TRANSACTIONS OF SOCIETY OF ACTUARIES
1950 VOL. 2 NO. 2

ACTUARIAL NOTE: THE VALUATION OF SELF-INSURED RETIREMENT PLANS

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INTRODUCTION

In recent years there has been a rapid spread of formal retirement plans among employers in the United States. The three most popular avenues for funding these employee retirement plans are the group annuity method, the individual contract method, and the self-insured method. Other methods, such as the group permanent method, are in use.

Under the group annuity method, increments of deferred annuity are purchased each month with respect to each participating employee. Each such increment is related to the employee's earnings in that month and if the employee continues in service to the date of his retirement, his retirement income is the aggregate of these increments. Under this method, all the terms and conditions of the plan, including the benefit formula and the premium rates, are set forth in detail in a contract between the Employer and the Insurance Company which is underwriting the plan.

Under the individual contract method, the benefit formula is set forth in a Trust Indenture. The Employer, the Trustee, and, sometimes, the participating employees are parties to this Indenture. This Trust Agreement provides that the Employer shall make payments to the Trustee and that the Trustee shall purchase policies of individual insurance on the life of each participating employee, and shall keep such policies in force. The funding is usually carried out by the purchase of annual premium retirement income policies, the amount of such policies being sufficient to provide each employee with the appropriate retirement benefit on the normal retirement date. The Insurance Company is not a party to the Trust Indenture, it merely furnishes its policies to fund the plan. The policies may have special ownership provisions; however, they are usually the same as those normally issued to individual risks.

Employee Retirement Plans which are funded using the facilities of an Insurance Company are often called "Insured Plans." The current cost of an insured plan depends on actual salaries (past and present), on current turnover, and on stated premium rates. The Employer's liability is not dependent upon future salary changes.

Under a self-insured plan, the Employer assumes the function of insurer. He sets up a trust and proceeds to define the terms of the plan. In general, the Employer has greater freedom of choice. This is reflected in the wide variation found among self-insured plans. The current cost and valuation liability of self-insured plans very often depend upon estimated future salaries as well as estimated future turnover.

The purpose of this paper is to sketch the mathematical functions which are needed in valuing the benefits of a self-insured plan, and to study certain problems that arise in connection with the valuation of these plans. Because of the variety of benefits, the functions derived herein may be looked upon as typical. In each problem the functions should be derived from first principles.

MATHEMATICAL FUNCTIONS

A. *The Basic Factors*

The basic functions required in handling self-insured plans are (1) an interest rate, (2) a service table, (3) capitalized value of decremental benefits, and (4) salary increase index numbers. In practice, expense rates may be needed. However, the investment expenses may be deducted in arriving at the net earned interest rate and the administrative expenses of the plan may be handled as a part of the Employer's overhead. The latter assumption has been made herein.

1. The interest rate required is that net rate which it is reasonably anticipated that the "trust *res*" will earn.

2. The form of the service table depends upon the benefits provided in the plan. The most general form of service table may be expressed as:

$$l_{[x]+t} = l_{[x]+t+1} + d_{[x]+t} + w_{[x]+t} + h_{[x]+t} + r_{[x]+t}$$

where $l_{[x]+t}$ is the number of employees in service at age $[x] + t$ out of a group of $l_{[x]}$ employees who entered service at age $[x]$.

$(d_{[x]+t} + w_{[x]+t} + h_{[x]+t} + r_{[x]+t})$ is the total number of employees who leave service between $[x] + t$ and $[x] + t + 1$ out of the $l_{[x]}$ who entered service at age $[x]$.

$d_{[x]+t}$ is the number dying while in service.

$w_{[x]+t}$ is the number voluntarily leaving service. The turnover rates depend largely upon the individual group and the plan's eligibility requirements.

$h_{[x]+t}$ is the number leaving service on account of disability.

$r_{[x]+t}$ is the number retiring from service. If the plan provides that retirement must be at $[x] + t$ in order to get full benefit, then $r_{[x]+t} = l_{[x]+t}$. If, on the other hand, retirement may occur over a period of ages at full accrual to date of retirement, then $r_{[x]+t}$ has values for several t and retirement is assumed to occur in the middle of the year of life, except perhaps at the compulsory age.

3. The service table is a multiple decrement table and the various decrements are distinguished because the benefits provided by the plan depend upon the mode of termination. Typical annuity functions required for the capitalized value of the decremental benefits include:

(a) Upon death—annuity certain ($\bar{a}_{\overline{n}|}$) or life annuity ($\bar{a}_y$) to widow

(b) Upon retirement or ill-health withdrawal
—life annuity ($\bar{a}_{x+1/2}$) to employee
—life annuity with n years certain ($\bar{a}_{\overline{n}|} + {}_{n|}\bar{a}_{x+1/2}$) to employee

(c) Upon voluntary withdrawal—a deferred life annuity (${}_{y-(x+1/2)|}\bar{a}_{x+1/2}$) to commence at normal retirement age y.

4. *Salary Increase Index Numbers:*

(a) A self-insured plan may be either an "average salary plan" or a "final salary plan." Under an "average salary plan," the retirement benefits are described as a percentage of average annual salary for each year of service. For example, under a 2% average salary plan the annual rate of the employee's retirement annuity is 2% of his salary in the first year of service, plus 2% of his salary in his second year of service, plus . . . Under a "final salary plan" the retirement benefits are described as a percentage of final annual salary (or final average salary) for each year of service. In either case, the contributions to the Trustee are expressed as a percentage of current payroll.

From these definitions it follows that a change, in the future, in the salary scale assumption will affect the financial status of a final salary plan more sharply than it will affect the status of an average salary plan. Under an average salary plan, accruals on account of past service are definitely ascertainable and only future accruals depend upon estimated salaries. Under a final salary plan, all benefits payable in the future to persons now in service must be estimated using salary scale functions.

The following symbols apply to a member who entered the plan at age $[x]$ and who is now $[x] + t$:

$(AS)_{[x]+t}$ is the actual annual rate of salary during the year of life $[x] + t$ to $[x] + t + 1$.

$(TPS)_{[x]+t}$ is the total salary credited to a member between $[x]$ and $[x] + t$, that is, total past salary.

$(ES)_{[x]+t+n}$ is the estimated salary during the year of life $[x] + t + n$ and $[x] + t + n + 1$.

$S_{[x]+t}$ is a salary scale function such that

$$(ES)_{[x]+t+n} = (AS)_{[x]+t} \cdot \frac{S_{[x]+t+n}}{S_{[x]+t}}.$$

The salary scale numbers will depend upon the characteristics of the participating employees. A flat salary scale suitable for an industrial group might run from 1.00 at age 25 to 1.50 at age 65; while a steep scale might run from 1.00 to 6.00.

