TRANSACTIONS OF SOCIETY OF ACTUARIES
1950 VOL. 2 NO. 3

VOL. II, PART TWO No. 3

TRANSACTIONS

JUNE, 1950

EXTRA PREMIUMS BASED ON THE NET AMOUNT AT RISK

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AN APPLICANT subject to extra mortality may be denied a standard insurance policy. The extra mortality might be due to one or more factors influencing the risk such as occupation, physical condition, avocation, or other condition known to increase the mortality hazard. Individual factors that are known to affect mortality can be grouped into broad categories depending on the incidence of the expected extra mortality. Some factors cause an extra mortality that is decreasing and temporary and may or may not be independent of age. Other factors cause an accelerated increase in the rate of mortality with advancing age. Still others cause extra mortality that is considered constant for all durations and ages. This paper develops an equitable and practical method of calculating and assessing the required extra premiums to give insurance protection to those applicants subject to the factor in this latter group which includes most of the accident hazards connected with vocations and avocations.

The symbol k is used hereafter to denote this rate of extra mortality—the number of extra deaths a year per person exposed to the specific hazard. Although k is assumed to be the same at all ages for any specific hazard being considered, it will vary as between one hazard and another.

Let us consider five elements that might properly enter into the calculation of the gross extra premium to provide insurance protection to an applicant subject to k.

(1) A rate of interest.
(2) A rate of expense for the extra coverage.
(3) A withdrawal rate which would include terminations of the specific hazard for all reasons including death other than death due to the specific hazard.
(4) The extra mortality rate k for the class of risks. Theoretically, a double decrement table should be used for this and the withdrawal rate. Practically, it would make no difference in the charge.

(5) An equitable method of determining and collecting the required extra premiums.

(1) and (2) are of little trouble as they are either known or readily approximated. (3), while unknown, does not necessarily enter the calculations. This will be discussed more fully below. (4) is troublesome. Often a small amount of data collected in the recent past must be used as a guide to future experience on a hazard that may be rapidly changing, *e.g.*, aviation risks. Thus the most important element, (4) above, may not be much more than a hopeful guess. Probably because of this not much attention has been paid to the fifth element which can be accurately evaluated. As will be shown in this paper the extra premium will vary by plan and perhaps by age if broad principles of equity are to be maintained.

When a person who is subject to the extra mortality rate k applies for insurance, it is necessary to evaluate the cost of this extra risk and the extra expenses incident thereto. A discussion of the handling of expenses is deferred until later in the paper. The net cost for insurance to cover the extra risk in policy year n can be expressed as $k[(1+i/2) - {}_n\mathrm{CV}_x^P]$ where ${}_n\mathrm{CV}_x^P$ is the nth year cash surrender value on a plan of insurance P issued at age x. As usual it is assumed that claims are evenly spread throughout the policy year. Also it is assumed that the cash surrender values are based on asset shares so that the face value of the policy less the cash surrender value is the true amount at risk. Thus, the net cost in any policy year to provide for the payment of the face amount of insurance in event of death of the insured due to the specific hazard covered by rate k will vary by plan of insurance, age of the insured and the duration of the policy. Yet in calculating the extra premium this variation in cost by age, plan and duration is seldom taken into account. The justification usually given for charging the extra premiums without refinement is that the risk cannot be accurately determined because of the limitations of available statistics. Thus a convenient and sufficient charge that meets competition has been considered reasonable and proper and refinements to increase equity by plan or age have been ignored.

The true net single premium necessary at issue to cover the extra hazard k for the duration of the policy is given by:

$$\mathrm{S.P.} = \sum_{n=1}^{m} k\, v^n \frac{l'_{x+n-1}}{l'_x}\left[\left(1+\frac{i}{2}\right) - {}_n\mathrm{CV}_x^P\right]$$

$$= k\, v \sum_{n=1}^{m} \frac{\mathrm{D}'_{x+n-1}}{\mathrm{D}'_x}\left[\left(1+\frac{i}{2}\right) - {}_n\mathrm{CV}_x^P\right]$$

where the values of l'_x and D'_x are obtained from a special mortality table constructed by increasing a standard q_x by k at all ages.

