Implementing Risk Appetite for Variable Annuities

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Abstract

The following paper starts by defining and discussing the nature of risk and its primary relationship to capital preservation. The paper then continues with a guide for implementing a company's risk appetite statement for a variable annuity product. A company's unique risk profile changes at the level of individual transactions. Because it is impractical to set limits and monitor risk at such a low level, companies group risk into larger classes in accordance with a chosen risk framework. One function of a risk appetite statement is to define the risk capital allocated to the variable annuity business by risk class. Risk capital is typically defined in terms of economic capital allocations at the corporate level. To implement the risk limits set forth by this statement, a company can 1) create a risk map from risk class to transaction type for each related control variable, 2) model the sensitivity of capital to each control variable in terms of risk appetite and 3) set the allowable range for each control variable in order to stay under the explicit limits of the risk appetite statement. All this must be done while maintaining compliance both with regulations and company best practices. Several potential models are discussed to model the capital sensitivity. The focus is on those elements that influence the liability side of the balance sheet.

Introduction

A risk represents the probability of losing capital. Typical risk statements take the form: “I’m willing to lose up to X percent of my capital.” Risk managers often further associate a confidence limit with this probability of loss. When this is done, the risk statement becomes “I’m willing to lose up to X percent of my capital with Y percent probability.” This corresponds exactly to the definition of economic capital. Thus, the probability of losing capital can be fully specified by detailing the amount of economic capital a firm is willing to hold. This does not yet complete the definition of risk for there is a second dimension: potential reward.

Imagine you are taking a trip and have to drive from Philadelphia to Boston in 15 hours. To do this, you have to cross over into the state of New York at some point. There are two potential bridges you can take: the George Washington or the Tappan Zee. The former takes 20 minutes to cross on average, but it could take seven hours when traffic is heavy. The latter is farther north and takes one hour to cross in good conditions and two hours in traffic. Which is the better choice? The potential gain in time you would get by crossing the Tappan Zee is six hours, when the Tappan Zee is empty and the George Washington is congested. Similarly, the potential loss from this route is 1.67 hours. The reward-to-loss ratio is thus 360 percent. You should take the Tappan Zee not as a shortcut but because it is a better risk-reward decision. Combining the probability of loss and the potential reward leads to a full understanding of risk. In our example above, if you think the probability of traffic on the George Washington is 15 percent, then the Tappan Zee would be a better choice as long as the chance of traffic is less than 33 percent. There is no logical reason a large risk must be commensurate with a high reward. In our driving example above, we could have analyzed a more circuitous route to Boston that would increase our risk without promising a time savings. Not all risk is opportunity; indeed, only risk that has a high reward ratio can be called opportunistic.

The overall risks of a firm are not independent of each other and are classified in a way that makes sense to the stakeholders involved. Each company has a unique mixture of risks that define it within a given risk framework. What actions can change a company’s unique risk profile? It is tempting to think that a company’s risks change with external events, but that is only a consequence of the decisions that allowed such events to be damaging. A company’s risk changes with every transaction it makes. Every policy sold, every benefit paid, every factor changed alters a firm’s mixture of risk. Because it would be impractical to monitor risk at a transaction level (this would be both very costly and intrusive), firms look to aggregate similar risks into larger risk classes for the purpose of quantization. For example, if a policy is mistakenly sold with a lower price than intended, it might be classified as pricing risk, which can be further summed up under underwriting risk.

A risk appetite statement outlines the risk for a company. It usually includes financial targets for the company, leverage and liquidity ratios, for example, as well as exposure or cash flow limits for each line of business. From a single line of business point of view, this can be translated into a set of economic capital limits by risk classification that can then be used to set operational limits for the business.

