices and the cost of living were merely a lead-and-lag phenomenon, but such is very definitely not the case.

Because such indirect adjustments of asset values do not properly stabilize the purchasing power of the annuity unit, some economists have considered the possibility of making a direct adjustment in the valuation of the Fund's assets to reflect the current inconsistency of the two indexes. The questions then become: How much adjustment can be made? and What is the financial effect of this adjustment? More important: How will this adjustment, made for the purpose of adjusting the current year's benefit payments, affect the amount of funds available to pay benefits in future years?

For example, take the case of a fund in which the annuity unit was

worth $10 a year ago and (based on current stock prices) is worth the same amount today, and assume that the cost of living has risen 15% during the year. Mr. Duncan's methods do not contemplate any adjustment for this latter fact, and by his formula the purchasing power of the annuity unit would fall about 13%. Some adjustment is desired which will minimize this abrupt change.

Various asset valuation adjustment formulas might increase (or even decrease) the calculated value of V_y^B by various amounts, but it is unlikely that any such adjustment would properly anticipate the other questions raised above. The rather obvious adjustment—a straight 15% increase in book values of assets—would probably weaken the position of the Fund to meet its subsequent benefit commitments, because the actual dollar market value of the assets used to pay current benefits would be less than the dollar amount of benefits paid unless there were an instantaneous increase of 15% in stock prices on the first day of the new year.

The remarks above seem to lead to an inference that the key to "proper" adjustment of V_y^B in recognition of the level of the cost of living lies in the valuation of the benefit obligations of the Annuity Fund. Since this is entirely a prospective valuation, it is necessarily based on predictions of things to come. Because such predictions involve many more uncontrollable and unreliable factors than mere conservative predictions of interest, mortality, and expense rates as are required for valuing fixed-dollar annuities backed by funds invested in high-grade debt instruments, the elements of judgment and luck will enter heavily into the ultimate accuracy of these valuations. Fortunately, the opportunity to readjust the predictions annually will minimize the extent of any irreparable financial damage which may be caused by erroneous predictions, but the success of the plan in stabilizing the purchasing power of annuity benefits will depend primarily upon the selection of realistic assumptions which may be maintained consistently from year to year in the light of actual experience.

The determination of the proper adjustment to be made at any time thus requires a translation of current conditions into assumptions as to future developments. In the example above, I have already mentioned that an assumption that stock prices will suddenly and immediately increase 15% justifies a 15% increase in the calculated value of V_y^B; in the opposite case in which it is assumed that the cost of living will continue at a level 15% above that considered "consistent" with current stock prices, no adjustment at all is justified.

Briefly, any adjustment which is made at all will have to be based on an assumption that the gap between the current level of the cost of living and

one consistent with current stock prices will be diminished or eliminated. This may happen as a result of abnormal dividend earnings, capital gains or losses, or changes in the cost of living. The extent to which each of these events takes place during each future year will determine the amount of adjustment which can be justified.

Again returning to our previous example, suppose that the managers of the Fund make the following assumptions:

That the cost of living will decrease 3% during the next year and 1% during the following year (thus justifying decreases in V_y^B at the end of these years) and remain constant thereafter

That dividend earnings on investments will be 5% in each of the next two years and 4% per year thereafter

That stock prices will increase 1% immediately at the start of the next year, that additional capital gains will be earned over the full year at the rate of 2% and over the following year at 1%, with no further capital gains thereafter

That mortality experience will follow the Standard Annuity Table

That the contractual expense rate e_y^B will be maintained indefinitely

Since the simpler formula (13) has been used with assumptions of mortality according to the Standard Annuity Table and investment returns at the rate of 4% per year in all years, the more elaborate assumptions listed above assume that the "gap" will be closed from 15% to $(115\% \times .97 \times .99) \div [(1.07/1.04) \times (1.06/1.04) \times 1.01] - 100\% = 4.3\%$ within a two-year period.

