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A PRESENT VALUE APPROACH TO PROFIT MARGINS AND DIVIDENDS

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OUTLINE OF PURPOSE

ACTUARIES know that nearly all actuarial calculations can be made either by accumulation or by discounting. Yet there is one field in which the present value approach seems to have received much less attention than it deserves. As compared with the traditional asset share method, the present value method has many advantages in estimating future profits and in obtaining dividend scales. This paper outlines and illustrates its use for both purposes.

The present value method is essentially a discounted asset share calculation. Each year's margin is clearly shown, and the effect of variations is readily ascertained. The method can be made to yield asset shares, as is done herein. These are a means, however, rather than an end, as the main point is the adequacy of the premium, together with any dividend scale. This adequacy can be determined directly, without reference to asset shares as such. The exact equivalence of the present value method and the customary asset share method, which will be established, is worth emphasizing, especially to those who might find the new approach more difficult to understand.

Because of the cumulative nature of the usual asset share calculation, any change in assumptions for a given year, such as initial expenses or second year persistency, requires reworking for all subsequent durations. The same applies if a clerk makes an error. Under the present value method, however, any change for a particular year does not affect the figures for any other year, but only the totals.

Finally, there seems to be a general belief that the contribution formula and an asset share approach produce different dividend scales. Included is a demonstration that, if the assumptions are the same, so are the results by both the contribution method and the present value method, which is a modified asset share process. Incidentally, although one frequently hears of "dividends by the asset share method," a search of the literature fails to reveal any earlier practical, step-by-step, numerical example.

TABLE 1

DETERMINATION OF PROFIT MARGINS OR MAXIMUM DIVIDENDS, AND DIVIDEND SCALE—AGE 35—GROSS PREMIUM $27.44

YEAR n	CASH VALUE $_nCV$ (1)	NET RISK 1,000−(1) (2)	MORTALITY RATE $q^d_{[x]+n-1}$ (3)	MORTALITY COST (2)×(3) (4)	DEDUCTIVE ITEMS: End of Year (1)+(4) (5)	DEDUCTIVE ITEMS: Beg. of Year (5)÷1.03 (6)	EXPENSES (BEGINNING OF YEAR) (7)
1	....	1,000.00	.00115	1.150	1.150	1.117	25.00
2	.03	999.97	.00145	1.450	1.480	1.437	3.00
3	17.56	982.44	.00169	1.660	19.220	18.660	"
4	35.32	964.68	.00210	2.026	37.346	36.258	"
5	53.29	946.71	.00256	2.424	55.714	54.091	"
6	71.47	928.53	.00322	2.990	74.460	72.291	"
7	89.84	910.16	.00352	3.204	93.044	90.334	"
8	108.41	891.59	.00386	3.442	111.852	108.594	"
9	127.15	872.85	.00424	3.701	130.851	127.040	"
10	146.05	853.95	.00467	3.988	150.038	145.668	"
11	165.10	834.90	.00514	4.291	169.391	164.457	"
12	184.29	815.71	.00566	4.617	188.907	183.405	"
13	203.59	796.41	.00623	4.962	208.552	202.478	"
14	223.00	777.00	.00688	5.346	228.346	221.695	"
15	242.48	757.52	.00759	5.750	248.230	241.000	"
16	262.04	737.96	.00838	6.184	268.224	260.412	1.35
17	281.65	718.35	.00925	6.645	288.295	279.898	"
18	301.28	698.72	.01021	7.134	308.414	299.431	"
19	320.92	679.08	.01126	7.646	328.566	318.996	"
20	340.55	659.45	.01240	8.177	348.727	338.570	"
Totals:							
By addition	3,174.02	16,825.98	....	86.787	3,260.807	3,165.832	73.75
By formula	....	16,825.98	....	....	3,260.807	3,165.832	....