(b) If, in the final salary plan, the age at retirement is $[x] + r$, then the retirement benefit is based upon the estimated final salary which may be expressed as

$$(\text{AS})_{[x]+t} \cdot \frac{S_{[x]+r}}{S_{[x]+t}}.$$

Sometimes to minimize the effects of salary changes immediately before retirement, the final average salary is used. Suppose that the benefit is based upon the average salary over the last five year period and that retirement is assumed to take place in the middle of the rth year of service. Let

$$Z_{[x]+r} = \tfrac{1}{10}\{S_{[x]+r} + 2\,(S_{[x]+r-1} + S_{[x]+r-2} + S_{[x]+r-3} + S_{[x]+r-4}) + S_{[x]+r-5}\},$$

then final average salary =

$$(\text{AS})_{[x]+t} \cdot \left(\frac{Z_{[x]+r}}{S_{[x]+t}}\right).$$

(c) Upon occasion the function ${}_{[x]}Z_{[x]+t}$ is used. For example, if the benefit upon disability is an annuity whose annual rate is 30% of the average annual salary to date of disability, then such annual rate may be expressed as

$$.30\,(\text{AS})_{[x]} \cdot \frac{{}_{[x]}Z_{[x]+t}}{S_{[x]}},$$

where $t + 1$ is the year in which disablement occurs and

$${}_{[x]}Z_{[x]+t} = \frac{1}{t}\,(S_{[x]} + S_{[x]+1} + \ldots + S_{[x]+t-1})$$

if salary in completed years only is considered.

If total salary is considered, then the numerator becomes:

$${}_{[x]}Z_{[x]+t+1/2} = \frac{1}{t+\frac{1}{2}}\,(S_{[x]} + S_{[x]+1} + \ldots + S_{[x]+t-1} + \tfrac{1}{2}S_{[x]+t}) .$$

Further comment is made below as to the use of this function.

B. *Contribution Functions*

1. The value as of age $[x] + t$ of a contribution of a unit to be made during the year $[x] + t + n$ to $[x] + t + n + 1$ by a member is:

$$\int_{n}^{n+1} v^{z} \cdot \frac{l_{[x]+t+z}}{l_{[x]+t}} \cdot dz$$

or

$$\frac{\bar{D}_{[x]+t+n}}{D_{[x]+t}}.$$

2. If the contribution rate is c, and $(AS)_{[x]+t}$ is the present rate of salary, then the present value of the contributions to be made between $[x]+t+n$ and $[x]+t+n+1$ is:

$$c\cdot(AS)_{[x]+t}\cdot\int_n^{n+1}\frac{S_{[x]+t+z}\cdot v^z\cdot l_{[x]+t+z}}{S_{[x]+t}\cdot l_{[x]+t}}\cdot dz$$

or

$$c\cdot(AS)_{[x]+t}\cdot\left\{\frac{{}^{S}\bar{D}_{[x]+t+n}}{{}^{S}D_{[x]+t}}\right\},$$

where

$${}^{S}\bar{D}_{[x]+t+n}=S_{[x]+t+n}\cdot\bar{D}_{[x]+t+n}$$

and

$${}^{S}D_{[x]+t}=S_{[x]+t}\cdot D_{[x]+t}\,.$$

3. If the contribution rate is c, and $(AS)_{[x]+t}$ is the present rate of salary, then the present value of all future contributions becomes

$$c\cdot(AS)_{[x]+t}\cdot\sum_{n=0}^{\infty}\frac{{}^{S}\bar{D}_{[x]+t+n}}{{}^{S}D_{[x]+t}}$$

or

$$c\cdot(AS)_{[x]+t}\cdot\left(\frac{{}^{S}\bar{N}_{[x]+t}}{{}^{S}D_{[x]+t}}\right).$$

4. It follows that the following functions may be built up using the $l_{[x]+t}$ columns of the service table:
(a) Commutation columns independent of salary

$$D_{[x]+t}\,;\quad N_{[x]+t}\,;\quad \bar{D}_{[x]+t}\,;\quad \bar{N}_{[x]+t}\,;\quad \bar{S}_{[x]+t}\,.$$

(b) Commutation columns involving the salary scale function

$${}^{S}D_{[x]+t}\,;\quad {}^{S}N_{[x]+t}\,;\quad {}^{S}\bar{D}_{[x]+t}\,;\quad {}^{S}\bar{N}_{[x]+t}\,;\quad {}^{S}\bar{S}_{[x]+t}\,.$$

C. *Simple Benefit Functions*

1. The value as of age $[x]+t$ of a unit payable in the event of retirement between $[x]+t+n$ and $[x]+t+n+1$ is

$$v^{n+1/2}\left(\frac{r_{[x]+t+n}}{l_{[x]+t}}\right)$$

or

$$\frac{C^{r}_{[x]+t+n}}{D_{[x]+t}},\qquad \text{where}\qquad C^{r}_{[x]+t+n}=v^{x+t+n+1/2}\,r_{[x]+t+n}\,.$$

2. The value as of age $[x] + t$ of a unit payable in event of withdrawal subsequent to $[x] + t$ is

$$\sum_{n=0}^{\infty} v^{n+1/2}\left(\frac{w_{[x]+t+n}}{l_{[x]+t}}\right)$$

or

$$\sum_{n=0}^{\infty} \frac{C^{w}_{[x]+t+n}}{D_{[x]+t}} = \frac{M^{w}_{[x]+t}}{D_{[x]+t}}.$$

3. In this manner, the following commutation functions may be defined:

for retirement: C^r; M^r; R^r
for disability: C^h; M^h; R^h
for withdrawal: C^w; M^w; R^w

4. The value as of age $[x] + t$ of a life annuity of one per annum payable in the event of retirement between $[x] + t + n$ and $[x] + t + n + 1$ is

$$\frac{v^{n+1/2} \cdot r_{[x]+t+n} \cdot \bar{a}_{[x]+t+n+1/2}}{l_{[x]+t}}$$

or

$$\frac{C^{ra}_{[x]+t+n}}{D_{[x]+t}}, \qquad \text{where} \qquad C^{ra}_{[x]+t+n} = C^{r}_{[x]+t+n} \cdot \bar{a}_{[x]+t+n+1/2}\,.$$

5. The value as of age $[x] + t$ of a life annuity of one per annum payable in the event of any subsequent disability is

$$\sum_{n=0}^{\infty} \frac{C^{ha}_{[x]+t+n}}{D_{[x]+t}}$$

or

$$\frac{M^{ha}_{[x]+t}}{D_{[x]+t}}.$$

6. In this manner, commutation columns comparable to those listed in 3 above may be derived.

D. *Salaried Benefit Functions—New Entrants*

1. *General Statement:*

The salaried benefit functions may be either "final salary functions" or "average salary functions." These terms are not to be confused with the terms introduced above, "final salary plan" and "average salary plan." The latter terms refer to the type of functions required to value the retirement benefits only. The point is that a plan which provides a final salary benefit on retirement may provide an average salary benefit upon withdrawal.

