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ACTUARIAL ASPECTS OF THE RAILROAD RETIREMENT SYSTEM

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THE past decade has witnessed a steadily increasing interest in and growth of pension programs designed to meet, in part, the financial problems besetting individuals as their working days grow to a close. This trend is not at all surprising as evidence mounts that the age group 65 and over will grow rapidly over the course of the next 50 years relative to the other age segments of the population. It is also worthy of comment that attention previously focused almost exclusively on life insurance as a means for meeting the needs of dependents left in a precarious financial position by the early decease of the breadwinner, has been broadened to include a growing realization of the value of monthly family income benefits as a supplementary tool for such purpose.

Impetus was originally given to latter-day pension developments by the enactment of the 1935 Social Security Act and its modifying amendments of 1939. The benefits provided by such legislation served as a spark and springboard for the institution of many company plans during the war years. And today the fires have been kindled by large unions ever since pensions became a legitimate subject for collective bargaining.

While much has been said, done, and written on the hows, whats, and whys of pensions, very little has appeared in print relating to the origin of the basic cost factors and to the actuarial formulas which underlie the calculation of benefit liabilities and the consequent estimates of amounts required to finance a particular plan. It is the purpose of this paper to fill in this gap in connection with the methods developed for analyzing the costs of one of the largest pension plans in the country (Railroad Retirement program). This plan provides for monthly employee annuities comparable in size to those of a staff retirement plan, lump-sum and monthly survivor insurance benefits similar to those contained in the Social Security Act, and a residual lump sum which guarantees that total benefits with respect to a covered employee's earnings will always exceed his con-

* The opinions expressed in this paper are those of the author and do not necessarily represent the official views of the Railroad Retirement Board.

tributions. Indicative of the size of this program, it might be mentioned that during the year 1948, 2.3 million employees received $4.9 billion in compensation credits; $568 million was collected in taxes from employers and employees; and $254 million was disbursed in benefit payments.

PROVISIONS OF THE RAILROAD RETIREMENT ACT

Before going into the actuarial methods used for cost purposes, it is convenient to present a thumb-nail sketch of the benefit and tax provisions of the 1937 Act and the 1946 and 1948 amendments.

Under the original 1937 Act, four different types of retirement annuity were recognized. These covered the following groups of employees:

(1) Individuals 65 or over regardless of service (normal retirements),
(2) Individuals between the ages of 60 and 65 who had completed 30 years of service (pre-normal retirements),
(3) Individuals between ages 60 and 65 who became totally and permanently disabled before completion of 30 years of service, and
(4) Individuals without regard to age who were totally and permanently disabled and who had completed 30 years of service.

For the second and third categories, the annuity was reduced by 1/180 for each month the employee was under age 65 when the annuity began to accrue. The normal monthly annuity was computed by multiplying the individual's "years of service" by 2 percent of the first $50 of average monthly compensation, 1½ percent of the next $100, and 1 percent of the remainder. For individuals who were employees on August 29, 1935, years of service include all service subsequent to December 31, 1936 (up to age 65) and service prior to 1937, but if the latter is included, the total creditable service cannot exceed 30 years. In addition, there was a minimum annuity for employees at age 65 who had completed 20 years of service. Such annuity was equal to $40 per month payable when the average monthly compensation was $50 or over.

In addition to the monthly employee annuity, there was available a lump-sum benefit under the 1937 Act equal to 4 percent of the employee's taxable compensation (after 1936) less the amounts paid out in retirement annuities.

The 1946 amendments to the 1937 Act introduced the following important changes:

(1) Reduced the service requirement from 30 to 10 years for total and permanent disability annuities under age 60.
(2) Introduced an occupational disability provision for individuals having a current connection with the railroad industry. A service requirement is necessary for this type of benefit under age 60 (20 years).

(3) Substituted a new monthly minimum annuity provision equal to $3 per year of service up to $50.
(4) Eliminated the lump-sum benefit of the 1937 Act (the excess of 4 percent of aggregate compensation after 1936 over the total employee annuity payments).
(5) Introduced a set of monthly survivor annuities and an insurance lump sum to dependents of deceased employees patterned after those of the 1939 social security amendments. Such benefits are based on combined railroad and social security coverage and average about 25 percent higher than under the latter Act.

The latest amendments of June 23, 1948, are significant in two respects:

(a) All retirement benefits—disability as well as nondisability—have been increased by 20 percent.
(b) The lump-sum benefit of the 1937 Act has been restored in a modified form. A residual benefit is presently provided which is equal to 4 percent of the employee's railroad earnings in 1937–46 and 7 percent thereafter (less the total amount already paid out in retirement and survivor benefits).

The reader is referred, for a more extensive treatment of the benefits provided under the 1937 Act and the 1946 amendments, to the writer's discussion of Mr. Immerwahr's paper "Problems in Federal Old-Age and Survivors Insurance." Such discussion appears beginning with page 90 in *TASA* XLVII. In that connection, note that the original version of H.R. 1362 was followed practically *in toto* for the 1946 amendments.

To fill out the picture, there is presented a table below which shows the tax rates which have applied since the railroad retirement system of benefits came into existence. These rates are applicable to railroad earnings up to $300 a month.

Calendar Years	Tax Rate on	
	Employees	Employers
1937–1939	2¾	2¾
1940–1942	3	3
1943–1945	3¼	3¼
1946	3½	3½
1947–1948	5¾	5¾
1949–1951	6	6
1952 and subsequent	6¼	6¼

ACTIVE SERVICE TABLES

Basic to the calculation of benefit liabilities under any employee retirement plan is the preparation of appropriate service tables. In view of the

different types of benefits and the various age and service requirements for eligibility, it has been considered necessary to build our own service tables by quinquennial ages at entry (18, 23, 28, etc.). Such tables are built on a multiple decrement basis which provides for (a) deaths in active service, (b) final withdrawals from service (other than by death or retirement), (c) immediate disability retirements, and (d) immediate age retirements.

Mortality Rates

To date, the information available to the Railroad Retirement Board on deaths of covered employees has been too incomplete and fragmentary to permit valid mortality investigations. Recourse has therefore been made to group life insurance experience for steam railroad employees. In connection with our third valuation (as of December 31, 1944) the group life insurance experience for 1939–41 was utilized. The data for this 3-year period were derived from the reports of the Committee on Group Mortality Investigations for 1932–41 and for 1932–38. The graduated mortality rates and the various functions derived therefrom are shown in Table 1 of this paper entitled "1944 Railway Employees Mortality Table." For the purposes of the Board's most recent valuation as of December 31, 1947, it was not considered necessary to prepare a new table. Instead, the calculations allowed for recent improved mortality by applying a 1-year rate-back in age.

Rates of Withdrawal

The determination of withdrawal rates in an industry-wide system such as ours has posed peculiarly difficult problems relative to appropriate definitions. The following facts had to be kept in mind. First, there is considerable in, out, and in again movement of railroad employees within a railroad and among railroads. Such movements generally encompass the relatively unskilled employees and/or those with low seniority. Second, although the data available to the Board permit a study of the yearly employment pattern of any particular employee—via a year-check code—information is not available relative to the month of the year of entry or of the year of exit. Third, our withdrawal tables must take into consideration the fact that coverage under the retirement benefit provisions of the system is all-inclusive. Theoretically, benefits vest on the basis of even one day's service. Fourth, it was decided to make implicit provision for re-entries in the basic active service tables without unduly complicating them. Finally, the emphasis placed upon seniority rights in the railroad industry precluded the use of withdrawal rates by attained age independent of the duration since entry.

TABLE 1

1944 Railway Employees Mortality Table Including Monetary Functions at 3% Interest

Age x	l_x	q_x	d_x	D_x	N_x	M_x	$a_x^{(12)}$
18.....	100,000	.00156	156	58,739.50	1,518,487.11	14,512.04	25.3095
19.....	99,844	.00162	162	56,939.63	1,459,747.61	14,423.02	25.0951
20.....	99,682	.00166	165	55,191.53	1,402,807.98	14,333.34	24.8754
21.....	99,517	.00172	171	53,495.26	1,347,616.45	14,244.58	24.6496
22.....	99,346	.00178	177	51,847.98	1,294,121.19	14,155.41	24.4182
23.....	99,169	.00185	183	50,248.14	1,242,273.21	14,065.69	24.1811
24.....	98,986	.00194	192	48,694.58	1,192,025.07	13,975.66	23.9379
25.....	98,794	.00204	202	47,184.61	1,143,330.49	13,883.97	23.6893
26.....	98,592	.00214	211	45,716.62	1,096,145.88	13,790.28	23.4353
27.....	98,381	.00226	222	44,290.04	1,050,429.26	13,695.24	23.1754
28.....	98,159	.00237	233	42,903.04	1,006,139.22	13,598.23	22.9098
29.....	97,926	.00249	244	41,554.51	963,236.18	13,499.29	22.6384
30.....	97,682	.00261	255	40,243.71	921,681.67	13,398.81	22.3608
31.....	97,427	.00274	267	38,969.53	881,437.96	13,296.77	22.0769
32.....	97,160	.00287	279	37,730.82	842,468.43	13,193.08	21.7867
33.....	96,881	.00302	293	36,526.66	804,737.61	13,087.87	21.4898
34.....	96,588	.00318	307	35,355.55	768,210.95	12,980.64	21.1865
35.....	96,281	.00336	324	34,216.63	732,855.40	12,871.48	20.8764
36.....	95,957	.00357	343	33,108.24	698,638.77	12,759.69	20.5600
37.....	95,614	.00380	363	32,029.06	665,530.53	12,644.82	20.2373
38.....	95,251	.00405	386	30,978.10	633,501.47	12,526.74	19.9083
39.....	94,865	.00432	410	29,954.00	602,523.37	12,404.90	19.5733
40.....	94,455	.00464	438	28,955.84	572,569.37	12,279.18	19.2322
41.....	94,017	.00498	468	27,982.09	543,613.53	12,148.80	18.8855
42.....	93,549	.00538	503	27,031.83	515,631.44	12,013.55	18.5333
43.....	93,046	.00581	541	26,103.40	488,599.61	11,872.45	18.1762
44.....	92,505	.00630	583	25,195.77	462,496.21	11,725.11	17.8144
45.....	91,922	.00683	628	24,307.76	437,300.44	11,570.95	17.4485
46.....	91,294	.00742	677	23,438.55	412,992.68	11,409.73	17.0785
47.....	90,617	.00804	729	22,587.10	389,554.13	11,240.95	16.7051
48.....	89,888	.00873	785	21,752.81	366,967.03	11,064.53	16.3282
49.....	89,103	.00948	845	20,934.75	345,214.22	10,880.04	15.9483
50.....	88,258	.01028	907	20,132.27	324,279.47	10,687.31	15.5657
51.....	87,351	.01118	977	19,345.01	304,147.20	10,486.42	15.1806
52.....	86,374	.01214	1,049	18,571.53	284,802.19	10,276.38	14.7937
53.....	85,325	.01320	1,126	17,811.59	266,230.66	10,057.36	14.4053
54.....	84,199	.01435	1,208	17,064.61	248,419.07	9,829.16	14.0159
55.....	82,991	.01560	1,295	16,329.89	231,354.46	9,591.46	13.6258
56.....	81,696	.01696	1,386	15,606.88	215,024.57	9,344.07	13.2359
57.....	80,310	.01842	1,479	14,895.26	199,417.69	9,087.02	12.8463
58.....	78,831	.01998	1,575	14,195.10	184,522.43	8,820.70	12.4573
59.....	77,256	.02168	1,675	13,506.28	170,327.33	8,545.33	12.0693
60.....	75,581	.02352	1,778	12,828.59	156,821.05	8,261.02	11.6826
61.....	73,803	.02554	1,885	12,161.92	143,992.46	7,968.00	11.2979
62.....	71,918	.02774	1,995	11,506.16	131,830.54	7,666.46	10.9157

TABLE 1—*Continued*

Age x	l_x	q_x	d_x	D_x	N_x	M_x	$a_x^{(m)}$
63.....	69,923	.03015	2,108	10,861.14	120,324.38	7,356.57	10.5367
64.....	67,815	.03279	2,224	10,226.91	109,463.24	7,038.68	10.1618
65.....	65,591	.03564	2,338	9,603.38	99,236.33	6,713.02	9.7918
66.....	63,253	.03870	2,448	8,991.35	89,632.95	6,380.70	9.4271
67.....	60,805	.04196	2,551	8,391.64	80,641.60	6,042.87	9.0681
68.....	58,254	.04539	2,644	7,805.40	72,249.96	5,701.05	8.7147
69.....	55,610	.04904	2,727	7,234.08	64,444.56	5,357.07	8.3668
70.....	52,883	.05294	2,800	6,678.96	57,210.48	5,012.65	8.0241
71.....	50,083	.05716	2,863	6,141.13	50,531.52	4,669.35	7.6867
72.....	47,220	.06176	2,916	5,621.40	44,390.39	4,328.49	7.3550
73.....	44,304	.06677	2,958	5,120.66	38,768.99	3,991.47	7.0294
74.....	41,346	.07224	2,987	4,639.60	33,648.33	3,659.56	6.7107
75.....	38,359	.07818	2,999	4,179.02	29,008.73	3,334.11	6.3998
76.....	35,360	.08463	2,993	3,740.10	24,829.71	3,016.91	6.0971
77.....	32,367	.09156	2,964	3,323.80	21,089.61	2,709.54	5.8033
78.....	29,403	.09898	2,910	2,931.48	17,765.81	2,414.03	5.5187
79.....	26,493	.10694	2,833	2,564.42	14,834.33	2,132.36	5.2430
80.....	23,660	.11546	2,732	2,223.50	12,269.91	1,866.13	4.9766
81.....	20,928	.12460	2,608	1,909.47	10,046.41	1,616.86	4.7197
82.....	18,320	.13434	2,461	1,622.82	8,136.94	1,385.82	4.4724
83.....	15,859	.14473	2,295	1,363.91	6,514.12	1,174.18	4.2344
84.....	13,564	.15581	2,113	1,132.55	5,150.21	982.54	4.0057
85.....	11,451	.16760	1,919	928.28	4,017.66	811.26	3.7864
86.....	9,532	.18021	1,718	750.21	3,089.38	660.23	3.5763
87.....	7,814	.19358	1,513	597.08	2,339.17	528.95	3.3760
88.....	6,301	.20776	1,309	467.45	1,742.09	416.71	3.1851
89.....	4,992	.22280	1,112	359.55	1,274.64	322.42	3.0034
90.....	3,880	.23866	926	271.32	915.09	244.67	2.8310
91.....	2,954	.25542	755	200.55	643.77	181.80	2.6683
92.....	2,199	.27298	600	144.94	443.22	132.03	2.5163
93.....	1,599	.29134	466	102.33	298.28	93.64	2.3732
94.....	1,133	.31020	351	70.39	195.95	64.68	2.2421
95.....	782	.32896	257	47.17	125.56	43.51	2.1202
96.....	525	.34718	182	30.75	78.39	28.47	2.0076
97.....	343	.36408	125	19.50	47.64	18.11	1.9014
98.....	218	.37962	83	12.03	28.14	11.21	1.7975
99.....	135	.39616	53	7.24	16.11	6.77	1.6834
100.....	82	.41840	34	4.27	8.87	4.01	1.5356
101.....	48	.45182	22	2.42	4.60	2.29	1.3591
102.....	26	.49902	13	1.28	2.18	1.22	1.1614
103.....	13	.56004	7	.62	.90	.59	.9099
104.....	6	1.00000	6	.28	.28		

The above considerations led us to conduct the necessary withdrawal investigations on a calendar-year basis, by year of entry, and by quinquennial ages at entry (central ages 18, 23, etc.). The actual withdrawals of a calendar year have been defined as individuals who worked in the year and who were "final" separations in that they performed no compensated employment in any subsequent year prior to the valuation date. Such withdrawals are exclusive of deaths and retirements in the year of last employment (the latter are identifiable in our punch-card records). The corresponding exposures by duration since entry (year of experience plus 1 minus year of entry) include in each instance all individuals who last worked in the particular calendar year or in any subsequent year.