The Variable Annuities Risk Appetite Statement

Liability-side transactions are the basis of economic loss, and economic loss is the basis of economic capital. All three of these items can be managed with a companywide risk appetite statement. This statement is an outline of the economic capital a firm is willing to allocate to each of its major risk classifications and a set of guidelines that set company financial policy. Each of these classifications is a sum over a large number of transactions performed by the company. These transactions and the risk changes associated with them can to some degree be controlled by the firm. Such statements are not static. The specific amount of economic capital allowed by risk class should be dynamic within itself and change as business data is realized, particularly sales. If the total allocation at the beginning of the year is $10 billion based on a yearly sales projection of $2 billion and $4 billion of variable annuities premium is sold in the first quarter of the subsequent year, then the yearly number should increase dramatically. Risk changes occur at the transaction level while the risk appetite statement outlines risk classes. To translate between the objectives set for a risk class and operational guidelines that run the business, a map is needed. At the root of this map are the transactions the business undertakes. For a variable annuities block, we can divide them into three classes:

- Premium related transactions
- Decrements
- Guaranteed minimum benefits (GMxBs)

To focus on the liability side of the business, we can divide these three major classes into individual controls that can be affected through changes in pricing and financing policy.

Premium related transactions

Management & Expense (M&E) fees
Premium bonus
Premium loads
Investment options for deposit split
Fund transfer limits and fees
Deposit, trail and renewal commissions

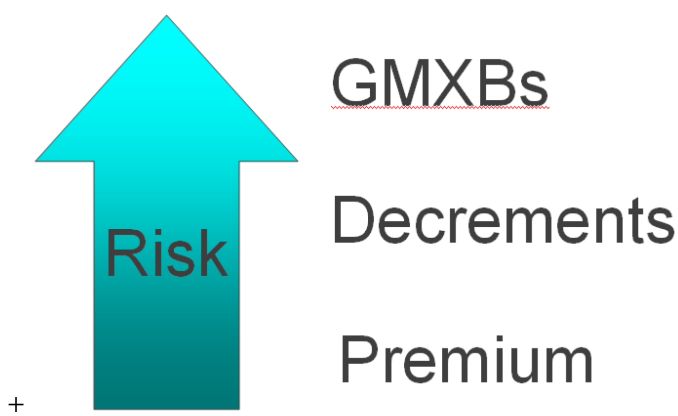
Decrements

Lockout period on annuitizations
Surrender charges
Withdrawal limits

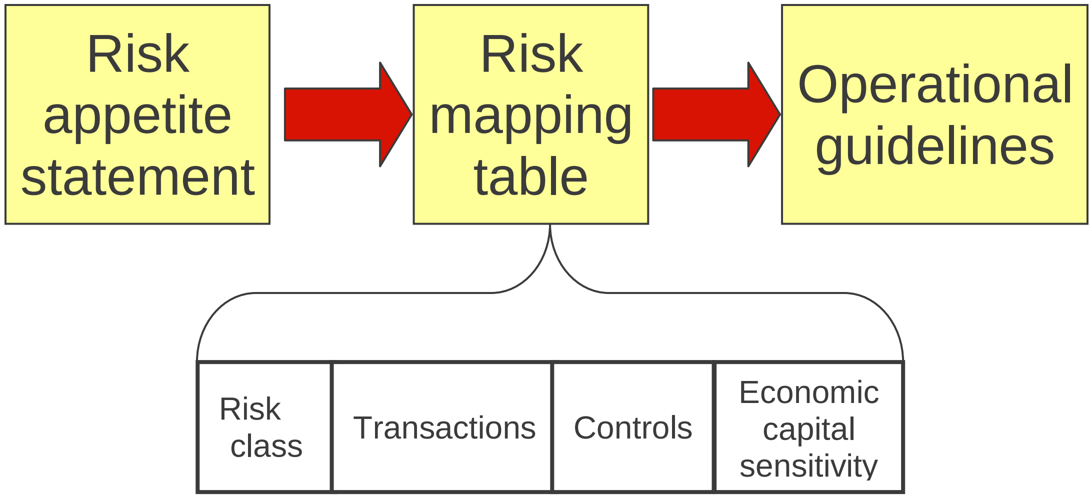
GMxBs

Charge percentage
Charge base
Charge timing
Deposit bonus rate
Guaranteed interest rates
Hedge ratios
Lifetime benefits
Maximum age provisions
Benefit payment frequency
Ratchet period
Ratchet maximum
Reinsurance fees
Rollup rate
Rollup cap
Rollup frequency
Waiting period

GMxBs are the riskiest class of transactions. Premium related transactions can affect the bottom line, but higher commissions and changes in management and expense (M&E) fees only tend to squeeze margins and have less loss potential. Decrements can cause financial distress and loss in a more material way than the premium transactions but are also unlikely to lead to the loss levels possible in the rider-related transactions. Most of the probability of bankruptcy among these transaction classes rests with these guarantees and the company's skill in mitigating them.