The calculation of net single premiums by the formula shown above entails considerable work as separate mortality tables are necessary for each value assigned to k and separate summations are required for each plan and age. However, by making the approximate assumption that $l'_{x+n-1}/l'_x = l_{x+n}/l_x$ a simple formula can be developed for obtaining the net single premiums. The error in this assumption will not be large for the usual values of k that will be encountered in life insurance.

Calculations of the premiums in this paper were made both with and without this assumption. However, only the approximate premiums are shown as they differ from the accurate ones by less than three percent in every instance.

If a company has based its scale of nonforfeiture values on the minimum allowed under the Standard Non-Forfeiture Law, the net single premium necessary at issue to cover the extra hazard k for the duration of the policy can now be conveniently expressed as:

$$\text{S.P.} = k\left[\frac{i}{2}\, a_{x:\overline{m}|} + d\,(\mathrm{I}a)_{x:\overline{m-1}|} + \mathrm{P}'_P(\mathrm{I}a)_{x:\overline{t-1}|}\right] \tag{1}$$

where:

k is the mortality rate for the specific hazard
m is the period of coverage
t is the premium paying period on the policy
P'_P is the modified net annual premium defined in the Standard Non-Forfeiture Law.

a_x and $(\mathrm{I}a)_x$ are based on Commissioners Standard Ordinary Mortality Table at interest rate i.

This formula is general in that it applies to any level premium t-payment m-year endowment policy. The derivation is as follows:

$$\text{S.P.} = \sum_{n=1}^{m} k\, v^n \frac{l'_{x+n-1}}{l'_x}\left[\left(1+\frac{i}{2}\right) - {}_n\mathrm{CV}^P_x\right].$$

If

$$\frac{l'_{x+n-1}}{l'_x} = \frac{l_{x+n}}{l_x},$$

then:

$$\text{S.P.} = \sum_{n=1}^{m} k\, v^n \frac{l_{x+n}}{l_x}\left[\left(1+\frac{i}{2}\right) - {}_n\mathrm{CV}^P_x\right]$$

$$= \sum_{n=1}^{m} \frac{k\mathrm{D}_{x+n}}{\mathrm{D}_x}\left[1+\frac{i}{2} - {}_n\mathrm{CV}^P_x\right]. \tag{2}$$

If the cash surrender values granted are the minimum allowed under the Standard Non-Forfeiture Law, ${}_nCV_x = A_{x+n:\overline{m-n}|} - P'_P\ddot{a}_{x+n:\overline{t-n}|}$ where P'_P is the modified net annual premium defined in that law, then:

$$\text{S.P.} = \sum_{n=1}^{m} \frac{kD_{x+n}}{D_x}\left[1 + \frac{i}{2} - A_{x+n:\overline{m-n}|}\right] + \sum_{n=1}^{t} \frac{kD_{x+n}}{D_x}\left[P'_P \cdot \ddot{a}_{x+n:\overline{t-n}|}\right]$$

$$= \sum_{n=1}^{m} \frac{kD_{x+n}}{D_x}\left[\frac{i}{2} + d\,\ddot{a}_{x+n:\overline{m-n}|}\right] + \sum_{n=1}^{t} \frac{kD_{x+n}}{D_x}\left[P'_P\,\ddot{a}_{x+n:\overline{t-n}|}\right]$$

$$= \sum_{n=1}^{m} \frac{k}{D_x}\left[\frac{i}{2}\,D_{x+n} + d\,(N_{x+n} - N_{x+m})\right]$$

$$+ \sum_{n=1}^{t} \frac{k}{D_x}\left[P'_P\,(N_{x+n} - N_{x+t})\right]$$

$$= \frac{k}{D_x}\Big[\frac{i}{2}\,(N_{x+1} - N_{x+m+1}) + d\,(S_{x+1} - S_{x+m+1} - m \cdot N_{x+m})$$

$$+ P'_P\,(S_{x+1} - S_{x+t+1} - t \cdot N_{x+t})\Big]$$

$$= k\left[\frac{i}{2}\,a_{x:\overline{m}|} + d\,(Ia)_{x:\overline{m-1}|} + P'_P\,(Ia)_{x:\overline{t-1}|}\right].$$