Taking the oversimplified case in which all participants are males aged 70, the use of the additional assumptions produces a special annuity value $\ddot{a}_x'^{(12)} = 7.994$ in place of the customary $\ddot{a}_x^{(12)} = 8.721$ used in Mr. Duncan's formula. The use of the additional discount factors consequently increases V_y^B from \$10 to \$10.91, a 9.1% increase.

The symbols and formulas used to introduce the additional elements into the calculation of V_y^B are as follows:

${}_{y+s}d_n'$ and ${}_{y+s}g_n'$ are defined as the current estimates, at the end of the year y, of the net monthly dividend rate d_n and capital gain rate g_n which will be realized in the nth month of year $y + s$.

$(1 + i_{y+s}')$ is the product of the 12 terms $(1 + {}_{y+s}d_n' + {}_{y+s}g_n')$ and v_{y+s}' is its reciprocal.

r_{y+s}' is the current estimate, at the end of year y, of the ratio of V_{y+s}^B to V_{y+s-1}^B. If the purchasing power of the annuity unit is to be kept constant, it is the ratio of the cost of living in year $y + s$ to that in year $y + s - 1$.

${}_{y+s}\ddot{a}'^{(12)}_{x+s:\overline{1}|}$ is used to represent a one-year annuity based on capital gains and dividends at the rates ${}_{y+s}g'_n$ and ${}_{y+s}d'_n$.

$$
\begin{aligned}
{}_y\ddot{a}'^{(12)}_x &= {}_y\ddot{a}'^{(12)}_{x:\overline{1}|} + v'_{y+1} \cdot r'_{y+1} \cdot p_x \cdot \left[{}_{y+1}\ddot{a}'^{(12)}_{x+1} \right] \\
&= {}_y\ddot{a}'^{(12)}_{x:\overline{1}|} + v'_{y+1} \cdot r'_{y+1} \cdot p_x \cdot \left[{}_{y+1}\ddot{a}'^{(12)}_{x+1:\overline{1}|} \right. \\
&\qquad + v'_{y+2} \cdot r'_{y+2} \cdot p_{x+1} \cdot \{ \cdots\cdots ({}_{y+k-1}\ddot{a}'^{(12)}_{x+k-1:\overline{1}|} \\
&\qquad \left. + v'_{y+k} \cdot r'_{y+k} \cdot p_{x+k-1} \cdot \ddot{a}^{(12)}_{x+k}) \} \right]
\end{aligned}
$$

where k is the number of years for which the special discount factors are used.

The formulas have been used to determine what adjustment in V^B_y may be justified on the basis of probable future experience. It would also be possible to use them to determine the future changes required if V^B_y is now set at a certain specified level, and certain conditions prevail. For example, it may be desired to limit changes in V^B_y or changes in the purchasing power of the annuity unit to a certain percentage, or to spread any adjustments over a period of years.

In the situation illustrated above where a 15% "gap" had developed, assume that there will be no changes in the cost of living or level of stock prices in the foreseeable future. Immediate adjustment of V^B_y according to Mr. Duncan's formula (13) would result in a 13% decrease in the purchasing power of the annuity unit. Suppose that it is desired to limit such changes to 5% in any given year. In the simplified case in which all participants are age 70 this means that V^B_y must be 95% of \$11.50 or \$10.93; to obtain this result ${}_y\ddot{a}'^{(12)}_x$ must be 7.983. On this basis, V^B_{y+1} will be 95% of \$10.93, or \$10.38, V^B_{y+2} will be 95% of \$10.38, or \$9.86, and the ultimate value of V^B for years after $y+2$ will be \$9.81. The ultimate dollar value of the annuity unit determined on this basis is, of course, slightly less than the purchasing power of a \$10 annuity unit which would have been developed by Mr. Duncan's formulas, but the decrease in purchasing power has been much more gradual. There is always the possibility of changes in conditions to make the later adjustments unnecessary.

The methods may also be used to compare the effects on the values of V^B_y in future years if experience during the coming year is not that now expected. In the original example above it could be anticipated currently that capital gains earned at the rate of 5% over the coming year would justify an increase in V^B_y for the following year of about 2.97%.