To translate the risk appetites statement into a set of business rules for the line, a map is needed. This map should list out the risk classifications defined in the risk appetite statement, the associated transactions that fall into those classes, the controls used to affect them and the sensitivity of economic capital to any change in those controls.



Since economic capital is related to loss, we need only observe the variables that create liabilities for the company in the creation of a map. One possible mapping is below with a generic risk class assignment based on transaction group.

Risk Class	Transaction	Control
A	GMxB hedging costs	Charge % X hedge ratio
	GMxB reinsurance costs	Reinsurance %
		Other treaty provisions
	GMxB fees	Charge %
		Charge base
		Charge timing
		Deposit bonus
	GMxBs	Guaranteed rate
		Benefit payment frequency
		Maximum age
		Ratchet period
		Ratchet maximum
		Rollup rate
		Rollup cap
		Rollup frequency
		Waiting period
B	Annuitizations	Lockout period
	Lapses	Surrender charges
		Withdrawal charges
C	Revenues	M&E fees
		Premium bonus
		Premium load
	Separate account loss	Investment options
		Fund transfer charges
	Commissions	Deposit commissions
		Asset trails
		Renewal commissions

Once the sensitivity of economic capital is established, each control can be assigned an upper limit to its value in a way that will not exceed the total capital allocated to the risk classification. For example, if we determine that the sensitivity of the GMxB rollup rate is $10 million in economic capital per 10 basis point increase in rollup rate and we wish to stay below $300 million in total capital for the transaction class, then the rollup rate should never be sold above 3 percent. In general, for a given transaction:

$$\text{Maximum Operational Limit} = \frac{\text{Transaction Class Limit}}{\text{EC Sensitivity}}$$

The total economic capital within each risk class will be the sum used over all transactions within that class. Thus the problem of implementing the risk appetite statement reduces to two core issues:

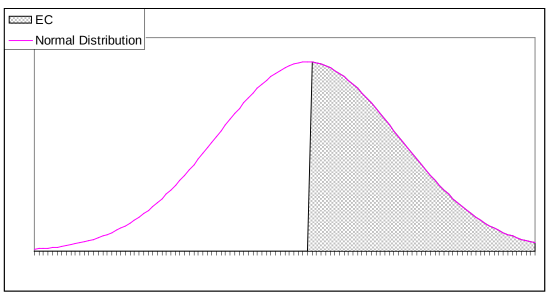
1. Determining the sensitivity of economic capital for each control
2. Setting the transaction class limit for each control

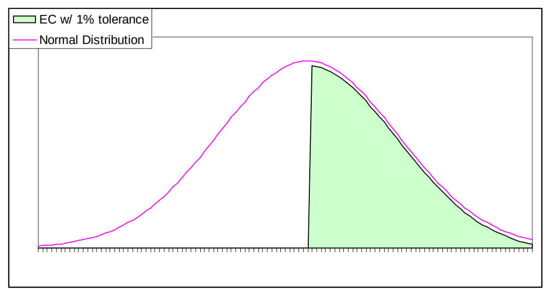
Determining the Sensitivities

The economic capital is the answer to the question "How much capital should I hold to cover 100 percent of my losses with X percent probability?" Though we need not be so specific, if we wish to answer the question "How much capital should I hold to cover 99 percent of my losses with X percent probability," we can consider a measure of economic capital as well, one in which a higher loss amount is tolerated. The capital needed is partially covered by the natural reserves the firm must hold. For any loss distribution, the reserves are sufficient to handle the average loss. Thus we can define economic capital as

$$EC = \int^{p} c(x-\mu) f(x) dx$$

Where p is the probability level desired in the measurement and c is the risk tolerance. The economic capital needed will differ based on the level of risk tolerance but should be substantial enough to cover the vast majority of losses.



