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ACTUARIAL NOTE: VALUATION OF REVERSIONARY INTERESTS INVOLVING TWO OR MORE LIVES FOR FEDERAL TAX PURPOSES

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UNDER Sections 811 and 1005 of the Internal Revenue Code, regulations are issued which relate to the valuation of annuities, life, remainder, and reversionary interests for Estate Tax purposes and to the valuation of annuities, life estates, remainders and reversions for Gift Tax purposes. As an example we can take Section 81.10 (i) (3) of Regulation 105 relating to Estate Taxes which reads in part as follows:

> All other future payments are to be discounted upon the basis of compound interest at the rate of 4 per cent a year. If the time of payment or of payments is dependent upon the continuation of, or upon the termination of a life or of lives, the Actuaries' or Combined Experience Table of Mortality, as extended, and established actuarial principles are to be used in the computation of the present worth. For the purpose of the computation the age of a person is to be taken as the age of that person at his nearest birthday.... If the time of payment or of payments is dependent upon the continuation of, or termination of more than one life, or there is a term certain concurrent with one or more lives, a special computation in accordance with the first two sentences of this paragraph is necessary.

In this field of valuation of interests for tax purposes, the actuary is frequently called upon to compute the proper value to be placed on a contingent or reversionary interest that is, or may be, subject to a Federal Tax. For the most part the calculation may be readily performed using published tables of values on the Actuaries' Mortality Table (Makehamized) at four percent interest, and it is not until the calculation involves contingent assurances on two or more lives that any difficulty is met. Any acceptable method in common use for determining $A^{1}_{xyz\ldots(m)}$ is quite tedious and technical, requiring calculation by highly trained personnel. The problem is quickly multiplied if the factor to be determined is, say,

$$A^{4}_{xyzu} = A_{x} - A^{1}_{xy} - A^{1}_{xz} - A^{1}_{xu} + A^{1}_{xyz} + A^{1}_{xyu} + A^{1}_{xzu} - A^{1}_{xyzu} \, .$$

This leads us to the immediate purpose of this paper which is to describe a method and furnish the tools whereby contingent assurances involving 2, 3 and 4 lives and based on the Actuaries' Table (Makehamized) at 4% can be determined by someone with little technical knowledge of the subject.

The method used was first described by Mr. A. W. Evans, F.I.A.,* and is based on the fact that, for a Makehamized Mortality Table, the same value of n satisfies both of the following equations:

$$\mu_{w+n} = \frac{1}{m}\left(\frac{\bar{A}_{xyz\ldots(m)}}{\bar{a}_{xyz\ldots(m)}}\right) = \frac{1}{m}\left(\frac{\bar{A}_{www\ldots(m)}}{\bar{a}_{www\ldots(m)}}\right) \quad (1)$$

$$\bar{A}^{1}_{xyz\ldots(m)} = \mu_{x+n}\cdot \bar{a}_{www\ldots(m)}\,. \quad (2)$$

The required procedure is first to find w corresponding to $x, y, z, \ldots$ (m) and then by the use of (1) determine the value for n corresponding to w. This leads to the ready determination of $x + n$ and μ_{x+n}. Then after calculating the value for $\bar{a}_{www\ldots(m)}$, $\bar{A}^{1}_{xyz\ldots(m)}$ follows from the multiplication indicated in (2).

The basic requirements for the practical use of the method for values of m less than 5 are tables giving:

(a) Force of mortality, μ_x.
(b) Values of n for 2, 3, and 4 lives.
(c) Continuous annuity values for 2, 3, and 4 lives.

These are given in Table I of the Appendix. (b) is nothing more than the determination of n in (1) corresponding to integral values of w; hence the value of n for nonintegral values of w may then be found by simple interpolation. (c) was calculated from values of immediate annuities by the formula:

$$\bar{a}_{xxx\ldots(m)} = a_{xxx\ldots(m)} + \tfrac{1}{2} - \tfrac{1}{12}(m\mu_x + \delta). \quad (3)$$

Also given in the Appendix are examples illustrating the method in some detail. Examples 1, 2, and 3 show a step by step calculation of $\bar{A}^{1}_{xyz\ldots(m)}$, where $m = 2$, 3, and 4, respectively.