Again I should like to point out my awareness that the determination of the extent to which special assumptions may be justified is the responsi-

bility of the investment men and the economists rather than the actuaries. The purpose of this discussion has been to outline actuarial approaches which may be used to attempt to stabilize the purchasing power of the annuity unit. I have not attempted to consider some of the related considerations such as the attitude of any state insurance commissioners toward the use of such assumptions or the practical problems amassed by time lags in the valuation process. It may very well be that such down-to-earth practical considerations may make the use of the devices mentioned above completely impossible for use in the determination of V_y^B.

I should like to commend Mr. Duncan for his presentation of some of the interesting special problems inherent in the development of this new form of retirement plan. My interest is aroused particularly by the fact that the opportunity to readjust the calculation of the Fund's dollar liability annually seems to make it possible to take the actuary off his leash of conservatism and let him run around in more realistic surroundings.

(AUTHOR'S REVIEW OF DISCUSSION)

ROBERT M. DUNCAN:

The discussions submitted have all added materially to the paper, bringing out several points not previously mentioned or fully explored, and we are indeed grateful for the additional light shed on some of the problems encountered in the use of unit annuities.

Mr. Hennington outlines the simpler procedures possible under a single unit system. While somewhat more complicated, the two unit system seemed preferable in our case for several reasons. First, since we wished to reflect actual mortality experience in the unit values applicable to annuity benefits currently being paid, it seemed more equitable to exclude this factor in determining the values credited to participants not yet receiving annuity benefits. Secondly, this system allows the more equitable crediting of monthly capital value changes in the premium-paying period, since most of our premiums are payable monthly, while allowing unit values to change only annually in the payout period, for the reasons given in the paper. Similarly, although it would have simplified the calculations to merge dividends with the capital gains, we felt that there were definite advantages in not doing so, in that the segregation would emphasize the interest earned by the fund in the premium accumulating period, as well as avoid the probable result of a series of equal premiums buying fewer and fewer units than if changes in capital values were the main cause of changes in accumulation unit values.

Although a reduction in a variable annuity benefit may possibly be

more acceptable under a noncontributory plan, I believe that this is at least open to question, since many employees look primarily to the employer as responsible for the amount of pension benefits, regardless of the method used to provide these benefits.

In addition to the Long Island Lighting Company plan mentioned by Mr. Hennington, interest in the variable annuity has been shown by several large corporations and at least one other sizable program is nearing completion.

Mr. Hennington and Mr. Greenwood raise the question as to how equity is to be preserved when the trend of actual mortality differs from the assumptions used in the operation. Mr. Greenwood's tables develop the cumulative effects of this kind of difference for extended periods by the use of mortality factors assumed to be inappropriate to the trend of actual experience, and also show how these effects might develop differently for male and female lives. If a thoroughly realistic set of mortality assumptions, by sex, is established at the levels of mortality existent at the initiation of the plan, and if projection of future mortality changes is also built into the periodic valuation procedures, it should be reasonable to expect no serious systematic divergence of mortality from assumptions for some time. Of course, it is inevitable that at some future date changes in the mortality assumptions will need to be made. Assuming that such change is made promptly as soon as a divergence in trend is apparent, so that the readjustment is small, practical equity will be retained by merely timing the change in mortality assumptions as of any valuation date. Although this changes the unit value, no change is necessary in the number of units for any individual. The effect is that participants at the same attained age, entering either before or after the change, will receive practically the same number of units for the same current "reserve" amount in the Annuity Fund. (The number would be exactly the same if annuity values under the new assumptions were a constant percentage of the former values.)

However, aside from any consideration as to the kinds of inequities that would be introduced by the use of closed groups, the use of only one annuity unit value at any given time has several advantages. Being based on current mortality trends, it provides a more consistent basis of equity with the other current changes in unit values, and produces a combined effect of equity of at least as high an order as is commonly available in the annuity field. The use of only one annuity group, besides being simpler to administer, avoids the wider mortality fluctuations in the closed groups as they become progressively smaller. Also, the use of current mortality assumptions avoids the cumulative mortality divergence in unit values,

and, if interest and expense assumptions are kept up to date, establishes the pattern of all unit value changes mainly by reason of capital gains or losses, which is a fundamental objective of the unit annuity.