In accordance with the definitions adopted, first-year withdrawals include one-half year of experience since entry; the second-year exits refer to a full year's withdrawals in the interval $\frac{1}{2}$ to $1\frac{1}{2}$ years after entry, and so on. It is also pertinent to re-emphasize that the corresponding exposures to withdrawal in a particular calendar year include individuals who worked in the year and those who re-entered in a subsequent year prior to the valuation date.

For the third valuation, the withdrawal investigations covered the calendar years 1938–43 inclusive. The actual to "expected" withdrawal ratios were then obtained on a calendar-year basis (the expected withdrawals refer to separation rates previously used in our first and second valuations). Using such ratios as a guide, a broad over-all modification was made, according to service-year duration since entry, in our previously used withdrawal rates. The resulting figures entitled "Rates of withdrawal for causes other than death, disability, or retirement" are reproduced in Table 2. In connection with the recent fourth valuation, much higher withdrawal rates have shown up on the basis of actual 1944–46 experience. Nevertheless, the third valuation withdrawal table was retained, in view of the obviously atypical experience represented by the years 1944–46. The retention of the third valuation rates was, of course, conservative in terms of effect on costs.

Disability Retirement Rates

In the last analysis, the disability rates which are utilized in a service table necessarily reflect the types of disability recognized under the particular plan, the service and age requirements for benefit eligibility, the adjudicative and administrative practices which are followed, and the manner in which relevant statistics have been gathered. Before the 1946 amendments to the Railroad Retirement Act, a monthly annuity benefit was provided for an employee only if he became totally and permanently

TABLE 2

RATES OF WITHDRAWAL FOR CAUSES OTHER THAN DEATH, DISABILITY, OR RETIREMENT

DURATION	CENTRAL AGE AT ENTRY										
	13	18	23	28	33	38	43	48	53	58	63
0	.49856	.50864	.51200	.50864	.49856	.48176	.45824	.42800	.39104	.34736	.29712
1	.20824	.21056	.20958	.20867	.20664	.20226	.19369	.18704	.17851	.16519	.14678
2	.15629	.15760	.15632	.15574	.15472	.15206	.14592	.14246	.13802	.12962	.....
3	.12362	.12449	.12327	.12286	.12225	.12038	.11563	.11349	.11079	.10484	.....
4	.09846	.09908	.09800	.09770	.09729	.09591	.09221	.09080	.08903	.08464	.....
5	.08180	.08228	.08133	.08109	.08080	.07973	.07667	.07565	.07441	.07098	.....
6	.06997	.07035	.06951	.06931	.06909	.06820	.06560	.06484	.06392	.06110	.....
7	.06113	.06144	.06069	.06052	.06035	.05960	.05734	.05673	.05602	.....	.....
8	.05428	.05454	.05386	.05370	.05357	.05292	.05092	.05042	.04986	.....	.....
9	.04881	.04903	.04840	.04826	.04816	.04758	.04580	.04539	.04491	.....	.....
10	.04434	.04452	.04394	.04383	.04374	.04323	.04162	.04126	.04087	.....	.....
11	.04061	.04078	.04026	.04014	.04007	.03959	.03811	.03782	.03748	.....	.....
12	.03747	.03762	.03713	.03703	.03696	.03653	.03517	.03492	.....	.....	.....
13	.03476	.03492	.03446	.03436	.03431	.03392	.03266	.03242	.....	.....	.....
14	.03244	.03257	.03213	.03204	.03199	.03164	.03046	.03026	.....	.....	.....
15	.03040	.03052	.03011	.03004	.02999	.02965	.02856	.02836	.....	.....	.....
16	.02861	.02871	.02832	.02825	.02820	.02790	.02686	.02669	.....	.....	.....
17	.02701	.02712	.02674	.02667	.02664	.02635	.02536	.....	.....	.....	.....
18	.02558	.02567	.02533	.02526	.02523	.02496	.02402	.....	.....	.....	.....
19	.02429	.02440	.02406	.02399	.02395	.02370	.02283	.....	.....	.....	.....
20	.02314	.02322	.02290	.02285	.02281	.02256	.02173	.....	.....	.....	.....
21	.02208	.02217	.02186	.02179	.02178	.02154	.02074	.....	.....	.....	.....
22	.02111	.02120	.02091	.02084	.02082	.02060	.....	.....	.....	.....	.....
23	.02023	.02030	.02003	.01998	.01994	.01974	.....	.....	.....	.....	.....
24	.01941	.01948	.01923	.01918	.01914	.01894	.....	.....	.....	.....	.....
25	.01867	.01873	.01848	.01843	.01841	.01821	.....	.....	.....	.....	.....
26	.01797	.01804	.01778	.01775	.01773	.01754	.....	.....	.....	.....	.....
27	.01732	.01739	.01715	.01710	.01708	.....	.....	.....	.....	.....	.....
28	.01673	.01680	.01656	.01652	.01649	.....	.....	.....	.....	.....	.....
29	.01617	.01624	.01600	.01596	.01595	.....	.....	.....	.....	.....	.....
30	.01566	.01571	.01549	.01545	.01543	.....	.....	.....	.....	.....	.....
31	.01516	.01522	.01499	.01496	.01494	.....	.....	.....	.....	.....	.....
32	.01470	.01476	.01454	.01451	.....	.....	.....	.....	.....	.....	.....
33	.01426	.01431	.01411	.....	.....	.....	.....	.....	.....	.....	.....
34	.01386	.01391	.01370	.....	.....	.....	.....	.....	.....	.....	.....
35	.01346	.01352	.01333	.....	.....	.....	.....	.....	.....	.....	.....
36	.01311	.01316	.01297	.....	.....	.....	.....	.....	.....	.....	.....
37	.01277	.01280	.....	.....	.....	.....	.....	.....	.....	.....	.....
38	.01243	.01248	.....	.....	.....	.....	.....	.....	.....	.....	.....
39	.01212	.01216	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	.01183	.01187	.....	.....	.....	.....	.....	.....	.....	.....	.....
41	.01154	.01158	.....	.....	.....	.....	.....	.....	.....	.....	.....
42	.01127	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
43	.01102	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
44	.01078	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
45	.01054	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
46	.01032	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

disabled for regular employment for hire—subject to a service requirement of 30 years before age 60 (no service requirement is needed thereafter).

Under the aforementioned amendments, the service requirement for the total and permanent disability annuity benefit under age 60 was reduced to 10 years effective January 1, 1947. In addition, a new type of disability benefit became payable after that date to individuals who had a recent attachment to the industry—subject to a service requirement of 20 years under age 60—if their permanent physical or mental condition was such as to make them disabled for work at their regular occupation. Further, the benefit formulas are alike under both types of disability. In effect, the result has been to establish an overlapping area within which it is immaterial whether the applicant is disabled for all gainful employment or only for his regular occupation (subject to the established recency of his attachment to the railroad industry). The similar areas apply to individuals with at least 20 years of service regardless of age, and to employees at ages 60 to 65 practically regardless of service. In view of the fact that certifications can be made more rapidly under the regular occupational provisions, the Board has followed the practice of adjudicating cases under this more liberal definition of disability wherever expedient. In consequence, the statistical data available on disability retirements in the common area permit of no clear differentiation between the truly totally and permanently disabled annuitants, on the one hand, and those only disabled for their regular occupation, on the other.

Taking the foregoing considerations into account, the latest valuation utilized composite tabular disability retirement rates at those points where an employee could be eligible for either type of disability annuity. One set of disability retirement rates was developed for individuals with 20 years of service regardless of age; a second set was used at ages 60–64 for individuals with less than 20 years of service. Finally, a third group of rates was established for cases in which an annuity could be obtained only on the basis of total and permanent disability. In each instance, the tabular disability rates followed the results of the 1947 disability investigation rather closely. The resulting rates are reproduced in Table 3.

It would appear that the rates as developed contain a substantial margin of conservatism in that the 1947 retirements included individuals who undoubtedly had become disabled in previous years but who could only retire on a disability annuity for the first time in that year. The available statistics for 1948 and 1949 substantiate this view.

It may be of interest to note how we obtain our crude rates of immediate disability retirement. The immediate disability retirements at age x in

a particular year include all individuals who retired at age x last birthday in the year, if they last worked in the year of retirement or in the previous calendar year. The corresponding exposures used in our disability investigations come from employees who worked in the immediately preceding calendar year and were alive and not retired at the end of that year, and the entrants and re-entrants in the calendar year of retirement. A full

TABLE 3

TABULAR RATES OF IMMEDIATE DISABILITY RETIREMENT USED IN THE FOURTH VALUATION

Age* x	Rates for 10–20 Years of Service,† Ages under 60	Rates for 20 or More Years of Service‡	Age* x	Rates for 10–20 Years of Service,† Ages under 60	Rates for 20 or More Years of Service‡
Under 38	.0010	.0010	52	.0140	.0175
38	.0014	.0014	53	.0164	.0195
39	.0019	.0020	54	.0189	.0218
40	.0024	.0027	55	.0213	.0244
41	.0029	.0035	56	.0237	.0271
42	.0034	.0043	57	.0260	.0298
43	.0039	.0052	58	.0283	.0325
44	.0044	.0062	59	.0305	.0351
45	.0049	.0073	60	.....	.0377
46	.0056	.0085	61	.....	.0403
47	.0064	.0097	62	.....	.0429
48	.0074	.0111	63	.....	.0455
49	.0086	.0125	64	.....	.0481
50	.0101	.0140			
51	.0120	.0157			

* Age x refers to the age interval x to $x + 1$.

† Refers to employees who can qualify only under the total and permanent disability provisions.

‡ Refers to employees who meet the age and service requirements for total and permanent as well as occupational disability retirement.

NOTE.—For ages 60–64 with less than 20 years of service a flat rate of 20.0 per thousand was used.

year of exposure is assigned to the appropriate members of the first group. The employees in the latter group are assumed to enter or re-enter in the middle of the calendar year of retirement, warranting only one-half year of exposure to retirement for the eligibles among them.

Before setting down the applicable exposure formula, certain prefatory remarks are in order. It should be pointed out that we obtain our actual immediate disability retirements from a separate file of punch cards maintained on a 100 percent basis, according to age last birthday. On the other hand, the basic source from which exposures are derived is a 4-percent work-history file maintained for employees with social security numbers

ending in 30, 35, 80, and 85. (Incidentally, this file was also used for withdrawal investigations.) Each year deaths and retirements are properly identified via a death and retirement code. The file also shows through a year-check code the years in which a particular employee worked. However, the punch cards of this file are maintained by year of birth. Thus, to obtain proper exposures at age x in calendar year of retirement y, it is necessary to average the appropriate data which come from the two corresponding successive years of birth.

The exposure formula follows:

$$E_x^y = \tfrac{1}{2}\left[e_{x-1}^{y-1} + e_x^{y-1} + \tfrac{1}{2}(r\,e)_{x-1}^{y} + \tfrac{1}{2}(r\,e)_x^{y} + \tfrac{1}{2}(n\,e)_{x-1}^{y} + \tfrac{1}{2}(n\,e)_x^{y}\right] \quad (1)$$

where

E_x^y represents the exposure to retirement in year y for the year of age x to $x + 1$,

e^{y-1} represents eligible employees working in year $y - 1$ who were alive and not retired at the end of that year,

$(re)^y$ represents eligible re-entrants in year y who had not worked in the preceding year,

$(ne)^y$ refers to eligible new entrants in the year of retirement.

In each instance the subscript refers to the attained age at the beginning of the year of retirement.

As used herein, the term "eligible employee" means any individual who could meet the age and service requirements for immediate disability retirement. For ages under 60 all re-entrants were excluded from the exposure on the grounds that they would not meet the necessary service requirements. At ages 60 and over, all entrants and re-entrants in the year of retirement are included with a half-year of exposure, while new entrants of the previous year are given a full year.

As the exposure formula is applied, it will be noted that not all the employees working the year prior to retirement would be included. To obtain the true eligibles, it was necessary to collate on social security number the 4 percent wage-history file previously referred to (which contained only cumulative earnings and service data since 1936) with a corresponding prior service file maintained by the Board. The pertinent information from the first file was then reproduced to the second. The unmatched cards of both files required no further work relative to the amount of total service performed by the employees they represented, except for a reproduction of the relevant information contained in the unmatched subsequent wage card into columns of a dummy card corresponding to those used in cases of employees with a prior service history (before 1937). In instances involving prior and subsequent service, the prior

service and the reproduced subsequent service were added mechanically and the resulting sum was punched into a blank field in the prior service card.

It will be noted that, in the process, a uniform file of cards was produced consisting of three subgroups: (a) individuals with prior service only, (b) those with subsequent service only, and (c) individuals with prior and subsequent service. Files (b) and (c) representing all individuals with subsequent service were then utilized to develop the "true" exposures to retirement on the basis of total service performed. An idea of the information that was punched into this composite file of cards identified as "7V" can be gathered from Appendix A. It refers to the statistics collected for the fourth valuation (as of December 31, 1947).

Immediate Age Retirement Rates

The method for deriving immediate age retirement rates parallels that adopted for immediate disability retirements. The definition of immediate age retirement in a particular year corresponds to the one used for disability retirement. The general exposure formula for retirement year y here takes the form:

$$E_x^y = \tfrac{1}{2}\left[e_{x-1}^{y-1} + e_x^{y-1} + \tfrac{1}{2}(r\,e)\,{}_{x-1}^{y} + \tfrac{1}{2}(r\,e)\,{}_{x}^{y} \right]. \qquad (2)$$

For ages in which a service requirement was involved (ages 60–64), reentrants are eliminated since to all intents and purposes none of them would meet the 30-year service requirement. At ages 65 and over, reentrants are assigned a half-year of exposure.

The experience examined for 1945–47 has indicated substantially lower retirement rates at ages 65 and over than those which prevailed prior to the war years. It is not clear, however, whether the recent trend toward low retirement rates at those ages will continue to prevail in the long run. The decision to retain the previous third valuation rates for our latest report reflects a conservative view and can be considered as something of a hedge against future improvement in mortality at these older ages. On the other hand, the opposite position was taken in connection with the tabular immediate age retirement rates for pre-normal retirements. Here, it was considered justifiable to pattern tabular immediate age retirement rates closely following the actual experience in 1947. We felt that the disability liberalizations of the 1946 amendments which first became effective in 1947 would contribute toward much lower pre-normal age retirements than those experienced before 1947.

It would appear desirable, regardless of the position taken in our own valuation calculations, to present the age-retirement story as it unfolded

in actual experience. The results of the Board's experience in the years 1945–47 have therefore been included in Table 4. There is also presented in Table 5 the rates actually adopted for the third and fourth valuations.