THE 5% RULE

A frequent source of valuations requiring the use of contingent assurances arises in connection with Federal Estate Taxes. In order to determine whether certain transfers of property effective at death may be excluded for tax purposes it is necessary to determine whether the decedent retained a reversionary interest that was less than 5% of the value of the property.

As an example of this 5% rule we can take Sec. 503 of the Revenue Act

* *JIA* LVI, 220.

of 1950, relating to Reversionary Interests in Case of Life Insurance, which reads as follows:

(a) Amendment of section 404 (c) of Revenue Act of 1942.—Effective with respect to estates of decedents dying after October 21, 1942, section 404(c) of the Revenue Act of 1942 is hereby amended by adding at the end thereof the following:

"For the purposes of the preceding sentence, the term 'incident of ownership' includes a reversionary interest only if (1) at some time after January 10, 1941, the value of such reversionary interest exceeded 5 per centum of the value of the policy, and (2) the reversionary interest arose by the express terms of the policy or other instrument and not by operation of law. As used in this subsection, the term 'reversionary interest' includes a possibility that the policy, or the proceeds of the policy, (A) may return to the decedent or his estate, or (B) may be subject to a power of disposition by him. The value of a reversionary interest at any time shall be determined (without regard to the fact of the decedent's death) by usual methods of valuation, including the use of tables of mortality and actuarial principles, pursuant to regulations prescribed by the Secretary. In determining the value of a possibility that the policy or proceeds thereof may be subject to a power of disposition by the decedent, such possibility shall be valued as if it were a possibility that such policy or proceeds may return to the decedent or his estate."

We see that here the problem has become even more troublesome in the case of life insurance because now to determine whether the insured retained an "incident of ownership" it may be necessary to make a number of valuations before one could say definitely that the reversionary interest retained by the decedent continued to be less than 5% at all times after January 10, 1941.

The usual case for valuation of a reversion under this rule is where the value of some property is to be transferred on the death of x to one or more beneficiaries, y, z, u, etc., but in the event that x should survive all beneficiaries, the value is to revert to x's estate upon his death. The present value of x's interest in a property that has a value of B dollars at the time of his death is

$$B \cdot \bar{A}^{m}_{xyz\ldots(m)}\,. \tag{4}$$

The total present value of the property is $B \cdot \bar{A}_x$. It follows that the proportionate interest of x in the property is

$$\frac{B \cdot \bar{A}^{m}_{xyz\ldots(m)}}{B \cdot \bar{A}_x} = \frac{\bar{A}^{m}_{xyz\ldots(m)}}{\bar{A}_x}\,. \tag{5}$$

Formula (5) can be stated in terms of contingent assurances payable on the first death as follows:

Two lives:

$$\frac{\bar{A}_x - \bar{A}^1_{xy}}{\bar{A}_x} \tag{6}$$

Three lives:

$$\frac{\bar{A}_x - \bar{A}^1_{xy} - \bar{A}^1_{xz} + \bar{A}^1_{xyz}}{\bar{A}_x} \tag{7}$$

Four lives:

$$\frac{\bar{A}_x - \bar{A}^1_{xy} - \bar{A}^1_{xz} - \bar{A}^1_{xu} + \bar{A}^1_{xyz} + \bar{A}^1_{xyu} + \bar{A}^1_{xzu} - \bar{A}^1_{xyzu}}{\bar{A}_x}. \tag{8}$$

This type of valuation is the most common type under the 5% rule and the problem in most cases resolves itself into that of determining whether the value as given by (6), (7) or (8) is greater or less than 5%. In order to provide the means for a ready determination of this question, Tables IV and V of the Appendix have been compiled which give the lowest age of x corresponding to y and z, or y alone, for which the value given by formulas (7) or (6) is less than 5%.