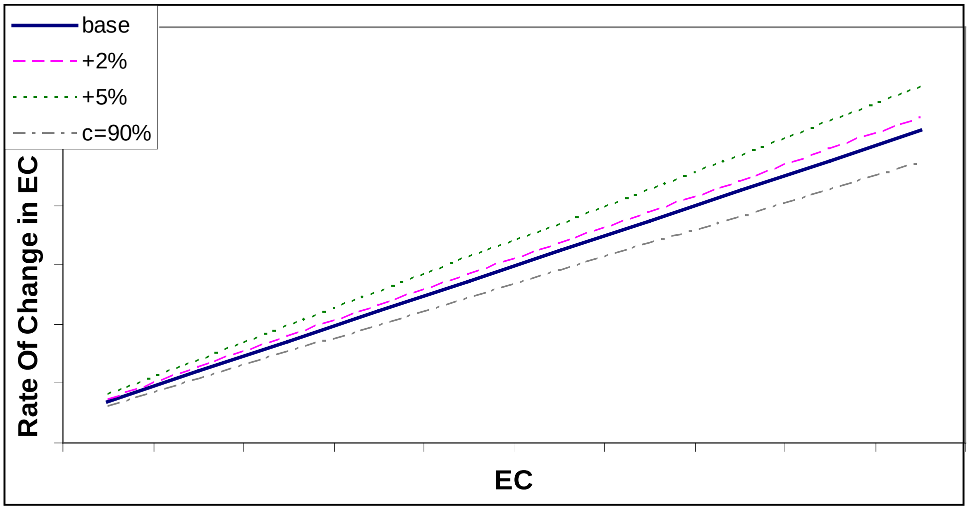


For each variable defined above, we can define a distribution that best fits the potential loss a company can sustain. From there, the goal is to determine the rate of change in economic capital as the underlying parameters of that distribution change. Then the parameters can be estimated with experience studies and the resulting distribution and economic sensitivity will be known. Ideally, the theoretical distribution will be combined with a predictive model on a weighted basis to improve the accuracy of this sensitivity for any individual company. Without loss of generality, several potential distributions are discussed below as models.

We will first consider the sensitivity of economic capital to the parameters of the loss distribution for a company with a normal loss distribution. Using Leibniz's rule, the change in EC based on small changes in the mean is

$$\frac{\partial}{\partial\mu}EC = \frac{\partial}{\partial\mu}\int_{\mu}^{p} c(x-\mu) f(x) dx = \int_{\mu}^{p} \frac{\partial}{\partial\mu} c(x-\mu) f(x) dx = c\int_{\mu}^{p} \frac{(x-\mu)^2}{\sigma^2} f(x) = \frac{c(\mu-p)}{\sigma^2}EC + \frac{c^2(\mu-p)}{\sqrt{2\pi\sigma^2}}$$

This shows that as the mean loss increases, not only will the economic capital needed increase but the rate of change will also increase. Consider a base case in which we wish to be 99 percent sure of covering 100 percent of all losses (c=1.0, p=0.99). The rate of change grows steeper as the mean increases. Additionally, if the risk tolerance is increased and c is dropped, then the rate of change will drop substantially at the higher levels of economic capital.