Formulas for Service Tables

The investigations discussed above indicate that except for the rate of mortality, all other decrement factors have been obtained as probabili-

TABLE 4

IMMEDIATE AGE RETIREMENTS, 1945–47*

Age† x	1945 Retirements			1946 Retirements			1947 Retirements		
	Exposure	Actual Retirements	Crude Probability	Exposure	Actual Retirements	Crude Probability	Exposure	Actual Retirements	Crude Probability
60	17,225	342	.0199	17,558	403	.0230	19,046	276	.0145
61	16,302	255	.0156	17,275	312	.0181	19,184	132	.0069
62	15,348	281	.0183	16,532	353	.0214	17,983	194	.0108
63	13,757	384	.0279	14,834	413	.0278	16,620	219	.0132
64	12,822	547	.0427	13,362	497	.0372	15,434	348	.0225
65	18,645	7,132	.3825	19,892	6,820	.3429	19,838	7,091	.3574
66	12,448	2,405	.1932	14,040	2,279	.1623	14,914	2,262	.1517
67	10,292	1,917	.1863	10,550	1,775	.1682	11,587	1,696	.1464
68	8,334	1,408	.1689	8,254	1,475	.1787	8,408	1,298	.1544
69	6,791	1,258	.1852	6,543	1,388	.2121	6,790	1,393	.2052
70	4,379	1,406	.3211	5,320	1,699	.3194	4,956	1,532	.3091
71	3,044	661	.2171	3,384	783	.2314	3,546	665	.1875
72	2,326	475	.2042	2,379	612	.2573	2,472	548	.2217
73	1,443	300	.2079	1,860	450	.2419	1,792	403	.2249
74	900	210	.2333	1,181	294	.2489	1,286	311	.2418
75	534	151	.2828	726	171	.2355	800	178	.2225
76 and over	828	269	.3249	1,248	425	.3405	1,533	398	.2596
Total 60–64	75,454	1,809	.0240	79,561	1,978	.0249	88,267	1,169	.0132
Total 65 and over	69,964	17,592	.2514	75,377	18,171	.2411	77,922	17,775	.2281

* Immediate retirements are defined as those with calendar year last worked the same as the year of accrual or one year before. The exposures correspond with this definition.

† Age x refers to the age interval x to $x + 1$.

ties rather than rates. If we denote q_x as the rate of mortality, q_x^w as either the probability of nondisability withdrawal or the probability of nondisability retirement, and q_x^i as the probability of disability retire-

ment, the formulas for the service table functions (prepared by quinquennial ages at entry from 18 to 63, inclusive) can be set down as follows:

$$d_x^s = l_x^s \cdot q_x [1 - \tfrac{1}{2}(q_x^w + q_x^i)] \tag{3}$$

$$i_x = l_x^s \cdot q_x^i \tag{4}$$

$$w_x = l_x^s \cdot q_x^w . \tag{5}$$

In this notation l_x^s represents the number of individuals in service at age x; the deaths in active service are shown as d_x^s; the disability retirements are shown as i_x; and w_x represents the exits from service among nondisabled lives. All exits refer to the year of age x to $x + 1$.

TABLE 5

TABULAR RATES OF IMMEDIATE AGE RETIREMENT ACCORDING TO THE THIRD AND FOURTH VALUATIONS*

Age† x	Rates of Third Valuation	Rates of Fourth Valuation	Age† x	Rates of Third Valuation	Rates of Fourth Valuation
60	.020	.015	68	.240	.240
61	.015	.007	69	.295	.295
62	.020	.010	70	.515	.515
63	.030	.013	71 and over	.370	.370
64	.045	.022			
65	.410	.410			
66	.210	.210			
67	.220	.220			

* Retirement rates as used here actually refer to probabilities of retirement.
† Age x refers to the age interval x to $x + 1$.

The following remarks are pertinent in connection with the manner in which the multiple decrement service tables are prepared. First, no disabilities are recognized before the age at which a disability retirement becomes possible. Second, nondisability retirements cannot occur, under age 65, before an individual has attained age 60 and completed 30 years of service. On the other hand, when nondisability retirements are recognized, nondisability withdrawals (before retirement) are omitted. Third, no disabilities are recognized at ages 65 and beyond. Finally, in view of the large proportion of retirements occurring exactly on the sixty-fifth and seventieth birthdays, formula (3) is modified at those ages to provide for one-fourth of a year of exposure to death in the year of retirement rather than half a year.

FORMULAS FOR COST CALCULATIONS

Once the service tables have been prepared, the next step in our procedure is to establish benefit values *per capita*. Three entirely different types of benefits must be considered: (a) retirement, (b) survivor, and (c) the guaranteed residual lump-sum. The methods involved and the sources of data utilized in the calculation of *per capita* values for each of these types of benefits are now taken up in order.

I. Retirement Benefits

The calculations for retirement benefits proceed in three phases. First, a set of monetary functions is derived for each of the service tables in which age, mode of exit, and eligibility for particular benefits are involved. Second, the monthly amount of the retirement benefit is computed, which is a function of the age at entry, the duration of service on the valuation date, and the age at which the individual leaves employment either as a withdrawal or an immediate retirement. The final step is to obtain the *per capita* values by a multiple accumulation of the monetary functions and the amount of benefits through an appropriate range of ages and the division of the resulting amount by the D function related to the employee's age at entry and duration of service on the date as of which the calculations are made.

A. MONETARY FUNCTIONS

The monetary functions used in connection with retirement benefits, which apply an interest factor of 3 percent throughout, follow.

$$^{wa}C_x^s = v^{x+1/2} \cdot w_x \cdot a_{x+1/2}^{(12)} \tag{6}$$

$$^{wda}C_x^s = v^{x+1/2} \cdot w_x \cdot {}_{66-x-1/2}\big| a_{x+1/2}^{(12)} \tag{7}$$

$$^{ia}C_x^s = v^{x+1/2} \cdot i_x \cdot a_{[x]}^{i(12)} \tag{8}$$

$$^{d}C_x^s = v^{x+1/2} \cdot d_x^s \tag{9}$$

$$D_x^s = v^x \cdot l_x^s . \tag{10}$$

The first of these formulas is applicable to age retirements. The annuity values included therein correspond to those used in the third valuation with a 1-year rate-back in age. The original annuity values were based on experience among nondisability retirements for the years 1938–41. However, the investigations which have been run for the years 1944–48 indicate that the third valuation rates are no longer sufficiently conservative. As of the present date, a 1-year rate-back in age seems adequate. The basic table, known as "The 1944 Railway Annuitants Mortality Table," is reproduced for ready reference as Table 6.

TABLE 6

1944 Railway Annuitants Mortality Table, Including Monetary Functions at 3% Interest

Age x	l_x	q_x	d_x	D_x	N_x	M_x	$a_x^{(m)}$
60...	100,000	.02051	2,051	16,973.30	211,738.60	10,806.20	11.9331
61...	97,949	.02162	2,118	16,140.92	194,765.30	10,468.19	11.5249
62...	95,831	.02363	2,264	15,332.00	178,624.38	10,129.39	11.1087
63...	93,567	.02652	2,481	14,533.76	163,292.38	9,777.71	10.6937
64...	91,086	.03022	2,753	13,736.32	148,758.62	9,403.58	10.2879
65...	88,333	.03452	3,049	12,933.10	135,022.30	9,000.44	9.8984
66...	85,284	.03908	3,333	12,123.04	122,089.20	8,567.07	9.5291
67...	81,951	.04341	3,557	11,309.98	109,966.16	8,107.11	9.1812
68...	78,394	.04712	3,694	10,503.93	98,656.18	7,630.47	8.8506
69...	74,700	.05020	3,750	9,717.42	88,152.25	7,149.90	8.5299
70...	70,950	.05295	3,757	8,960.77	78,434.83	6,676.28	8.2114
71...	67,193	.05588	3,755	8,239.14	69,474.06	6,215.64	7.8905
72...	63,438	.05942	3,769	7,552.10	61,234.92	5,768.57	7.5666
73...	59,669	.06374	3,803	6,896.54	53,682.82	5,332.97	7.2423
74...	55,866	.06891	3,850	6,268.95	46,786.28	4,906.25	6.9215
75...	52,016	.07491	3,897	5,666.88	40,517.33	4,486.77	6.6081
76...	48,119	.08166	3,929	5,089.64	34,850.45	4,074.59	6.3056
77...	44,190	.08894	3,930	4,537.92	29,760.81	3,671.11	6.0165
78...	40,260	.09642	3,882	4,013.92	25,222.89	3,279.28	5.7422
79...	36,378	.10376	3,775	3,521.24	21,208.97	2,903.51	5.4815
80...	32,603	.11082	3,613	3,063.93	17,687.73	2,548.76	5.2312
81...	28,990	.11772	3,413	2,645.05	14,623.80	2,219.12	4.9870
82...	25,577	.12504	3,198	2,265.66	11,978.75	1,916.77	4.7454
83...	22,379	.13350	2,988	1,924.64	9,713.09	1,641.74	4.5050
84...	19,391	.14330	2,779	1,619.09	7,788.45	1,392.24	4.2687
85...	16,612	.15380	2,555	1,346.65	6,169.36	1,166.96	4.0396
86...	14,057	.16450	2,312	1,106.34	4,822.71	965.87	3.8175
87...	11,745	.17608	2,068	897.46	3,716.37	789.22	3.5993
88...	9,677	.18976	1,836	717.90	2,818.91	635.80	3.3849
89...	7,841	.20613	1,616	564.76	2,101.01	503.57	3.1785
90...	6,225	.22314	1,389	435.30	1,536.25	390.56	2.9875
91...	4,836	.23940	1,158	328.32	1,100.95	296.25	2.8116
92...	3,678	.25610	942	242.43	772.63	219.93	2.6453
93...	2,736	.27378	749	175.09	530.20	159.65	2.4865
94...	1,987	.29248	581	123.45	355.11	113.11	2.3348
95...	1,406	.31222	439	84.81	231.66	78.06	2.1898
96...	967	.33303	322	56.63	146.85	52.35	2.0514
97...	645	.35494	229	36.67	90.22	34.04	1.9186
98...	416	.37798	157	22.96	53.55	21.40	1.7906
99...	259	.40213	104	13.88	30.59	12.99	1.6622
100...	155	.42742	66	8.07	16.71	7.58	1.5289
101...	89	.45384	40	4.50	8.64	4.25	1.3783
102...	49	.48140	24	2.40	4.14	2.28	1.1833
103...	25	.51008	13	1.19	1.74	1.14	.9205
104...	12	1.00000	12	.55	.55	.....	.....

Formula 7 is used in connection with withdrawals who leave service before they are eligible to retire. For such individuals we use an average deferred retirement age of 66. That age is used rather than 65 since under the Railroad Retirement Act an individual who withdraws from the railroad industry must relinquish rights with his last outside employer before he can become eligible for an annuity. For mortality before retirement, the rates of Table 1 are used with a 1-year rate-back; after retirement, the mortality rates are those assigned for the ordinary immediate age retirements.

The type of annuity value used in formula 8, for immediate disability retirements, varies according to the amount of service creditable toward a railroad retirement annuity. The annuity values used in any particular instance represent a makeshift in that, as of the time the latest valuation was under way, only a year's experience was available relative to mortality after disability under the Act as amended in 1946. They were decided upon on the basis of actual to expected mortality ratios for three broad classes of disability retirement, using the 1944 Disabled Railway Employees Select Mortality Table as the standard of measurement. The latter table was originally designed as a valuation basis under the previous total and permanent disability provisions of the Railroad Retirement Act. It was developed on the basis of mortality of disability annuitants who last worked in 1937 or in subsequent years. The period of the investigation ran from the 1937 to the 1944 anniversary. Since most plans containing a disability feature have been related to a total and permanent disability definition, it appears desirable to present the mortality rates developed in this table, the auxiliary functions, and the corresponding annuity values at 3 percent (Tables 7a–7e). The data used in such functions are all related to age last birthday at accrual.

Our empirical investigations have indicated the use of the annuity values from the 1944 Disabled Railway Employees Select Mortality Table with a 1-year rate-back in the tabular age at disability and a 1-year advance in duration ($a^{i\,(12)}_{[x-1]+1}$) for individuals with less than 20 years of service who retired before age 60. For disability retirements with 20 or more years of service, or after age 60, the same table is used, except that there is a 3-year rate-back in the tabular age at disability and a 3-year advance in duration ($a^{i\,(12)}_{[x-3]+3}$). The first group, it will be noted, is truly permanently and totally disabled; the second involves a mixture of total and permanent disabilities, on the one hand, and disability for regular occupation on the other. It would appear that such annuity values should be sufficiently adequate for present purposes. As experience develops un-

TABLE 7a

1944 Disabled Railway Employees Select Mortality Table

Mortality Rates—$q^{i}_{[x]+t}$

Age at Disability $[x]$	Duration since Disability								Ultimate Section			
	0	1	2	3	4	5	6	7				
	$q^{i}_{[x]}$	$q^{i}_{[x]+1}$	$q^{i}_{[x]+2}$	$q^{i}_{[x]+3}$	$q^{i}_{[x]+4}$	$q^{i}_{[x]+5}$	$q^{i}_{[x]+6}$	$q^{i}_{[x]+7}$	Attained Age x	q^{d}_{x}	Attained Age x	q^{d}_{x}
30	.203	.106	.067	.050	.043	.037	.032	.029	38	.026	73	.099
31	.198	.103	.066	.048	.042	.037	.033	.029	39	.026	74	.103
32	.194	.101	.065	.047	.042	.037	.033	.029	40	.026	75	.106
33	.190	.100	.064	.046	.042	.038	.034	.030	41	.026	76	.110
34	.186	.099	.064	.046	.042	.038	.034	.030	42	.026	77	.115
35	.182	.098	.064	.047	.043	.039	.035	.031	43	.027	78	.122
36	.178	.097	.065	.048	.043	.039	.035	.031	44	.027	79	.132
37	.174	.096	.066	.049	.044	.039	.035	.031	45	.027	80	.144
38	.170	.096	.067	.049	.044	.039	.035	.031	46	.027	81	.159
39	.167	.096	.068	.050	.044	.039	.035	.031	47	.028	82	.174
40	.164	.097	.069	.050	.044	.039	.035	.031	48	.029	83	.192
41	.162	.099	.072	.052	.046	.040	.035	.031	49	.030	84	.211
42	.160	.101	.075	.053	.047	.042	.036	.032	50	.031	85	.236
43	.158	.103	.079	.055	.048	.043	.037	.034	51	.033	86	.266
44	.158	.104	.083	.057	.050	.044	.039	.036	52	.035	87	.303
45	.156	.107	.086	.060	.052	.046	.042	.039	53	.039	88	.347
46	.155	.109	.090	.064	.054	.048	.045	.043	54	.044	89	.396
47	.154	.112	.093	.067	.056	.050	.048	.048	55	.049	90	.455
48	.154	.113	.093	.071	.058	.053	.051	.052	56	.053	91	.532
49	.154	.114	.094	.074	.060	.055	.054	.055	57	.056	92	.634
50	.153	.114	.095	.076	.063	.058	.057	.057	58	.058	93	.734
51	.153	.114	.096	.077	.065	.060	.059	.059	59	.060	94	.857
52	.153	.114	.098	.078	.067	.062	.060	.061	60	.062	95	1.000
53	.153	.114	.097	.078	.069	.062	.062	.063	61	.064		
54	.151	.113	.096	.079	.070	.064	.064	.065	62	.066		
55	.150	.113	.095	.079	.071	.065	.066	.067	63	.068		
56	.147	.112	.095	.079	.072	.067	.068	.069	64	.070		
57	.144	.111	.094	.079	.074	.069	.070	.071	65	.073		
58	.139	.111	.092	.080	.076	.072	.072	.074	66	.076		
59	.133	.107	.091	.081	.078	.074	.075	.077	67	.079		
60	.127	.106	.091	.082	.079	.076	.078	.080	68	.082		
61	.122	.102	.091	.082	.080	.079	.081	.083	69	.086		
62	.119	.102	.092	.083	.082	.082	.084	.086	70	.089		
63	.116	.101	.093	.084	.084	.085	.087	.089	71	.092		
64	.115	.102	.094	.086	.086	.088	.090	.092	72	.095		