The values in Table IV are given for integral ages of one beneficiary and quinquennial ages of the other. In a given valuation if neither beneficiary's age is quinquennial, one could either interpolate for one of the ages or use the next higher quinquennial age for one of the beneficiaries to arrive at the lowest age, x. In case of the more usual combinations of one or two beneficiaries, these tables will enable one to tell by simple references to the tables whether the reversionary interest of x is greater or less than 5%. Even where there are more than two beneficiaries many cases may still be resolved by entering Table IV with the ages of the two youngest beneficiaries. Then if x's interest is less than 5%, it will continue to be less than 5% with the inclusion of more beneficiaries.

Generally, the value of a reversionary interest in any given valuation will be considerably less or considerably greater than 5%. If x's interest is so close to 5% that his relative interest is not readily apparent by an inspection of these tables, a detailed calculation of his interest should be made. In this connection if x's interest appears to be just over 5%, it might be advisable to make a valuation on some revised basis using more modern tables and interest rates. This might well indicate that x's interest is less than 5%. The taxpayer then has the privilege of filing his tax return on the basis of a value computed by an actuary, but he may have to convince the Commissioner or a court that the computation is based on a better mortality table or more appropriate rate of interest.

Example 4 of the Appendix gives an example of a calculation involving the use of formula (7). Also values of $\bar{A}_x$ are given in Table II and additions to be made to the younger of two lives to obtain two equal ages are given in Table III for ready determination of w in the case of two lives.

APPENDIX

Example 1

In formula (2) for two lives,

$$\bar{A}^{1}_{xy} = \mu_{x+n} \cdot \bar{a}_{ww} ,$$

take $x = 65$ and $y = 40$. The difference in the ages is 25, and entering Table III, we find the addition to the younger age to be 18.464, hence $w = 58.464$. Using straight-line interpolation between the values of $\bar{a}_{58:58}$ and $\bar{a}_{59:59}$ in Table I,

$$\bar{a}_{ww} = 7.70793 .$$

Similarly, using Table I for two lives and interpolating between the values of n for integral ages 58 and 59, we find, for $w = 58.464$,

$$n = 7.338 .$$

Then

$$x + n = 65 + 7.338$$

$$= 72.338 ,$$

and

$$\mu_{x+n} = .07663 .$$

Consequently, we have

$$\bar{A}^{1}_{65:40} = .07663\,(7.70793)$$

$$= .59066 .$$

This compares closely with the value of .59057 obtained by the use of the formula

$$c^{x}\,\frac{\frac{1}{m}\,\bar{A}_{ww\ldots(m)} + \log_e s \cdot \bar{a}_{ww\ldots(m)}}{c^{w}} - \log_e s \cdot \bar{a}_{ww\ldots(m)} . \qquad (9)$$

In fact the value is much closer to the value obtained by (9) than that obtained by Hardy's approximate integration formula

$$\int_0^{\infty} f(x)\,dx = n\{.28f(0) + 1.62f(n) + 2.2f(3n) + 1.62f(5n) + .56f(6n) + 1.62f(7n)\} \qquad (10)$$

which produces a value of .59095. This fact is not surprising since the method used is exact, as is formula (9), within the accuracy of the basic values used.

Example 2

Take $x = 65$, $y = 60$ and $z = 35$. Since

$$\mu_w = \frac{1}{m}(\mu_x + \mu_y + \ldots m \text{ terms}),$$

we have, using the column for the force of mortality, μ_x, in Table I:

$$\mu_w = \tfrac{1}{3}(\mu_{65} + \mu_{60} + \mu_{35})$$

$$= .02700 \, .$$

Since μ_w lies between μ_{58} and μ_{59}, we have

$$w = 58 + \frac{.00136}{.00181}$$

$$= 58.751 \, .$$

Following the same procedure as for two lives, only using the columns headed "Three Lives" in Table I, we have

$$\bar{a}_{www} = 6.14591$$

and

$$n = 5.819 \, .$$

Hence,

$$x + n = 70.819$$

and

$$\mu_{x+n} = .06758 \, .$$

Finally we have

$$\bar{A}^{\,1}_{65:60:35} = .06758\,(6.14591)$$

$$= .41534 \, ,$$

which is even closer to the value of .41533 obtained from (9).