Salaried benefit functions fall into three separate categories:

(a) Final salary functions, such as those needed to value the following death benefit: 10% of final salary for each completed year of service, subject to a minimum of one times final salary and to a maximum of three times final salary.

(b) Average salary functions—without interest, such as those needed to value the following disability benefit: Life annuity in event of disability whose annual rate is 2% of average salary for each completed year of service, subject to a minimum benefit of 30% of average salary.

(c) Average salary functions—with interest, such as those needed to value the following withdrawal benefit: Lump sum benefit in event of withdrawal equal to the aggregate of the employee's contributions accumulated at 3%, where the employee contributed 5% of salary while in service.

2. *Final Salary Functions:*

(a) Flat percentage benefit

The value at entry $[x]$ of a retirement annuity, whose annual rate is 45% of final salary, is

$$.45\,(AS)_{[x]} \sum_{t=0}^{\infty} \frac{S_{[x]+t} \cdot v^{t+1/2} \cdot r_{[x]+t} \cdot \bar{a}_{[x]+t+1/2}}{S_{[x]} \cdot l_{[x]}},$$

where $r_{[x]+t}$ is the number retiring between $[x]+t$ and $[x]+t+1$. Using ${}^{S}C^{ra}_{[x]+t}$ and ${}^{S}M^{ra}_{[x]}$ we get

$$.45\,(AS)_{[x]} \cdot \left(\frac{{}^{S}M^{ra}_{[x]}}{{}^{S}D_{[x]}}\right),$$

where

$${}^{S}C^{ra}_{[x]+t} = S_{[x]+t} \cdot v^{x+t+1/2} \cdot r_{[x]+t} \cdot \bar{a}_{[x]+t+1/2}\,.$$

If the plan provided for retirement at and only at $[x]+N$, then the value would have been

$$.45\,(AS)_{[x]} \cdot \frac{S_{[x]+N} \cdot v^{N} \cdot r_{[x]+N} \cdot \bar{a}_{[x]+N}}{S_{[x]} \cdot l_{[x]}}$$

and it would be convenient to define

$${}^{S}M^{ra}_{[x]} = {}^{S}C^{ra}_{[x]+N} = S_{[x]+N} \cdot v^{x+N} \cdot r_{[x]+N} \cdot \bar{a}_{[x]+N}\,.$$

Again, the value at entry $[x]$ of a retirement annuity, whose annual rate is 45% of final *average* salary, is

$$.45\,(AS)_{[x]} \sum_{t=0}^{\infty} \frac{Z_{[x]+t} \cdot v^{t+1/2} \cdot r_{[x]+t} \cdot \bar{a}_{[x]+t+1/2}}{S_{[x]} \cdot l_{[x]}},$$

where $Z_{[x]+t}$ is the appropriate final average salary function.

This may be expressed as:

$$.45\,(AS)_{[x]}\cdot\frac{{}^{Z}M^{ra}_{[x]}}{{}^{S}D_{[x]}}.$$

(b) Percentage times service benefit

A plan provides for a retirement annuity of 2% of final salary for each completed year of service at any time after the completion of 20 years service and attainment of age 55. The value of this benefit as of $[x]$ is

$$.02\,(AS)_{[x]}\cdot\sum_{t=0}^{\infty}\frac{t\cdot S_{[x]+t}\cdot v^{t+1/2}\cdot r_{[x]+t}\cdot \bar{a}_{[x]+t+1/2}}{S_{[x]}\cdot l_{[x]}},$$

where retirements are assumed to be spread over the year and where

$$r_{[x]+t}=0\begin{cases}\text{for } t<20\\ \text{or}\\ \text{for } x+t<55\,.\end{cases}$$

This may be written

$$.02\,(AS)_{[x]}\cdot\frac{{}^{S}R^{ra}_{[x]+1}}{{}^{S}D_{[x]}}$$

or

$$.02\,(AS)_{[x]}\left\{\frac{20\,{}^{S}M^{ra}_{[x]+20}+{}^{S}R^{ra}_{[x]+21}}{{}^{S}D_{[x]}}\right\}.$$

(c) Percentage times service benefit, subject to maximum and minimum

A plan provides for a death benefit equal to 10% of final salary for each completed year of service, provided that in no event shall the benefit be less than one times final salary, nor greater than three times final salary.

Let

$$K_{[x]+t}=\begin{cases}1.00 \text{ for } t<10\\ .10t \text{ for } 10\leqq t\leqq 30\\ 3.00 \text{ for } t>30\end{cases}$$

Then the value as of $[x]$ of such death benefit is

$$(AS)_{[x]}\sum_{t=0}^{\infty}\frac{K_{[x]+t}\cdot S_{[x]+t}\cdot v^{t+1/2}\cdot d_{[x]+t}}{S_{[x]}\cdot l_{[x]}}$$

or

$$(AS)_{[x]}\cdot\frac{\sum_{t=0}^{\infty}{}^{KS}C^{d}_{[x]+t}}{{}^{S}D_{[x]}},\quad \text{where}\quad {}^{KS}C^{d}_{[x]+t}=K_{[x]+t}\cdot S_{[x]+t}\cdot C^{d}_{[x]+t}$$

or

$$(AS)_{[x]} \cdot \frac{{}^{KS}M^{d}_{[x]}}{{}^{S}D_{[x]}} .$$

3. *Average Salary Functions—without Interest:*

(a) Retirement Annuity

Where average salary functions are involved, the salary of each future year $(ES)_{[x]+t}$ is considered separately. Each such annual salary gives rise to benefits which may be entered upon in any subsequent year. The present value of all future service benefits is derived by means of a double summation.

First, the estimated salary during the $(t+1)$th year and retirements during the $(t+n+1)$th year are considered. Then a summation is made with respect to n. This produces the product

$${}^{S'}M = S_{[x]+t} \cdot M .$$

Then a summation is made with respect to t.