TABLE 7b

1944 Disabled Railway Employees Select Mortality Table

Number Living—$l^i_{[x]+t}$

Age at Disability [x]	Duration since Disability								Ultimate Section			
	0	1	2	3	4	5	6	7				
	$l^i_{[x]}$	$l^i_{[x]+1}$	$l^i_{[x]+2}$	$l^i_{[x]+3}$	$l^i_{[x]+4}$	$l^i_{[x]+5}$	$l^i_{[x]+6}$	$l^i_{[x]+7}$	Attained Age x	l^i_x	Attained Age x	l^i_x
30	100,000	79,700	71,252	66,478	63,154	60,438	58,202	56,340	38	54,706	73	8,668
31	96,162	77,122	69,178	64,612	61,511	58,928	56,748	54,875	39	53,284	74	7,810
32	92,795	74,793	67,239	62,868	59,913	57,397	55,273	53,449	40	51,899	75	7,006
33	89,926	72,840	65,556	61,360	58,537	56,078	53,947	52,113	41	50,550	76	6,263
34	87,063	70,869	63,853	59,766	57,017	54,622	52,546	50,759	42	49,236	77	5,574
35	84,727	69,307	62,515	58,514	55,764	53,366	51,285	49,490	43	47,956	78	4,933
36	82,123	67,505	60,957	56,995	54,259	51,926	49,901	48,154	44	46,661	79	4,331
37	79,679	65,815	59,497	55,570	52,847	50,522	48,552	46,853	45	45,401	80	3,759
38	77,237	64,107	57,953	54,070	51,421	49,158	47,241	45,588	46	44,175	81	3,218
39	75,041	62,509	56,508	52,665	50,032	47,831	45,966	44,357	47	42,982	82	2,706
40	72,840	60,894	54,987	51,193	48,633	46,493	44,680	43,116	48	41,779	83	2,235
41	71,314	59,761	53,845	49,968	47,370	45,191	43,383	41,865	49	40,567	84	1,806
42	69,825	58,653	52,729	48,774	46,189	44,018	42,169	40,651	50	39,350	85	1,425
43	68,444	57,630	51,694	47,610	44,991	42,831	40,989	39,472	51	38,130	86	1,089
44	67,175	56,561	50,679	46,473	43,824	41,633	39,801	38,249	52	36,872	87	799
45	65,995	55,700	49,740	45,462	42,734	40,512	38,648	37,025	53	35,581	88	557
46	64,777	54,737	48,771	44,382	41,542	39,299	37,413	35,729	54	34,193	89	364
47	63,262	53,520	47,526	43,106	40,218	37,966	36,068	34,337	55	32,689	90	220
48	61,260	51,826	45,970	41,695	38,735	36,488	34,554	32,792	56	31,087	91	120
49	58,950	49,872	44,187	40,033	37,071	34,847	32,930	31,152	57	29,439	92	56
50	56,420	47,788	42,340	38,318	35,406	33,175	31,251	29,470	58	27,790	93	20
51	53,718	45,499	40,312	36,442	33,636	31,450	29,563	27,819	59	26,178	94	5
52	51,044	43,234	38,305	34,551	31,856	29,722	27,879	26,206	60	24,607	95	1
53	48,131	40,767	36,120	32,616	30,072	27,997	26,261	24,633	61	23,081		
54	45,231	38,401	34,062	30,792	28,359	26,374	24,686	23,106	62	21,604		
55	42,420	36,057	31,983	28,945	26,658	24,765	23,185	21,627	63	20,178		
56	39,648	33,820	30,032	27,179	25,032	23,230	21,674	20,200	64	18,806		
57	36,980	31,655	28,141	25,496	23,482	21,744	20,244	18,827	65	17,490		
58	34,411	29,628	26,339	23,916	22,003	20,331	18,867	17,509	66	16,213		
59	31,776	27,550	24,602	22,363	20,552	18,949	17,547	16,231	67	14,981		
60	29,349	25,622	22,906	20,822	19,115	17,605	16,267	14,998	68	13,798		
61	26,965	23,675	21,260	19,325	17,740	16,321	15,032	13,814	69	12,667		
62	24,911	21,947	19,708	17,895	16,410	15,064	13,829	12,667	70	11,578		
63	22,915	20,257	18,211	16,517	15,130	13,859	12,681	11,578	71	10,548		
64	21,128	18,698	16,791	15,213	13,905	12,709	11,591	10,548	72	9,578		

TABLE 7c

1944 Disabled Railway Employees Select Mortality Table
Commutation Functions at 3%—$D^{i}_{[x]+t}$
($l^{i}_{[30]} = 100{,}000$)

Age at Disability [x]	Duration since Disability								Ultimate Section			
	0	1	2	3	4	5	6	7				
	$D^{i}_{[x]}$	$D^{i}_{[x]+1}$	$D^{i}_{[x]+2}$	$D^{i}_{[x]+3}$	$D^{i}_{[x]+4}$	$D^{i}_{[x]+5}$	$D^{i}_{[x]+6}$	$D^{i}_{[x]+7}$	Attained Age x	D^{i}_{x}	Attained Age x	D
30	41,198.7	31,879.0	27,669.8	25,063.9	23,117.2	21,478.6	20,081.6	18,872.9	38	17,791.8	73	1,001.8
31	38,463.5	29,949.3	26,081.9	23,650.9	21,860.0	20,332.0	19,009.6	17,846.8	39	16,824.6	74	876.4
32	36,035.7	28,198.9	24,612.5	22,342.2	20,671.9	19,227.0	17,976.2	16,876.7	40	15,910.0	75	763.3
33	33,904.4	26,662.7	23,297.5	21,171.2	19,608.9	18,238.0	17,034.0	15,975.6	41	15,045.1	76	662.5
34	31,869.0	25,185.6	22,031.3	20,020.6	18,543.4	17,247.1	16,108.3	15,107.3	42	14,227.2	77	572.4
35	30,110.5	23,913.1	20,941.5	19,030.3	17,607.7	16,359.7	15,263.9	14,300.6	43	13,453.7	78	491.8
36	28,335.1	22,613.0	19,824.8	17,996.4	16,633.5	15,454.6	14,419.3	13,509.3	44	12,709.1	79	419.2
37	26,691.1	21,404.7	18,786.4	17,035.4	15,728.7	14,598.8	13,620.9	12,761.4	45	12,005.8	80	353.3
38	25,119.5	20,242.0	17,765.9	16,092.7	14,858.6	13,790.9	12,867.1	12,055.2	46	11,341.4	81	293.6
39	23,694.5	19,162.6	16,818.4	15,218.0	14,036.1	13,027.8	12,155.2	11,388.1	47	10,713.7	82	239.7
40	22,329.6	18,123.8	15,889.0	14,361.8	13,246.3	12,294.6	11,471.0	10,747.1	48	10,110.5	83	192.2
41	21,225.0	17,268.5	15,105.8	13,609.9	12,526.5	11,602.2	10,813.6	10,131.3	49	9,531.2	84	150.8
42	20,176.6	16,454.7	14,361.9	12,897.7	11,858.4	10,971.9	10,204.9	9,551.0	50	8,976.0	85	115.5
43	19,201.5	15,696.8	13,669.9	12,223.2	11,214.4	10,365.1	9,630.4	9,003.8	51	8,444.4	86	85.7
44	18,296.6	14,956.9	13,011.2	11,583.8	10,605.4	9,781.7	9,078.9	8,470.7	52	7,928.0	87	61.1
45	17,451.7	14,300.3	12,398.1	11,001.8	10,040.4	9,241.1	8,559.1	7,960.9	53	7,427.5	88	41.3
46	16,630.7	13,643.7	11,802.5	10,427.6	9,476.0	8,703.3	8,044.3	7,458.4	54	6,929.9	89	26.2
47	15,768.6	12,951.8	11,166.2	9,832.8	8,906.8	8,163.2	7,529.2	6,959.1	55	6,432.1	90	15.4
48	14,824.9	12,176.5	10,486.1	9,233.9	8,328.5	7,616.9	7,003.1	6,452.4	56	5,938.7	91	8.1
49	13,850.3	11,376.2	9,785.8	8,607.6	7,738.6	7,062.4	6,479.5	5,951.2	57	5,460.1	92	3.7
50	12,869.8	10,583.3	9,103.7	7,998.9	7,175.7	6,527.7	5,970.1	5,465.9	58	5,004.1	93	1.3
51	11,896.5	9,782.9	8,415.1	7,385.7	6,618.5	6,008.1	5,483.1	5,009.4	59	4,576.6	94	.3
52	10,975.1	9,025.1	7,763.3	6,798.5	6,085.6	5,512.6	5,020.2	4,581.5	60	4,176.6	95	.1
53	10,047.3	8,262.2	7,107.2	6,230.8	5,577.5	5,041.4	4,591.1	4,181.0	61	3,803.5		
54	9,167.0	7,556.0	6,507.1	5,711.1	5,106.6	4,610.8	4,190.0	3,807.6	62	3,456.4		
55	8,346.9	6,888.2	5,932.0	5,212.1	4,660.5	4,203.4	3,815.7	3,460.1	63	3,134.2		
56	7,574.2	6,272.7	5,407.9	4,751.6	4,248.8	3,828.0	3,467.6	3,137.7	64	2,836.1		
57	6,858.8	5,700.1	4,919.8	4,327.5	3,869.6	3,478.8	3,144.5	2,839.2	65	2,560.8		
58	6,196.4	5,179.7	4,470.6	3,941.1	3,520.3	3,158.0	2,845.3	2,563.5	66	2,304.7		
59	5,555.2	4,676.1	4,054.1	3,577.9	3,192.3	2,857.6	2,569.1	2,307.2	67	2,067.5		
60	4,981.5	4,222.2	3,664.7	3,234.3	2,882.7	2,577.6	2,312.3	2,069.9	68	1,848.8		
61	4,443.5	3,787.8	3,302.3	2,914.3	2,597.4	2,320.0	2,074.6	1,850.9	69	1,647.8		
62	3,985.5	3,409.0	2,972.1	2,620.1	2,332.7	2,079.0	1,852.9	1,647.8	70	1,462.3		
63	3,559.4	3,054.9	2,666.3	2,347.9	2,088.1	1,857.0	1,649.6	1,462.3	71	1,293.4		
64	3,186.2	2,737.6	2,386.8	2,099.5	1,863.1	1,653.3	1,463.9	1,293.4	72	1,140.2		

TABLE 7d

1944 DISABLED RAILWAY EMPLOYEES SELECT MORTALITY TABLE
COMMUTATION FUNCTIONS AT 3%—$N^i_{[x]+t}$
($l^i_{[30]} = 100{,}000$)

AGE AT DISABILITY [x]	DURATION SINCE DISABILITY								ULTIMATE SECTION			
	0	1	2	3	4	5	6	7				
	$N^i_{[x]}$	$N^i_{[x]+1}$	$N^i_{[x]+2}$	$N^i_{[x]+3}$	$N^i_{[x]+4}$	$N^i_{[x]+5}$	$N^i_{[x]+6}$	$N^i_{[x]+7}$	Attained Age x	N^i_x	Attained Age x	N^i_x
30	474,251.2	433,052.5	401,173.5	373,503.7	348,439.8	325,322.6	303,844.0	283,762.4	38	264,889.5	73	6,375.7
31	444,291.7	405,828.2	375,878.9	349,797.0	326,146.1	304,286.1	283,954.1	264,944.5	39	247,097.7	74	5,373.9
32	416,214.2	380,178.5	351,979.6	327,367.1	305,024.9	284,353.0	265,126.0	247,149.8	40	230,273.1	75	4,497.5
33	390,255.4	356,351.0	329,688.3	306,390.8	285,219.6	265,610.7	247,372.7	230,338.7	41	214,363.1	76	3,734.2
34	365,430.6	333,561.6	308,376.0	286,344.7	266,324.1	247,780.7	230,533.6	214,425.3	42	199,318.0	77	3,071.7
35	342,618.1	312,507.6	288,594.5	267,653.0	248,622.7	231,015.0	214,655.3	199,391.4	43	185,090.8	78	2,499.3
36	320,423.1	292,098.0	269,475.0	249,650.2	231,653.8	215,020.3	199,565.7	185,146.4	44	171,637.1	79	2,007.5
37	299,555.4	272,864.3	251,459.6	232,673.2	215,637.8	199,909.1	185,310.3	171,689.4	45	158,928.0	80	1,588.3
38	279,714.1	254,594.6	234,352.6	216,586.7	200,494.0	185,635.4	171,844.5	158,977.4	46	146,922.2	81	1,235.0
39	261,081.5	237,387.0	218,224.4	201,406.0	186,188.0	172,151.9	159,124.1	146,968.9	47	135,580.8	82	941.4
40	243,330.3	221,000.7	202,876.9	186,987.9	172,626.1	159,379.8	147,085.2	135,614.2	48	124,867.1	83	701.7
41	227,039.4	205,814.4	188,545.9	173,440.1	159,830.2	147,303.7	135,701.5	124,887.9	49	114,756.6	84	509.5
42	211,702.5	191,525.9	175,071.2	160,709.3	147,811.6	135,953.2	124,981.3	114,776.4	50	105,225.4	85	358.7
43	197,254.5	178,053.0	162,356.2	148,686.3	136,463.1	125,248.7	114,883.6	105,253.2	51	96,249.4	86	243.2
44	183,590.2	165,293.6	150,336.7	137,325.5	125,741.7	115,136.3	105,354.6	96,275.7	52	87,805.0	87	157.5
45	170,830.4	153,378.7	139,078.4	126,680.3	115,678.5	105,638.1	96,397.0	87,837.9	53	79,867.0	88	96.4
46	158,636.0	142,005.3	128,361.6	116,559.1	106,131.5	96,655.5	87,952.2	79,907.9	54	72,449.5	89	55.1
47	146,797.3	131,028.7	118,076.9	106,910.7	97,077.9	88,171.1	80,007.9	72,478.7	55	65,519.6	90	28.9
48	135,209.8	120,384.9	108,208.4	97,722.3	88,488.4	80,159.9	72,543.0	65,539.9	56	59,087.5	91	13.5
49	124,000.4	110,150.1	98,773.9	88,988.1	80,380.5	72,641.9	65,579.5	59,100.0	57	53,148.8	92	5.4
50	113,383.8	100,514.0	89,930.7	80,827.0	72,828.1	65,652.4	59,124.7	53,154.6	58	47,688.7	93	1.7
51	103,283.9	91,387.4	81,604.5	73,189.4	65,803.7	59,185.2	53,177.1	47,694.0	59	42,684.6	94	.4
52	93,869.9	82,894.8	73,869.7	66,106.4	59,307.9	53,222.3	47,709.7	42,689.5	60	38,107.0	95	.1
53	84,969.9	74,912.6	66,660.4	59,553.2	53,322.4	47,744.9	42,703.5	38,112.4	61	33,931.4		
54	76,784.1	67,617.1	60,061.1	53,554.0	47,842.9	42,736.3	38,125.5	33,935.5	62	30,127.9		
55	69,190.4	60,843.5	53,955.3	48,023.3	42,811.2	38,150.7	33,947.3	30,131.6	63	26,671.5		
56	62,225.8	54,651.6	48,378.9	42,971.0	38,219.4	33,970.6	30,142.6	26,675.0	64	23,537.3		
57	55,839.5	48,980.7	43,280.6	38,360.8	34,033.3	30,163.7	26,684.9	23,540.4	65	20,701.2		
58	50,015.3	43,818.9	38,639.2	34,168.6	30,227.5	26,707.2	23,549.2	20,703.9	66	18,140.4		
59	44,625.2	39,070.0	34,393.9	30,339.8	26,761.9	23,569.6	20,712.0	18,142.9	67	15,835.7		
60	39,713.4	34,731.9	30,509.7	26,845.0	23,610.7	20,728.0	18,150.4	15,838.1	68	13,768.2		
61	35,210.2	30,766.7	26,978.9	23,676.6	20,762.3	18,164.9	15,844.9	13,770.3	69	11,919.4		
62	31,170.7	27,185.2	23,776.2	20,804.1	18,184.0	15,851.3	13,772.3	11,919.4	70	10,271.6		
63	27,494.8	23,935.4	20,880.5	18,214.2	15,866.3	13,778.2	11,921.2	10,271.6	71	8,809.3		
64	24,199.7	21,013.5	18,275.9	15,889.1	13,789.6	11,926.5	10,273.2	8,809.3	72	7,515.9		