Example 3

Take $x = 65$, $y = 60$, $z = 45$, $u = 35$. We have

$$\mu_w = .02338 \, ,$$

hence

$$w = 56.589 \, .$$

Using the columns headed "Four Lives" in Table I, we have

$$\bar{a}_{wwww} = 5.70383$$

and

$$n = 5.429 \, .$$

Hence,

$$x + n = 70.429$$

and

$$\mu_{x+n} = .06548 \, .$$

Finally, we have

$$A^{\ 1}_{65:60:45:35} = .06548\,(5.70383)$$

$$= .37349 \, .$$

The value obtained from (9) is .37340.

Example 4

To determine

$$\frac{\bar{A}^{\ 3}_{65:48:45}}{\bar{A}_{65}} = \frac{\bar{A}_{65} - \bar{A}^{\ 1}_{65:48} - \bar{A}^{\ 1}_{65:45} + \bar{A}^{\ 1}_{65:48:45}}{\bar{A}_{65}} \, ,$$

we can set up the following table:

x	y	z	m	w	n	$x+n$	μ_{x+n}	$\bar{a}_{ww\ldots(m)}$	$\bar{A}^{\ 1}_{xyz\ldots(m)}$
65	48	....	2	59.507	7.026	72.026	.07462	7.40649	.55267
65	....	45	2	59.037	7.165	72.165	.07552	7.54200	.56957
65	48	45	3	56.424	6.446	71.446	.07118	6.76022	.48119

Then using the value of $\bar{A}_{65}$ from Table II, we have

$$\frac{\bar{A}^{\ 3}_{65:48:45}}{\bar{A}_{65}} = \frac{.67171 - .55267 - .56957 + .48119}{.67171} = \frac{.03066}{.67171}$$

$$= 4.56\% \, .$$

The corresponding percentage for $x = 64$ is 5.02%.

TABLE I

ACTUARIES' MORTALITY TABLE (MAKEHAMIZED) AT 4%

AGE x	μ_x	TWO LIVES		THREE LIVES		FOUR LIVES	
		n	$\mathring{a}_{xx}$	n	$\mathring{a}_{xxx}$	n	$\mathring{a}_{xxxx}$
10	.00697	26.080	17.29206	22.706	15.36779	20.143	13.86171
11	.00700	25.640	17.20977	22.316	15.29022	19.786	13.79019
12	.00702	25.231	17.12408	21.895	15.20948	19.467	13.71569
13	.00705	24.808	17.03361	21.550	15.12383	19.176	13.63621
14	.00708	24.414	16.94032	21.182	15.03510	18.824	13.55401
15	.00711	24.000	16.84224	20.773	14.94195	18.474	13.46704
16	.00715	23.563	16.74050	20.400	14.84480	18.100	13.37656
17	.00719	23.143	16.63418	20.038	14.74311	17.800	13.28137
18	.00723	22.714	16.52344	19.654	14.63678	17.455	13.18194
19	.00728	22.282	16.40785	19.276	14.52597	17.120	13.07777
20	.00733	21.872	16.28792	18.931	14.41030	16.760	12.96897
21	.00739	21.429	16.16356	18.531	14.29039	16.423	12.85599
22	.00745	21.022	16.03346	18.171	14.16464	16.103	12.73715
23	.00751	20.609	15.89856	17.800	14.03449	15.759	12.61384
24	.00759	20.200	15.75771	17.410	13.89758	15.438	12.48377
25	.00767	19.780	15.61279	17.048	13.75691	15.114	12.35056
26	.00776	19.382	15.46196	16.667	13.61052	14.771	12.21160
27	.00786	18.982	15.30531	16.283	13.45840	14.410	12.06661
28	.00796	18.557	15.14380	15.935	13.30081	14.071	11.91684
29	.00808	18.152	14.97625	15.560	13.13767	13.738	11.76092
30	.00821	17.742	14.80265	15.200	12.96843	13.413	11.59951
31	.00835	17.333	14.62246	14.836	12.79331	13.080	11.43227
32	.00850	16.944	14.43811	14.459	12.61286	12.760	11.25988
33	.00867	16.538	14.24672	14.106	12.42638	12.418	11.08183
34	.00886	16.138	14.04955	13.742	12.23407	12.115	10.89776
35	.00906	15.736	13.84674	13.389	12.03632	11.770	10.70935
36	.00928	15.337	13.63770	13.050	11.83299	11.439	10.51413
37	.00953	14.958	13.42277	12.675	11.62408	11.125	10.31541
38	.00979	14.558	13.20251	12.322	11.40873	10.806	10.10978
39	.01008	14.174	12.97577	11.989	11.18819	10.488	9.89920
40	.01040	13.791	12.74334	11.632	10.96315	10.172	9.68445
41	.01075	13.400	12.50521	11.288	10.73291	9.862	9.46500
42	.01114	13.029	12.26267	10.962	10.49827	9.547	9.24173
43	.01156	12.650	12.01394	10.609	10.25786	9.240	9.01335
44	.01202	12.272	11.76054	10.272	10.01406	8.942	8.78197
45	.01252	11.907	11.50176	9.952	9.76603	8.626	8.54716
46	.01307	11.537	11.23816	9.613	9.51360	8.328	8.30841
47	.01368	11.177	10.97096	9.291	9.25857	8.044	8.06777
48	.01434	10.818	10.69926	8.974	8.99991	7.752	7.82431
49	.01506	10.462	10.42363	8.652	8.73864	7.457	7.57883
50	.01586	10.115	10.14504	8.343	8.47512	7.177	7.33213
51	.01673	9.765	9.86300	8.041	8.20971	6.909	7.08394
52	.01768	9.422	9.57850	7.731	7.94299	6.624	6.83560
53	.01872	9.092	9.29197	7.429	7.67557	6.355	6.58733
54	.01987	8.754	9.00364	7.139	7.40784	6.097	6.33953