Mr. Myers summarizes the data for the unit annuity in the form used in Table 6. This indicates a superiority, from the purchasing power standpoint, of the unit annuity over both the fixed annuity and the combined annuity for the periods covered. While the figures shown would undoubtedly raise the question as to the advisability of requiring that not more than 50% of premiums be allowed to go into the unit annuity, it is obvious that the historical data, although showing this superiority in the past, are only retrospective and too many unpredictable forces are impinging on the economy to justify the assumption that this pattern would necessarily hold in other circumstances. The combination method, moreover, has several other advantages, including the very important one of stabilization, already referred to in the paper and discussions.

Mr. Myers mentions the possible effects of basing the tables on premiums varying with salary scales or the cost of living, rather than on the level premiums used in the paper. In the preliminary stages of preparation we tested the patterns produced by using approximate changes in salary levels of college professors for the seventy calendar years involved in the study. After making allowance for the additional "dollar cost averaging" effect from monthly premiums (our predominant mode of premium payment) over the annual premiums used in the tables, we found that varying the premiums by salary levels introduced practically no relative effect on the sizes of the fixed annuity and unit annuity. We therefore decided to complete the calculations by the simpler method, especially as it allowed easy comparison of the data for the various periods.

Several comments and suggestions have been made as to methods of graduation of the fluctuations in the market values of common stocks. We tried numerous linear compound formulas, using periods of up to four years, our feeling being that any longer periods would introduce too many serious practical problems, including lags and inequities, even though a powerful smoothing effect could be achieved. One factor, for example, is the fact that the average annuitant does not have too many years for the graduation to operate after his retirement, particularly in our case, where ages 68 to 70 are fairly commonly elected for the start of annuity payments. After running off several "model office" studies with moderate to very little "backstopping" for the fund as a whole, we uncovered various disappointing aspects of weighted averages moving in the "wrong" directions in relation to cost of living and current stock price changes. In addition, the lag also introduced raises some rather serious practical problems

in individual equities. For example, one problem is the treatment as between maturities of deferred annuities and single premium immediate annuities. We also found that the compounding did not work well in periods when successive stock price changes were in the same direction, since the effects were mainly to defer rather than to diminish the maximum or minimum points. In periods of rapidly fluctuating prices, however, the results were quite good.

Mr. Moran has suggested an interesting approach to the problem of providing better correlation between the unit annuity and the cost of living changes. While any improvement in correlating with the cost of living is undoubtedly to be desired, I am wondering if the economists and investment men might quail from the responsibility for fixing the constants for the additional factors introduced, particularly if the projections extended over any appreciable period. Also, I suspect that many serious practical questions of individual equity and expense of administration might be raised in addition to the other difficulties mentioned by Mr. Moran.

Professor Nesbitt also inquires as to the consistency of formulas (11) and (12). These can be shown to be exactly equivalent by making use of the relation

$$V^{A}_{y+1} = V^{A}_{y}\left[\prod_{n=1}^{12}(1+g_n)\right]$$

which, incidentally, is a good check on the numerical value obtained by formula (9).

This relation of successive accumulation unit values can be derived from the basic operations of the Accumulation Fund. Dealing only with dollar values, it is evident that:

Accumulation Fund at end of year = Accumulation Fund at beginning of year, with net capital gains to end of year
+ Net dividends, with net capital gains to end of year
+ Net premiums, with net capital gains to end of year
− Termination values, with net capital gains to end of year

or

$$F^A_{y+1} = F^A_y \Big[\prod_{n=1}^{12} (1 + g_n) \Big] + \gamma^A_{y+1} + \Sigma\beta^P_{y+1} - \Sigma T^{PA}_{y+1}.$$