TABLE 7e

1944 Disabled Railway Employees Select Mortality Table
Annuity Values at 3%—$a^{i(12)}_{[x]+t}$

Age at Disability [x]	Duration since Disability								Ultimate Section			
	0	1	2	3	4	5	6	7				
	$a^{i(12)}_{[x]}$	$a^{i(12)}_{[x]+1}$	$a^{i(12)}_{[x]+2}$	$a^{i(12)}_{[x]+3}$	$a^{i(12)}_{[x]+4}$	$a^{i(12)}_{[x]+5}$	$a^{i(12)}_{[x]+6}$	$a^{i(12)}_{[x]+7}$	Attained Age x	$a^{i(12)}_{x}$	Attained Age x	$a^{i(12)}_{x}$
30.....	10.9696	13.0426	13.9569	14.3604	14.5311	14.6047	14.5888	14.4937	38	14.3466	73	5.8225
31.....	11.0093	13.0088	13.8698	14.2483	14.3781	14.4242	14.3957	14.3038	39	14.1450	74	5.5901
32.....	11.0084	12.9403	13.7591	14.1107	14.2138	14.2476	14.2070	14.1027	40	13.9318	75	5.3505
33.....	10.9688	12.8235	13.6095	13.9304	14.0037	14.0219	13.9806	13.8765	41	13.7063	76	5.0948
34.....	10.9249	12.7024	13.4555	13.7608	13.8205	13.8248	13.7698	13.6518	42	13.4679	77	4.8247
35.....	10.8370	12.5268	13.2393	13.5229	13.5784	13.5793	13.5212	13.4012	43	13.2159	78	4.5402
36.....	10.7670	12.3751	13.0511	13.3305	13.3852	13.3713	13.2985	13.1634	44	12.9634	79	4.2472
37.....	10.6813	12.2062	12.8435	13.1165	13.1681	13.1518	13.0631	12.9121	45	12.6959	80	3.9539
38.....	10.5936	12.0358	12.6494	12.9170	12.9518	12.9190	12.8136	12.6458	46	12.4128	81	3.6647
39.....	10.4770	11.8463	12.4336	12.6930	12.7232	12.6725	12.5493	12.3638	47	12.1132	82	3.3857
40.....	10.3555	11.6522	12.2267	12.4781	12.4903	12.4217	12.2807	12.0770	48	11.8085	83	3.1092
41.....	10.1551	11.3768	11.9400	12.2020	12.2177	12.1545	12.0075	11.7852	49	11.4984	84	2.8369
42.....	9.9508	11.0979	11.6483	11.9186	11.9230	11.8493	11.7055	11.4755	50	11.1813	85	2.5639
43.....	9.7312	10.8016	11.3352	11.6226	11.6269	11.5420	11.3876	11.1482	51	10.8563	86	2.2961
44.....	9.4924	10.5096	11.0127	11.3133	11.3147	11.2289	11.0626	10.8240	52	10.5323	87	2.0360
45.....	9.2471	10.1839	10.6760	10.9728	10.9796	10.8896	10.7208	10.4907	53	10.2125	88	1.7924
46.....	8.9970	9.8664	10.3341	10.6362	10.6583	10.5639	10.3918	10.1721	54	9.9129	89	1.5614
47.....	8.7678	9.5749	10.0328	10.3312	10.3576	10.2593	10.0846	9.8733	55	9.6446	90	1.3349
48.....	8.5788	9.3450	9.7775	10.0413	10.0831	9.9823	9.8170	9.6157	56	9.4079	91	1.1250
49.....	8.4112	9.1408	9.5519	9.7966	9.8453	9.7440	9.5794	9.3891	57	9.1923	92	.9178
50.....	8.2684	8.9557	9.3368	9.5631	9.6076	9.5158	9.3618	9.1831	58	8.9882	93	.7660
51.....	8.1402	8.7998	9.1557	9.3679	9.4007	9.3092	9.1567	8.9792	59	8.7848	94	.7916
52.....	8.0113	8.6432	8.9735	9.1820	9.2039	9.1130	8.9618	8.7759	60	8.5825		
53.....	7.9143	8.5264	8.8376	9.0162	9.0186	8.9289	8.7597	8.5739	61	8.3794		
54.....	7.8344	8.4071	8.6884	8.8355	8.8271	8.7270	8.5575	8.3709	62	8.1749		
55.....	7.7477	8.2913	8.5539	8.6721	8.6443	8.5345	8.3550	8.1666	63	7.9681		
56.....	7.6738	8.1709	8.4043	8.5018	8.4536	8.3325	8.1509	7.9598	64	7.7575		
57.....	7.5996	8.0513	8.2555	8.3227	8.2533	8.1290	7.9445	7.7495	65	7.5422		
58.....	7.5300	7.9180	8.1013	8.1281	8.0449	7.9153	7.7348	7.5347	66	7.3293		
59.....	7.4914	7.8136	7.9420	7.9381	7.8416	7.7063	7.5203	7.3219	67	7.1176		
60.....	7.4305	7.6843	7.7836	7.7584	7.6488	7.4999	7.3078	7.1099	68	6.9054		
61.....	7.3823	7.5809	7.6280	7.5826	7.4518	7.2880	7.0959	6.8981	69	6.6918		
62.....	7.2793	7.4328	7.4581	7.3985	7.2536	7.0828	6.8911	6.6918	70	6.4826		
63.....	7.1829	7.2934	7.2896	7.2160	7.0567	6.8779	6.6850	6.4826	71	6.2693		
64.....	7.0535	7.1342	7.1154	7.2063	6.8597	6.6721	6.4760	6.2693	72	6.0500		

der the program in the future, an attempt will be made to develop more appropriate mortality rates and annuity values after disability.

B. AMOUNT OF ANNUITY

Consider an employee entering service at age k whose age on the valuation date is $k + t$ and who withdraws subsequently at age x. Then the average monthly compensation, from which the amount of annuity is derived, is obtained as indicated in formula 11.

$$\bar{S}_x^t = \frac{P^p_{k+t} \cdot \bar{S}^p_{k+t} + S^{1937\text{-}47}_{k+t} + \sum_{k+t}^{x-1} S_r \cdot f_r}{P^p_{k+t} + P^{1937\text{-}47}_{k+t} + \sum_{k+t}^{x-1} f_r} \tag{11}$$

where P^p_{k+t} equals creditable service months prior to 1937,

$\bar{S}^p_{k+t}$ is the applicable average prior monthly compensation (1924–31),

$P^{1937\text{-}47}_{k+t}$ represents the service months rendered in the calendar years 1937–47 inclusive,

$S^{1937\text{-}47}_{k+t}$ is the total compensation in the period 1937–47,

S_r is the assumed future compensation per service month in the year of age r to $r + 1$, and

f_r represents the assumed future service months in the year of age r to $r + 1$.

For new entrants after the valuation date, the average monthly compensation takes on the simplified form

$$\bar{S}_x = \frac{\sum_{k}^{x-1} S_r \cdot f_r}{\sum_{k}^{x-1} f_r}. \tag{12}$$

As indicated by formula 11, earnings for the period 1924–31 are assigned to all creditable service prior to 1937. Such prior service average is contained in the valuation card along with the actual service rendered through the valuation date and the total compensation in the period after 1936.

It remains to show the basis adopted for the assumed future compensation per service month in the year of age r to $r + 1$ (S_r) together with the corresponding assumed future service months f_r for that year of age. As a

starting point, salary scales were prepared by age at original entry, on the basis of 1947 earnings per service month. Such salary scales were then adjusted for hourly wage-rate increases provided through wage negotiations in 1948 and 1949. The adjustments took into account the fact that wages are not creditable beyond the ceiling of $300 per month. Then, after appropriate graduation of the modified salary scales, it was assumed that the future course of earnings by age at entry and duration would parallel the 1947 experience after adjustment.

The pattern of assumed future service months is in accordance with that which resulted as a by-product of the withdrawal rate investigations. This service-months pattern was originally based on the withdrawal study dealing with employees who first entered railroad service in the years 1938–43 and on supplementary studies in connection with employees who entered service prior to 1937. It was considered desirable to use the service patterns prior to 1944 because the results for 1944–47 were too much affected by the dislocations of the recent war to consider them other than atypical.

For ready reference the future service patterns in the nth service year after entry presently being used are shown in the table below.

Employees	Service in the nth Year after Entry										
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th and Over
Continuing.....	6	7	7.5	8	8.5	9	9.5	10	10.5	11	11.5
Withdrawing...	3	3.5	3.75	4	4.25	4.5	4.75	5	5.25	5.5	5.75

The credited months of service for the employee are generally taken as

$$P^t_x = P^p_{k+t} + P^{1937\text{-}47}_{k+t} + \sum_{k+t}^{x-1} f_r + 1.75 \qquad (13)$$

where P^t_x cannot exceed 360 if P^p is used. The summation does not extend beyond the year in which the employee reaches age 65 or June 30, 1937, if later. For new entrants after the valuation date, the formula reduces to

$$P_x = \sum_{k}^{x-1} f_r + 1.75 . \qquad (14)$$

The adjustment of 1.75 months is made in view of the fact that under the Act if an employee will have had not less than 54 months of service, an ultimate fraction of 6 months or more is taken as a year; when the excess

months are 5 or less, the months credited are the same as the number of months actually rendered.

The calculation of the corresponding retirement benefit is presently obtained by taking 2.4 percent of the first \$50 of the average monthly compensation, 1.8 percent of the next \$100, and 1.2 percent of the remainder, and multiplying the resulting annuity factor by the total creditable years of service. Because of the bent nature of the annuity factor per year of service, the formula for the employee annuity benefit varies in accordance with the range within which the average monthly compensation $\bar{S}_x^t$ falls. For a typical instance involving an average monthly compensation over \$150, the employee's monthly retirement annuity becomes

$$ {}^{e}B_x^t = \tfrac{1}{12}\left[(1.20 + .012\bar{S}_x^t) \cdot P_x^t\right]. \tag{15} $$

C. PRESENT VALUE OF BENEFITS

The present values at age $k + t$ of the various types of retirement benefit can now be determined in terms of dollars *per capita*, as indicated in the formulas below.

Nondisability retirements

$$ \frac{\sum_{\alpha}^{64} {}^{wa}C_x^s \left(\frac{2x - 99}{30} \cdot {}^{e}B_{x+1/2}^t\right) + \sum_{65}^{\omega} {}^{wa}C_x^s \cdot {}^{e}B_{x+1/2}^t}{D_{k+t}^s} \tag{16} $$

where α is the earliest possible pre-normal retirement age. When $k + t > 65$ the first summation obviously vanishes and the second starts at $k + t$.

Deferred withdrawal annuities

$$ \frac{\sum_{k+t}^{\alpha-1} {}^{wda}C_x^s \cdot {}^{e}B_{x+1/2}^t}{D_{k+t}^s}. \tag{17} $$

Disability retirements

$$ \frac{\sum_{\beta}^{64} {}^{ia}C_x^s \cdot {}^{e}B_{x+1/2}^t}{D_{k+t}^s}. \tag{18} $$

In connection with the formula for disability retirements, if $k + t < 60$, β equals the age at which 10 years of service is obtained or age 60, whichever is earlier. Where $60 \leq k + t < 65$, β equals $k + t$.

II. Survivor Insurance Benefits

It has been found convenient, for purposes of the survivor insurance benefits, to break up the calculations as follows. First, we determine what is called the value of the insurance benefits per dollar of basic amount, according to the employee's age and mode of exit from service. (The term "basic amount" is similar in concept to the primary insurance benefit of the Social Security Act.) This factor is then attached to the applicable "discounted" exits from the service table. The next step involves the calculation of the corresponding basic amount upon which the survivor insurance benefits are calculated. Finally, the *per capita* values according to the type of exit from service are obtained by a multiple accumulation through the range of ages affected, with the corresponding basic amounts, the resulting aggregate product being then divided by the D function related to the age at entry and duration of service as of the date on which the calculation is being made. The formulas indicating the methodology are herewith presented.

A. INSURANCE VALUES

The values of the insurance benefits per dollar of basic amount take on the following forms for age x last birthday, according to the mode and type of exit from railroad service.

Deaths among nondisability annuitants

$$ {}^{wa}_{iB}A_{x+1/2} = \frac{\sum_{\gamma=x+1}^{\omega} v^{\gamma} \cdot d_{\gamma-1/2}\,(9 \cdot {}_{w}K_{\gamma} + 9 \cdot {}_{w}K_{\gamma} + 6 \cdot {}_{c}K_{\gamma} + 8 \cdot {}_{i}K_{\gamma})}{D_{x+1/2}} \tag{19} $$

where

$$ {}_{wc}K_{\gamma} = {}_{wc}\phi_{\gamma} \cdot a^{r(12)}_{w:\overline{18-x}|} $$

$$ {}_{w}K_{\gamma} = {}_{wc}\phi_{\gamma} \cdot {}_{65-w}| a^{r(12)}_{w} + {}_{yw}\phi_{\gamma} \cdot {}_{65-w'}| a^{r(12)}_{w'} + {}_{aw}\phi_{\gamma} \cdot a^{r(12)}_{w''} $$

$$ {}_{c}K_{\gamma} = {}_{c}\phi_{\gamma} \cdot n_{\gamma} \cdot a^{(12)}_{\overline{18-w}|} $$

$$ {}_{i}K_{\gamma} = {}_{i}\phi_{\gamma} \,. $$

Here, γ is the average exact age at death of the employee; ${}_{wc}\phi_{\gamma}$ is the probability of leaving a widow with eligible children; ${}_{yw}\phi_{\gamma}$ is the probability of leaving a widow under age 65 with no children eligible for a child's benefit; ${}_{aw}\phi_{\gamma}$ is the corresponding probability of leaving a widow age 65 or over; ${}_{c}\phi_{\gamma}$ is the probability of leaving eligible children; n_{γ} is the average number of children per surviving family with children; and ${}_{i}\phi_{\gamma}$ represents the probability that the deceased employee leaves no survivor eligible for immediate monthly benefits. w denotes the average age of

widows eligible for a current insurance benefit (those having one or more children eligible for a child's benefit); w' is the average age of widows under 65 without eligible children; and w'' denotes the average age of widows 65 and over. In each instance, these are average exact ages corresponding to the employee's age at death γ. Similarly, z is the age of the youngest child and u is the average age of the children represented in monthly benefits. $a^{r(12)}$ is an annuity value based upon the Revised American Remarriage Table presented by Mr. A. M. Niessen which is contained in *RAIA* XXXVIII. Finally, $a^{(12)}_{\overline{u}|}$ is an annuity certain payable monthly. It will be noted that mortality of children is disregarded.