TABLE I—*Continued*

Age x	μ_x	Two Lives		Three Lives		Four Lives	
		n	δ_{xx}	n	δ_{xxx}	n	δ_{xxxx}
55.......	.02112	8.426	8.71402	6.852	7.14015	5.834	6.09277
56.......	.02249	8.109	8.42386	6.562	6.87325	5.574	5.84751
57.......	.02400	7.792	8.13261	6.289	6.60668	5.327	5.60357
58.......	.02564	7.478	7.84234	6.026	6.34268	5.088	5.36285
59.......	.02745	7.176	7.55267	5.750	6.08067	4.852	5.12474
60.......	.02942	6.880	7.26434	5.490	5.82151	4.615	4.89041
61.......	.03159	6.580	6.97780	5.238	5.56510	4.390	4.65976
62.......	.03396	6.295	6.69334	5.000	5.31251	4.176	4.43334
63.......	.03656	6.024	6.41223	4.749	5.06460	3.968	4.21215
64.......	.03940	5.742	6.13451	4.511	4.82127	3.751	3.99616
65.......	.04252	5.472	5.86044	4.287	4.58264	3.551	3.78530
66.......	.04593	5.217	5.59061	4.072	4.34956	3.360	3.58030
67.......	.04967	4.968	5.32652	3.857	4.12283	3.177	3.38195
68.......	.05377	4.715	5.06690	3.644	3.90190	3.005	3.18957
69.......	.05825	4.473	4.81381	3.443	3.68793	2.824	3.00431
70.......	.06317	4.245	4.56629	3.254	3.48021	2.655	2.82532
71.......	.06855	4.027	4.32582	3.072	3.27997	2.494	2.65362
72.......	.07445	3.804	4.09221	2.893	3.08689	2.342	2.48904
73.......	.08091	3.592	3.86536	2.717	2.90087	2.200	2.33107
74.......	.08799	3.391	3.64640	2.552	2.72254	2.065	2.18074
75.......	.09574	3.199	3.43515	2.394	2.55204	1.931	2.03759
76.......	.10423	3.020	3.23119	2.247	2.38861	1.802	1.90100
77.......	.11353	2.836	3.03562	2.108	2.23308	1.680	1.77165
78.......	.12372	2.659	2.84834	1.973	2.08529	1.563	1.64941
79.......	.13488	2.495	2.66848	1.840	1.94442	1.457	1.53341
80.......	.14711	2.339	2.49709	1.714	1.81104	1.357	1.42412
81.......	.16050	2.192	2.33397	1.595	1.68521	1.261	1.32150
82.......	.17517	2.055	2.17853	1.485	1.56607	1.175	1.22474
83.......	.19124	1.920	2.03101	1.382	1.45384	1.095	1.13388
84.......	.20884	1.786	1.89165	1.284	1.34847	1.018	1.04911
85.......	.22812	1.663	1.75933	1.197	1.24905	.947	.96926
86.......	.24924	1.549	1.63451	1.116	1.15583	.882	.89458
87.......	.27238	1.436	1.51775	1.033	1.06940	.813	.82569
88.......	.29772	1.341	1.40666	.967	.98726	.767	.76030
89.......	.32548	1.245	1.30325	.894	.91137	.720	.69989
90.......	.35589	1.153	1.20661	.823	.84093	.675	.64402
91.......	.38920	1.085	1.11439	.784	.77358	.669	.59043
92.......	.42569	1.001	1.03024	.723	.71275	.645	.54197
93.......	.46566	.943	.94956	.705	.65410	.673	.49509
94.......	.50945	.879	.87533	.691	.59986	.728	.45114
95.......	.55741	.739	.81188	.598	.55381	.723	.41310
96.......	.60994	.649	.74974	.546	.50932	.782	.37609
97.......	.66749	.729	.68209	.663	.46147	1.029	.33679
98.......	.73053	1.026	.60889	1.025	.40924	1.536	.29414
99.......	.79958	....	.56480	....	.38251	....	.26788
100.......	.87522	....	.41096	....	.29292	....	.20874