But

$$F^A_{y+1} = f^A_{y+1} (A^C_{y+1} - L^C_{y+1}) = V^A_{y+1} \Sigma N^{PA}_{y+1}$$

and hence from formula (9), which defines V^A_{y+1}, we obtain

$$V^A_{y+1} \Sigma N^{PA}_y = F^A_y \Big[\prod_{n=1}^{12} (1 + g_n) \Big] = V^A_y \Sigma N^{PA}_y \Big[\prod_{n=1}^{12} (1 + g_n) \Big]$$

and

$$V^A_{y+1} = V^A_y \Big[\prod_{n=1}^{12} (1 + g_n) \Big].$$

The reconciliation of formulas (11) and (12) can then be shown by breaking down formula (11) into its component parts in the following manner, taking all values at the end of year $y + 1$:

Let (a) = Value, at end of year, of units held at beginning of year and in force end of year

(b) = Value, at end of year, of units held at beginning of year and terminated during year

(c) = Value of net premiums paid, with net capital gains to end of year, by survivors at end of year

(d) = Value of net premiums paid, with net capital gains to end of year, by terminators during year

(e) = Value of net dividends, with net capital gains to end of year, on units held at beginning of year and in force end of year

(f) = Value of net dividends, with net capital gains to end of year, on units held at beginning of year and terminated during year

(g) = Value of net dividends, with net capital gains to end of year, on premiums paid during year by survivors at end of year

(h) = Value of net dividends, with net capital gains to end of year, on premiums paid during year by terminators during year

Accumulation Fund at end of year $= (a) + (c) + (e) + (g)$
$= V^A_{y+1} \Sigma N^{PA}_{y+1}.$ (A)

But

$$(a) + (b) = V^{A}_{y+1}\Sigma' N^{PA}_{y}\,, \text{ where } \Sigma' \text{ denotes sum at end of year } y$$

$$(c) + (d) = \Sigma\beta^{P}_{y+1}$$

$$(e) + (f) + (g) + (h) = \gamma^{A}_{y+1}$$

$$(b) + (d) + (f) + (h) = \Sigma T^{PA}_{y+1}, \text{ since } V^{A}_{y+1} = V^{A}_{y}\left[\prod_{n=1}^{12}(1+g_n)\right].$$

Subtracting the last expression from the sum of the three prior expressions, we obtain

$$(a) + (c) + (e) + (g) = V^{A}_{y+1}\Sigma' N^{PA}_{y} + \Sigma\beta^{P}_{y+1} + \gamma^{A}_{y+1} - \Sigma T^{PA}_{y+1}\,.$$

Using formula (A) above, we arrive at the relationship given in formula (11) in the paper, whereby

$$\begin{aligned} V^{A}_{y+1}\Sigma N^{PA}_{y+1} &= (a) + (c) + (e) + (g) \\ &= V^{A}_{y+1}\Sigma' N^{PA}_{y} + \Sigma\beta^{P}_{y+1} + \gamma^{A}_{y+1} - \Sigma T^{PA}_{y+1}\,. \end{aligned} \tag{B}$$

Turning now to formula (12) in the paper, and summing it for all survivors, we have

$$V^{A}_{y+1}\Sigma N^{PA}_{y+1} = V^{A}_{y+1}\Sigma N^{SPA}_{y} + \Sigma a^{SP}_{y+1} + V^{A}_{y}\Sigma N^{SPA}_{y} \times\left[\prod_{n=1}^{12}(1+g_n+d_n) - \prod_{n=1}^{12}(1+g_n)\right]$$

where the superscript "S" refers only to survivors at the end of year $y+1$.

But

$$V^{A}_{y+1}\Sigma N^{SPA}_{y} = (a)$$

$$\Sigma a^{SP}_{y+1} = (c) + (g)$$

$$V^{A}_{y}\Sigma N^{SPA}_{y}\left[\prod_{n=1}^{12}(1+g_n+d_n) - \prod_{n=1}^{12}(1+g_n)\right] = (e)\,.$$

From the summation of formula (12), therefore, $V^{A}_{y+1}\Sigma N^{PA}_{y+1} = (a) + (c) + (g) + (e)$, as in formula (B) above, and hence the consistency of formulas (11) and (12) is established.