(i) The value at entry $[x]$ of the increment of retirement annuity accruing in year $[x]+t$ to $[x]+t+1$ under a 2% plan in the event the employee were to retire in year $[x]+t+n$ to $[x]+t+n+1$ is:

$$.02\,(AS)_{[x]} \cdot \left(\frac{S_{[x]+t}}{S_{[x]}}\right) \cdot \left\{\frac{v^{t+n+1/2} \cdot r_{[x]+t+n} \cdot \bar{a}_{[x]+t+n+1/2}}{l_{x]}}\right\}$$

or

$$.02\,(AS)_{[x]} \cdot \left(\frac{S_{[x]+t}}{{}^{S}D_{[x]}}\right) \cdot \left(C^{ra}_{[x]+t+n}\right) .$$

(ii) The value at entry $[x]$ of the increment of retirement annuity accruing in year $[x]+t$ to $[x]+t+1$ under a 2% plan in event of any retirement subsequent to $[x]+t+1$ is:

$$.02\,(AS)_{[x]} \cdot \frac{S_{[x]+t}}{{}^{S}D_{[x]}} \cdot M^{ra}_{[x]+t+1} = .02\,(AS)_{[x]} \left\{\frac{{}^{S'}M^{ra}_{[x]+t}}{{}^{S}D_{[x]}}\right\} ,$$

where the prime (S') distinguishes this M symbol from those defined as the summation of ${}^{S}C^{ra}_{[x]+t}$.

This is the formula under a plan that provides a 2% benefit for each *completed* year of service.

(iii) The value at entry $[x]$ of the increment of retirement annuity accruing in the year $[x]+t$ to $[x]+t+1$ under a 2% plan that provides benefit for each year of service, whether or not such year is complete, is

$$.02\,(AS)_{[x]} \cdot \left(\frac{S_{[x]+t}}{{}^{S}D_{[x]}}\right) \cdot \left\{\tfrac{1}{2} C^{ra}_{[x]+t} + M^{ra}_{[x]+t+1}\right\}$$

or

$$.02\,(\text{AS})_{[x]}\left\{\frac{{}^{S'}\overline{\text{M}}^{ra}_{[x]+t}}{{}^{S}\text{D}_{[x]}}\right\}.$$

(iv) Now for the second summation:

The value at entry $[x]$ of an annuity, entered into upon retirement, whose annual rate is 2% of the average salary for each year of service is:

$$.02\,(\text{AS})_{[x]}\cdot\sum_{t=0}^{\infty}\frac{{}^{S'}\overline{\text{M}}^{ra}_{[x]+t}}{{}^{S}\text{D}_{[x]}}$$

or

$$.02\,(\text{AS})_{[x]}\cdot\frac{{}^{S'}\overline{\text{R}}^{ra}_{[x]}}{{}^{S}\text{D}_{[x]}}.$$

It may be well to restate this as a double summation:

$$\text{Value at }[x] = .02\,(\text{AS})_{[x]}\cdot\sum_{t=0}^{\infty}\frac{S_{[x]+t}}{{}^{S}\text{D}_{[x]}}\left\{\tfrac{1}{2}\text{C}^{ra}_{[x]+t}+\sum_{n=1}^{\infty}\text{C}^{ra}_{[x]+t+n}\right\}.$$

(v) The value at entry $[x]$ of an annuity entered into upon retirement, whose annual rate is 2% of average salary for each *completed* year of service is:

$$.02\,(\text{AS})_{[x]}\sum_{t=0}^{\infty}\frac{S_{[x]+t}}{{}^{S}\text{D}_{[x]}}\left\{\sum_{n=1}^{\infty}\text{C}^{ra}_{[x]+t+n}\right\}$$

and is written

$$.02\,(\text{AS})_{[x]}\cdot\frac{{}^{S'}\text{R}^{ra}_{[x]}}{{}^{S}\text{D}_{[x]}}\qquad\text{without the bar.}$$

(b) Return of Contributions

An employee retirement plan provides that each participating employee shall contribute c times his salary and in the event of withdrawal from service the employee shall receive the return of his contributions without interest.

(i) The value at entry $[x]$ of the return of the contributions made in the year $[x]+t$ to $[x]+t+1$ in the event of withdrawal is:

$$c\cdot(\text{AS})_{[x]}\cdot\frac{S_{[x]+t}}{{}^{S}\text{D}_{[x]}}\left\{\tfrac{1}{2}\text{C}^{w}_{[x]+t}+\sum_{n=1}^{\infty}\text{C}^{w}_{[x]+t+n}\right\}$$

or

$$c\cdot(\text{AS})_{[x]}\left\{\frac{{}^{S'}\overline{\text{M}}^{w}_{[x]+t}}{{}^{S}\text{D}_{[x]}}\right\}.$$

(ii) The value at entry $[x]$ of the return of contribution provision in its entirety is:

$$c \cdot (\mathrm{AS})_{[x]} \cdot \left(\frac{1}{{}^{S}\mathrm{D}_{[x]}}\right) \left\{ \sum_{t=0}^{\infty} {}^{S'}\overline{\mathrm{M}}^{w}_{[x]+t} \right\}$$

or

$$c \cdot (\mathrm{AS})_{[x]} \cdot \left\{ \frac{{}^{S'}\overline{\mathrm{R}}^{w}_{[x]}}{{}^{S}\mathrm{D}_{[x]}} \right\}.$$

(c) Deferred Annuity Vesting

A 2% average salary plan provides that, in the event of withdrawal after having completed 15 years of service, the employee retains a paid-up annuity (deferred to age R) whose annual rate is 2% of average salary for each year of service.

(i) The value at entry $[x]$ of the increment of deferred annuity which accrues in the year of life $[x] + t$ to $[x] + t + 1$ in event of withdrawal is:

$$.02\,(\mathrm{AS})_{[x]} \cdot \frac{S_{[x]+t}}{{}^{S}\mathrm{D}_{[x]}} \cdot \left\{ \tfrac{1}{2}\, {}^{K}\mathrm{C}^{wa}_{[x]+t} + \sum_{n=1}^{\infty} {}^{K}\mathrm{C}^{wa}_{[x]+t+n} \right\},$$

where

$${}^{K}\mathrm{C}^{wa}_{[x]+m} = K_{[x]+m} \cdot \mathrm{C}^{w}_{[x]+m} \cdot {}_{\overline{R-x-m-1/2|}}\bar{a}_{[x]+m+1/2}$$

and

$$K_{[x]+m} = \begin{cases} 0 \text{ for } m < 15 \\ 1 \text{ for } m \geqq 15\,. \end{cases}$$

(ii) The value at entry $[x]$ of this withdrawal benefit is

$$.02\,(\mathrm{AS})_{[x]} \cdot \left(\frac{1}{{}^{S}\mathrm{D}_{[x]}}\right) \cdot \left\{ \sum_{t=0}^{\infty} S_{[x]+t} \cdot {}^{K}\overline{\mathrm{M}}^{wa}_{[x]+t} \right\}$$

or

$$.02\,(\mathrm{AS})_{[x]} \left\{ \frac{{}^{S'K}\overline{\mathrm{R}}^{wa}_{[x]}}{{}^{S}\mathrm{D}_{[x]}} \right\}.$$

(d) Disability annuity subject to minimum—the K Method

A $1\frac{1}{2}$% average salary plan provides that in the event of disability the former member shall be entitled to an annuity whose annual rate is $1\frac{1}{2}$% of average salary for each completed year of service subject to a minimum annuity of 30% of average salary.