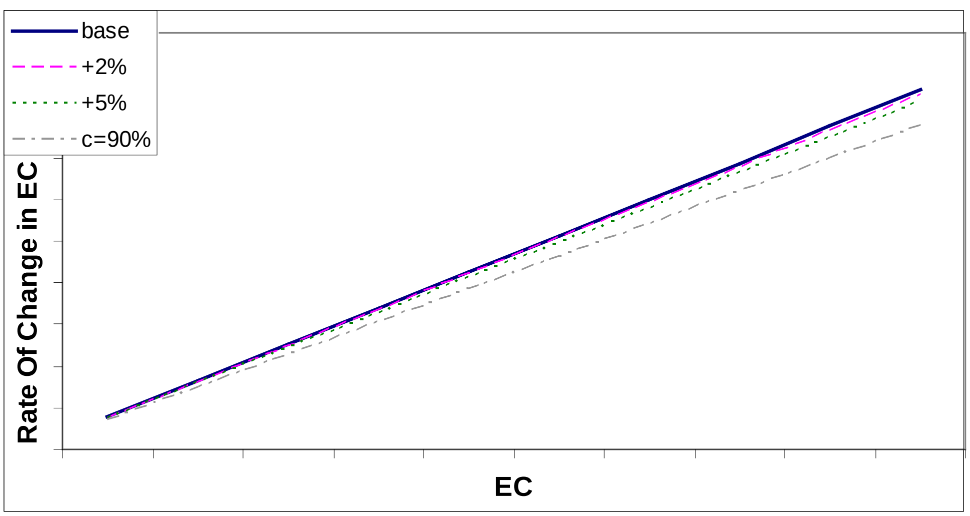


Performing the same calculation on the change in economic capital needed as the standard deviation changes results in another linear form,

$$\frac{\partial}{\partial \sigma} EC = \frac{\partial}{\partial \sigma} \int_{\mu}^{p} c(x-\mu) f(x) dx = \int_{\mu}^{p} \frac{\partial}{\partial \sigma} c(x-\mu) f(x) dx = c \int_{\mu}^{p} \frac{(x-\mu)^3}{\sigma^3} f(x) =$$

$$c \frac{2p\mu + 2\sigma^2 - p^2 - \mu^2}{\sigma^3} EC + \frac{c(2p\mu - p^2 - \mu^2)}{\sigma^2 \sqrt{2\pi}}$$

In this case, the rate of change in capital is much less sensitive to changes in the standard deviation.

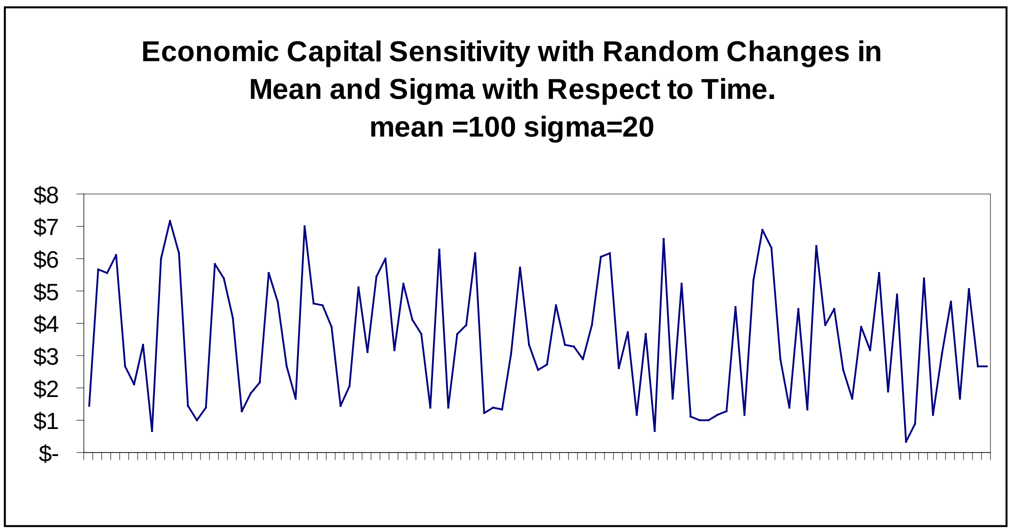


The total rate of change in the economic capital over time is

$$\frac{d}{dt}EC = \frac{\partial EC}{\partial \mu}\frac{\partial \mu}{\partial t} + \frac{\partial EC}{\partial \sigma}\frac{\partial \sigma}{\partial t}$$

Thus, if we assume normality, the problem of deriving sensitivity reduces to one in which we must estimate the mean and standard deviation of the loss distribution using internal data. Estimates are also needed for $\partial\mu/\partial t$ and $\partial\sigma/\partial t$, though it may be more feasible to assume that the mean and standard deviation change uniformly over time. The total change in economic capital will increase in a linear fashion if the estimated change in $\partial\mu/\partial t$ and $\partial\sigma/\partial t$ over time increases.