TABLE II

ACTUARIES' MORTALITY TABLE (MAKEHAMIZED) AT 4%

Age x	$\bar{A}_x$	Age x	$\bar{A}_x$	Age x	$\bar{A}_x$	Age x	$\bar{A}_x$
10	.218298	35	.347930	60	.609495	80	.839456
11	.221461	36	.355812	61	.621926	81	.848595
12	.224760	37	.363889	62	.634389	82	.857420
13	.228218	38	.372225	63	.646853	83	.865905
14	.231784	39	.380820	64	.659296	84	.874049
15	.235527	40	.389634	65	.671711	85	.881870
16	.239400	41	.398649	66	.684068	86	.889343
17	.243424	42	.407927	67	.696337	87	.896445
18	.247617	43	.417496	68	.708514	88	.903264
19	.251980	44	.427207	69	.720560	89	.909710
20	.256493	45	.437198	70	.732472	90	.915822
21	.261199	46	.447445	71	.744212	91	.921676
22	.266069	47	.457862	72	.755769	92	.927138
23	.271124	48	.468500	73	.767136	93	.932380
24	.276405	49	.479360	74	.778260	94	.937283
25	.281849	50	.490428	75	.789144	95	.941758
26	.287486	51	.501682	76	.799787	96	.946325
27	.293327	52	.513113	77	.810142	97	.951360
28	.299378	53	.524705	78	.820210	98	.956939
29	.305644	54	.536466	79	.829992	99	.962341
30	.312126	55	.548364				
31	.318846	56	.560387				
32	.325773	57	.572541				
33	.332949	58	.584782				
34	.340333	59	.597105				