TABLE 1—*Continued*

Year n	Persistency and Discount Factor* $v^{n-1}\,{}_{n-1}p'_{[x]}$ (8)	Margin (or Crude Dividend): Beg. of Year $27.44+(1)_{n-1}-(6)-(7)$ (9)	Margin (or Crude Dividend): End of Year $1.03\times(9)$ (10)	Margin (or Crude Dividend): Value at Issue $(8)\times(9)$ (11)	Dividend Scale: Value at Issue (11) Smoothed (12)	Dividend Scale: End of Year $1.03\times(12)\div(8)$ (13)	Value at Issue of Margin (After Div.) $(11)-(12)$ (14)
1	1.00000	1.323	1.363	1.323	....	....	1.323
2	.77670	23.003	23.693	17.866	3.115	4.13	14.751
3	.69375	5.810	5.984	4.031	"	4.62	.916
4	.61966	5.742	5.914	3.558	"	5.18	.443
5	.55950	5.669	5.839	3.172	"	5.73	.057
6	.50518	5.439	5.602	2.748	"	6.35	− .367
7	.46103	5.576	5.743	2.571	"	6.96	− .544
8	.42075	5.686	5.857	2.392	"	7.63	− .723
9	.38398	5.810	5.984	2.231	"	8.36	− .884
10	.35416	5.922	6.100	2.097	"	9.06	−1.018
11	.32666	6.033	6.214	1.971	"	9.82	−1.144
12	.30446	6.135	6.319	1.868	"	10.54	−1.247
13	.28376	6.252	6.440	1.774	"	11.31	−1.341
14	.26448	6.335	6.525	1.675	"	12.13	−1.440
15	.24651	6.440	6.633	1.588	"	13.02	−1.527
16	.22975	8.158	8.403	1.874	"	13.96	−1.241
17	.21414	8.232	8.479	1.763	3.114	14.98	−1.351
18	.19958	8.309	8.558	1.658	"	16.07	−1.456
19	.18601	8.374	8.625	1.558	"	17.24	−1.556
20	.17337	8.440	8.693	1.463	"	18.50	−1.651
Totals:							
By addition	8.20343	142.688	146.968	59.181	59.181	195.59	0.000
By formula	....	142.688	146.969	....	....	....	0.000

* See Table 7.

PRESENT VALUE METHOD

Table 1 illustrates the method, which has been labeled a "present value" method for lack of a more distinctive name. As far as Column 10, it closely resembles Marshall's formula,[1] and needs little explanation. The major departure from the traditional asset share calculation is the compounding of persistency and discount factors, in place of accumulating funds from year to year. This, of course, produces identical results, as will be shown more fully later.

At first glance, it would seem from Table 1 that amounts at risk based on cash values, instead of reserves, have been used. This makes no difference. The best proof is the equivalence of the results to those by a method which bases mortality cost on the full face amount.

This does provide, however, for having only the cash value in hand each year.[2] A list of other assumptions appears in the Appendix.

This method, in various stages of its development, has been employed by one large company for a number of years, especially for the analysis of profits.

NONPARTICIPATING COMPANY

A nonparticipating company would be interested in carrying Table 1 only through Column 11, and would dispense with Column 10. The figure of \$59.18 in Column 11 is the present value of profits for the first twenty years of the policy, if the assumptions are realized. Profits beyond the twentieth year are ignored; but a profit test could be run for a longer period, if desired. Alternatively, a Hoskins type of prospective valuation formula[3] could be utilized to estimate the effect of ignoring them.

Since the gross premium of \$27.44, used in Table 1, is a participating one, the company will usually prefer a lower one. Suppose that they feel that profits with a present value of \$10 are sufficient. A reduced gross premium to correspond would be obtained as follows:

$$27.44 - \frac{59.18 - 10.00}{7.02} = 20.43 .$$

(The denominator of 7.02 is the effect on discounted profits of a unit change in the gross premium, per Table 8.) Hence, as brought out by Hoskins, it is immaterial what gross premium is taken for the original calculation.

[1] *Actuarial Studies No. 6*, p. 138.

[2] For further discussion of this point, see *ibid.*, pp. 139–140.

[3] Cf. *TASA* XXX, 153–154.

PARTICIPATING COMPANY

Column 10 of Table 1, the crude dividends, clearly requires adjustment. It could, of course, be worked on directly. After this was done, however, further calculation and testing would be necessary.

It is much more logical to operate on Column 11. This has been done in Column 12, in the simplest sort of fashion. Taking the first-year dividend as zero, we divided the sum of Column 11 by 19. Minor adjustments were made, so that Column 12 has the same sum. This means that all anticipated surplus will be paid out in twenty years. The operation indicated in the heading of Column 13 gives the dividend scale. Since Column 8 decreases steadily, division of a constant by it produces an increasing scale.

More elaborate graduations of Column 11 could be performed. A scale with a steeper slope might be wanted; or, conversely, a flatter scale. Some contribution to a contingency fund might be deemed wise. In any event, it is easier to work with Column 11 than with Column 10.

The final column of Table 1 gives discounted profit margins after paying dividends.