It is conceivable that the rate of change in a variable's mean loss can be different than the rate associate with the standard deviation; these changes may become quite complex. If we select $\partial\mu/\partial t$ and $\partial\sigma/\partial t$ as uniform random from values between 0 and 10 percent, the resulting sensitivity of the economic capital becomes volatile.



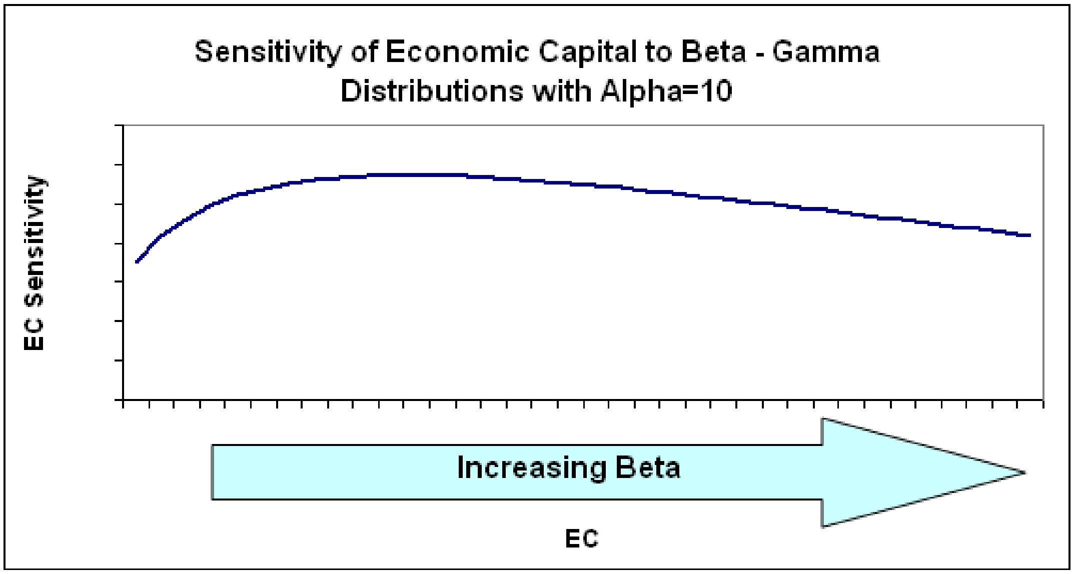
We can perform the same analysis on the more general gamma distribution below:

$$f(x) = x^{\alpha-1}e^{-\beta x}\frac{\beta^{\alpha}}{\Gamma(\alpha)}$$

Where α defines the shape of the distribution and β defines the scale. Because the parameters themselves are difficult to fit to an experience study, it is easier to instead hold α fixed and modify β as new information on the mean and deviation of losses becomes available. This leads to a focus on the sensitivity of economic capital with respect to β.

$$\frac{\partial}{\partial \beta}EC = \frac{1}{\beta}\frac{e^{\alpha}p^{\alpha}(1+p\beta-\alpha) - e^{p\beta}\left[\alpha/\beta\right]^{\alpha}}{e^{p\beta}\left[\alpha/\beta\right]^{\alpha} - e^{\alpha}p^{\alpha}}EC$$

Unlike the normal distribution, this sensitivity is nonlinear. Holding all parameters constant and setting the α to10 while increasing the β shows an accelerated change in economic capital that levels off and decreases over time.