TABLE III

ACTUARIES' MORTALITY TABLE (MAKEHAMIZED): ADDITION TO YOUNGER AGE TO OBTAIN TWO EQUIVALENT AGES

Diff. in Ages	Addition	Diff. in Ages	Addition	Diff. in Ages	Addition	Diff. in Ages	Addition
1	.511	16	10.689	31	24.026	46	38.558
2	1.046	17	11.507	32	24.972	47	39.544
3	1.602	18	12.339	33	25.923	48	40.531
4	2.181	19	13.182	34	26.878	49	41.519
5	2.782	20	14.037	35	27.836	50	42.508
6	3.405	21	14.904	36	28.799	51	43.498
7	4.049	22	15.780	37	29.764	52	44.489
8	4.713	23	16.666	38	30.732	53	45.481
9	5.398	24	17.561	39	31.703	54	46.473
10	6.102	25	18.464	40	32.676	55	47.466
11	6.824	26	19.375	41	33.652	56	48.460
12	7.564	27	20.293	42	34.630	57	49.454
13	8.322	28	21.217	43	35.609	58	50.449
14	9.096	29	22.148	44	36.591	59	51.444
15	9.885	30	23.084	45	37.574	60	52.440

TABLE IV

LOWEST AGE, x, FOR WHICH $\dfrac{\bar{A}_x - \bar{A}^1_{xy} - \bar{A}^1_{xz} + \bar{A}^1_{xyz}}{\bar{A}_x}$ IS LESS THAN 5%

ACTUARIES' TABLE (MAKEHAMIZED) AT 4%

y	z														
	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
10.....	33	35	37	40	42	45	48	51	54	57	60	62	65	66	68
11.....	33	36	37	40	43	45	49	51	54	57	60	62	65	66	68
12.....	34	36	37	41	43	46	49	52	54	57	60	63	65	67	68
13.....	34	36	38	41	43	46	49	52	55	58	60	63	65	67	68
14.....	34	37	38	41	44	46	49	52	55	58	60	63	65	67	68
15.....	35	37	39	41	44	46	49	52	55	58	61	63	65	67	68
16.....	36	37	39	42	44	47	50	53	55	58	61	63	65	67	69
17.....	36	37	40	42	45	47	50	53	56	58	61	63	65	67	69
18.....	36	38	40	42	45	47	50	53	56	59	61	63	66	68	69
19.....	37	38	40	42	45	48	50	53	56	59	61	64	66	68	69
20.....	37	39	41	43	46	48	51	53	56	59	61	64	66	68	69
21.....	37	40	41	44	46	48	51	54	56	59	62	64	66	68	70
22.....	38	40	41	44	46	49	51	54	57	59	62	64	66	68	70
23.....	39	41	42	44	47	49	52	55	57	60	62	65	67	69	70
24.....	40	41	42	45	47	50	52	55	57	60	62	65	67	69	70
25.....	40	41	43	45	47	50	52	55	58	60	63	65	67	69	71
26.....	40	42	44	45	48	50	53	55	58	61	63	65	68	69	71
27.....	41	42	44	46	48	51	53	56	58	61	63	66	68	69	71
28.....	41	42	45	46	49	51	54	56	59	61	64	66	68	70	71
29.....	42	43	45	47	49	52	54	56	59	61	64	66	68	70	72
30.....	42	44	46	47	49	52	54	57	59	62	64	66	69	71	72
31.....	43	45	46	48	50	52	55	57	60	62	65	67	69	71	72
32.....	44	45	47	48	50	53	55	57	60	62	65	67	69	71	73
33.....	44	46	47	49	51	53	55	58	61	63	65	68	70	72	73
34.....	45	46	47	49	51	54	56	58	61	63	66	68	70	72	74
35.....	45	46	48	50	52	54	57	59	61	64	66	68	70	72	74
36.....	46	47	48	50	52	54	57	59	62	64	66	69	71	73	74
37.....	46	47	49	51	53	55	57	60	62	64	67	69	71	73	75
38.....	47	48	50	52	53	55	58	60	62	65	67	70	72	74	75
39.....	48	49	50	52	54	56	58	61	63	65	68	70	72	74	76
40.....	48	49	51	52	54	57	59	61	63	66	68	70	73	75	76
41.....	49	50	51	53	55	57	59	61	64	66	69	71	73	75	77
42.....	50	51	52	53	55	57	60	62	64	67	69	71	73	75	77
43.....	50	51	52	54	56	58	60	62	65	67	69	72	74	76	78
44.....	51	52	53	54	56	58	60	63	65	67	70	72	74	76	78
45.....	51	52	53	55	57	59	61	63	66	68	70	73	75	77	79
46.....	52	53	54	56	57	59	61	64	66	68	71	73	75	77	79
47.....	53	53	55	56	58	60	62	64	66	69	71	74	76	78	80
48.....	53	54	55	57	58	60	62	65	67	69	72	74	76	78	80
49.....	54	55	56	57	59	61	63	65	67	70	72	75	77	79	81