EQUIVALENCE TO OLDER METHODS

Table 2, through Column 11, is simply a rearrangement of Table 1, assuming the dividend scale already fixed. The last three columns produce asset shares. These are included primarily for comparison with Table 3, as the question of whether or not the plan is self-supporting has already been answered.

In common with Tables 3 and 4, Table 2 has been shown in detail only through the fifth year, to save space. The results through the twentieth year are tabulated in Table 5, in a form to facilitate comparison.

Table 3 starts with precisely the same figures as Table 2, but employs the classical approach. From Table 5, it is seen that asset shares by both methods are the same, except for negligible differences which would diminish further if more decimal places were carried.

An algebraic demonstration of equivalence could be given. But this demonstration by arithmetic will be preferred by most busy actuaries, as they can spot-check a line or two, and, if still skeptical, have a relatively untrained clerk verify the remaining computations.

Exactly the same is true of crude dividends by two methods. Table 4 shows the calculation by the usual contribution method. Since, as already mentioned, cash values, and not reserves, are being built up, we need something else in place of the net level premium. In the illustration, the cash values are minimum ones, and we use the adjusted premium. Table 5 compares the results with those from Table 1.

ADVANTAGES OF PRESENT VALUE METHOD FOR DIVIDEND PURPOSES

The method of Table 4 requires that the cash values used be obtainable by the Standard Nonforfeiture Method. The "nonforfeiture factor" would appear in the heading of Column 6. No such restriction applies to the method of Table 1, where the cash values could be purely arbitrary.

TABLE 2

PROFIT MARGINS AND THEIR RELATION TO ASSET SHARES (AFTER DIVIDENDS)—AGE 35—GROSS PREMIUM $27.44

Year n	Cash Value $_nCV$ (1)	Net Risk 1,000−(1) (2)	Mortality Rate $q^d_{[x]+n-1}$ (3)	Mortality Cost (2)×(3) (4)	Dividend (5)
1	....	1,000.00	.00115	1.150	....
2	.03	999.97	.00145	1.450	4.13
3	17.56	982.44	.00169	1.660	4.62
4	35.32	964.68	.00210	2.026	5.18
5	53.29	946.71	.00256	2.424	5.73

Year n	Deductive Items		Expenses (Beginning of Year) (8)	Margin (Beg. of Year) $27.44+(1)_{n-1}-(7)-(8)$ (9)
	End of Year (1)+(4)+(5) (6)	Beg. of Year (6)÷1.03 (7)		
1	1.150	1.117	25.00	1.323
2	5.610	5.447	3.00	18.993
3	23.840	23.146	3.00	1.324
4	42.526	41.287	3.00	.713
5	61.444	59.654	3.00	.106

Year n	Persistency and Discount Factor $v^n{}_np^s_{[x]}$ (10)	Value at Issue of Margin $(9)\times(10)_{n-1}$ (11)	Asset Share Less Cash Value		Asset Share $_nRV$ (1)+(13) (14)
			Value at Issue $(11)+(12)_{n-1}$ (12)	Current Value (12)÷(10) (13)	
1	.77670	1.323	1.323	1.703	1.703
2	.69375	14.752	16.075	23.171	23.201
3	.61966	.919	16.994	27.425	44.985
4	.55950	.442	17.436	31.164	66.484
5	.50518	.059	17.495	34.631	87.921

This also explains why Table 4 would produce dividends different from those of Table 1 for the first two durations. For identical results, it would be necessary to employ, at durations zero and one, the negative cash values produced by the Guertin formula. But the impracticality of actually using negative values is obvious.

The method of Table 1 also lends itself more readily to direct adjustment for certain modern practices. Immediate payment of death claims is now almost universal. Also, death claim expense may be more easily related to claims than to premiums. The first can be provided for by re-

TABLE 3

TRADITIONAL ASSET SHARE CALCULATION—AGE 35
GROSS PREMIUM $27.44

Year n	Mortality Rate $q^{d}_{[x]+n-1}$ (1)	Lapse Rate $q^{w}_{[x]+n-1}$ (2)	Cash Value ${}_nCV$ (3)	Mortality Cost 1,000×(1) (4)
1	.00115	.19885	......	1.15
2	.00145	.07855	.03	1.45
3	.00169	.07831	17.56	1.69
4	.00210	.06790	35.32	2.10
5	.00256	.06744	53.29	2.56