(i) Let

$$K_{[x]+m} = \begin{cases} .30 & \text{for } m = 0 \\ .30/m & \text{for } 1 \leqq m \leqq 20 \\ .015 & \text{for } 20 < m\,. \end{cases}$$

Then the value at entry $[x]$ of the increment of benefit accruing in the year $[x]+t$ to $[x]+t+1$ is:

$$(AS)_{[x]}\left(\frac{S_{[x]+t}}{{}^{S}D_{[x]}}\right)\left\{\sum_{n=1}^{\infty} K_{[x]+t+n}\cdot C^{ha}_{[x]+t+n}\right\}$$

or

$$(AS)_{[x]}\cdot\left\{\frac{{}^{S'K}M^{ha}_{[x]+t}}{{}^{S}D_{[x]}}\right\}.$$

(ii) The value at entry $[x]$ of the disability benefit is

$$(AS)_{[x]}\left\{\sum_{t=0}^{\infty}\frac{{}^{S'K}M^{ha}_{[x]+t}}{{}^{S}D_{[x]}}\right\}$$

or

$$(AS)_{[x]}\left(\frac{{}^{S'K}R^{ha}_{[x]}}{{}^{S}D_{[x]}}\right).$$

(e) Disability annuity subject to minimum—the Z Method

(i) The value at entry $[x]$ of the disability benefit described in the preceding subdivision (d) may be written in a somewhat different form using the function ${}_{[x]}Z_{[x]+t}$, where

$${}_{[x]}Z_{[x]+t}=\frac{1}{t}\left(S_{[x]}+S_{[x]+1}+S_{[x]+2}+\ldots+S_{[x]+t-1}\right).$$

(ii) The value as of $[x]$ of the disability benefit payable in the event disability occurs in the year of life $[x]+t$ to $[x]+t+1$ is

$$\frac{(AS)_{[x]}}{{}^{S}D_{[x]}}.30\left({}_{[x]}Z_{[x]+t}\right)\cdot C^{ha}_{[x]+t},\qquad\text{where}\qquad t\leq 20\text{, and}$$

$$\frac{(AS)_{[x]}}{{}^{S}D_{[x]}}.015t\cdot\left({}_{[x]}Z_{[x]+t}\right)\cdot C^{ha}_{[x]+t},\qquad\text{where}\qquad t>20$$

(iii) The value as of $[x]$ of the disability benefit is therefore

$$(AS)_{[x]}\cdot\frac{\left\{.30\sum_{t=1}^{20}{}_{[x]}Z_{[x]+t}\cdot C^{ha}_{[x]+t}\right\}+\left\{.015\sum_{t=21}^{\infty}t\cdot{}_{[x]}Z_{[x]+t}\cdot C^{ha}_{[x]+t}\right\}}{{}^{S}D_{[x]}}.$$

It may be shown that the above expression is equivalent to that derived using the K method (see subdivision d). It is merely necessary to substitute the S expression for ${}_{[x]}Z_{[x]+t}$, expand, and collect terms.

The Z expression derived above is suitable for evaluating the benefits of new entrants. If an attempt is made to apply it to participants with

N years of service, the function ${}_{[x]+N}Z_{[x]+N+t}$ must be introduced. It would seem that this would tend to be more cumbersome than the K method.

4. *Average Salary Functions—with Interest:*

An employee retirement plan provides that each participant shall contribute 5% of his salary and in the event of withdrawal from service these contributions shall be returned, together with interest at j percent per annum compounded annually.

In the following the valuation rate of interest is i and it is assumed that $j \neq i$ although this is not necessary.

(a) The value at entry $[x]$ of the return of the contributions made in the year $[x]+t$ to $[x]+t+1$ is:

$$.05\,(AS)_{[x]}\cdot\frac{S_{[x]+t}}{S_{[x]}}\cdot\left\{\frac{1}{2}\,\frac{C^{w}_{[x]+t}}{D_{[x]}}+\sum_{n=1}^{\infty}(1+j)^{n}\,\frac{C^{w}_{[x]+t+n}}{D_{[x]}}\right\};$$

multiplying numerator and denominator by $(1+j)^{x+t}$ we get

$$.05\,(AS)_{[x]}\cdot\left(\frac{1}{{}^{S}D_{[x]}}\right)\left(\frac{S_{[x]+t}}{(1+j)^{x+t}}\right)\left\{\tfrac{1}{2}\,(1+j)^{x+t}C^{w}_{[x]+t}+\sum_{n=1}^{\infty}(1+j)^{x+t+n}\cdot C^{w}_{[x]+t+n}\right\};$$

and if

$${}^{j}C^{w}_{[x]+t}=(1+j)^{x+t}\cdot C^{w}_{[x]+t}$$

and

$${}^{jS'}\overline{M}^{w}_{[x]+t}=\frac{S_{[x]+t}}{(1+j)^{x+t}}\left\{\tfrac{1}{2}\,{}^{j}C^{w}_{[x]+t}+\sum_{n=1}^{\infty}{}^{j}C^{w}_{[x]+t+n}\right\}$$

then the value becomes:

$$.05\,(AS)_{[x]}\left\{\frac{{}^{jS'}\overline{M}^{w}_{[x]+t}}{{}^{S}D_{[x]}}\right\}.$$

(b) The value at entry $[x]$ of the return of contribution benefit is:

$$.05\,(AS)_{[x]}\cdot\sum_{t=0}^{\infty}\frac{{}^{jS'}\overline{M}^{w}_{[x]+t}}{{}^{S}D_{[x]}}$$

or

$$.05\,(AS)_{[x]}\cdot\left\{\frac{{}^{jS'}\overline{R}^{w}_{[x]}}{{}^{S}D_{[x]}}\right\}.$$

E. *Salaried Benefit Functions—after N years Service*

1. *Final Salary Functions:*

The principal difference between the New Entrant formulae and the formulae for an employee with N years of service is the use of $(\mathrm{AS})_{[x]+N}$ instead of $(\mathrm{AS})_{[x]}$. Consider the following example as typical.