The mortality table functions $d_{\gamma-1/2}$ and $D_{x+1/2}$ for formula 19 are in accordance with Table 6 of this paper with a 1-year rate-back in age. Further, the coefficients which are attached to the insurance benefits per dollar of basic amount ${}^{\bullet}B^{t}_{x+1/2}$ arise in view of the fact that (1) the widow's survivor insurance benefit on a yearly basis is equal to 9 times the basic amount; (2) the corresponding child's benefit is equal to 6 times the basic amount; and (3) the insurance lump sum (as distinguished from the guaranteed residual benefit) is equal to 8 times the basic amount.

Attention is called to the fact that for formula 19, as well as succeeding ones, family composition factors are required as of the point of death of the employee. Most of the available statistics present this story on the living side. In the absence of better data, we previously used the results of the Richmond Family Composition Study as a basis for survivor cost calculations. There was reason to believe, however, that actual family composition at the point of death, in relation to cases involving entitlement to survivor benefits, would be significantly different. Pertinent statistics are now available. Tables 8 and 8a summarize the results of Board studies on the basis of survivor benefit awards through 1948 with respect to earnings of employees who died after 1946. The experience covers more than 33,000 employee deaths. Tables 9 and 10 compare the data based on railroad retirement experience with similar data obtained from other sources.

Reference to Table 9 supports the preconceived notion that there exists a very substantial difference in the proportion of married persons as between deceased and living individuals. On the other hand, the proportions married in the RRB and OASI experiences are substantially higher than those for the deaths among U.S. white males in 1943. The differences appear to arise from two causes: (1) the increase in the marital rates from 1943 to 1947 and (2) the fact that there would be a natural bias in favor of deceased married employees in cases involving survivor benefit awards. After due consideration of the results shown in Tables 8–10, the

TABLE 8

FAMILY COMPOSITION DATA BASED ON RAILROAD RETIREMENT BOARD SURVIVOR BENEFIT AWARDS, 1947–48

(Marital and Parental Status)

AGE OF EMPLOYEE AT DEATH*	PROPORTION MARRIED (PERCENT)				PROPORTION EX-MARRIED WITH CHILDREN (PERCENT)	PROPORTION SINGLE (PERCENT)
	Total	With Children	Young Widows without Children	Aged Widows		
Under 20	12.4	7.2	5.2	†	†	87.6
20–24	33.0	20.9	12.1	†	1.1	65.9
25–29	61.8	45.9	15.9	†	4.8	33.4
30–34	67.1	50.9	16.2	†	6.6	26.3
35–39	68.0	48.3	19.7	†	6.0	26.0
40–44	72.7	45.0	27.7	†	3.5	23.8
45–49	73.9	34.6	39.3	†	2.7	23.4
50–54	78.5	24.9	52.8	0.8	1.6	19.9
55–59	79.2	14.9	61.1	3.2	1.2	19.6
60–64	78.3	7.7	57.8	12.8	.6	21.1
65–69	77.3	3.7	41.4	32.2	.3	22.4
70–74	72.6	1.4	22.0	49.2	.1	27.3
75–79	63.3	1.0	10.9	51.4	†	36.7
80–84	50.5	.7	5.6	44.2	†	49.5
85 and over	35.6	.2	2.5	32.9	†	64.4

* Age last birthday. † Less than .05 percent.

TABLE 8a

FAMILY COMPOSITION DATA BASED ON RAILROAD RETIREMENT BOARD SURVIVOR BENEFIT AWARDS, 1947–48

(Average Ages of Survivors and Average Number of Children)

AGE OF EMPLOYEE AT DEATH*	AVERAGE AGES OF WIDOWS†				AVERAGE AGES OF CHILDREN†		AVERAGE NUMBER OF CHILDREN PER FAMILY WITH CHILDREN
	All	Under 65 with Children	Under 65 without Children	Age 65 and over	All	Youngest Child	
Under 20	19.2	18.3	20.1	.....	0.9	0.1	1.4
20–24	22.6	22.1	23.0	.....	1.8	1.0	1.4
25–29	26.1	25.6	26.9	.....	3.6	2.3	1.8
30–34	30.3	30.1	31.4	.....	5.8	4.0	2.2
35–39	34.9	34.3	36.3	.....	8.2	6.4	2.1
40–44	39.5	38.7	41.1	.....	10.3	8.6	2.0
45–49	44.2	42.7	45.4	.....	11.4	10.4	1.8
50–54	49.2	46.7	50.0	67.1	12.5	11.9	1.7
55–59	53.6	49.5	53.9	67.0	13.2	12.6	1.6
60–64	58.3	51.4	57.5	66.2	13.6	13.3	1.5
65–69	62.0	52.0	59.0	66.9	13.6	13.4	1.5
70–74	65.6	52.0	59.2	68.9	13.6	13.4	1.5
75–79	69.7	52.0	59.2	72.2	13.6	13.4	1.5
80–84	73.1	52.0	59.2	75.2	13.6	13.4	1.5
85 and over	76.2	52.0	59.2	77.6	13.6	13.4	1.5

* Age last birthday. † Exact age on the date of the employee's death.

conclusion was drawn that our own survivor benefit award experience would be most valid and suitable for the population covered under the Railroad Retirement Act.

TABLE 9

FAMILY COMPOSITION DATA BASED ON RAILROAD RETIREMENT BOARD EXPERIENCE COMPARED WITH SIMILAR DATA DERIVED FROM OTHER SOURCES
(Marital and Parental Status)

AGE OF MALE*	PROPORTION MARRIED (PERCENT)					PROPORTION OF MARRIED WHO HAVE CHILDREN (PERCENT)		
	RRB 1947–48	OASI 1947	U.S. Deaths 1943 W.M.	RFC Urban	U.S. 1947	RRB 1947–48	OASI 1947	RFC Urban
Under 20	12.4	8.5	.....	0.7	7.0	58.3	42.1	31.6
20–24	33.0	28.7	27.4	22.3	36.7	63.4	53.5	46.4
25–29	61.8	52.6	53.0	59.2	70.0	74.3	69.8	59.0
30–34	67.1	67.7	65.6	76.1	81.9	75.8	76.8	70.1
35–39	68.0	74.3	68.2	81.8	85.4	71.1	77.0	74.2
40–44	72.7	75.1	70.6	84.2	86.4	61.8	67.4	72.5
45–49	73.9	76.3	73.2	84.4	86.5	46.7	53.5	61.5
50–54	78.6	78.0	72.7	82.9	84.8	31.7	38.8	46.2
55–59	79.2	77.6	71.7	81.8	82.8	18.8	25.1	31.2
60–64	78.3	73.7	69.5	78.0	79.5	9.8	13.1	17.4
65–69	77.3	71.4	65.1	72.6	76.4	4.8	5.5	8.6
70–74	72.6	67.4	58.9	65.3	68.9	1.9	2.7	4.1
75–79	63.3	64.1	50.9	56.4	59.4	1.6	1.5	1.8
80–84	50.5	53.8	41.2	46.0	48.5	1.3	.6	1.0
85 and over	35.6	.....	.....	32.8	34.9	.6	.....	.8

* Age last birthday, except for the OASI awards which relate to the age attained in the year of death.

Sources: 1. Railroad Retirement Board (RRB) survivor benefit statistics, awards in 1947–48.
2. Substantive Statistics for OASI awards in 1947.
3. Unpublished data of the U.S. Public Health Service on U.S. deaths in 1943, white males (W.M.).
4. Richmond Family Composition Study (RFC), 1935–36; data supplied by the Social Security Administration.
5. Bureau of the Census, Series P-20, No. 10.

Withdrawals before retirement

$$ {}_{iB}^{w}A_{x+1/2} = \frac{1}{D_{x+1/2}} \times \left[\begin{array}{l} v^{x+1} \cdot d_{x+1/2}(9 \cdot {}_{wc}K_{x+1} + 9 \cdot {}_{w}K_{x+1} + 6 \cdot {}_{c}K_{x+1} + 8 \cdot {}_{is}K_{x+1}) \\ + \frac{1}{2} v^{x+2} \cdot d_{x+3/2}(9 \cdot {}_{wc}K_{x+2} + 9 \cdot {}_{w}K_{x+2} + 6 \cdot {}_{c}K_{x+2} + 8 \cdot {}_{is}K_{x+2}) \end{array} \right]. \quad (20) $$

Here, $d_{x+1/2}$, $d_{x+3/2}$, and $D_{x+1/2}$ refer to Table 1 of this paper with a 1-year rate-back in age. It will be noted further in connection with formula 20

that the insurance functions relate only to the mortality experience for 1½ years after withdrawal. The reason for this procedure is explained below.

Unlike the retirement benefits, which relate solely to railroad earnings,

TABLE 10

FAMILY COMPOSITION DATA BASED ON RAILROAD RETIREMENT BOARD EXPERIENCE COMPARED WITH SIMILAR DATA DERIVED FROM OTHER SOURCES
(Average Ages of Survivors and Average Number of Children)

Age of Male*	Average Age of Wife†			Average Age of All Children†		Average Age of Youngest Child†		Average Number of Children per Family with Children		
	RRB 1947–48	OASI 1944	RFC Urban	RRB 1947–48	RFC Urban	RRB 1947–48	RFC Urban	RRB 1947–48	OASI 1947	RFC Urban
Under 20	19.2	18.9	18.8	.9	.9	.1	.8	1.4	1.2	1.1
20–24	22.6	22.0	21.8	1.8	2.1	1.0	1.6	1.4	1.4	1.3
25–29	26.1	25.1	25.5	3.6	3.8	2.3	2.8	1.8	1.6	1.6
30–34	30.3	29.8	29.6	5.8	6.0	4.0	4.4	2.2	2.0	1.9
35–39	34.9	34.1	34.0	8.2	8.7	6.4	6.6	2.1	2.2	2.3
40–44	39.5	38.7	38.6	10.3	10.6	8.6	8.6	2.0	2.0	2.4
45–49	44.2	43.6	43.2	11.4	11.7	10.4	10.2	1.8	1.7	2.3
50–54	49.2	48.2	47.8	12.5	12.6	11.9	11.6	1.7	1.6	2.0
55–59	53.6	52.6	51.9	13.2	13.2	12.6	12.6	1.6	1.5	1.8
60–64	58.3	57.2	56.8	13.6	13.6	13.3	13.3	1.5	1.3	1.7
65–69	62.0	61.6	61.1	13.6	13.5	13.4	13.2	1.5	1.3	1.6
70–74	65.6	65.5	65.6	13.6	13.3	13.4	13.1	1.4	1.1	1.6
75–79	69.7	69.2	69.5	.....	13.0	.....	12.7	.....	.....	1.7
80–84	73.1	.....	72.8	.....	13.2	.....	13.3	.....	.....	1.4
85 and over	76.2	.....	75.8	.....	15.3	.....	15.3	.....	.....	1.0

* Age last birthday, except for the OASI awards which relate to the age attained in the year of death.

† Exact age on the date of employee's death or on the date of enumeration as the case may be.

Sources: 1. Railroad Retirement Board (RRB) survivor benefit statistics, awards in 1947–48.
2. Substantive Statistics for OASI awards in 1944 and 1947.
3. Richmond Family Composition Study (RFC), 1935–36; data supplied by the Social Security Administration.

the survivor insurance benefits provided under the Railroad Retirement Act are based on combined social security and railroad retirement coverage. The same now holds true in connection with survivor benefits payable under the Social Security Act. The adjudicating agency for a particular employee death involving earnings under both systems is decided on the basis of the recency of connection with the railroad industry at the time the employee died. In accordance with the provisions of the amendments governing these cases of dual coverage, a procedure is to be worked out in the future calling for yearly reimbursements to the railroad retirement account when the Railroad Retirement Board is the adjudicating

agency, and similar reimbursements to the old-age and survivors insurance account when the Social Security Administration takes over the survivor benefits. The procedure has not been worked out as yet. Furthermore, the only information presently available to us relative to dual coverage involves instances in which the Railroad Retirement Board is the adjudicating agency. The most recent calculations have therefore been made on the rough and ready assumption that the reimbursements between the two agencies will wash out. Subject to the limitations of such an assumption, the calculations relating to deaths among withdrawals need only concern themselves with deaths among withdrawals within the period in which a current connection with the railroad industry is retained. Since for deaths prior to retirement a current connection is generally deemed to exist only if the deceased individual had at least 12 months of railroad service in the last 30 calendar months preceding death, it is clear that such connection with the railroad industry would definitely be severed, in most instances, 18 months after covered employment ceased.

Disability retirements

$$ {}^{a}_{i}\mathrm{A}_{x+1/2} = \frac{\sum_{\gamma=x+1}^{\omega} v^{\gamma} \cdot d^{i}_{[x]+(\gamma-x-1)} (9 \cdot {}_{wc}K_{\gamma} + 9 \cdot {}_{w}K_{\gamma} + 6 \cdot {}_{c}K_{\gamma} + 8 \cdot {}_{is}K_{\gamma})}{1.015 \mathrm{D}^{i}_{[x]}} . \quad (21) $$

The functions $d^{i}_{[x]+(\gamma-x-1)}$ and $\mathrm{D}^{i}_{[x]}$ are in accordance with the remarks made for $a^{i(12)}_{[x]}$ in connection with formula 8.

Deaths in active service

$$ {}^{d}_{i}\mathrm{A}_{x+1/2} = (9 \cdot {}_{wc}K_{x+1/2} + 9 \cdot {}_{w}K_{x+1/2} + 6 \cdot {}_{c}K_{x+1/2} + 8 \cdot {}_{is}K_{x+1/2}) . \quad (22) $$

The K functions for formulas 20–22 have the same meaning in each instance as that assigned for nondisability retirements.

B. AMOUNT OF SURVIVOR ANNUITY

The average monthly remuneration (similar in concept to "average monthly wage" as used under the Social Security Act) and the corresponding basic amount, from which survivor insurance benefits are determined, are calculated as indicated in formulas 23 and 24. Consider first the average monthly remuneration $\bar{R}^{t}_{x}$ with respect to an exit from railroad service at age x among employees aged $k + t$ on the valuation date (December 31, 1947).

$$ \bar{R}^{t}_{x} = \frac{{}^{w}S_{k+t} + {}^{c}S^{1937\text{-}47}_{k+t} + \sum_{k+t}^{x-1} {}^{c}S_{r}}{3 \text{ (elapsed quarters)}} \quad (23) $$

where ${}^{w}S_{k+t}$ denotes the assumed creditable social security wages prior to entry into railroad service,

${}^{c}S_{k+t}^{1937-47}$ represents total creditable railroad compensation (as adjusted for survivor benefits) in the period 1937–47,

${}^{s}S_{r}$ denotes the corresponding assumed future creditable railroad compensation in the year of age r to $r+1$.

The elapsed quarters are calculated in much the same manner as for Social Security Act adjudications, except that quarters after an annuity becomes payable are excluded. Two remarks are in order. First, combined railroad compensation and social security wages are not creditable beyond $3,000 in any year. Second, the assumed future creditable railroad compensation scales were derived from those used for retirement benefits as appropriately modified for a $3,000 ceiling on creditable compensation for survivor benefit purposes.

As formula 23 stands, it will be noted that no explicit provision is made for social security earnings after entry into railroad service. However, prior thereto, a substantial social security wage history has been predicated for individuals who entered after 1936. In such instances, they were assigned social security credits in calendar year 1937 and subsequent, roughly equivalent to the earnings of employees covered under that Act who had fewer than four quarters of coverage. The assumed annual social security earnings prior to entry into railroad service start with $500 in 1937 and then advance by successive stages to $925 in 1947. Prior social security wages were assumed for all new entrants after 1947 at the rate of $1,000 per calendar year. It is believed that the type of approximation to social security wages utilized herein makes sufficient allowance for that part of the survivor insurance benefit handled by the Railroad Retirement Board which is based on social security coverage.