Year n	Surrender Cost (2)×(3) (5)	Dividend (6)	Total Payts. to Policyholders (4)+(5)+(6) (7)	Expenses (Beginning of Year) (8)
1	......	......	1.150	25.00
2	.002	4.13	5.582	3.00
3	1.375	4.62	7.685	3.00
4	2.398	5.18	9.678	3.00
5	3.594	5.73	11.884	3.00

Year n	Annual Contrib. to Asset Share 1.03[27.44−(8)] −(7) (9)	Fund for Those Entering Year 1.03×$(12)_{n-1}$ +(9) (10)	Persistency Factor $p^{s}_{[x]+n-1}$ 1−(1)−(2) (11)	Asset Share ${}_nRV$ (10)÷(11) (12)
1	1.363	1.363	.80	1.704
2	19.591	21.346	.92	23.202
3	17.488	41.386	.92	44.985
4	15.495	61.830	.93	66.484
5	13.289	81.768	.93	87.923

TABLE 4

DIVIDENDS BY THREE-FACTOR FORMULA (CONTRIBUTION METHOD)—AGE 35
GROSS PREMIUM $27.44—ADJUSTED PREMIUM $22.036

Year n	Cash Value $_nCV$ (1)	Net Risk 1,000−(1) (2)	Mortality Rate: Tabular q (3)	Mortality Rate: Experience q' (4)	Mortality Rate: Difference (3)−(4) (5)
1	....	1,000.00	.00459	.00115	.00344
2	.03	999.97	.00486	.00145	.00341
3	17.56	982.44	.00515	.00169	.00346
4	35.32	964.68	.00546	.00210	.00336
5	53.29	946.71	.00581	.00256	.00325

Year n	Initial Fund $(1)_{n-1}+22.036$ (6)	Expenses (Beginning of Year) (7)	Loading Profit: Beg. of Year 27.44−22.036−(7) (8)	Loading Profit: End of Year 1.03×(8) (9)
1	22.036	25.00	*	*
2	22.036	3.00	*	*
3	22.066	3.00	2.404	2.476
4	39.596	3.00	2.404	2.476
5	57.356	3.00	2.404	2.476

Year n	Interest Profit .005×(6) (10)	Mortality Profit (2)×(5) (11)	Crude Dividend (9)+(10)+(11) (12)
1	*	*	*
2	*	*	*
3	.110	3.399	5.985
4	.198	3.241	5.915
5	.287	3.077	5.840

* Formula does not apply.

placing 1,000 by $1{,}000(1 + i)^{1/2}$ in the heading of Column 2, and the second by adding a constant there.

FOR PROFIT MARGIN PURPOSES

The outstanding advantage, that a change or error does not invalidate all figures at higher durations, has already been mentioned.

OTHER VARIATIONS

The foregoing tables are based on the conservative assumption that dividends are payable to all who enter the policy year and pay the annual premium, whether or not they survive to the end of the year. Rieder[4] refines this by having no dividends payable at the end of the year of death. Marshall[5] grants a pro-rata dividend at death. The corresponding adjustments to Table 1 are division of Column 9 by $1 - q^d$ and $1 - \frac{1}{2}q^d$,

TABLE 5

SUMMARY, EXTENSION, AND COMPARISON
(ARITHMETICAL DEMONSTRATION OF EQUIVALENCE OF METHODS)

Year n	Value at Issue of Margin (After Dividend)			Asset Share			Crude Dividend		
	Table 1	Table 2	Difference	Table 3	Table 2	Difference	Table 1	Table 4	Difference
1......	1.323	1.323	....	1.704	1.703	.001	1.363	......	....
2......	14.751	14.752	−.001	23.202	23.201	.001	23.693	......	....
3......	.916	.919	−.003	44.985	44.985	.....	5.984	5.985	−.001
4......	.443	.442	.001	66.484	66.484	.....	5.914	5.915	−.001
5......	.057	.059	−.002	87.923	87.921	.002	5.839	5.840	−.001
6......	− .367	− .367	....	108.623	108.622	.001	5.602	5.601	.001
7......	− .544	− .544	....	129.256	129.255	.001	5.743	5.738	.005
8......	− .723	− .725	.002	149.714	149.712	.002	5.857	5.861	−.004
9......	− .884	− .885	.001	169.432	169.430	.002	5.984	5.982	.002
10......	−1.018	−1.018	....	188.777	188.773	.004	6.100	6.100	....
11......	−1.144	−1.144	....	207.185	207.181	.004	6.214	6.213	.001
12......	−1.247	−1.248	.001	225.047	225.043	.004	6.319	6.324	−.005
13......	−1.341	−1.342	.001	242.246	242.239	.007	6.440	6.438	.002
14......	−1.440	−1.439	−.001	258.635	258.629	.006	6.525	6.526	−.001
15......	−1.527	−1.529	.002	274.060	274.053	.007	6.633	6.625	.008
16......	−1.241	−1.240	−.001	290.134	290.124	.010	8.403	8.407	−.004
17......	−1.351	−1.352	.001	305.021	305.009	.012	8.479	8.484	−.005
18......	−1.456	−1.456	....	318.531	318.516	.015	8.558	8.552	.006
19......	−1.556	−1.556	....	330.455	330.437	.018	8.625	8.625	....
20......	−1.651	−1.651	....	340.566	340.544	.022	8.693	8.694	−.001