A plan provides for a death benefit equal to twice final salary. The value as of $[x] + N$ of such death benefit is

$$2 \cdot (\mathrm{AS})_{[x]+N} \cdot \sum_{t=0}^{\infty} \frac{S_{[x]+N+t} \cdot v^{t+1/2} \cdot d_{[x]+N+t}}{S_{[x]+N} \cdot l_{[x]+N}}$$

or

$$2 \cdot (\mathrm{AS})_{[x]+N} \cdot \left\{ \frac{{}^{S}\mathrm{M}^{d}_{[x]+N}}{{}^{S}\mathrm{D}_{[x]+N}} \right\}.$$

2. *Average Salary Functions—without Interest:*

Where average salary functions are used, the value of a benefit at $[x] + N$ must be considered in two parts. One part arises from the total past salary, namely $(\mathrm{TPS})_{[x]+N}$; the other part arises from future expected salaries.

Consider one example:

A $1\frac{1}{2}\%$ average salary plan provides that in the event of disability the former member shall be entitled to an annuity whose annual rate is $1\frac{1}{2}\%$ of average salary for each completed year of service, subject to a minimum annuity of 30% of average salary.

(a) Let

$$K_{[x]+m} = \begin{cases} .30 & \text{for } m = 0 \\ .30/m & \text{for } 1 \leqq m \leqq 20 \\ .015 & \text{for } 20 < m , \end{cases}$$

then the value as of $[x] + N$ of the future benefits that have already accrued is

$$(\mathrm{TPS})_{[x]+N} \cdot \sum_{t=0}^{\infty} \frac{K_{[x]+N+t} \cdot v^{t+1/2} \cdot h_{[x]+N+t} \cdot \bar{a}_{[x]+N+t+1/2}}{l_{[x]+N}}$$

or

$$(\mathrm{TPS})_{[x]+N} \cdot \sum_{t=0}^{\infty} \frac{{}^{K}\mathrm{C}^{ha}_{[x]+N+t}}{\mathrm{D}_{[x]+N}}$$

or

$$(\mathrm{TPS})_{[x]+N} \cdot \left(\frac{{}^{K}\mathrm{M}^{ha}_{[x]+N}}{\mathrm{D}_{[x]+N}} \right).$$

(b) The value as of $[x] + N$ of the future benefits that will accrue from future salary is

$$(\mathrm{AS})_{[x]+N} \left\{ \frac{1}{{}^{S}\mathrm{D}_{[x]+N}} \cdot \sum_{t=0}^{\infty} S_{[x]+N+t} \left(\sum_{m=1}^{\infty} {}^{K}\mathrm{C}^{ha}_{[x]+N+t+m} \right) \right\}$$

or

$$(\mathrm{AS})_{[x]+N}\left(\frac{1}{{}^{S}\mathrm{D}_{[x]+N}}\right)\left({}^{S'K}\mathrm{R}^{ha}_{[x]+N}\right).$$

(c) Consequently, the present value of this disability benefit with respect to a member $[x]+N$ is

$$(\mathrm{TPS})_{[x]+N}\cdot\left(\frac{{}^{K}\mathrm{M}^{ha}_{[x]+N}}{\mathrm{D}_{[x]+N}}\right)+(\mathrm{AS})_{[x]+N}\left(\frac{{}^{S'K}\mathrm{R}^{ha}_{[x]+N}}{{}^{S}\mathrm{D}_{[x]+N}}\right).$$

3. *Average Salary Functions—with Interest:*

An employee retirement plan provides that each participant shall contribute 4% of his salary and in the event of withdrawal from service these contributions shall be returned together with interest at j percent per annum compounded annually.

(a) Let

$$(\mathrm{TPS})^{j}_{[x]+N}=\sum_{t=0}^{N-1}(\mathrm{AS})_{[x]+t}\div(1+j)^{x+t+1/2},$$

then the present value of future benefits which arise out of past salaries is

$$.04\,(\mathrm{TPS})^{j}_{[x]+N}\cdot\sum_{t=0}^{\infty}\frac{(1+j)^{x+N+t+1/2}\cdot v^{t+1/2}\cdot w_{[x]+N+t}}{l_{[x]+N}}$$

or

$$.04\,(\mathrm{TPS})^{j}_{[x]+N}\cdot\left\{\frac{{}^{j}\mathrm{M}^{w}_{[x]+N}}{\mathrm{D}_{[x]+N}}\right\},$$

where

$${}^{j}\mathrm{M}^{w}_{[x]+N}=\sum_{t=0}^{\infty}(1+j)^{x+N+t+1/2}\cdot\mathrm{C}^{w}_{[x]+N+t}.$$

(b) The future service liability may be expressed as follows:

$$.04\,(\mathrm{AS})_{[x]+N}\left(\frac{1}{{}^{S}\mathrm{D}_{[x]+N}}\right)\left\{\sum_{t=0}^{\infty}\frac{S_{[x]+N+t}}{(1+j)^{x+N+t}}\left(\tfrac{1}{2}\,{}^{j}\mathrm{C}^{w}_{[x]+N+t}+\sum_{m=1}^{\infty}{}^{j}\mathrm{C}^{w}_{[x]+N+t+m}\right)\right\}.$$

Let the summation be defined as ${}^{jS'}\overline{\mathrm{R}}^{w}_{[x]+N}$, the liability becomes:

$$.04\,(\mathrm{AS})_{[x]+N}\cdot\left\{\frac{{}^{jS'}\overline{\mathrm{R}}^{w}_{[x]+N}}{{}^{S}\mathrm{D}_{[x]+N}}\right\}.$$

(c) The present value of this withdrawal benefit is

$$.04\left\{ (\mathrm{TPS})^{\,j}_{[x]+N}\cdot\left(\frac{{}^{i}\mathrm{M}^{w}_{[x]+N}}{\mathrm{D}_{[x]+N}}\right)+(\mathrm{AS})_{[x]+N}\cdot\left(\frac{{}^{jS'}\overline{\mathrm{R}}^{w}_{[x]+N}}{{}^{S}\mathrm{D}_{[x]+N}}\right)\right\}$$

VALUATION OF A PARTICULAR PLAN

A. *Description of Plan*

In order to illustrate the manner in which the functions derived above are applied in the valuation of a self-insured plan, consider the following plan which an Employer has just decided to install. It is assumed that the Employer has not had a formal plan in operation in the past. The following are the essential provisions of this illustrative plan:

1. *Eligibility.*—An employee with three years or more of service is eligible to participate, provided he has reached age 30.
2. *Contributions.*—Each participating employee shall contribute 4% of his salary toward the cost of funding the plan.
3. *Retirement.*—Upon attainment of age 65 retirement is compulsory. The employee is entitled to a life annuity of 2% of average salary for each year of service in plan.
4. *Ill-health Retirement.*—In the event of total and permanent disability prior to age 65, the employee is entitled to a life annuity of 2% of average salary for each year of service, subject to a minimum benefit of 30% of average salary.
5. *Withdrawal.*—In event of withdrawal from the plan the employee's contributions are returned to him together with interest at 3% compounded annually.
6. *Death Benefit:*

In event of death, while in service, the benefit shall be twice final salary.