The basic amount ${}^{s}B_{x}^{t}$ is determined from the average monthly remuneration $\bar{R}_{x}^{t}$ in accordance with the following range of earnings.

$${}^{s}B_{x}^{t} = .40\bar{R}_{x}^{t}(1+.01n_{x}^{t}) \text{ for } \bar{R}_{x}^{t} \leq 75 \tag{24}$$

$${}^{s}B_{x}^{t} = (22.50+.10\bar{R}_{x}^{t})(1+.01n_{x}^{t}) \text{ for } 75 < \bar{R}_{x}^{t} \leq 250 \tag{24a}$$

where n_{x}^{t} is equal to the number of "increment years"—those years after 1936 in which the employee earned $200 or more in compensation and wages.

C. PRESENT VALUE OF BENEFITS

The values of the various survivor benefits in dollars *per capita* with respect to an employee age $k+t$ on the valuation date are indicated in formulas 25–28.

Nondisability retirements

$$\frac{\sum_{a}^{\omega} \left(v^{x+1/2} w_x \cdot {}^{wa}_{iB}A_{x+1/2} \right) \cdot {}^{s}B^{t}_{x+1/2}}{D^{s}_{k+t}} . \tag{25}$$

Withdrawals before retirement

$$\frac{\sum_{k+t}^{a-1} \left(v^{x+1/2} w_x \cdot {}^{w}_{iB}A_{x+1/2} \right) \cdot {}^{s}B^{t}_{x+1/2}}{D^{s}_{k+t}} . \tag{26}$$

Disability retirements

$$\frac{\sum_{\beta}^{64} \left(v^{x+1/2} i_x \cdot {}^{ia}_{iB}A_{x+1/2} \right) \cdot {}^{s}B^{t}_{x+1/2}}{D^{s}_{k+t}} . \tag{27}$$

Deaths in active service

$$\frac{\sum_{k+t}^{\omega} \left(v^{x+1/2} d_x \cdot {}^{d}_{iB}A_{x+1/2} \right) \cdot {}^{s}B^{t}_{x+1/2}}{D^{s}_{k+t}} . \tag{28}$$

In connection with the above four formulas, it will be noted that after the insurance factors per dollar of basic amount are tied to the respective commutation functions to which they relate, the procedure for developing the survivor insurance present values *per capita* follows closely the one for deriving the *per capita* values for retirement benefits.

III. Residual Lump-sum Benefit

A. GROSS RESIDUAL BENEFIT

Prior to reduction on account of employee retirement benefits on the one hand, and benefits to survivors with respect to the earnings of such employees on the other, the gross residual benefit applicable to an individual who entered railroad service at age k, t years prior to the valuation date, and who withdrew from the industry at age x is determined from the relation

$$R^{t}_{x} = .04 S^{1937\text{-}46}_{k+t} + .07 \left(S^{1947}_{k+t} + \sum_{k+t}^{x-1} S_r \cdot f_r \right) \tag{29}$$

where R^{t}_{x} equals the gross residual amount,

$S^{1937\text{-}46}$ equals creditable railroad compensation from 1937 through 1946 (subject only to the maximum of $300 per month on such compensation),

S_{k+t}^{1947} represents the corresponding railroad compensation in 1947, and

S_r and f_r have the same meanings as assigned in formula 11.

B. INSURANCE VALUES

As for the ordinary survivor insurance benefits, it is first convenient to determine insurance values according to the type of survivor benefits payable and the manner of exit from the railroad industry. As presented in the formulas below, such insurance values have been obtained as of the point of separation and relate to the value of the net residual payments.

Five different types of situations are considered in connection with employee deaths in active service: Those involving (1) widows with eligible children (*wc*), (2) widows under 65 without children (*yw*), (3) widows 65 and over (*aw*), (4) children only (*co*), and (5) cases in which the employee dies single (*s*). The corresponding insurance values for deaths in active service (*d*) are shown in formulas 30–34.

$$
{}^{d}_{wc}\mathrm{A}^{t(R)}_{x+1/2} = {}_{wc}\phi_{x+1/2}
$$

$$
\times \left\{ \begin{array}{l} [R - {}^{s}B(6n \cdot \overline{18-u} + 9 \cdot \overline{18-z})]\,[\bar{M}^{r}_{[y]+18-z} - \frac{1}{2}(\bar{M}^{r}_{65} + \bar{M}^{r}_{65+h})] \div \bar{D}^{r}_{[y]} \\ + [R - {}^{s}B(6n \cdot \overline{18-u} + \tfrac{9}{2} \cdot \overline{18-z})]\,[\, v^{18-z} \cdot (l^{r}_{[y]} - l^{r}_{[y]+18-z})] \div l^{r}_{[y]} \end{array} \right\} \tag{30}
$$

$$
{}^{d}_{yw}\mathrm{A}^{t(R)}_{x+1/2} = {}_{yw}\phi_{x+1/2}\{[R - 8 \cdot {}^{s}B]\,[\bar{M}^{r}_{[y]} - \tfrac{1}{2}(\bar{M}^{r}_{65} + \bar{M}^{r}_{65+h})] \div \mathrm{D}^{r}_{[y]}\} \tag{31}
$$

$$
{}^{d}_{aw}\mathrm{A}^{t(R)}_{x+1/2} = {}_{aw}\phi_{x+1/2}\,[\tfrac{1}{2}R\,(\bar{M}^{r}_{[y]} - \bar{M}^{r}_{[y]+h}) \div \mathrm{D}^{r}_{[y]} \tag{32}
$$

$$
{}^{d}_{co}\mathrm{A}^{t(R)}_{x+1/2} = {}_{co}\phi_{x+1/2} \cdot v^{18-z}\,[R - {}^{s}B\,(6n \cdot \overline{18-u})] \tag{33}
$$

$$
{}^{d}_{s}\mathrm{A}^{t(R)}_{x+1/2} = {}_{s}\phi_{x+1/2}\,(R - 8 \cdot {}^{s}B)\,. \tag{34}
$$

As used in the above formulas, ${}_{co}\phi_{x+1/2}$ refers to the probability that the deceased employee leaves children only at the time of his death; ${}_{s}\phi_{x+1/2}$ is the probability that he will die single (leaving no wife or eligible child); R and ${}^{s}B$ denote the gross residual sum and the basic amount respectively applicable to the age of the employee at death $x+\frac{1}{2}$ who was age $k+t$ on the valuation date; y refers to the widow's average age in accordance with the type of applicable survivor insurance benefit payable at the time the employee died.

In connection with formula 30, two situations are considered. In the first, provision is made for the payment of the residual benefit if the widow is still alive when the youngest child reaches age 18. At that point, the gross residual benefit R has been reduced by the current insurance

benefits paid out to the widow and children. Thereafter, the second part of the numerator provides for the payment of the then remaining net residual amount if she should die or remarry before 65, with a further allowance of one-half of such sum for a period of h years, if the widow survives to age 65 and has not remarried in the interim.

In the formula under discussion, the period h (in years) is equal to the net residual amount remaining after the current insurance benefits will have been completed, divided by 9 times the basic amount from which the widow's insurance benefit is computed. The allowance of one-half of this net residual amount for the period of h years after 65 is an approximation to the steadily decreasing residual amounts available during that period—at the end of which no further benefits are payable.

The second part of this formula deals with the situation in which the widow dies or remarries before the youngest child reaches age 18. The type of approximation made here is similar to that for the situation above where the widow survives unmarried to age 65. It is assumed in obtaining the net residual payment, in the present instance, that the widow will draw benefits for one-half the period from the time the employee dies to the time the youngest child reaches age 18. The net remaining residual sum in this instance is considered payable at the time such youngest child is no longer eligible for benefits.

In formula 31 the gross residual benefit is first reduced by the lump-sum insurance benefit, which becomes payable immediately at death. Thereafter, the treatment is similar to that used in the first part of formula 30. In this instance, of course, h is equal to $(R - 8^{e}B)/9^{e}B$.

In formula 32, recognition is taken of the fact that the widow's monthly insurance benefit becomes payable immediately at death. And during the period in which there is a remaining residual benefit, the same type of approximation is made that one-half of the sum available at the time the employee dies will be paid. For this type of family composition at the time the employee dies, $h = R/9^{e}B$.

In formula 33, where the employee leaves children only, the gross residual benefit would be reduced by the survivor benefits payable to the children until the youngest child reaches 18, and becomes available at that time. Finally, in formula 34, the residual benefit is payable immediately. It equals the gross residual amount minus the insurance lump sum.

For withdrawals before retirement, the combined insurance value becomes

$$\Sigma\, {}^{c}\mathrm{A}^{t(R)}_{x+1/2} = v^{m-x}\, {}_{66-x-1/2}q_{x+1/2}\left({}^{w}_{wc}\mathrm{A}^{t(R)}_{m+1/2} + {}^{w}_{yw}\mathrm{A}^{t(R)}_{m+1/2} + {}^{w}_{aw}\mathrm{A}^{t(R)}_{m+1/2} + {}^{w}_{cw}\mathrm{A}^{t(R)}_{m+1/2} + {}^{w}_{s}\mathrm{A}^{t(R)}_{m+1/2}\right) \tag{35}$$

where $m + \frac{1}{2}$ is the average age of death between the point of withdrawal from railroad service and age 66 (at which time such withdrawals are assumed to retire on a railroad annuity or a primary insurance benefit under the Social Security Act); ${}_{66-x-1/2}q_{x+1/2}$ represents the probability of death during that period. The insurance functions on the right-hand side are derived in the same fashion as in formulas 30–34, except that the basic amount is replaced at all times by the social security primary benefit, and that the insurance lump sum of 8 times the basic amount is replaced by 6 times the primary benefit.

One further point should be made in connection with this formula. Theoretically, a portion would still have to be calculated for the possible residual benefit payments in the event of death after future retirement age 66. The effect on costs would be so negligible as not to warrant the additional detail involved. Similarly, the use of an average age at death from withdrawal to age 66 in the formula itself is highly questionable in a theoretical sense. However, here again the use of a more accurate methodology would hardly be warranted in terms of cost.

Formulas 36 and 37 present the insurance values of deaths with respect to nondisability retirements. Only two types of family composition are considered of particular consequence for cost purposes: (1) those cases in which the nondisability annuitant leaves a widow without children under age 65; and (2) those in which he dies single. The other types of family composition are purposely disregarded since immediate monthly insurance benefits would be payable in such instances. Since the employee retirement annuity as well as the monthly insurance survivor benefits are subtractive items from the gross residual benefits, the remaining net residual amount, if any, would generally be very small. Similar considerations warrant the decision to ignore the residual payments once a widow's insurance benefit becomes payable. The two formulas follow:

$$ {}^{wa}_{yw}A^{t(R)}_{x+1/2} = v^{h/2} \cdot {}_hq_{x+1/2} \left[{}_{yw}\phi_{x+1/2+h/2} \left(\tfrac{1}{2}R - 8 \cdot {}^eB\right) \left(\bar{M}^r_{[y]} - \bar{M}^r_{65}\right) \div D^r_{[y]} \right] \quad (36) $$

$$ {}^{wa}_{e}A^{t(R)}_{x+1/2} = v^{h/2} \cdot {}_hq_{x+1/2} \cdot {}_e\phi_{x+1/2+h/2} \left(\tfrac{1}{2}R - 8 \cdot {}^eB\right). \quad (37) $$

In each instance, the determination of the period h is now related to the employee's retirement benefit rather than to the basic amount; it takes the form $h = R \div 12^eB$. It should be noted further that the probabilities refer to an age at death halfway between the age of retirement $x + \frac{1}{2}$ and the age h years later, after which the residual benefit would disappear if the employee were still alive. Here again, the formulas involve rather rough approximations, but are warranted in the sense that more precise formulas could produce, at best, only a negligible change in costs.

The same types of formulas were utilized in connection with disability retirements, except that in the determination of the probability of death during the h year period, the appropriate mortality rates for disabled lives were utilized.

C. PRESENT VALUE OF BENEFITS

The corresponding *per capita* values according to the mode of exit from railroad service for an employee aged $k + t$ on the valuation date are presented in formulas 38–41. It should be noted that the limits of the summations for each of the withdrawal categories are the same as those shown for retirement and survivor insurance benefits.

Deaths in active service

$$\frac{\sum_{k+t}^{\omega} v^{x+1/2} \cdot d_x \left(\Sigma^d A^{t(R)}_{x+1/2}\right)}{D^s_{k+t}}. \tag{38}$$

Withdrawals before retirement

$$\frac{\sum_{k+t}^{a-1} v^{x+1/2} \cdot w_x \left(\Sigma^w A^{t(R)}_{x+1/2}\right)}{D^s_{k+t}}. \tag{39}$$

Nondisability retirements

$$\frac{\sum_{a}^{\omega} v^{x+1/2} \cdot w_x \left({}^{wa}_{yw}A^{t(R)}_{x+1/2} + {}^{wa}_{s}A^{t(R)}_{x+1/2}\right)}{D^s_{k+t}}. \tag{40}$$

Disability retirements

$$\frac{\sum_{\beta}^{64} v^{x+1/2} \cdot i_x \left({}^{ia}_{yw}A^{t(R)}_{x+1/2} + {}^{ia}_{s}A^{t(R)}_{x+1/2}\right)}{D^s_{k+t}}. \tag{41}$$

IV. Value of 1 Percent of Payroll

In order to determine the value of the various types of benefits as a percent of payroll, it is necessary to obtain the present values of 1 percent of future earnings. The formula used in this connection is

$$\frac{.01 \sum_{k+t}^{\omega} f_x \cdot S_x \cdot D^s_{x+1/2}}{D^s_{k+t}}. \tag{42}$$

V. Summary Calculations

After all *per capita* present values have been determined, the total liability with respect to all present employees and the normal cost for new entrants are computed by means of formulas 43 and 44 shown below.

Value of future benefits to present employees

$$\sum_k \sum_t (PB)_{k+t} \cdot n_{k+t} \tag{43}$$

where k denotes the central age at entry; t denotes the duration on the valuation date; n_{k+t} is the number of present employees in the age at entry k and duration-group t; and $(PB)_{k+t}$ is the consolidated *per capita* value of all benefits with respect to the particular group of present employees.

Normal cost for new entrants

$$\frac{\sum_k n_k \cdot (PB)_k}{\sum_k n_k \cdot (PC)} \tag{44}$$

where k denotes the central age at entry; n_k is the number of new entrants at age k, $(PB)_k$ is the consolidated individual single premium per new entrant coming in at age k, and $(PC)_k$ is the present value of 1 percent of future compensation per new entrant.

VI. General

The actuarial formulas which have been developed relate primarily to the calculation of benefits for the employees "in active service" on a particular valuation date (December 31, 1947). An individual is considered to be in active service as of the end of a year if he worked in that year and was alive and not retired at the end of the year. This definition is convenient in that individuals who leave the industry before the end of a year (other than deaths or retirements) cannot be readily identified until full information is available with respect to compensation and service for the succeeding calendar year. The actual processing of the relevant statistics occurs too late in time to be utilized for valuation purposes. To correspond with the definition of employees in active service, the inactives are considered to include those employees alive and not retired as of the valuation date who last worked one or more calendar years prior to the valuation year.

In view of the heavy first year turnover, the new entrants among the active employees are considered separately. Thereafter, the years of entry

are grouped. For the active employee population in the valuation year 1947, the year of entry groupings considered are 1947, 1942–46, 1937–41, 1932–36, etc. These groupings have been classified as central durations 0, 3, 8, 13, etc. This breakdown has the added convenience that it permits a clean separation of the active employees with subsequent service alone from those with prior and subsequent service in combination. Thereafter, the active service census is further subdivided by central age at entry (18, 23, 28, etc.). The central ages at entry have been noted by k in the formulas developed in this paper; t has been used to represent the durations 0, 3, 8, etc.