[4] *RAIA* XXX, 261.

[5] See footnote 1.

respectively, before proceeding further. The same adjustments could be made to the results of Table 4.

Annual premiums have been assumed throughout. Wells and Laing,[6] in their Appendix, indicate a technique for taking some account of premiums payable more frequently.

REVISING AN EXISTING DIVIDEND SCALE

As often as not, the actuary's problem is to revise a dividend scale that has been in existence for several years. This presents additional difficulties, especially where the experience has been less favorable than the original assumptions. For one thing, it is highly desirable that the next dividend to be paid be at least as large as the preceding one.

Table 6 illustrates one solution to such a problem, for one year of issue only, now ten years old. (The problem where several years of issue are involved is beyond the scope of this paper.) From the margins under current assumptions, the dividends already paid are deducted. The remainder is to be distributed over future years.

In the first trial, this is done, as in Table 1, by working with the present value, and assigning the same amount to each year. The result is rejected, for the obvious reason already stated. In practice, Columns 7 and 8 would be done on scratch paper.

Chiefly by inspection, we write $9.50 in Column 10 for the second trial. The corresponding figure in Column 9 is reached by reversing the formula. The total of Column 6 is carried over to Column 9. The balance of Column 9 is filled in to form an arithmetic progression, whose first term and sum are already known. The resulting revised scale is satisfactory; but if it had not been, further experimenting would have been in order.

Taking the present and revised scales of Table 6 together, we may compare the combined scale with the dividend scale from Table 1. Since both scales are now based on the same assumptions, and have the same present value, their main difference is in steepness of slope.

When time does not permit as much thoroughness, crude dividends for one or more years may be computed by either the present value method or the contribution method, without requiring all those for previous years. This would be a temporary expedient only, as a rough test of an existing scale, and perhaps resorted to if the original assumptions were unknown. It tacitly assumes the equity of all earlier dividends.

It is a pleasure to record my gratitude to Mr. Harry M. Sarason for his helpful suggestions.

[6] *RAIA* XXIX, 265.

TABLE 6

REVISION OF DIVIDEND SCALE—AGE 35

Year n	Present Dividend Scale		Margin (Beg. of Year) Table 1, Col. 9	Persistency and Discount Factor Table 1, Col. 8	Surplus Yet to Be Allocated		Revised Dividend Scale First Trial		Revised Dividend Scale Second Trial	
	End of Year (1)	Beg. of Year (1)÷1.03 (2)	(3)	(4)	Beg. of Year (3)−(2) (5)	Value at Issue (4)×(5) (6)	Value at Issue (6) Smoothed (7)	End of Year 1.03×(7) ÷(4) (8)	Value at Issue (6) Smoothed (9)	End of Year 1.03×(9) ÷(4) (10)
1	...	...	1.323	1.00000	1.323	1.323	...	...	...	...
2	5.00	4.854	23.003	.77670	18.149	14.096	...	...	...	...
3	5.50	5.340	5.810	.69375	.470	.326	...	...	...	...
4	6.00	5.825	5.742	.61966	−.083	−.051	...	...	...	...
5	6.50	6.311	5.669	.55950	−.642	−.359	...	...	...	...
6	7.00	6.796	5.439	.50518	−1.357	−.686	...	...	...	...
7	7.50	7.282	5.576	.46103	−1.706	−.787	...	...	...	...
8	8.00	7.767	5.686	.42075	−2.081	−.876	...	...	...	...
9	8.50	8.252	5.810	.38398	−2.442	−.938	...	...	...	...
10	9.00	8.738	5.922	.35416	−2.816	−.997	...	...	...	...
11	...	...	6.033	.32666	6.033	1.971	2.824	8.90	3.013	9.50
12	...	...	6.135	.30446	6.135	1.868	2.824	...	2.971	10.05
13	...	...	6.252	.28376	6.252	1.774	2.824	...	2.929	10.63
14	...	...	6.335	.26448	6.335	1.675	2.824	...	2.887	11.24
15	...	...	6.440	.24651	6.440	1.588	2.824	11.80	2.845	11.89
16	...	...	8.158	.22975	8.158	1.874	2.824	...	2.803	12.57
17	...	...	8.232	.21414	8.232	1.763	2.824	...	2.761	13.28
18	...	...	8.309	.19958	8.309	1.658	2.825	...	2.720	14.04
19	...	...	8.374	.18601	8.374	1.558	2.825	...	2.678	14.83
20	...	...	8.440	.17337	8.440	1.463	2.825	16.78	2.636	15.66
Totals:										
By addition	63.00	61.165	142.688	8.20343	81.523	28.243	28.243	...	28.243	123.69
By formula	...	61.165	...	...	81.523	...	...	...	...	...