In event of death following retirement, either for age or for ill-health, the benefit shall be

(a) twice final salary in first year after severance
(b) one and one-half final salary in second year
(c) once final salary in third year
(d) one-half final salary thereafter.

NOTE: This plan is illustrative only.

B. *New Entrants*

1. The value as of $[x]$ of the plan's benefits to a new entrant may be written as

$$\frac{(\mathrm{AS})_{[x]}}{{}^{S}\mathrm{D}_{[x]}}\left\{.02\,{}^{S'}\mathrm{R}^{ra}_{[x]}+{}^{S'K}\mathrm{R}^{ha}_{[x]}+.04\,{}^{iS'}\overline{\mathrm{R}}^{w}_{[x]}+2\cdot{}^{S}\mathrm{M}^{d}_{[x]}+{}^{S}\mathrm{M}^{rB}_{[x]}+{}^{S}\mathrm{M}^{hB}_{[x]}\right\},$$

where all the symbols, except the last two, are defined along the lines set down earlier in this paper. These latter two are defined as follows:

$$ {}^{S}\mathrm{M}^{hB}_{[x]} = \sum_{t=0}^{\infty} {}^{S}\mathrm{C}^{h}_{[x]+t} \cdot B_{[x]+t} $$

$$ {}^{S}\mathrm{M}^{rB}_{[x]} = S_{[x]+t} \cdot v^{x+t} l_{[x]+t} \cdot B_{[x]+t-1/2} \qquad \text{for} \qquad [x]+t = 65 $$

where

$$ B_{[x]+t} = \tfrac{1}{2}\left\{ \mathrm{A}_{x+t+1/2} + \mathrm{A}^{\;\;1}_{\overline{x+t+1/2:\overline{3}|}} + \mathrm{A}^{\;\;1}_{\overline{x+t+1/2:\overline{2}|}} + \mathrm{A}^{\;\;1}_{\overline{x+t+1/2:\overline{1}|}} \right\} . $$

2. Since the value as of $[x]$ of the employee's contributions is

$$ .04\,(\mathrm{AS})_{[x]} \left\{ \frac{{}^{S}\overline{\mathrm{N}}_{[x]}}{{}^{S}\mathrm{D}_{[x]}} \right\} , $$

it follows that the value as of $[x]$ of the Employer's contributions is $(\mathrm{AS})_{[x]} \cdot {}^{S}V^{Er}_{[x]}$, where

$$ {}^{S}V^{Er}_{[x]} = \frac{1}{{}^{S}\mathrm{D}_{[x]}} \Big\{ .02\,{}^{S'}\mathrm{R}^{ra}_{[x]} + {}^{S'K}\mathrm{R}^{ha}_{[x]} + .04\,{}^{iS'}\overline{\mathrm{R}}^{w}_{[x]} + 2\,{}^{S}\mathrm{M}^{d}_{[x]} + {}^{S}\mathrm{M}^{rB}_{[x]} + {}^{S}\mathrm{M}^{hB}_{[x]} - .04\,{}^{S}\overline{\mathrm{N}}_{[x]} \Big\} . $$

3. If the Employer's share of the cost is to be funded as a percentage of salary during the employee's service, the annual rate of such contribution may be expressed as:

$$ c^{Er}_{[x]} = \frac{1}{{}^{S}\overline{\mathrm{N}}_{[x]}} \Big\{ .02 \cdot {}^{S'}\mathrm{R}^{ra}_{[x]} + {}^{S'K}\mathrm{R}^{ha}_{[x]} + .04\,{}^{iS'}\overline{\mathrm{R}}^{w}_{[x]} + 2 \cdot {}^{S}\mathrm{M}^{d}_{[x]} + {}^{S}\mathrm{M}^{rB}_{[x]} + {}^{S}\mathrm{M}^{hB}_{[x]} - .04\,{}^{S}\overline{\mathrm{N}}_{[x]} \Big\} . $$

C. *Service prior to the Effective Date of the Plan*

1. A new plan usually makes some special provision for those persons who are in the employ on the effective date of the plan. For example, the retirement age may be adjusted for older employees.

For the present purposes, it will be assumed that only two modifications are required in the plan described above. These are:

(a) provision that the retirement annuity accruals for each year of service prior to the effective date of the plan shall be 1% of salary as of the inception of the plan, and

(b) provision that past service disability benefits shall be based upon salary as of the inception of the plan.

2. Suppose that an employee now aged $[x] + N$ would have satisfied the

eligibility provisions of the plan N years ago at age x and that his annual rate of salary as of the inception of the plan is

$$(\mathrm{IS})_{[x]+N}\,.$$

The value as of $[x]+N$ of the retirement and disability benefits that accrued on account of past service is:

$$(\mathrm{IS})_{[x]+N}\left\{\frac{N}{\mathrm{D}_{[x]+N}}\right\}(.01\mathrm{M}^{ra}_{[x]+N}+{}^{K}\mathrm{M}^{ha}_{[x]+N})\,.$$

3. The value as of $[x]+N$ of the Employer's share of the cost of the plan, with respect to a present employee whose annual rate of salary is $(\mathrm{IS})_{[x]+N}$, is:

$$(\mathrm{IS})_{[x]+N}\left\{{}^{S}V^{Er}_{[x]+N}+N\cdot\frac{.01\mathrm{M}^{ra}_{[x]+N}+{}^{K}\mathrm{M}^{ha}_{[x]+N}}{\mathrm{D}_{[x]+N}}\right\}.$$

4. If the expression set forth above were evaluated for each eligible employee, the total would represent the unfunded cost of all past and future service benefits. This may be called the value of expected future employer contributions.

Two methods by which this "liability" may be funded will now be described.