While explicit attention has previously been given to the methods for calculating benefits for present active employees, the modifications of the actuarial formulas applicable to new entrants after the valuation date are apparent.

GENERAL CONSIDERATIONS RELATED TO FINAL CALCULATIONS

In valuations of plans such as ours in which the final results must strike a balance between a realistic and conservative approach, it is necessary to make a series of general assumptions and to introduce certain broad adjustments to the figures which emerge from the basic calculations themselves. Some of the major assumptions and adjustments are considered below.

Active Employees

In connection with retirement annuities, account must be taken of the minimum annuity provisions. A direct allowance was made for the minimum in the worksheets themselves which developed the amounts of the monthly retirement benefit. The use of group averages for the monthly compensation, however, does not make a sufficient allowance for the cost of these benefits. By the same token, the use of such averages for calculating the benefits under the regular annuity formula overstates costs in so far as the ratio of the annuity factor per year of service to the monthly compensation decreases as the amount of monthly compensation increases. It is felt that the necessary adjustments on both grounds would tend to offset each other.

Another factor which must be considered is the effect of the work clause provisions of the Act in reducing costs. The monthly benefits of annuitants who retire under the age provisions of the Act are suspended for any month in which they return to railroad employment or go back to work for their last employer—if such last employer was not covered under the Act. On the other hand, monthly annuity benefits continue to

be payable under all other conditions of outside work. It is clear that a very wide area exists for work after retirement which is not subject to the penalties of the work clause. In actual operation, the reported returns to service have been so few as not to warrant any reductive adjustment in cost for the nondisability annuitants.

When disability annuities are considered, the Act provides that "an employee in receipt of such annuity, who earns more than $75 in service for hire, or in self-employment, in each of any six consecutive calendar months, shall be deemed to cease to be so disabled in the last of such six months." Furthermore, satisfactory proof must be made from time to time, as prescribed by the Board, of the continuance of a disability (according to the standards applied in the establishment of such disability) until the employee attains the age of 65. Thus, even though a disability annuitant does not have his annuity suspended for a month in which he works outside of railroad coverage, a legitimate reduction appears to be warranted to take into account possible recoveries from disability. In view of the foregoing, it has been considered proper to reduce the gross calculated cost of disability annuities payable before the attainment of age 65 by 10 percent.

The formulas developed for survivor insurance benefits reflect the conservative assumption that employees who have a current connection with the railroad industry will die completely insured. Further, no provision is made therein for nonfiling or on account of the various deductions from survivor insurance annuities because of the survivor-benefit work clauses. Nor do such formulas provide for the lag between the earliest date a particular type of monthly survivor benefit could theoretically be paid and the date the annuity actually begins to accrue. The Act itself provides, in addition, that benefits are to be reduced by whatever other railroad retirement or social security benefit the survivor is receiving in his own right. In view of the foregoing considerations, the following broad reductive adjustments were made in the basic calculations: 10 percent for the widows' insurance benefits; 25 percent in benefits to widowed mothers; $12\frac{1}{2}$ percent for children; and 15 percent in the lump-sum insurance benefits.

Inactive Employees

With respect to the various categories of inactive employees, the already vested retirement annuity benefits were calculated on the basis of deferment to age 66, taking into account the actual prior and subsequent service and compensation. The mortality table used for this purpose was Table 1, with a 1-year rate-back in age.

Theoretically, some allowance should be made with respect to survivor benefits arising from the earnings of employees in this group. However, in accordance with the methodology discussed for active employees, concern would only have to be with those inactive employees who retained a current connection with the railroad industry at death. When it is remembered that according to definition all inactive employees as of a valuation date are at least a year removed from the date of last employment, it is felt that the calculation of survivor benefits with respect to inactive employees would hardly be worth the effort.

One further point should be mentioned in passing in connection with re-entrants among the inactive population after the valuation date. Survivor benefits and additional retirement credits are taken into account implicitly in the costs for the new entrant group as a whole in final cost calculations.

Benefits to Retired Employees

The problems involved in valuing liabilities with respect to individuals already on the rolls offer relatively little difficulty. The numbers involved and the amounts of benefits are derived from in-force tabulations modified to an accrued basis as of the valuation date. For age annuities and pensions appropriate annuity values can be applied by attained age. With respect to individuals who have retired on a disability annuity, tabulations have been necessary by age at accrual and duration since retirement. The present worth of the resulting liabilities is then established by applying the appropriate select annuity values according to the time disability retirement took place.

Residual Payments

The formulas relating to the residual benefit indicate the complexity of the calculations required for valuing this type of benefit. As a practical expedient, we have found it useful to make the necessary calculations on the basis of one central age at entry. For this purpose entry age 28, which appears typical in many respects, was chosen. Then a moderate reduction was made to take into account the fact that while the average age of new entrants is about 28 the entry-age distribution of present active employees is weighted more heavily toward the younger ages. A further moderate reduction in the calculated gross liabilities is warranted in that a certain amount of nonfiling can be expected for this type of benefit.

New Entrants

Inherent in the cost calculations as developed in the final stage is the assumption that although the numbers of individuals entering as new employees after a valuation date will vary from year to year, their age

distribution will remain relatively fixed. The 1946–47 experience has served as the basis for the typical age distribution of new entrants used in latest calculations. As indicated previously, age 28 is the average age for the distribution. The assumption of a fixed age distribution is convenient in that the retirement costs developed for new entrants, when expressed as a percent of payroll, remain unchanged regardless of the year of entry after a valuation date or of the number of employees involved.

The situation is somewhat different in connection with survivor insurance and residual benefits. The introduction of an increasing scale of prior social security wages, by calendar year before 1948, produces differing normal costs for such survivor benefits according to the year of entry after the valuation date. In the latest valuation, calculations were made for entries in 1948, 1953, 1958, 1963, and 1968. It was found that the normal cost leveled off in 1968; furthermore, the changes involved were quite small. In the interest of expediency, only one rate was used in the final calculations—the normal rate for survivor insurance and residual benefits developed for the year 1958.

A final point should be mentioned relative to adjustments made for benefits on the retirement level. As a practical matter, we know that there will be considerably less than 100 percent filing at the time retirement age is reached for individuals who have negligible earnings and service credits. Not only would the benefits available be extremely small, but also they would be deferred for many years into the future. The realistic approach has therefore been taken to assume that there would be only 25 percent filing in cases of individuals leaving service in the same calendar year as that in which they entered; 50 percent would file among the withdrawals in the second calendar year; 75 percent in the third; 100 percent filing has been assumed thereafter. The over-all effect of this modification is to reduce the retirement cost for new entrants to 97½ percent of what it would have been had complete filing been assumed in the calculations. The same type of reduction has been made for those active employees who entered for the first time in the valuation year, as well as for employees in the inactive census with minor amounts of service credits.

FINANCING THE BENEFITS

The benefits provided under the railroad retirement system are financed by carrier and employee taxes which are collected by the United States Treasury. Such monies, less appropriations for yearly administrative expenses, are ultimately transferred to the Railroad Retirement Account. The funds of such account are utilized to pay benefits with respect

to certifications to the Treasury; that portion which is not needed is invested in special bonds, bearing interest at 3 percent.

At present, employees and employers pay 6 percent apiece on compensation up to $300 a month; these rates will advance to 6¼ percent apiece in 1952 and subsequent years. This practice of splitting the costs down the middle between employee and employer has been followed ever since 1937. Further, no allocation of funds has ever been made for the amortization of the prior service or accrued liabilities as compared with those arising currently.

It appears to have been the intent of Congress to finance the benefits of the railroad retirement system on the basis of an effective tax rate which together with interest on the invested funds of the Account would be sufficient throughout the years to meet all benefit payments as they become due. In accordance with the method of financing which has been provided by Congress, the respective valuations of the Railroad Retirement Act have always been prepared with the prime consideration in mind of determining the required level tax rate which would be adequate into perpetuity. Subject to the reasonableness of the assumptions, although such a theoretical level tax rate would not produce reserves sufficient to cover accrued liabilities as of a particular date, it would nevertheless be adequate to meet all benefit payments throughout the years on an open-end basis.

The method of financing the railroad retirement system is not peculiar unto itself. A similar type of financing was originally provided for under the Social Security Act. Also, the taxing provisions of the amendment to that Act (H.R. 6000), passed by the House in the last session of Congress, are essentially of the same nature. There is also reason to believe that original plans to amortize the accrued liabilities of the United States Civil Service Retirement system by the year 1998 are gradually being superseded in favor of a system of financing similar in principle to that adopted for the railroad retirement system. The three plans have the common feature that they are handled by the Federal government, so that in the last analysis the Federal government is a guarantor that benefits will be paid; its taxing power assures adequate financing. A word of caution is in order, however. While this method for meeting benefit liabilities may be considered proper for these giant plans, serious dangers would be involved in its applicability to a private pension plan for a single company in which the assumption of permanency of existence is highly untenable.

The required level tax rate discussed above cannot be determined without recourse to assumptions as to the course of future payrolls. Such future

payrolls, as they relate to the railroad industry, are of major importance in view of the large net liability incurred as a result of (1) the crediting of service rendered prior to 1937 by individuals who were employees or were in an employment relation on August 29, 1935, (2) the benefit liberalizations introduced by the 1946 and 1948 amendments, and (3) previous inadequacies of the tax rates. When expressed as a percentage of payroll, the relative cost of servicing this liability together with the additional subsequent service obligations (over that for new entrants), due to the advanced age of employees as of the valuation date, varies inversely with the size of the future payrolls assumed.

An idea of how radically ideas have changed concerning future payrolls in the industry can be gathered from the fact that an equivalent future level payroll of $2 billion was assumed for the first valuation as of December 1938; the second valuation, prepared three years later, hiked this figure to $2.5 billion; while the third valuation predicated a future level payroll of $3.5 billion. Since the preparation of the third valuation, the far-reaching changes in the nation's economy and their corollary influence on railroad payrolls have already made the $3.5 billion payroll assumption inapplicable. On the basis of revised estimates prepared by the Board's economic staff, the equivalent level future payroll assumed for the fourth valuation was $4.6 billion. As a point of reference, the actual covered payroll for 1949 was closer to $5 billion.

Once the equivalent future level payroll is established, it is a relatively simple matter to obtain the net level cost of the system of benefits provided by the Railroad Retirement Act. This latter figure can be obtained either through a normal rate-accrued liability approach or else on the basis of the value of all future liabilities under the system after the valuation date. The second approach is the more direct and is exemplified in the summary of the level cost calculations of the fourth valuation (shown in Table 11).

A final note is in order. In interpreting such figure, it should be recognized, of course, that it is virtually impossible to develop a precise cost figure with respect to a system in which there has been such a great variability in the basic factors. At best, any single figure which finally emerges can only be looked upon as a "most reasonable" one within the range of the true costs of the system.

TABLE 11

SUMMARY OF LEVEL COST CALCULATIONS
(Dollar Figures in Millions)

Item	Retirement Benefits	Other Benefits	Total
a) Present value of benefits with respect to employees who entered before 1948	$10,463.8	$2,745.6	$13,209.4
b) Funds on hand, accrual basis			$ 1,159.5
c) Present value of 1 percent of total future payrolls			$ 1,556.3
d) Present value of 1 percent of future payrolls for present employees			$ 592.9
e) Present value of 1 percent of payrolls for future entrants (*c*–*d*)			$ 963.4
f) Normal tax rate for future entrants	5.788%	2.084%	7.872%
g) Total present value of future benefits ($a+e \cdot f$)	$16,040.0	$4,753.3	$20,793.3
h) Gross level cost of benefits as a percent of payroll ($g \div c$)	10.306%	3.954%	13.361%*
i) Reduction in level cost on account of reserve, ($b \div c$)			.745%
j) Cost of administration as a percent of payroll			.100%
k) Net level cost as of December 31, 1947 ($h-i+j$)			12.72%

* Discrepancy of .001 between the total and the sum of its components is due to rounding.

APPENDIX A

Outline of 1947 Valuation Card

Item Number	Card Columns	Item	Codes
1	1–2	Identification	7V
2	3–4	Year of birth	Last 2 digits of year of birth. Code 39 is used for unknown years of birth
3	5–13	Social security account number	
4	14	Sex	0—male 3—female "X" punch indicates active service death or retirement
5	15	Source code	1—final prior service adjudication 2—preliminary adjudication, Form AA-15 present, Forms AA-2P incomplete 3—preliminary adjudication, one or more AA-2P's present, AA-15 not present 4—preliminary adjudication, no AA-2P's or AA-15 present 5—preliminary adjudication, all AA-2P's and AA-15 present
6	16	Employment relation status on 8-29-35	1—employment relation on the basis of 6 or more months of service 2—employment relation other—final basis 3—employment relation other—preliminary basis 4—employment relation denied—final basis 5—employment relation denied—preliminary basis "X" punch indicates a former employment relation denial
7	17–19	Months of prior service (unadjusted)	
8	20–22	Months of prior service (adjusted)	
9	23–25	Average prior service monthly compensation, 1924–31	
10	26–27	Year of original entry	(Code 48 is used for unknown year of entry prior to 1937)
11	28–30	1937–47 service months	
12	31–35	1937–47 compensation	
13	36–37	1947 service months	

APPENDIX A—*Continued*

Item Number	Card Columns	Item	Codes
14	38–41	1947 compensation	
15	42–43	1946 service months	"X" punch in col. 42 indicates service performed but amount of service and compensation not shown
16	44–47	1946 compensation	
17	48–49	1945 service months	"X" punch in col. 48 indicates service performed but amount of service and compensation not shown
18	50–53	1945 compensation	
19	54–55	1944 service months	"X" punch in col. 54 indicates service performed but amount of service and compensation not shown
20	56–59	1944 compensation	
21	60–62	Occupation for year last worked after 1936	ICC and REA occupational codes
22	63–65	Total service months (total of adjusted prior and subsequent service, items 8 and 11)	
23	66–67	Quarters of coverage	
24	68	Insurance status code	1—completely, but not permanently, insured 1-X—permanently insured 2—partially insured 3—uninsured
25	69–70	Year of entry after 1936	
26	71–72	Year last worked after 1936	"X" punch in col. 72 indicates continuous service since entry. In the case of accounts with both prior and subsequent service, classification of continuous service is made if the employee worked continuously from 1937
27	73–74	Years of service after 1936	Years in which service was performed (not necessarily completed years of service)
28	75–78 75 76 77 78	1937–47 year check code 1937–39 service pattern 1940–42 service pattern 1943–45 service pattern 1946–48 service pattern	Years worked in each 3-year period 0 or blank—no service in period 1—service in first year only 2—service in second year only 3—service in first and second years 4—service in third year only 5—service in first and third years 6—service in second and third years 7—service in all three years

APPENDIX A—*Continued*

Item Number	Card Columns	Item	Codes
29	79	Service and employee status code	0—working in 1947, alive and not retired, prior and subsequent service 1—last working in 1947, death or retirement, prior and subsequent service 2—last working in 1947, alive and not retired, subsequent service only 3—last working in 1947, death or retirement, subsequent service only 4—last working in 1944–46, alive and not retired, prior and subsequent service 5—last working in 1937–43, alive and not retired, prior and subsequent service 6—last working in 1944–46, alive and not retired, subsequent service only 7—last working in 1937–43, alive and not retired, subsequent service only 8—last working in 1944–46, death or retirement, prior and subsequent service 9—last working in 1944–46, death or retirement, subsequent service only X—prior service only, "alive and not retired"
30	80	Death and retirement code	Code "X" means death, "Y" means retirement. The second punch in the column refers to years as indicated Blank—37–41 6— 42 7— 43 8— 44 9— 45 0— 46 1— 47