TABLE 7

PERSISTENCY AND DISCOUNT FACTORS—AGE 35

YEAR n	PERSISTENCY FACTOR		DISCOUNT FACTOR v^{n-1}	PERSISTENCY AND DISCOUNT FACTOR $v^{n-1}{}_{n-1}\overset{s}{p}_{[x]}$ (2)×(3)
	Annual $\overset{s}{p}_{[x]+n-1}$	Compound ${}_{n-1}\overset{s}{p}_{[x]}$ $(1)_{n-1}\times(2)_{n-1}$		
	(1)	(2)	(3)	(4)
1.......	.80	1.00000	1.00000	1.00000
2.......	.92	.80000	.97087	.77670
3.......	.92	.73600	.94260	.69375
4.......	.93	.67712	.91514	.61966
5.......	.93	.62972	.88849	.55950
6.......	.94	.58564	.86261	.50518
7.......	.94	.55050	.83748	.46103
8.......	.94	.51747	.81309	.42075
9.......	.95	.48642	.78941	.38398
10.......	.95	.46210	.76642	.35416
11.......	.96	.43900	.74409	.32666
12.......	.96	.42144	.72242	.30446
13.......	.96	.40458	.70138	.28376
14.......	.96	.38840	.68095	.26448
15.......	.96	.37286	.66112	.24651
16.......	.96	.35795	.64186	.22975
17.......	.96	.34363	.62317	.21414
18.......	.96	.32988	.60502	.19958
19.......	.96	.31668	.58739	.18601
20.......	.96	.30401	.57029	.17337
1.......	....	....	....	1.00000
$\sum_{2}^{15}$.......	....	....	....	6.20058
$\sum_{16}^{20}$.......	....	....	....	1.00285
Total...	....	....	....	8.20343

APPENDIX

After reading some of the comments about assumptions that were plainly labeled "purely for illustrative purposes," one is a little reluctant to draw the same fire. Hence the following assumptions underlying the tables are listed with some diffidence:

1. Age at Issue 35
2. Mortality Company experience table
3. Persistency Company experience
4. Interest 3%
5. Expenses Assessed at beginning of policy year, as a percentage of gross premium plus a constant, both varying somewhat by duration
6. Cash Values CSO $2\frac{1}{2}$% minimum values
7. Reserves These do not enter at all
8. Premiums Payable annually
9. Deaths and withdrawals are treated as though occurring only on anniversaries. (As far as deaths are concerned, this is classical theory.) This means that a dividend is payable for each premium received, except the first.

Table 7 shows the mechanics of compounding the persistency factors, and bringing in discount.

Table 8 is self-explanatory. A similar technique could be utilized to

TABLE 8

EFFECT OF $1 CHANGE IN GROSS PREMIUM—AGE 35

Year n	Persistency and Discount Factor $v^{n-1}{}_{n-1}p^{s}_{[x]}$ Table 7 (1)	Percentage Expenses (2)	Effect on Profit of $1 Change in Gross Premium (1)×[1−(2)] (3)
1	1.00000	.525	.47500
$\sum_{2}^{15}$	6.20058	.100	5.58052
$\sum_{16}^{20}$	1.00285	.040	.96274
Total	8.20343	....	7.01826

show the effect of altering expense factors, without calling for a complete recalculation. For instance, it may be that part of the constant expense is assessed per policy, and then divided by average size. Thus the impact of a change in the minimum amount, and consequently in the anticipated average size, is easily studied.

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