D. *The Aggregate Cost Method*

1. The value as of the effective date of the Plan of all future salaries to present eligible employees is

$$\Sigma\,(\mathrm{IS})_{[x]+N}\cdot\frac{{}^{S}\overline{\mathrm{N}}_{[x]+N}}{{}^{S}\mathrm{D}_{[x]+N}}.$$

2. The annual rate of Employer contribution under the aggregate cost method will be

$$c^{Er}_{\mathrm{Agg}}=\frac{\Sigma\,(\mathrm{IS})_{[x]+N}\left\{{}^{S}V^{Er}_{[x]+N}+\frac{N}{\mathrm{D}_{[x]+N}}\left(.01\mathrm{M}^{ra}_{[x]+N}+{}^{K}\mathrm{M}^{ha}_{[x]+N}\right)\right\}}{\Sigma\,(\mathrm{IS})_{[x]+N}\left\{\frac{{}^{S}\overline{\mathrm{N}}_{[x]+N}}{{}^{S}\mathrm{D}_{[x]+N}}\right\}},$$

and this accrual rate multiplied by the compensation payable in a given year to employees in service is the annual cost for that year.

3. In subsequent actuarial valuations the same procedure is repeated except that the value of the assets of the Trust is deducted from the numerator in deriving the Employer's accrual rate, c^{Er}_{Agg}.

E. *The Entry Age Normal Cost Method*

1. The Entry Age Normal Cost Method assumes "that the plan had been in effect from the beginning of the service of each then included em-

ployee and that such costs for prior years had been paid and all assumptions as to interest, mortality, time of payment, etc., had been fulfilled." If it is further assumed that all new entrants are aged $[x]$, then the Normal Cost accrual rate is $c^{Br}_{[x]}$ as defined in subdivision 3 of Section B above.

2. The Supplementary Cost as of the effective date of the plan is

$$\Sigma(\text{IS})_{[x]+N}\left\{{}^{S}V^{Br}_{[x]+N}+\frac{N}{\text{D}_{[x]+N}}\left(.01\text{M}^{ra}_{[x]+N}+{}^{K}\text{M}^{ha}_{[x]+N}\right)\right\} - c^{Br}_{[x]}\Sigma(\text{IS})_{[x]+N}\left\{\frac{{}^{S}\overline{\text{N}}_{[x]+N}}{{}^{S}\text{D}_{[x]+N}}\right\}.$$

3. Under the Entry Age Normal Cost Method the Employer contributes each year the sum of:

(a) the product of the Normal Cost accrual rate and the compensation payable to employees in service in that year, and

(b) a portion, usually not in excess of 10%, of the Supplementary Cost.

INTEGRATION WITH SOCIAL SECURITY

When the employees of an Employer are covered for old age benefits by some formal governmental plan, such as Federal OASI or the Railroad Retirement Act, the benefit and contribution provisions of the Employer's Retirement Plan are usually modified to avoid duplication of benefits and costs. In making this modification the objective is to produce an integrated and correlated retirement system, that is, a system under which no employee can receive a greater benefit in proportion to salary than a lower paid employee with similar age and service history.

In the United States the principal governmental plan is Federal OASI. When a plan is to be integrated with OASI, it is common to provide that a reduced accrual rate shall apply to the first \$3,000 of annual salary because the OASI benefits and contributions are based on that portion of the employee's salary at the present time. If the salary base for OASI were to be increased by legislation, the breaking point in the Employer's benefit formula would be increased accordingly.

The annual primary benefit under OASI may be stated as the sum of (a) 40% of the first \$600; (b) 10% of the next \$2,400; and (c) N% of the sum of (a) and (b), where N is the number of years of coverage. For purposes of integration, 150% of the primary benefit is taken, it being presumed that the employee's wife is alive and entitled to benefit.

Since the OASI formula is a "bent" one, it is only necessary to determine how much the OASI benefit is for a person whose average annual salary is \$3,000. For such an employee 150% of the primary benefit may be expressed as 24% + .24% N (or approximately 25% + ¼% N) times average annual salary. If such an employee enters the Employer's plan and OASI at age 30, then the OASI benefit would provide approximately 33¾% of \$3,000 upon retirement after 35 years of service at age 65. This is approximately equal to a retirement benefit of 1% of average salary (based on the first \$3,000 only) for each year of service.

It follows that one method for integrating the 2% Plan (described in Section A above) with Federal OASI is to provide that:

(a) the future service retirement accruals shall be 1% of the first \$3,000 salary in each year, plus 2% of any salary in excess of \$3,000,

(b) the disability accruals shall be 1% of the first \$3,000 of salary in each year, plus 2% of any salary in excess of \$3,000, and

(c) the employee contributions in each year shall be 2% of the first \$3,000 of salary, plus 4% of any salary in excess of \$3,000.

These modifications create a valuation problem. However, if an initial valuation of benefits and employee contributions is made on the basis of a full 2% plan, the true value may be obtained by deducting the value of the integration adjustment.

Integration with Social Security (OASI) has the effect of introducing a steeper salary scale function. If the Entry Age Normal Cost Method is used, and the Employer's contribution is applied to salary in proportion to retirement accruals, this means that the value of $c_{[x]}^{Er}$ is higher for the Integrated Plan than it was for the full 2% Plan.

EFFECT OF CHANGES IN SALARY SCALE

A. *Normal Cost Accrual Rate*

The qualitative effect of a change in salary scale upon the Normal Cost accrual rate, $c_{[x]}^{Er}$, may be derived by general reasoning.

A constant percentage change in $S_{[x]+t}$ does not affect the value of $c_{[x]}^{Er}$. This is true for both average salary plans and final salary plans, because both the numerator and denominator are multiplied by the same factor.

A flat increase in the value $S_{[x]+t}$ reduces the slope of the curve and thereby decreases the value of $c_{[x]}^{Er}$; while a flat decrease in the value of $S_{[x]+t}$ increases the value of $c_{[x]}^{Er}$. This result is obtained by considering the plan as an N-year endowment with premiums that increase according to the salary scale function. If the salary scale function is flattened, the contributions tend to be made earlier in the service of the employee and therefore will earn more interest. If the salary scale function is made steeper, the reverse is true. These conclusions apply to both average salary plans and final salary plans.

B. *Status of a Mature Fund*

Suppose that a plan which has been in operation for a number of years has just been valued by the Entry Age Normal Cost Method and that the valuation derived a new value for $c^{v}_{[x]}$. Suppose further that the valuation shows that there is no supplementary cost.

Immediately after the valuation the salary scale is subjected to an abrupt change which affects new entrants and present members in the same manner. The effect of this change upon the fund depends upon

1. whether the original salary scale was steep or flat,
2. whether the change in the original salary scale takes the form of a level change or a percentage change, and
3. whether the plan is an average salary plan or a final salary plan.

It is an interesting actuarial exercise to derive by general reasoning for each of the eight possible permutations the qualitative result. In some cases, the fund's position is improved; in others it is weakened. In many of the cases the result is indeterminate.

Of course, in order to make a quantitative evaluation in any particular case it is necessary to make a complete actuarial re-evaluation.