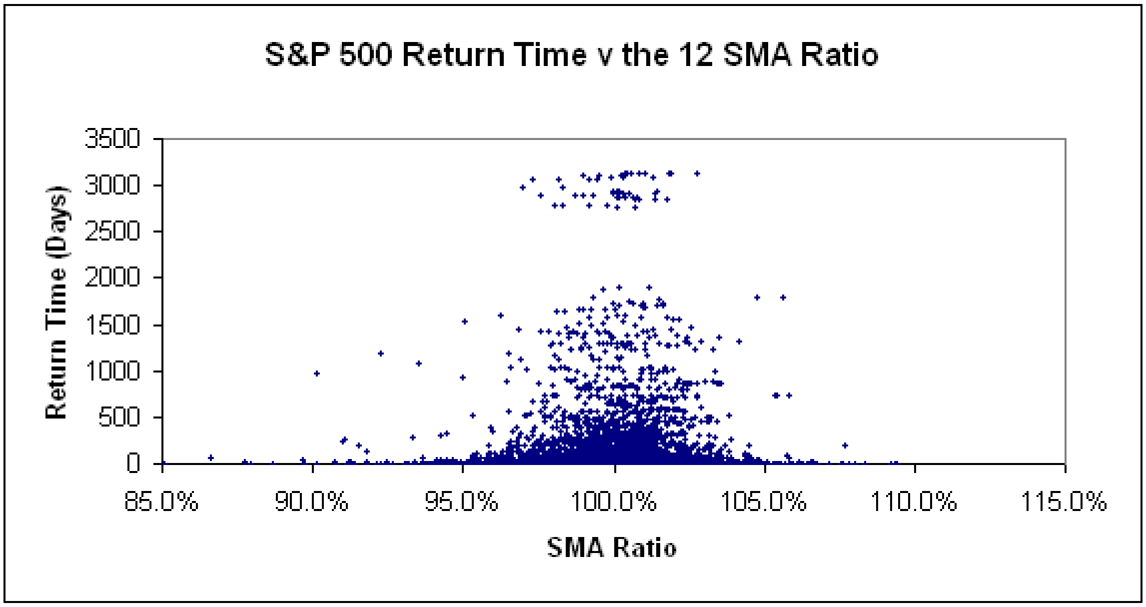


This shows that if losses follow a gamma distribution, then any efforts to decrease the mean and standard deviation of the loss distribution can have a marginally lower decrease in the economic capital required at the higher capital levels. The optimal point for a company to be at might be the point at which sensitivity is maximized in this distribution, depending on the costs involved with lowering the beta parameter.

Transaction Class Limits

In general, the transaction class limits should be based on the maximum loss possible. For the major classes discussed earlier, the highest limits should relate to the GMxBs, then decrements, then premium. This is a company-specific exercise but it should be noted that these limits should be flexible as the underlying readings of these transactions change. For example, if the transaction limit on GMxB claims is set to an annual level of 100 million when the rider's election rate is 20 percent, it should be set to change automatically if the election rate jumps to 40 percent.

The most difficult aspect of this exercise is to determine the hedging losses that can occur. Hedging is a balance of the Greeks hedged, accounting treatment and execution. All of these dimensions have a marked aspect on the final cost of hedging. The rebalance timing is also critical. For example, the S&P 500 index tends to return to its current value much faster when the ratio of the current closing price to the 12-month simple moving average (SMA) is farther from 100 percent. In fact, the return time distribution is fairly normal.[1] Since the market tends to repeat itself quite often, daily rebalancing may result in overhedging and a more flexible schedule could be better.



1. The chart shows the ratio of closing price to SMA value vs. the return time. For example, if the market closes at 1,000 on a particular day and is 102 percent of the SMA and then the market closes at 1,010, 1,007 and 999 over the next three days, the return time is three days. This represents a point (102 percent, 3) in the chart.

Completing the Implementation

Once the limits and sensitivities are set, the implementation is reduced to combining them into a set of rules for the block. If we continue with the procedure outlined above with some hypothetical numbers, you might see the following for a specific product.