TABLE IV—*Continued*

y	z														
	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
50.....	54	55	56	58	59	61	63	66	68	70	73	75	77	79	81
51.....	55	56	57	58	60	62	64	66	68	71	73	76	78	80	82
52.....	56	56	57	59	60	62	64	67	69	71	74	76	78	80	82
53.....	56	57	58	59	61	63	65	67	69	72	74	77	79	81	83
54.....	57	57	58	60	61	63	65	67	70	72	75	77	79	81	83
55.....	57	58	59	60	62	64	66	68	70	73	75	78	80	82	84
56.....	58	59	59	61	62	64	66	68	71	73	76	78	80	83	85
57.....	58	59	60	61	63	65	67	69	71	74	76	79	81	83	85
58.....	59	59	60	62	63	65	67	69	72	74	77	79	81	84	86
59.....	59	60	61	62	64	66	68	70	72	75	77	80	82	84	86
60.....	60	61	61	63	64	66	68	70	73	75	78	80	82	85	87
61.....	60	61	62	63	65	67	69	71	73	76	78	81	83	85	87
62.....	61	62	62	64	65	67	69	71	74	76	79	81	83	86	88
63.....	61	62	63	64	66	67	70	72	74	77	79	82	84	86	88
64.....	62	62	63	65	66	68	70	72	75	77	80	82	85	87	89
65.....	62	63	64	65	66	68	70	73	75	78	80	83	85	87	90
66.....	63	63	64	65	67	69	71	73	76	78	81	83	86	88	90
67.....	63	64	65	66	67	69	71	74	76	79	81	84	86	88	91
68.....	64	64	65	66	68	70	72	74	76	79	82	84	87	89	91
69.....	64	65	66	67	68	70	72	74	77	79	82	85	87	90	92
70.....	65	65	66	67	69	70	73	75	77	80	82	85	88	90	92
71.....	65	65	66	68	69	71	73	75	78	80	83	86	88	91	93
72.....	65	66	67	68	69	71	73	76	78	81	83	86	89	91	93
73.....	66	66	67	68	70	72	74	76	79	81	84	86	89	92	94
74.....	66	67	68	69	70	72	74	76	79	82	84	87	90	92	95
75.....	66	67	68	69	71	72	75	77	79	82	85	87	90	93	95
76.....	67	67	68	69	71	73	75	77	80	82	85	88	91	93	96
77.....	67	68	68	70	71	73	75	78	80	83	85	88	91	94	96
78.....	68	68	69	70	72	73	76	78	81	83	86	89	91	94	97
79.....	68	68	69	70	72	74	76	78	81	84	86	89	92	95	97
80.....	68	68	69	71	72	74	76	79	81	84	87	90	92	95	98

TABLE V

LOWEST AGE, x, FOR WHICH $\frac{\bar{A}_x - \bar{A}^1_{xy}}{\bar{A}_x}$ IS LESS THAN 5%: ACTUARIES'—4%

y	x	y	x	y	x	y	x
10	71	25	73	40	80	55	90
11	71	26	74	41	81	56	91
12	71	27	74	42	81	57	92
13	71	28	74	43	82	58	93
14	71	29	75	44	82	59	94
15	71	30	75	45	83	60	95
16	71	31	76	46	84	61	95
17	72	32	76	47	84	62	96
18	72	33	76	48	85	63	97
19	72	34	77	49	86	64	99
20	72	35	77	50	87	65–80	100
21	72	36	78	51	87		
22	73	37	78	52	88		
23	73	38	79	53	89		
24	73	39	79	54	90		