Most companies are using cash surrender values that are higher than the minimum allowed by law. Some companies use a set of modified cash values that can be defined by a formula as follows:

$${}_nCV_x = A_{x+n:\overline{m-n}|} - {}^mP_{x:\overline{t}|}\ddot{a}_{x+n:\overline{t-n}|} - C \cdot \ddot{a}_{x+n:\overline{s-n}|}$$

where C varies for different plans of insurance. The modified portion runs for s years or the premium paying period if less. With such cash values the net single premium to provide coverage in event of death due to the specific hazard for the duration of the policy is given by:

$$\text{S.P.} = k\left[\frac{i}{2}\,a_{x:\overline{m}|} + d\,(Ia)_{x:\overline{m-1}|} + {}^mP_{x:\overline{t}|} \cdot (Ia)_{x:\overline{t-1}|} + C \cdot (Ia)_{x:\overline{s-1}|}\right] \quad (3)$$

where:

k is the mortality rate for the specific hazard
m is the coverage period
t is the premium paying period on the policy
s is the period during which there is a surrender charge
${}^mP_{x:\overline{t}|}$ is the level net annual premium for the policy

Proof of this formula is as follows:

From formula (2) above

$$\begin{aligned}
\text{S.P.} &= \sum_{n=1}^{m} \frac{kD_{x+n}}{D_x}\left[1+\frac{i}{2} - A_{x+n:\overline{m-n}|} + {}^{m}P_{x:\overline{t}|}\cdot \ddot{a}_{x+n:\overline{t-n}|} + C\cdot \ddot{a}_{x+n:\overline{s-n}|}\right] \\
&= \sum_{n=1}^{m} \frac{kD_{x+n}}{D_x}\left[\frac{i}{2} + d\cdot \ddot{a}_{x+n:\overline{m-n}|}\right] + \sum_{n=1}^{t} \frac{kD_{x+n}}{D_x}\left[{}^{m}P_{x:\overline{t}|}\cdot \ddot{a}_{x+n:\overline{t-n}|}\right] \\
&\qquad + \sum_{n=1}^{s} \frac{kD_{x+n}}{D_x}\left[C\cdot \ddot{a}_{x+n:\overline{s-n}|}\right] \\
&= k\left[\frac{i}{2}\, a_{x:\overline{m}|} + d\,(Ia)_{x:\overline{m-1}|} + {}^{m}P_{x:\overline{t}|}\cdot (Ia)_{x:\overline{t-1}|} + C\cdot (Ia)_{x:\overline{s-1}|}\right].
\end{aligned}$$

In order to provide for net annual extra premiums rather than a net single premium let $F^{P}_{x:\overline{r}|}$ equal the annual extra premium payable for r years on plan P at age x at issue. Then

$$\left.\begin{aligned}
F^{P}_{x:\overline{r}|}\cdot \ddot{a}_{x:\overline{r}|} = k\Big[\frac{i}{2}\, a_{x:\overline{m}|} &+ d\,(Ia)_{x:\overline{m-1}|} + {}^{m}P_{x:\overline{t}|}(Ia)_{x:\overline{t-1}|} \\
&+ C\cdot (Ia)_{x:\overline{s-1}|}\Big].
\end{aligned}\right\} \quad (4)$$

In order to see the application of this principle and to aid in further discussion consider the extra premium applicable to scheduled airline pilots. For the year 1948, the mortality rate of all pilots employed in sched-

TABLE 1

SCHEDULED AIRLINE PILOTS

NET SINGLE AVIATION EXTRA PREMIUM PER $1,000

PLAN	AGE AT ISSUE		
	25	35	45
Whole Life	$46.13	$39.11	$31.82
20 P. L.	37.31	31.37	26.02
5 P. L.	30.84	23.68	17.18
Single P. L.	28.92	21.36	14.40
20 End't	22.25	21.92	21.17
10 End't	11.21	11.15	11.02

uled flying per pilot employed was .0025. Although this rate is admittedly below previous years, for this illustration assume that this rate will continue in the future. We can then calculate an array of net single premiums for this coverage. Using formula (3) and reasonable values for C and s, such an array is given in Table 1. When we consider equivalent practical annual extra premiums certain facts should be considered.