Risk Class	Economic Capital Allocation (Millions)	Transaction	Control	Transaction Limit (Millions)	Sensitivity (Millions)	Maximum Limit
A	100	GMxB hedging costs	Charge % X hedge ratio	20	10 per 1%	2%
		GMxB reinsurance costs	Reinsurance %	10	1 per 10%	100%
			Other treaty provisions	0	0	NA
		GMxB fees	Charge %	4	1 per 1%	4%
			Charge base	2	+2 for benefit base	benefit base
			Charge timing	2	+2 for semiannual	semiannual
			Deposit bonus	4	+2 per 1%	2%
		GMxBs	Guaranteed rate	15	5 per 1%	3%
			Benefit payment frequency	3	+3 for monthly	monthly
			Maximum age	10	1 per year after age 65	75 years
			Ratchet period	5	1 per year	5 years
			Ratchet maximum	5	1 per 1000	5,000
			Rollup rate	5	1 per 1%	5%
			Rollup cap	5	1 per 10% of deposit	50% of deposit
			Rollup frequency	5	1 per months less than 1 year	2 months
			Waiting period	5	1 per year	5 years
B	50	Annuitizations	Lockout period	10	2 per year	10 years
		Lapses	Surrender charges	20	2 per 1%	10%
			Withdrawal charges	20	2 per 1%	10%
C	15	Revenues	M&E fees	2	1 per 3%	6%
			Premium bonus	1	1 per 3%	3%
			Premium load	2	1 per 3%	6%
		Separate account loss	Investment options available	2	complex	2
			Fund transfer charges	2	complex	2
		Commissions	Deposit commissions	2	1 per 3%	6%
			Asset trails	2	1 per 10 bps	20 bps
			Renewal commissions	2	1 per 2%	4%

In the example above, an increase of 1 percent in the average GMxB charge leads to a $1 million increase in economic capital. With a transaction limit of $4 million, the average charge is thus limited to 4 percent. Similarly, each increase of 1 percent in the product of GMxB charge percentage and hedge ratio leads to an economic capital cost of $10 million. This limits the product to 2 percent, or the hedging ratio to 50 percent assuming maximum charges. The separate account controls are specific to a company and product and beyond the scope of this paper. This control would require a list of all funds and transfer charges as well as their economic capital sensitivity embedded in the chart to stay within the maximum limit.

The Risk Management Company

Some companies will have a natural advantage in the implementation of the risk limits above due to the way they are structured. In particular, there are several principles successful companies follow as they go through this and similar risk-related processes:

1. **Risk management is on the agenda at the highest level**. The board and senior management's dedication to a risk management policy creates a culture in which risk is a priority.
2. **Compliance is an all-encompassing term**. Compliance is not the same as staying within a company's legal limitations. It also means complying with the best risk management practices in the industry.
3. **Check the implementation**. A company willing to repeat the implementation exercise above will ensure the limits being set make sense.
4. **Stay in compliance**. Once a company's standard for compliance is set, it is beneficial to have an audit function that ensures that standard is being maintained below the executive level.
5. **Observe at the lowest level**. When the chief risk officer or other high level executive is part of the audit team at the operational level of the company, then the operational limits set in the risk management implementation are more likely to be adhered to.

Summary

In this paper, we have explored the nature of risk appetite conversion into business practices. We have taken as a given the idea that these statements can be created through the normal give and take of the business process and are an effective outline of a company's business plan in terms of risk. We continued by focusing on the sensitivity of economic capital as the key step in the risk appetite implementation and subsequently looked at an example of how the final parameters can be set. The major themes are:

- A risk represents the probability of loss with an associated reward and can be defined by economic capital.
- Risk is transaction based.
- The risk appetite statement is set at the corporate level and should include an allocation of economic capital for the line of business.
- Implementing this statement means setting operational limits for all variables the company controls.
- These limits are the quotient of the economic capital limit for the associated transaction and the current sensitivity of this capital.
- While the transaction limits can be set through decision and study, the sensitivities require more sophisticated techniques. The normal and gamma loss distributions are illustrated above.
- The results can be encapsulated in an implementation table that sets the necessary limits explicitly.

Lastly, it should be noted that the management of risk and the management of a company are extremely complicated, and turning a risk appetite statement into a usable operational framework is a difficult process. The work of the actuary is not simply the realization of the knowledge tested throughout the exam process. It is the ability to work through the difficult and often tedious jobs necessary in the modern business world without becoming discouraged. The problem above and in a subsequent paper is one example of the work actuaries have been trained to do for centuries, of which we should all be thankful.

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