The period over which the aviation extra premium is to be collected should not be longer than the premium paying period of the policy. Thus r should be less than or equal to t in formula (4). The premium charged should be sufficient so that there will be no loss on termination of the aviation coverage. A level premium is being calculated to cover a decreasing risk. Thus if level premiums are payable for the duration of the coverage, early premiums are too small and later premiums redundant. Any early termination results in a loss. Early terminations are expected and at an undetermined and uncontrolled rate. Therefore, a correct procedure might be to make the first premium, and thus later premiums, sufficient but to reduce the payment period. There will be no loss on termination if $F^{P}_{x:\overline{r}|}$ is equal to or greater than k.

In formula (4) let $F^{P}_{x:\overline{r}|} = k$ and solve for r, the payment period of the aviation extra. Remembering that r is less than or equal to t, the values of r under these conditions are shown in Table 2.

TABLE 2

MAXIMUM NUMBER OF YEARS (r) DURING WHICH NET AVIATION EXTRA PREMIUMS ARE COLLECTED

Plan	Age at Issue		
	25	35	45
Whole Life	26 years	22 years	17 years
20 P. L.	20 "	16 "	13 "
5 P. L.	5 "	5 "	5 "
Single P. L.	1 year	1 year	1 year
20 End't.	11 years	11 years	10 years
10 End't.	5 "	5 "	5 "

For example, if the aviation extra premium charged is just sufficient to cover the risk during the first policy year on a 10 Year Endowment, then, after five such extra premiums have been collected, enough has been collected to cover the extra risk for the balance of the duration of the policy. At age 35 sixteen such premiums are all that is necessary on a 20 Payment Life Policy.

This discussion has been restricted to net extra premiums. Suitable loadings can be added to cover the expenses involved in adding this benefit. Most of the expenses are incurred at issue. If a constant extra is added to the net annual premium at issue there will be a small loss on early terminations. These losses could be absorbed in the company's miscellaneous gains and losses. However, a more equitable procedure would be to add the present value of the entire expected future expenses applicable to the benefit to the net single premium for the benefit as shown in the first table above. Then let $F^{P}_{x:\overline{r}|}$ be equal to or greater than $k + c$ where c is the initial cost of adding the benefit to the policy. The maximum period for collecting the extra premiums would be reduced depending on the value given to c.

In order to observe how this method would appear in practice hypothetical gross rates have been calculated for scheduled airline pilots. First year expenses per $1,000 have been assumed to be $.70 and renewal expenses $.25 for nine years only. These are arbitrary but will illustrate the problem.

Gross single extra premium per $1,000 = net single extra premium per $1,000 + $.70 + $.25 $\cdot a_{x:\overline{9}|}$.

TABLE 3

SCHEDULED AIRLINE PILOTS
GROSS SINGLE AVIATION EXTRA PREMIUMS PER $1,000

PLAN	AGE AT ISSUE		
	25	35	45
Whole Life	$48.79	$41.76	$34.41
20 P. L.	39.97	34.01	28.61
5 P. L.	33.51	26.32	19.78
Single P. L.	31.58	24.00	16.99
20 End't.	24.92	24.57	23.77
10 End't.	13.87	13.79	13.62

The minimum gross annual aviation extra premium per $1,000 is $2.50 + $.70 = $3.20. Thus if $3.20 is collected the first year there will be no loss on terminations. Table 4 gives the minimum gross annual aviation extra premiums and the maximum number of years during which the premiums need be collected.

Implicit in this proposed system for calculating aviation extra premiums is the assumption that the mortality rate, k, remains constant for the duration of the policy. While this is unlikely, it is probably the only practical assumption that can be made.

It is not realistic to assume an increasing rate of mortality as generally there will be both less hazardous flying and a reduced amount of flying with advancing age. Only on a few will there be an increased hazard.

It is unnecessary to assume a decreasing rate of mortality as a reduced rating or removal of rating can be allowed an insured where the risk has decreased.

It is to be noted that the system now generally used for charging aviation extra premiums has the effect of assuming an increasing aviation

TABLE 4

SCHEDULED AIRLINE PILOTS

Plan	Age at Issue					
	25		35		45	
	Min. Gross Ann. Prem.	Max. Yrs. Payable	Min. Gross Ann. Prem.	Max. Yrs. Payable	Min. Gross Ann. Prem.	Max. Yrs. Payable
Whole Life....	$ 3.20	20	$ 3.20	17	$ 3.20	14
20 P. L........	3.20	16	3.20	13	3.20	11
5 P. L.........	7.08	5	5.58	5	4.23	5
Single P. L....	31.58	1	24.00	1	16.99	1
20 End't......	3.20	9	3.20	9	3.20	9
10 End't......	3.20	5	3.20	5	3.20	5

TABLE 5

Policy Year	10 Year Endowment	Whole Life
1............	.0026	.0025
2............	.0034	.0029
3............	.0039	.0030
4............	.0045	.0030
5............	.0053	.0031
6............	.0064	.0031
7............	.0084	.0032
8............	.0123	.0032
9............	.0230	.0033
10............	.2360	.0033

mortality rate which also varies both by plan of insurance and by age of the insured at issue.

To illustrate further the unrealistic assumptions underlying the usual basis of charging such extras, Table 5 shows the effective mortality rate that is being provided for by a charge of $3.20 per $1,000 for each of the first 10 years on a pilot age 25 at issue.

The effective mortality rate for the entire 10 year period is .0057 for the 10 Year Endowment and .0031 for the Whole Life Policy. The calculation of the premium was based on a rate of .0025.

The proposed plan eliminates such inequitable treatment as between plans and ages by making the same rate of extra premium charge payable for a variable payment period. A proper charge is thus provided to cover the same rate of extra mortality for the total period of each contract.

TABLE 6

NUMBER OF YEARS FOR WHICH EXTRA PREMIUM IS PAYABLE

Plan of Insurance	Age at Issue						
	25–27	28–30	31–33	34–36	37–39	40–42	43–45
Whole Life......	20	19	18	17	16	15	14
30 P. L..........	17	17	16	15	15	14	13
20 P. L..........	16	15	14	13	12	12	11
15 P. L..........	15	14	13	12	11	11	10
10 P. L..........	10*	10*	10*	10	10	9	9
5 P. L..........	5*	5*	5*	5*	5*	5*	5*
Single P. L.......	1*	1*	1*	1*	1*	1*	1*
E. at 65.........	16	15	14	13	12	11	10
30 End't........	13	13	13	13	13	12	12
20 End't........	9	9	9	9	9	9	9
10 End't........	5	5	5	5	5	5	5

* Multiply the Extra Premium in the first table by the following Factor to obtain Annual Extra Premium applicable to Plans shown.

Plan	Age at Issue						
	25–27	28–30	31–33	34–36	37–39	40–42	43–45
10 P. L..........	1.3	1.2	1.1	....	....	....	....
5 P. L...........	2.2	2.1	1.9	1.7	1.6	1.5	1.3
Single P. L.......	9.9	9.1	8.3	7.5	6.8	6.0	5.3

This discussion can be applied to all occupational hazards for which a constant extra rate of mortality can be assumed. Scheduled airline pilots merely provided an illustration of the method proposed.

It may be held that a serious objection to the proposed plan is that, if the annual extra premium is the minimum level amount that can be charged, $k + c$, then the payment period for the extra premium will vary by both plan of insurance and age of the insured. Variation of the payment period by plan cannot be avoided. Variation by age is not great with most plans and on some there will be no variation by age at all.

One method of handling these occupational extra premiums would be to set up two tables. The first table would show for each occupation a basic gross annual extra premium per $1,000 (*e.g.*, Pilots, scheduled air-line—$3.20). The second table (Table 6) would give the number of years over which the extra premium would be collected for each plan of insurance and for limited payment plans the increase in the annual extra premium. A sample of this second table is shown above.

Another refinement that could be adopted would be to calculate extra premiums so that they could be automatically removed at attained age 65. This could be done on a basis of either a continuing risk or an expiring risk depending on the occupation involved.